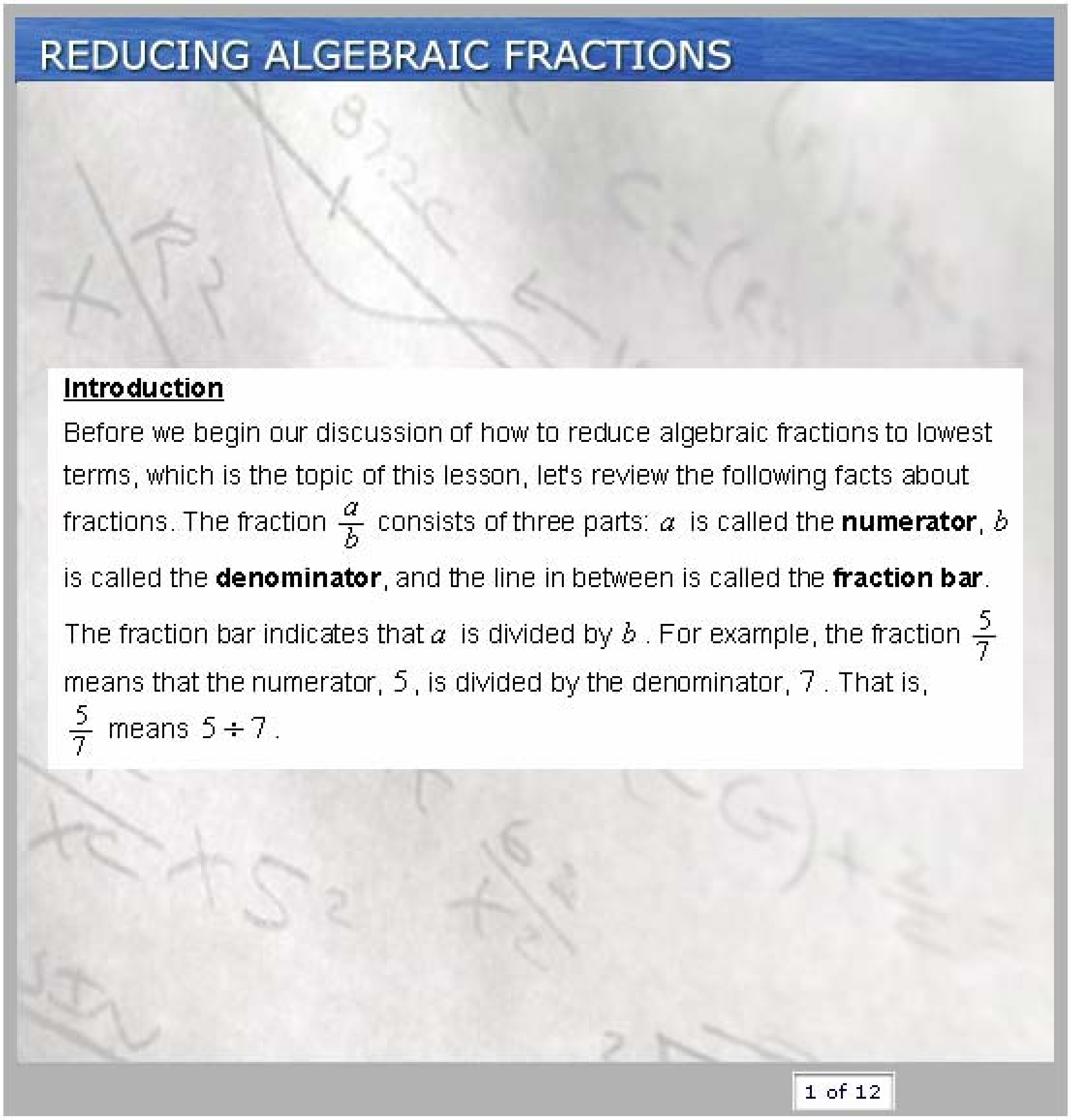
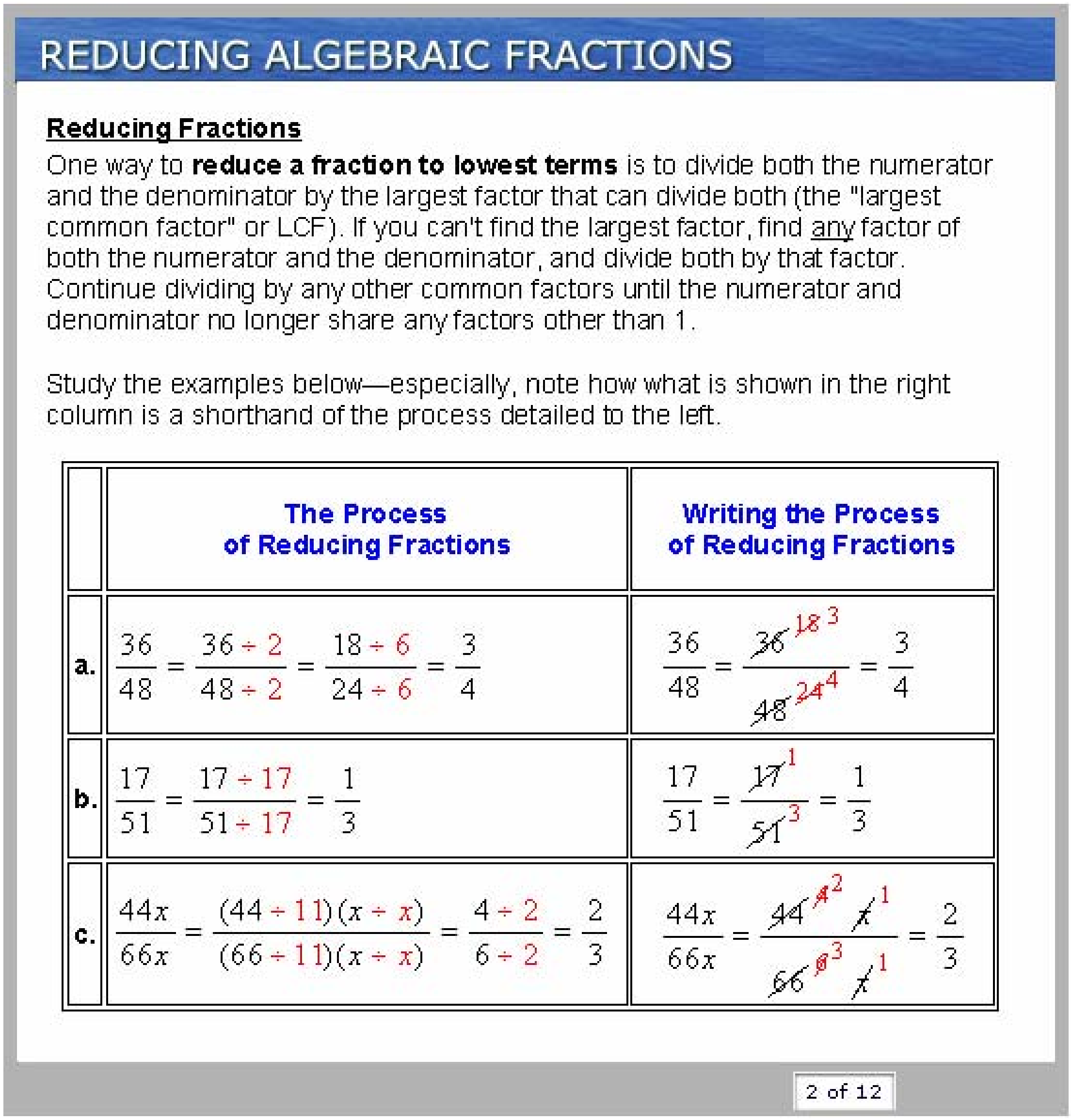


REDUCING ALGEBRAIC FRACTIONS

Introduction

Before we begin our discussion of how to reduce algebraic fractions to lowest terms, which is the topic of this lesson, let's review the following facts about fractions. The fraction $\frac{a}{b}$ consists of three parts: a is called the **numerator**, b is called the **denominator**, and the line in between is called the **fraction bar**. The fraction bar indicates that a is divided by b. For example, the fraction $\frac{5}{7}$ means that the numerator, 5, is divided by the denominator, 7. That is, $\frac{5}{7}$ means $5 \div 7$.



REDUCING ALGEBRAIC FRACTIONS

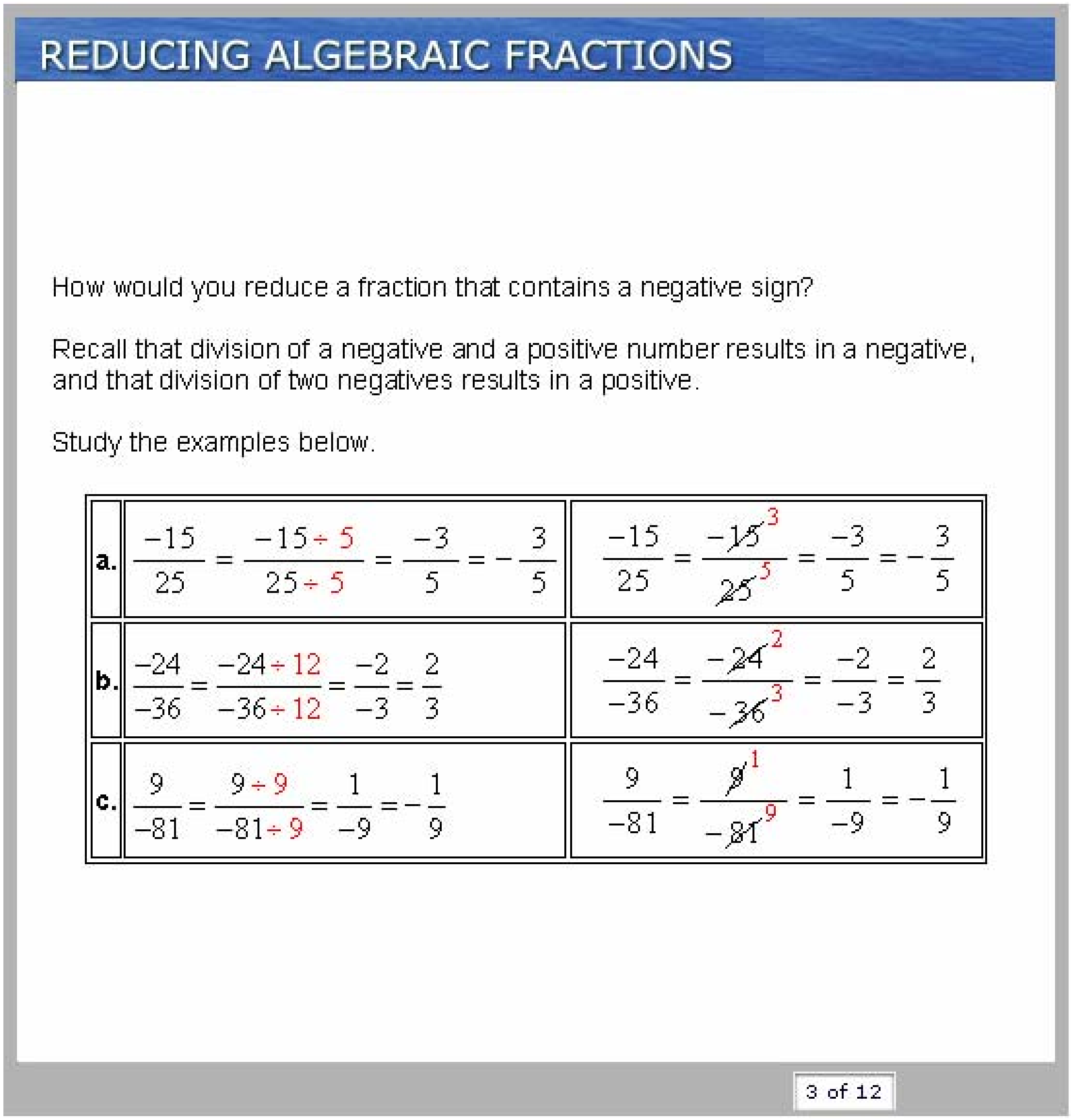
Reducing Fractions

One way to **reduce a fraction to lowest terms** is to divide both the numerator and the denominator by the largest factor that can divide both (the "largest common factor" or LCF). If you can't find the largest factor, find any factor of both the numerator and the denominator, and divide both by that factor. Continue dividing by any other common factors until the numerator and denominator no longer share any factors other than 1.

Study the examples below—especially, note how what is shown in the right column is a shorthand of the process detailed to the left.

	The Process of Reducing Fractions	**Writing the Process of Reducing Fractions**
a.	$\frac{36}{48} = \frac{36 \div 2}{48 \div 2} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$	$\frac{36}{48} = \frac{\cancel{36}^{\cancel{18}\,3}}{\cancel{48}^{\cancel{24}\,4}} = \frac{3}{4}$
b.	$\frac{17}{51} = \frac{17 \div 17}{51 \div 17} = \frac{1}{3}$	$\frac{17}{51} = \frac{\cancel{17}^{1}}{\cancel{51}^{3}} = \frac{1}{3}$
c.	$\frac{44x}{66x} = \frac{(44 \div 11)(x \div x)}{(66 \div 11)(x \div x)} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$	$\frac{44x}{66x} = \frac{\cancel{44}^{\cancel{4}\,2}\ \cancel{x}^{1}}{\cancel{66}^{\cancel{6}\,3}\ \cancel{x}^{1}} = \frac{2}{3}$

2 of 12



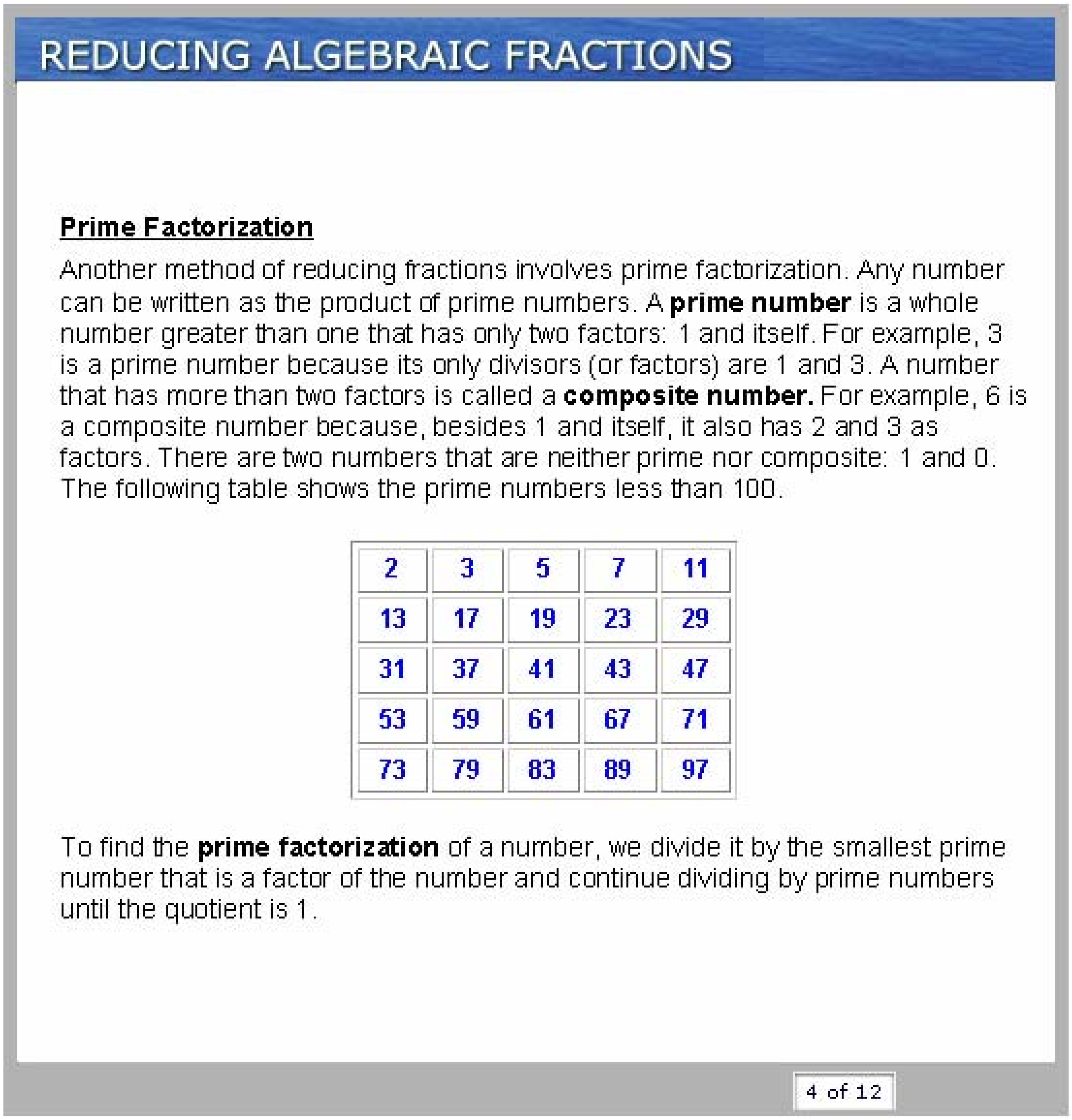
REDUCING ALGEBRAIC FRACTIONS

How would you reduce a fraction that contains a negative sign?

Recall that division of a negative and a positive number results in a negative, and that division of two negatives results in a positive.

Study the examples below.

a.	$\frac{-15}{25}=\frac{-15\div 5}{25\div 5}=\frac{-3}{5}=-\frac{3}{5}$	$\frac{-15}{25}=\frac{-\cancel{15}^{3}}{\cancel{25}^{5}}=\frac{-3}{5}=-\frac{3}{5}$
b.	$\frac{-24}{-36}=\frac{-24\div 12}{-36\div 12}=\frac{-2}{-3}=\frac{2}{3}$	$\frac{-24}{-36}=\frac{-\cancel{24}^{2}}{-\cancel{36}^{3}}=\frac{-2}{-3}=\frac{2}{3}$
c.	$\frac{9}{-81}=\frac{9\div 9}{-81\div 9}=\frac{1}{-9}=-\frac{1}{9}$	$\frac{9}{-81}=\frac{\cancel{9}^{1}}{-\cancel{81}^{9}}=\frac{1}{-9}=-\frac{1}{9}$



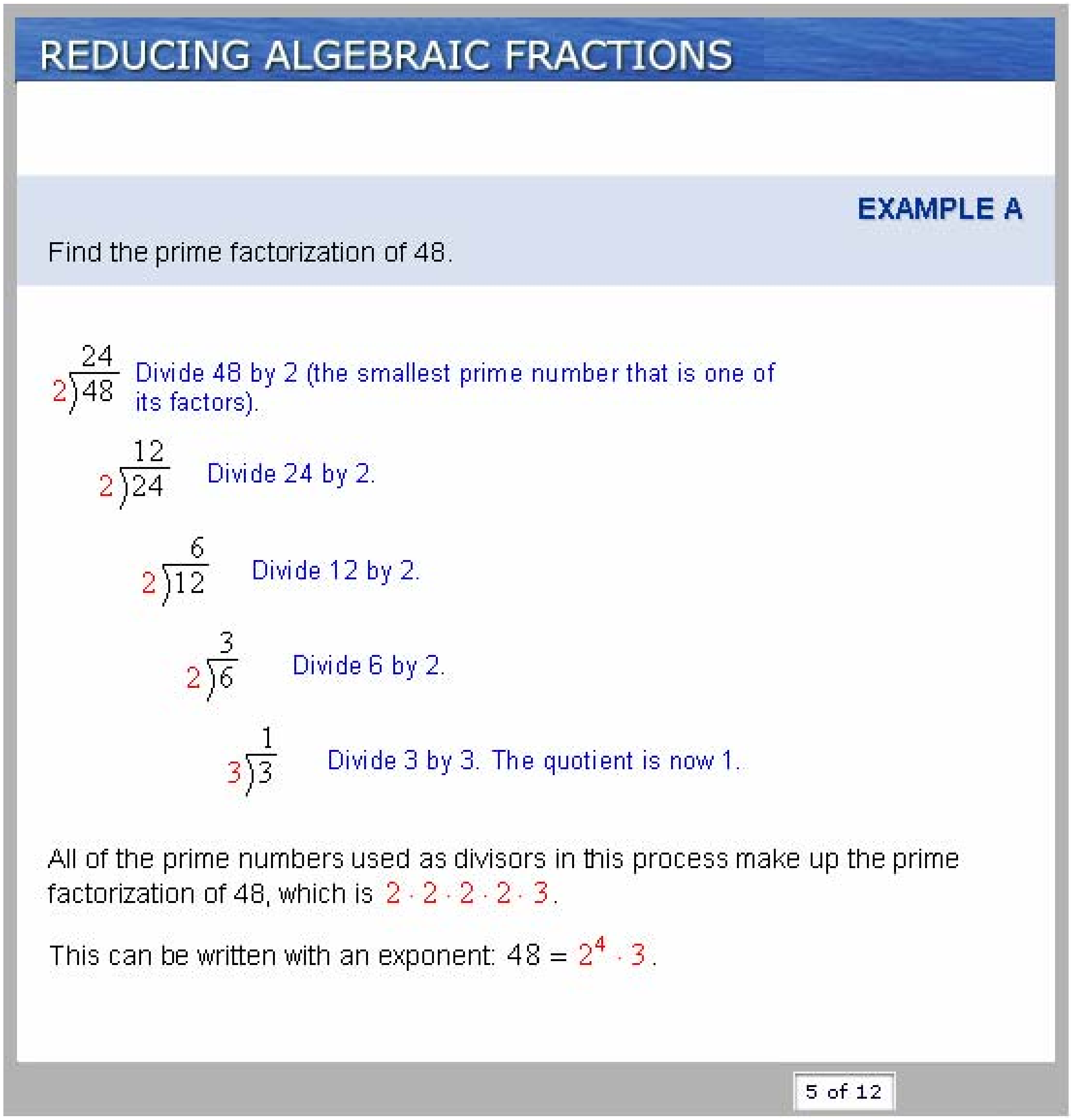
REDUCING ALGEBRAIC FRACTIONS

Prime Factorization

Another method of reducing fractions involves prime factorization. Any number can be written as the product of prime numbers. A **prime number** is a whole number greater than one that has only two factors: 1 and itself. For example, 3 is a prime number because its only divisors (or factors) are 1 and 3. A number that has more than two factors is called a **composite number.** For example, 6 is a composite number because, besides 1 and itself, it also has 2 and 3 as factors. There are two numbers that are neither prime nor composite: 1 and 0. The following table shows the prime numbers less than 100.

2	3	5	7	11
13	17	19	23	29
31	37	41	43	47
53	59	61	67	71
73	79	83	89	97

To find the **prime factorization** of a number, we divide it by the smallest prime number that is a factor of the number and continue dividing by prime numbers until the quotient is 1.



REDUCING ALGEBRAIC FRACTIONS

EXAMPLE A

Find the prime factorization of 48.

$\begin{array}{r}24\\ 2\overline{)48}\end{array}$ Divide 48 by 2 (the smallest prime number that is one of its factors).

$\begin{array}{r}12\\ 2\overline{)24}\end{array}$ Divide 24 by 2.

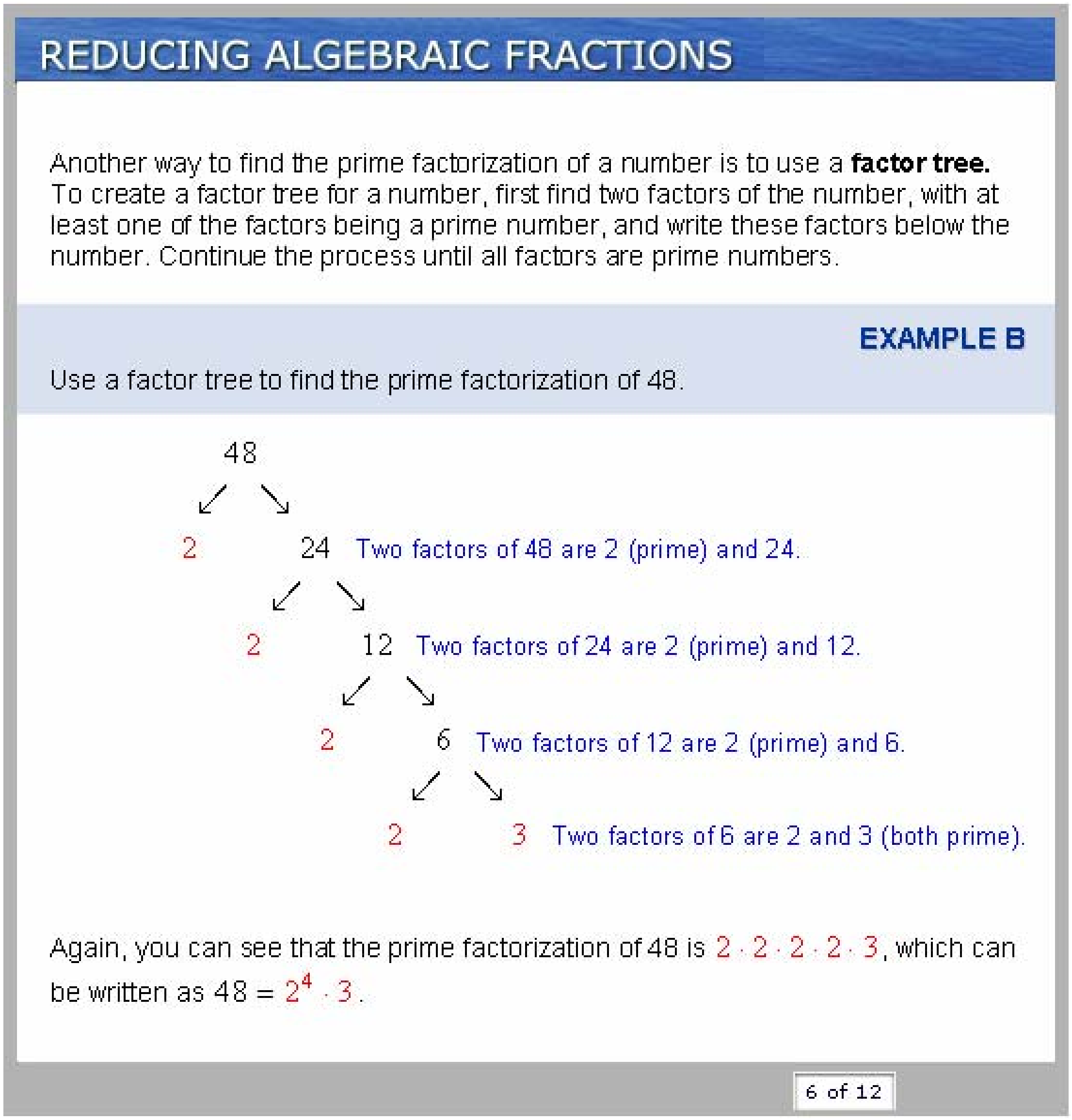
$\begin{array}{r}6\\ 2\overline{)12}\end{array}$ Divide 12 by 2.

$\begin{array}{r}3\\ 2\overline{)6}\end{array}$ Divide 6 by 2.

$\begin{array}{r}1\\ 3\overline{)3}\end{array}$ Divide 3 by 3. The quotient is now 1.

All of the prime numbers used as divisors in this process make up the prime factorization of 48, which is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$.

This can be written with an exponent: $48 = 2^4 \cdot 3$.



REDUCING ALGEBRAIC FRACTIONS

Another way to find the prime factorization of a number is to use a **factor tree.** To create a factor tree for a number, first find two factors of the number, with at least one of the factors being a prime number, and write these factors below the number. Continue the process until all factors are prime numbers.

EXAMPLE B

Use a factor tree to find the prime factorization of 48.

48

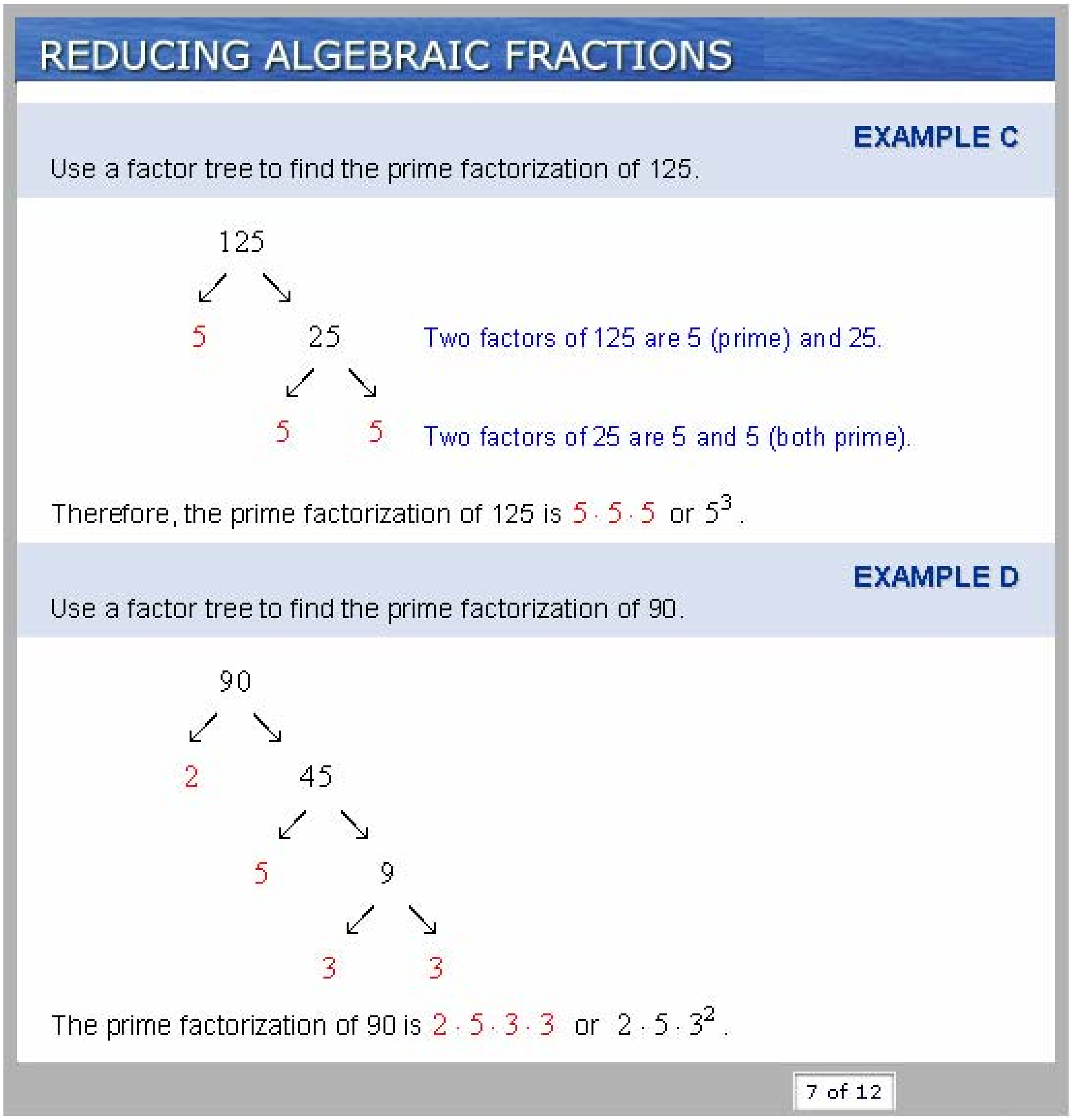
2 24 Two factors of 48 are 2 (prime) and 24.

2 12 Two factors of 24 are 2 (prime) and 12.

2 6 Two factors of 12 are 2 (prime) and 6.

2 3 Two factors of 6 are 2 and 3 (both prime).

Again, you can see that the prime factorization of 48 is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, which can be written as $48 = 2^4 \cdot 3$.



REDUCING ALGEBRAIC FRACTIONS

EXAMPLE C

Use a factor tree to find the prime factorization of 125.

125

5 25 Two factors of 125 are 5 (prime) and 25.

5 5 Two factors of 25 are 5 and 5 (both prime).

Therefore, the prime factorization of 125 is $5 \cdot 5 \cdot 5$ or 5^3.

EXAMPLE D

Use a factor tree to find the prime factorization of 90.

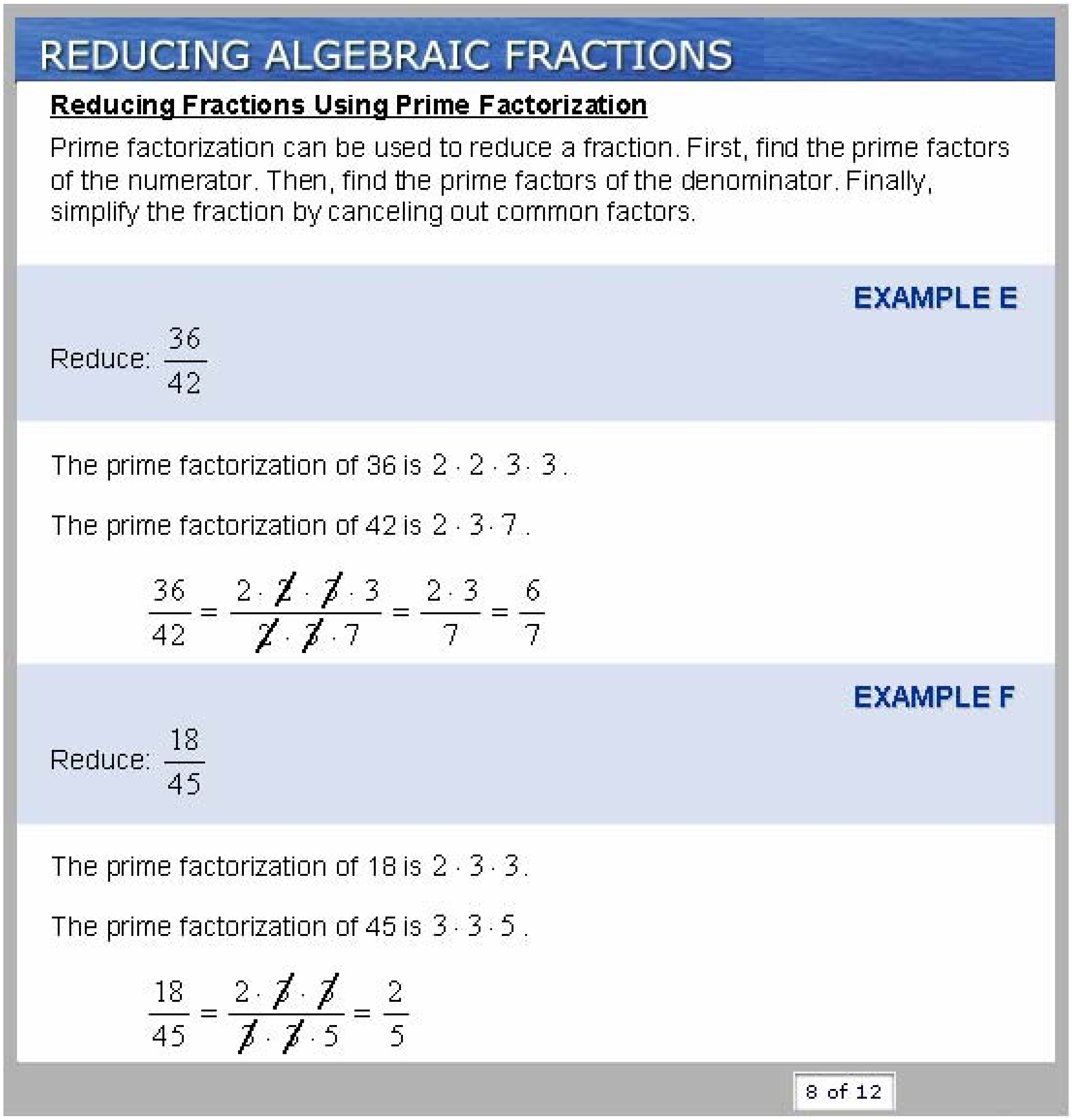
90

2 45

5 9

3 3

The prime factorization of 90 is $2 \cdot 5 \cdot 3 \cdot 3$ or $2 \cdot 5 \cdot 3^2$.



REDUCING ALGEBRAIC FRACTIONS

Reducing Fractions Using Prime Factorization

Prime factorization can be used to reduce a fraction. First, find the prime factors of the numerator. Then, find the prime factors of the denominator. Finally, simplify the fraction by canceling out common factors.

EXAMPLE E

Reduce: $\frac{36}{42}$

The prime factorization of 36 is $2 \cdot 2 \cdot 3 \cdot 3$.

The prime factorization of 42 is $2 \cdot 3 \cdot 7$.

$$\frac{36}{42} = \frac{2 \cdot \cancel{2} \cdot \cancel{3} \cdot 3}{\cancel{2} \cdot \cancel{3} \cdot 7} = \frac{2 \cdot 3}{7} = \frac{6}{7}$$

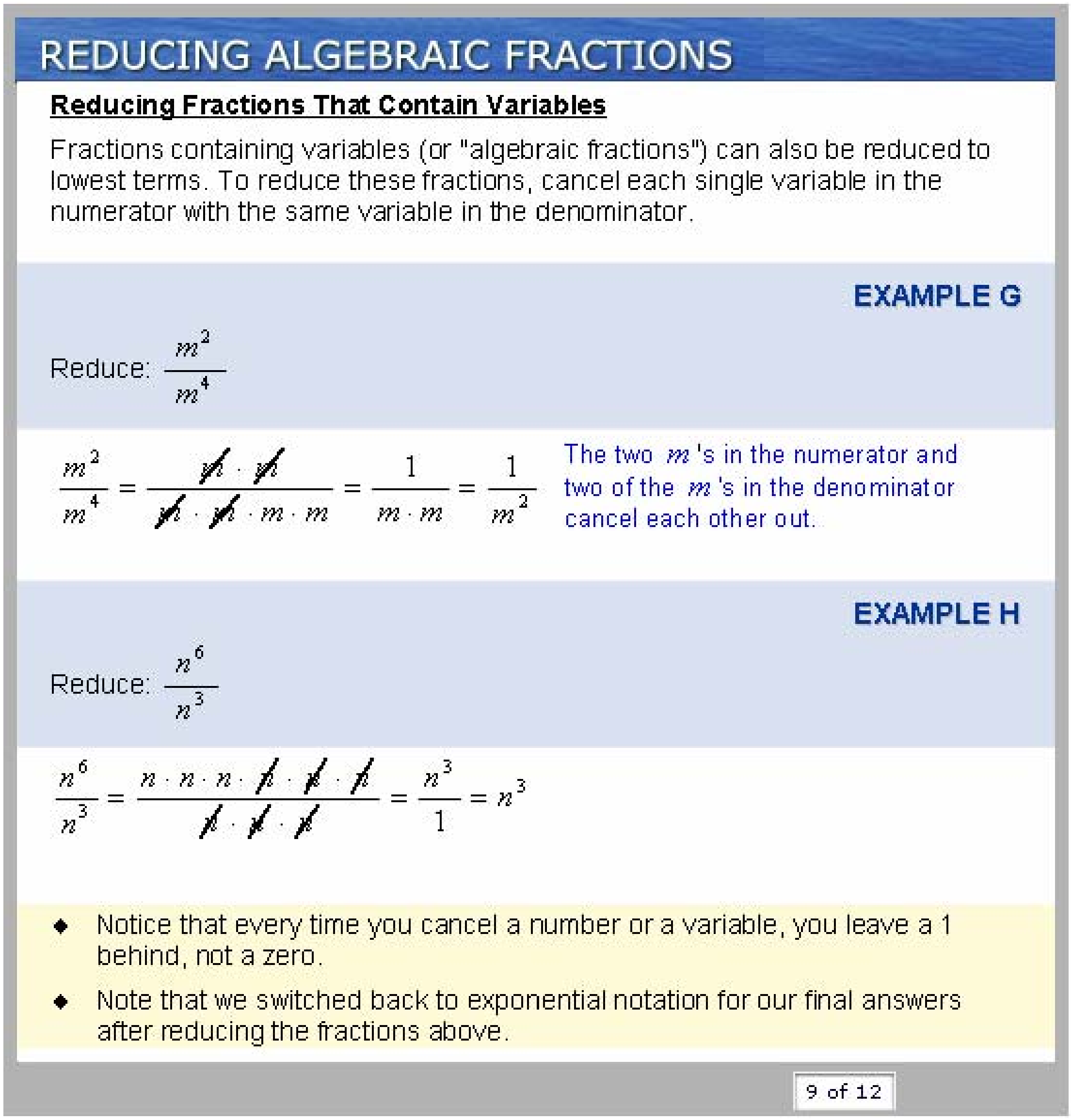
EXAMPLE F

Reduce: $\frac{18}{45}$

The prime factorization of 18 is $2 \cdot 3 \cdot 3$.

The prime factorization of 45 is $3 \cdot 3 \cdot 5$.

$$\frac{18}{45} = \frac{2 \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot 5} = \frac{2}{5}$$



REDUCING ALGEBRAIC FRACTIONS

Reducing Fractions That Contain Variables

Fractions containing variables (or "algebraic fractions") can also be reduced to lowest terms. To reduce these fractions, cancel each single variable in the numerator with the same variable in the denominator.

EXAMPLE G

Reduce: $\frac{m^2}{m^4}$

$$\frac{m^2}{m^4} = \frac{\cancel{m}\cdot\cancel{m}}{\cancel{m}\cdot\cancel{m}\cdot m\cdot m} = \frac{1}{m\cdot m} = \frac{1}{m^2}$$

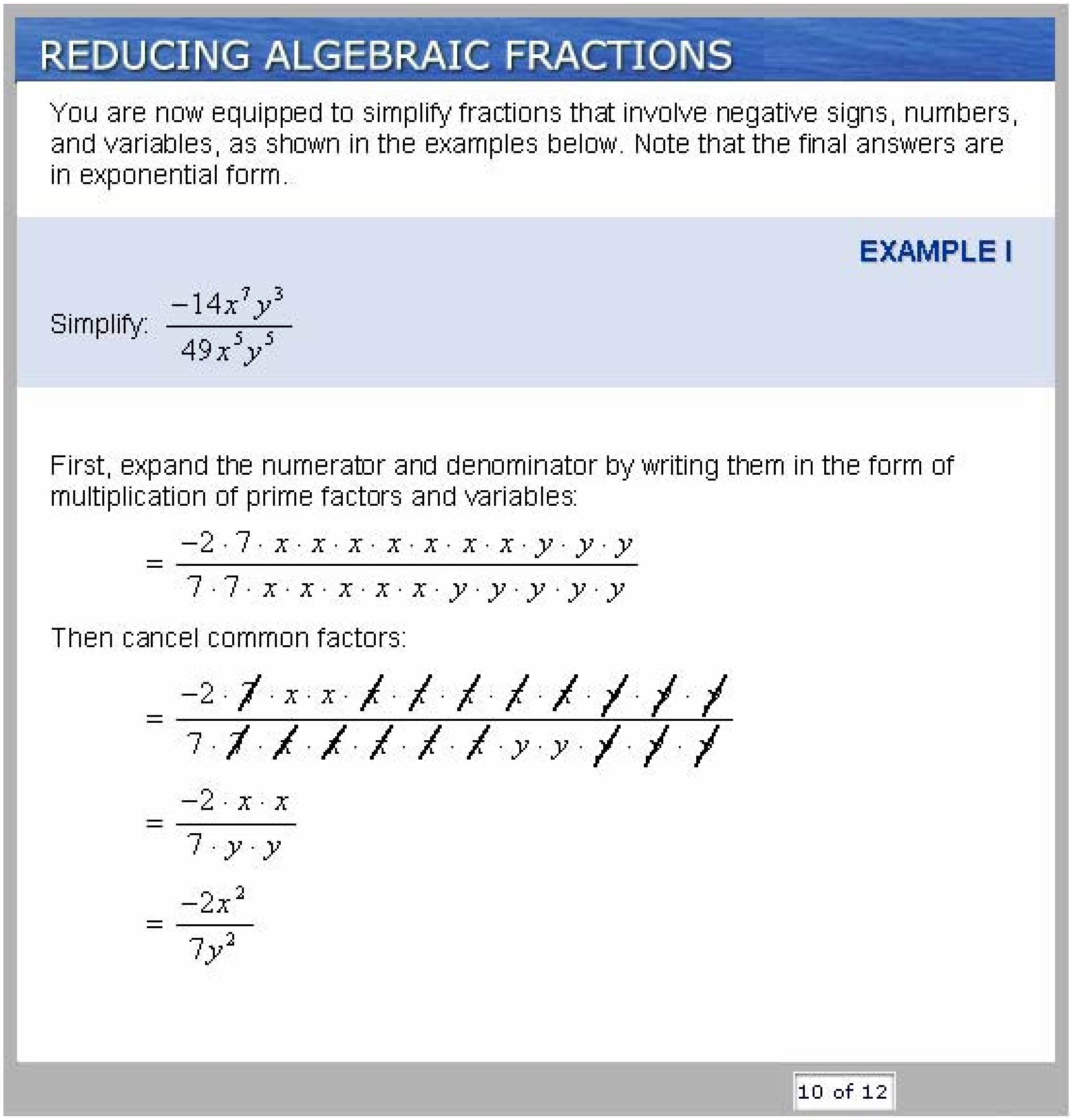
The two m's in the numerator and two of the m's in the denominator cancel each other out.

EXAMPLE H

Reduce: $\frac{n^6}{n^3}$

$$\frac{n^6}{n^3} = \frac{n\cdot n\cdot n\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}}{\cancel{n}\cdot\cancel{n}\cdot\cancel{n}} = \frac{n^3}{1} = n^3$$

- Notice that every time you cancel a number or a variable, you leave a 1 behind, not a zero.
- Note that we switched back to exponential notation for our final answers after reducing the fractions above.



REDUCING ALGEBRAIC FRACTIONS

You are now equipped to simplify fractions that involve negative signs, numbers, and variables, as shown in the examples below. Note that the final answers are in exponential form.

EXAMPLE I

Simplify: $\frac{-14x^7y^3}{49x^5y^5}$

First, expand the numerator and denominator by writing them in the form of multiplication of prime factors and variables:

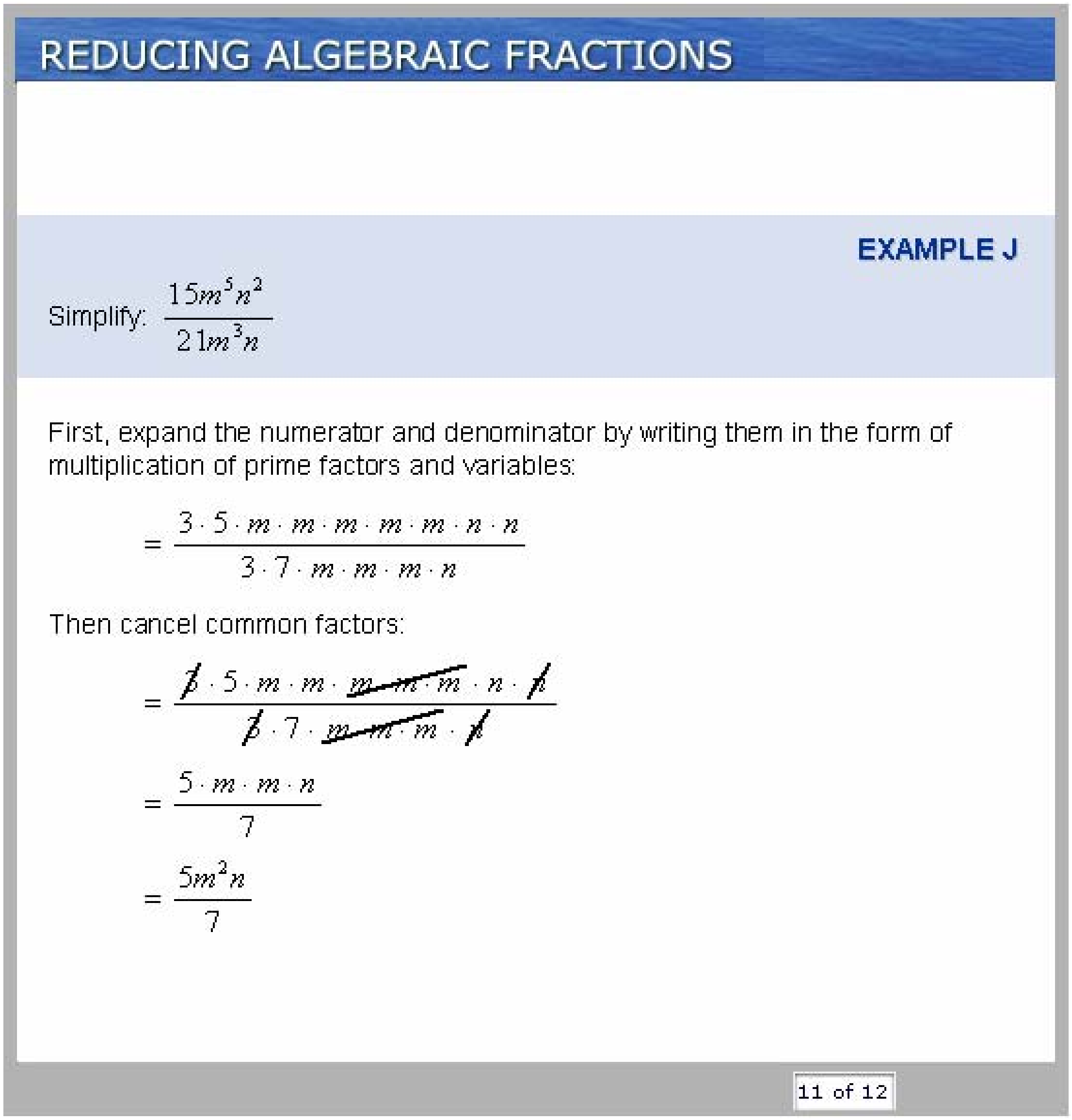
$$=\frac{-2\cdot 7\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y}{7\cdot 7\cdot x\cdot x\cdot x\cdot x\cdot x\cdot y\cdot y\cdot y\cdot y\cdot y}$$

Then cancel common factors:

$$=\frac{-2\cdot \cancel{7}\cdot x\cdot x\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{y}\cdot \cancel{y}\cdot \cancel{y}}{7\cdot \cancel{7}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot y\cdot y\cdot \cancel{y}\cdot \cancel{y}\cdot \cancel{y}}$$

$$=\frac{-2\cdot x\cdot x}{7\cdot y\cdot y}$$

$$=\frac{-2x^2}{7y^2}$$



REDUCING ALGEBRAIC FRACTIONS

EXAMPLE J

Simplify: $\frac{15m^5n^2}{21m^3n}$

First, expand the numerator and denominator by writing them in the form of multiplication of prime factors and variables:

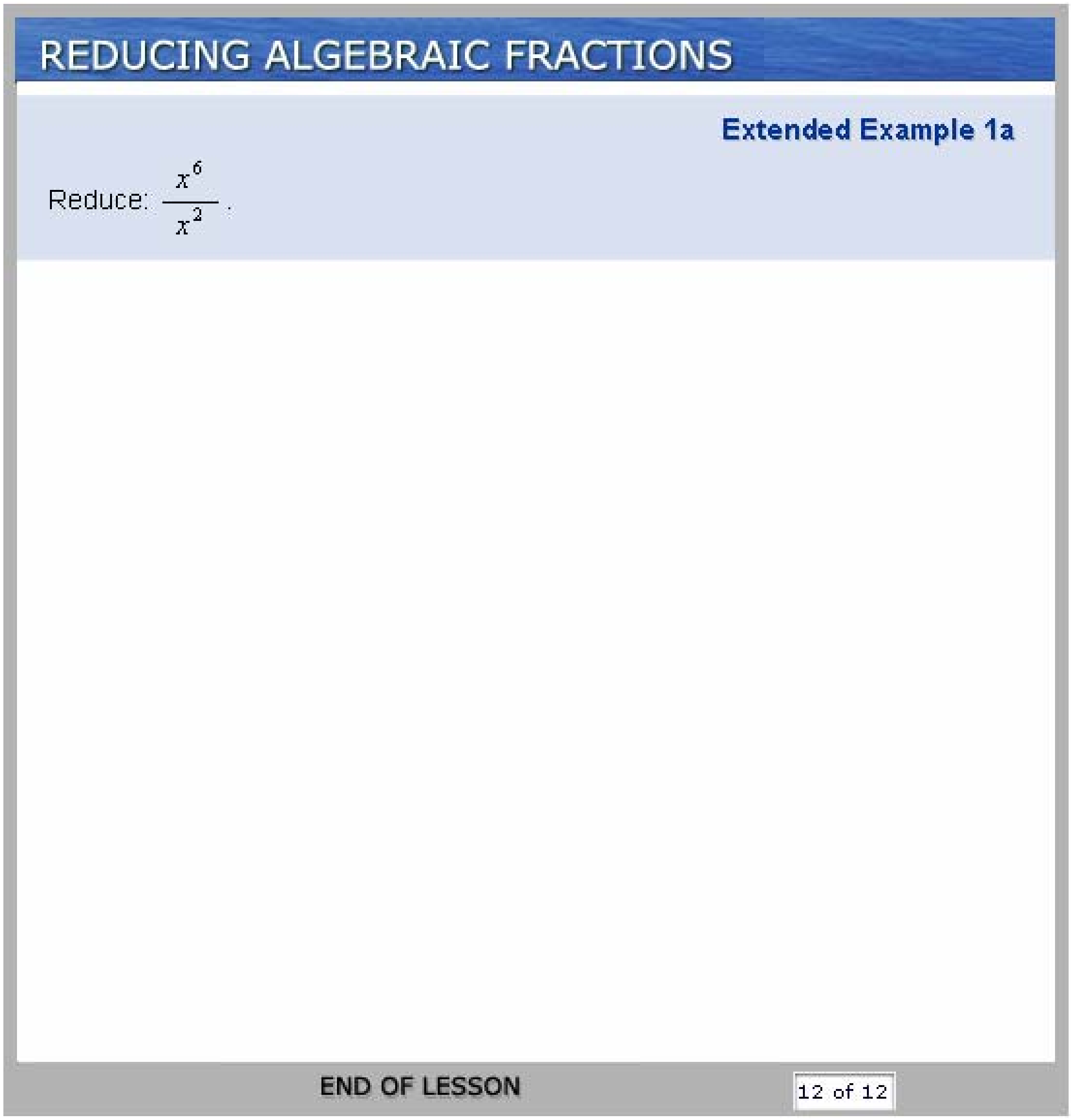
$$= \frac{3\cdot 5\cdot m\cdot m\cdot m\cdot m\cdot m\cdot n\cdot n}{3\cdot 7\cdot m\cdot m\cdot m\cdot n}$$

Then cancel common factors:

$$= \frac{\cancel{3}\cdot 5\cdot m\cdot m\cdot \cancel{m\cdot m\cdot m}\cdot n\cdot \cancel{n}}{\cancel{3}\cdot 7\cdot \cancel{m\cdot m\cdot m}\cdot \cancel{n}}$$

$$= \frac{5\cdot m\cdot m\cdot n}{7}$$

$$= \frac{5m^2n}{7}$$



REDUCING ALGEBRAIC FRACTIONS

Extended Example 1a

Reduce: $\frac{x^6}{x^2}$.

Reduce the fraction to lowest terms.

$$\frac{18}{-45} =$$

Find the prime factorization for the number.

116

Find the prime factorization for the number.

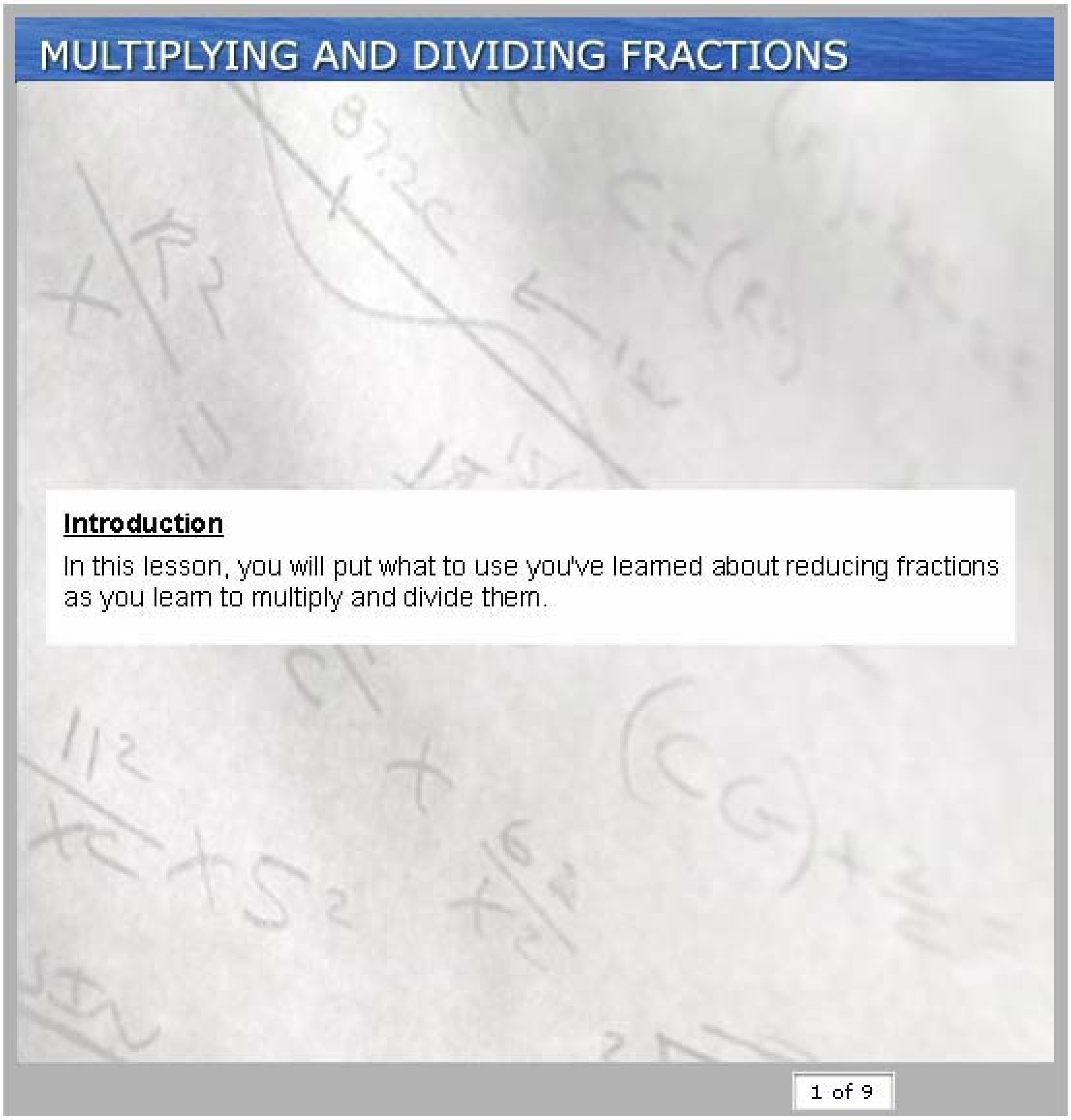
840

Reduce the algebraic fraction to lowest terms.

$$\frac{p^9q^4r^7}{p^5q^{10}r} =$$

Reduce the algebraic fraction to lowest terms.

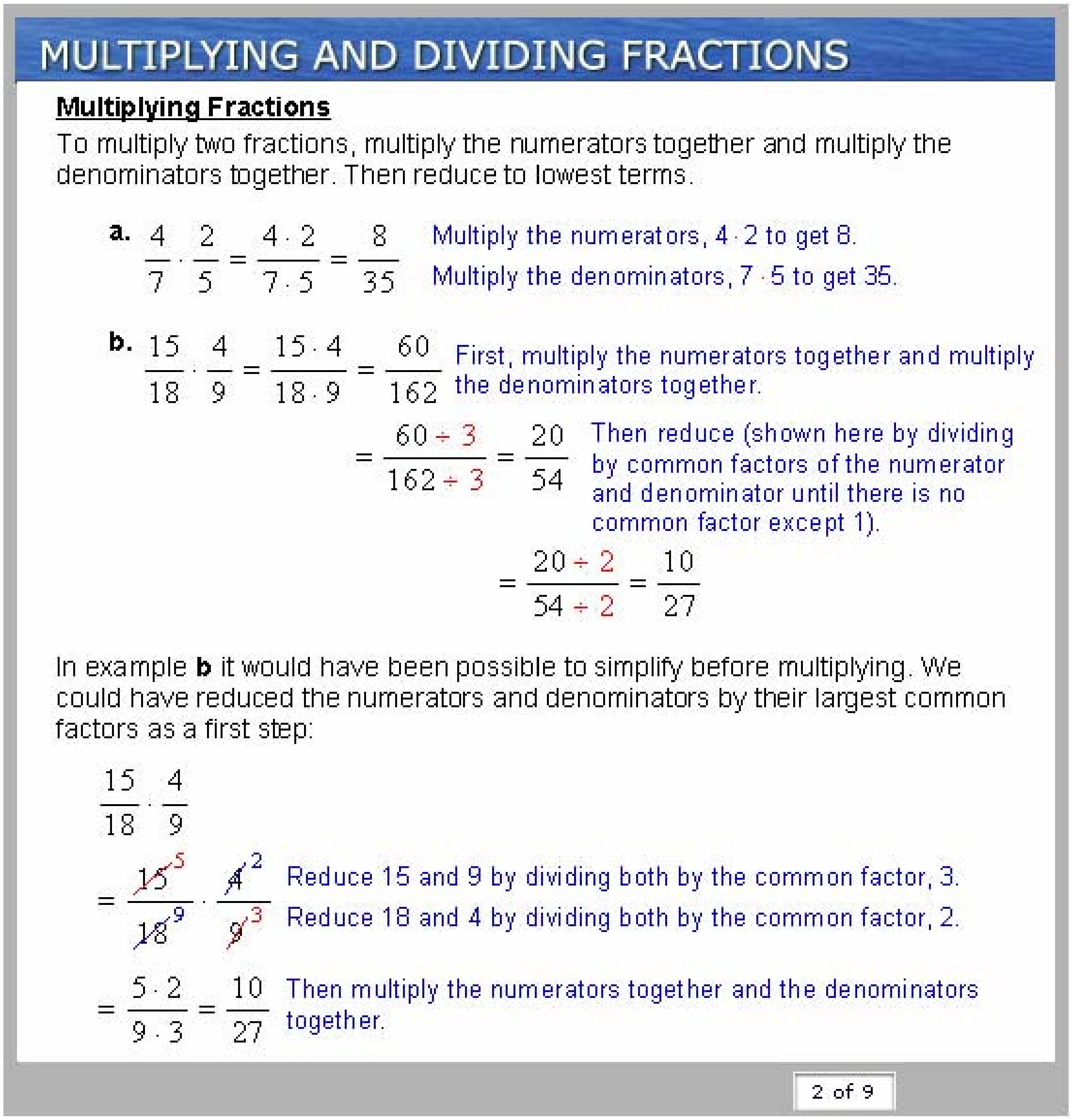
$$\frac{14x^3y^8z^4}{-28x^6y^4z^3} =$$



MULTIPLYING AND DIVIDING FRACTIONS

Introduction

In this lesson, you will put what to use you've learned about reducing fractions as you learn to multiply and divide them.



MULTIPLYING AND DIVIDING FRACTIONS

Multiplying Fractions

To multiply two fractions, multiply the numerators together and multiply the denominators together. Then reduce to lowest terms.

a. $\frac{4}{7} \cdot \frac{2}{5} = \frac{4 \cdot 2}{7 \cdot 5} = \frac{8}{35}$ Multiply the numerators, $4 \cdot 2$ to get 8. Multiply the denominators, $7 \cdot 5$ to get 35.

b. $\frac{15}{18} \cdot \frac{4}{9} = \frac{15 \cdot 4}{18 \cdot 9} = \frac{60}{162}$ First, multiply the numerators together and multiply the denominators together.

$= \frac{60 \div 3}{162 \div 3} = \frac{20}{54}$ Then reduce (shown here by dividing by common factors of the numerator and denominator until there is no common factor except 1).

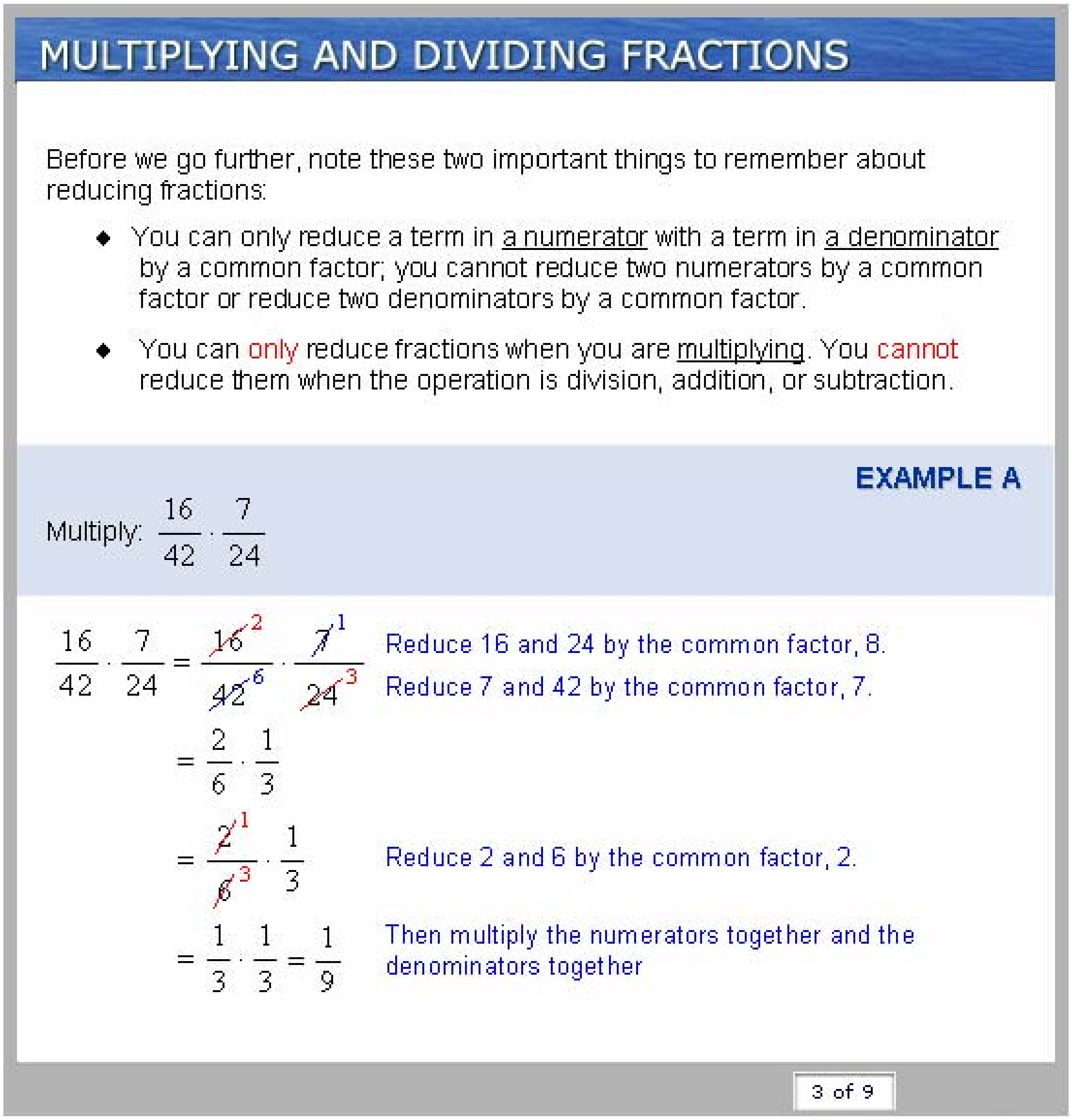
$= \frac{20 \div 2}{54 \div 2} = \frac{10}{27}$

In example **b** it would have been possible to simplify before multiplying. We could have reduced the numerators and denominators by their largest common factors as a first step:

$$\frac{15}{18} \cdot \frac{4}{9}$$

$= \frac{\cancel{15}^{5}}{\cancel{18}^{9}} \cdot \frac{\cancel{4}^{2}}{\cancel{9}^{3}}$ Reduce 15 and 9 by dividing both by the common factor, 3. Reduce 18 and 4 by dividing both by the common factor, 2.

$= \frac{5 \cdot 2}{9 \cdot 3} = \frac{10}{27}$ Then multiply the numerators together and the denominators together.



MULTIPLYING AND DIVIDING FRACTIONS

Before we go further, note these two important things to remember about reducing fractions:

- You can only reduce a term in a numerator with a term in a denominator by a common factor; you cannot reduce two numerators by a common factor or reduce two denominators by a common factor.
- You can only reduce fractions when you are multiplying. You cannot reduce them when the operation is division, addition, or subtraction.

EXAMPLE A

Multiply: $\frac{16}{42} \cdot \frac{7}{24}$

$$\frac{16}{42} \cdot \frac{7}{24} = \frac{\cancel{16}^{2}}{\cancel{42}^{6}} \cdot \frac{\cancel{7}^{1}}{\cancel{24}^{3}}$$

Reduce 16 and 24 by the common factor, 8.

Reduce 7 and 42 by the common factor, 7.

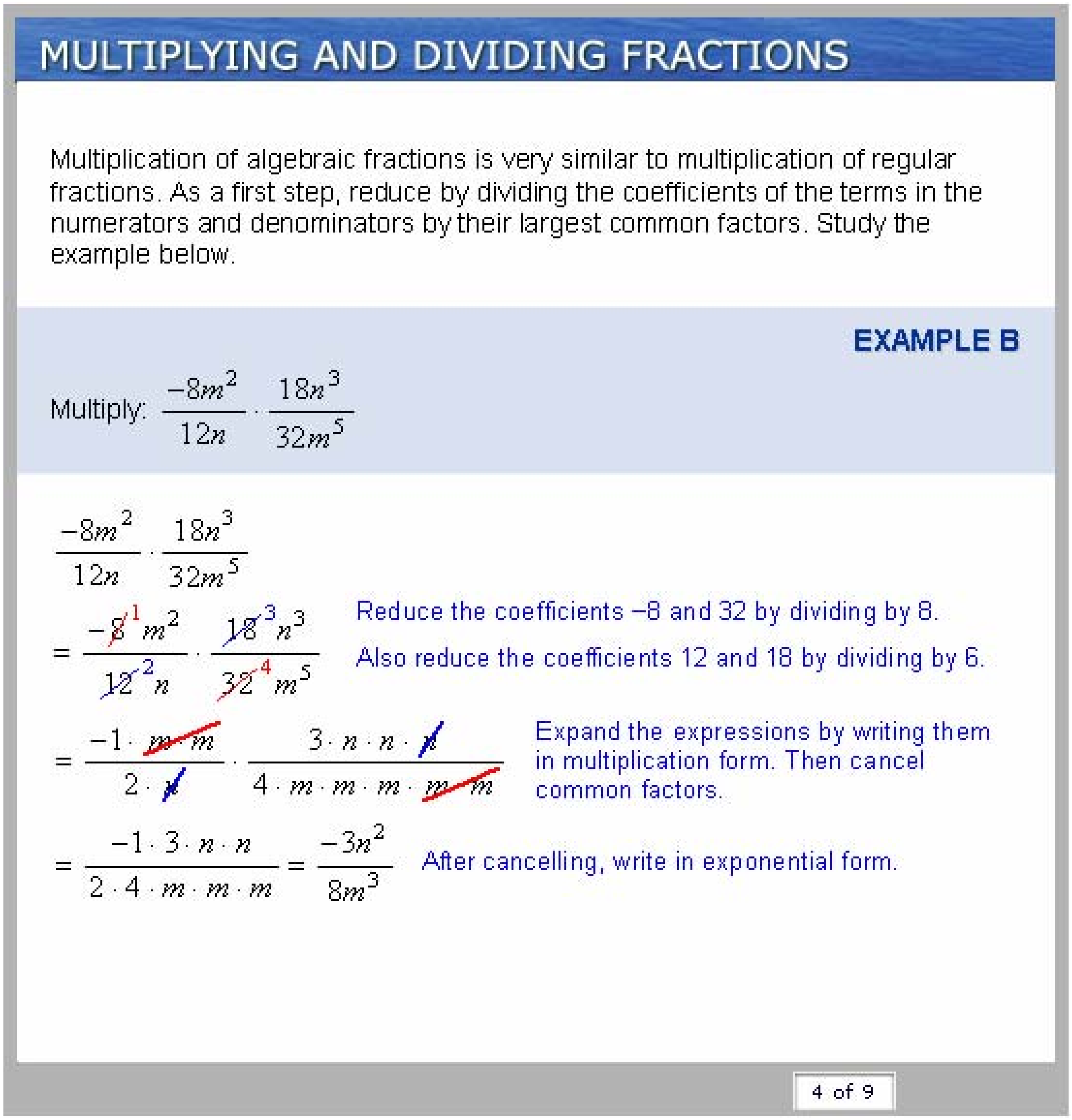
$$= \frac{2}{6} \cdot \frac{1}{3}$$

$$= \frac{\cancel{2}^{1}}{\cancel{6}^{3}} \cdot \frac{1}{3}$$

Reduce 2 and 6 by the common factor, 2.

$$= \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

Then multiply the numerators together and the denominators together



MULTIPLYING AND DIVIDING FRACTIONS

Multiplication of algebraic fractions is very similar to multiplication of regular fractions. As a first step, reduce by dividing the coefficients of the terms in the numerators and denominators by their largest common factors. Study the example below.

EXAMPLE B

Multiply: $\frac{-8m^2}{12n} \cdot \frac{18n^3}{32m^5}$

$$\frac{-8m^2}{12n} \cdot \frac{18n^3}{32m^5}$$

$$= \frac{-\cancel{8}^{1} m^2}{\cancel{12}^{2} n} \cdot \frac{\cancel{18}^{3} n^3}{\cancel{32}^{4} m^5}$$

Reduce the coefficients –8 and 32 by dividing by 8.

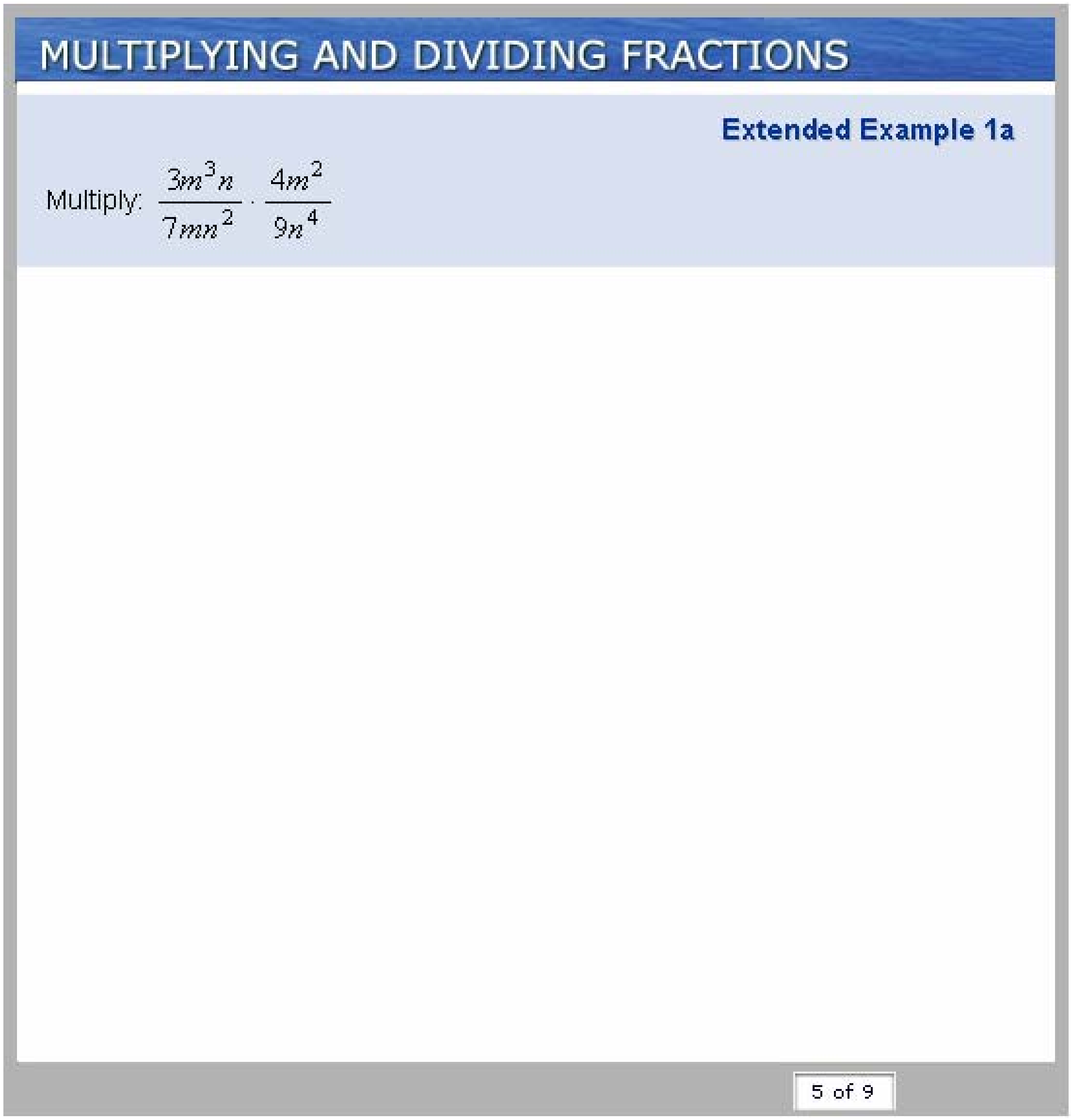
Also reduce the coefficients 12 and 18 by dividing by 6.

$$= \frac{-1 \cdot \cancel{m \cdot m}}{2 \cdot \cancel{n}} \cdot \frac{3 \cdot n \cdot n \cdot \cancel{n}}{4 \cdot m \cdot m \cdot m \cdot \cancel{m \cdot m}}$$

Expand the expressions by writing them in multiplication form. Then cancel common factors.

$$= \frac{-1 \cdot 3 \cdot n \cdot n}{2 \cdot 4 \cdot m \cdot m \cdot m} = \frac{-3n^2}{8m^3}$$

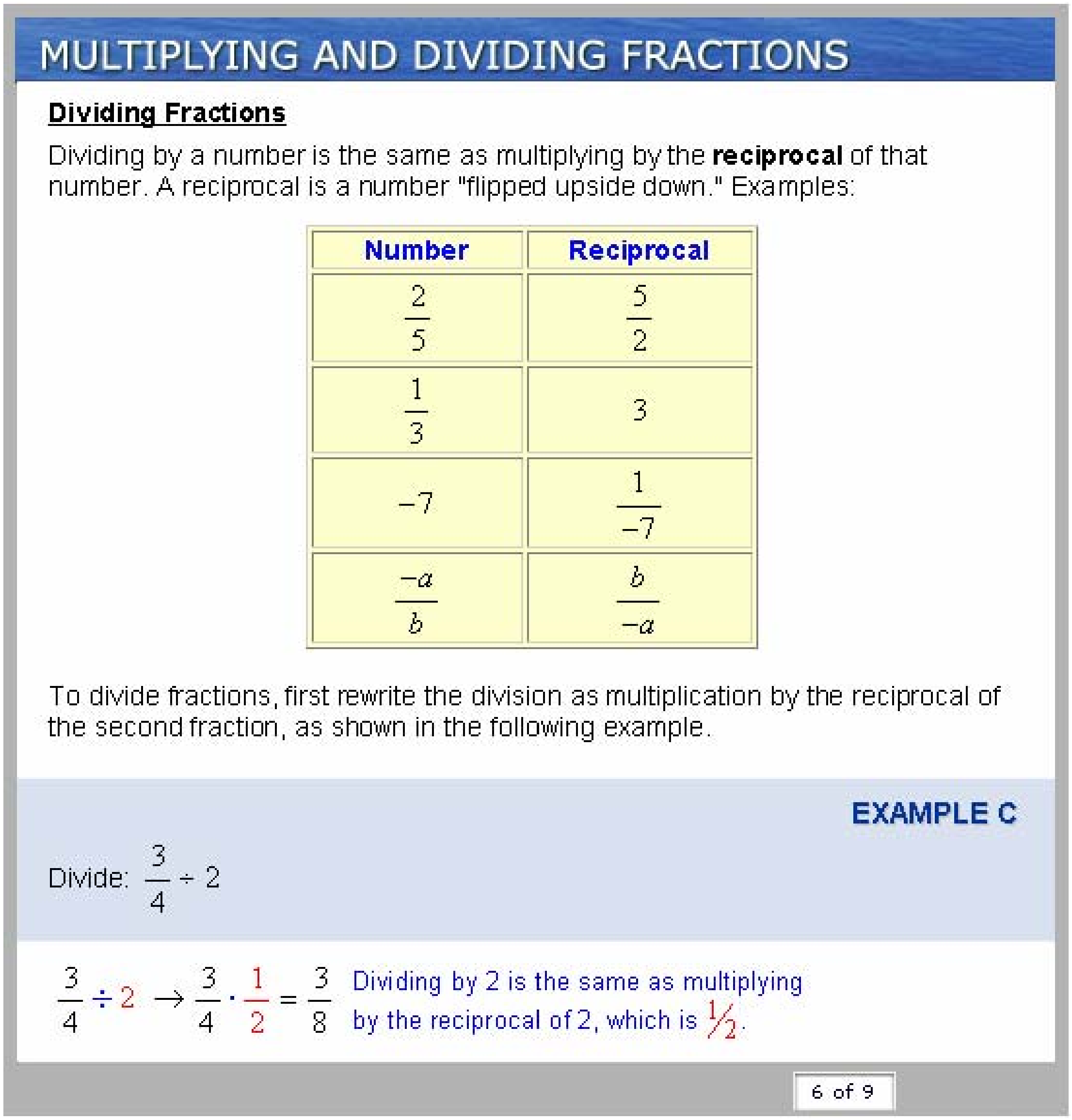
After cancelling, write in exponential form.



MULTIPLYING AND DIVIDING FRACTIONS

Extended Example 1a

Multiply: $\frac{3m^3n}{7mn^2} \cdot \frac{4m^2}{9n^4}$



MULTIPLYING AND DIVIDING FRACTIONS

Dividing Fractions

Dividing by a number is the same as multiplying by the **reciprocal** of that number. A reciprocal is a number "flipped upside down." Examples:

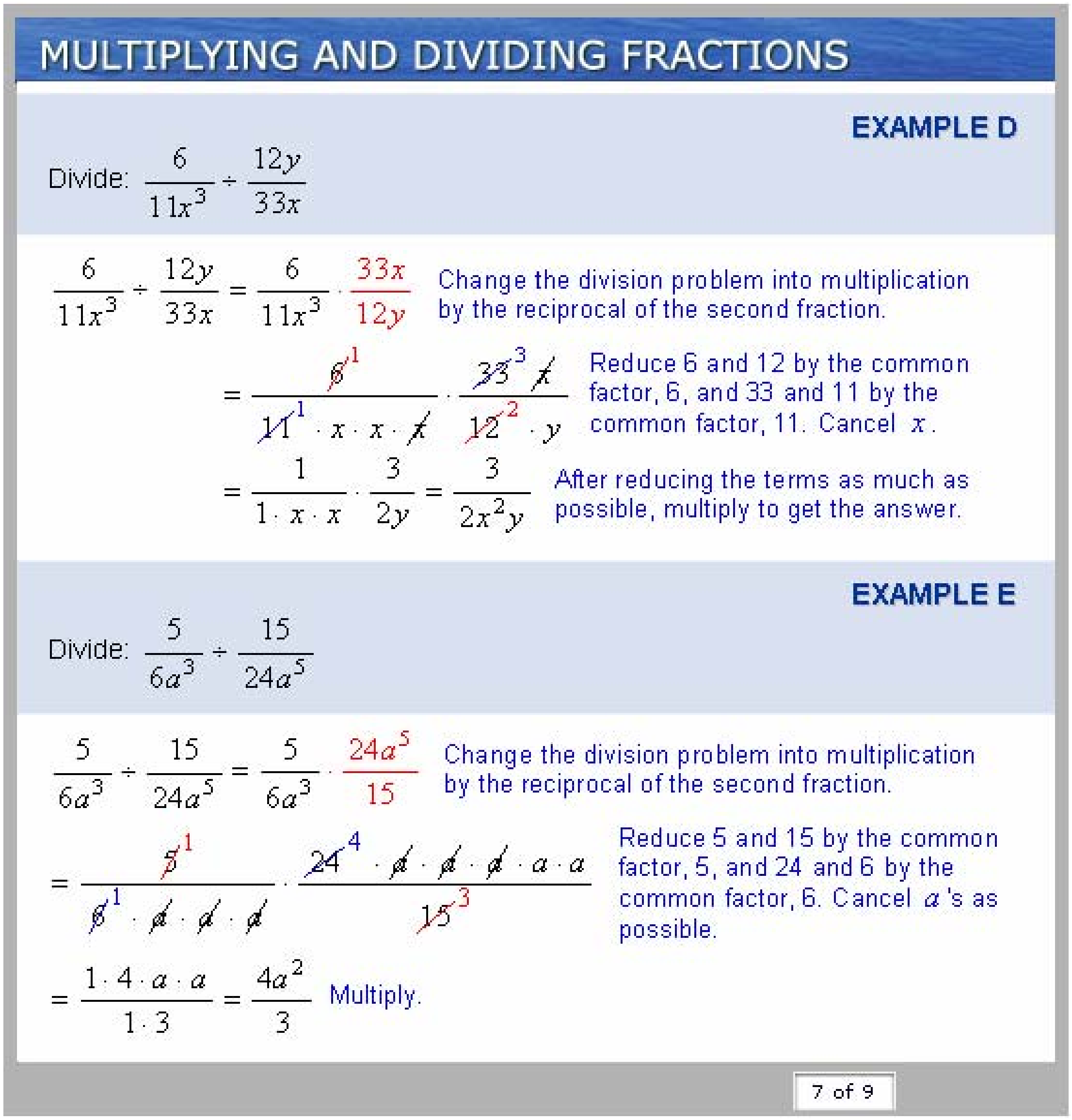
Number	Reciprocal
$\frac{2}{5}$	$\frac{5}{2}$
$\frac{1}{3}$	3
-7	$\frac{1}{-7}$
$\frac{-a}{b}$	$\frac{b}{-a}$

To divide fractions, first rewrite the division as multiplication by the reciprocal of the second fraction, as shown in the following example.

EXAMPLE C

Divide: $\frac{3}{4} \div 2$

$\frac{3}{4} \div 2 \rightarrow \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$ Dividing by 2 is the same as multiplying by the reciprocal of 2, which is $1/2$.



MULTIPLYING AND DIVIDING FRACTIONS

EXAMPLE D

Divide: $\frac{6}{11x^3} \div \frac{12y}{33x}$

$\frac{6}{11x^3} \div \frac{12y}{33x} = \frac{6}{11x^3} \cdot \frac{33x}{12y}$ Change the division problem into multiplication by the reciprocal of the second fraction.

$= \frac{\cancel{6}^{1}}{\cancel{11}^{1} \cdot x \cdot x \cdot \cancel{x}} \cdot \frac{\cancel{33}^{3}\,\cancel{x}}{\cancel{12}^{2} \cdot y}$ Reduce 6 and 12 by the common factor, 6, and 33 and 11 by the common factor, 11. Cancel x.

$= \frac{1}{1 \cdot x \cdot x} \cdot \frac{3}{2y} = \frac{3}{2x^2y}$ After reducing the terms as much as possible, multiply to get the answer.

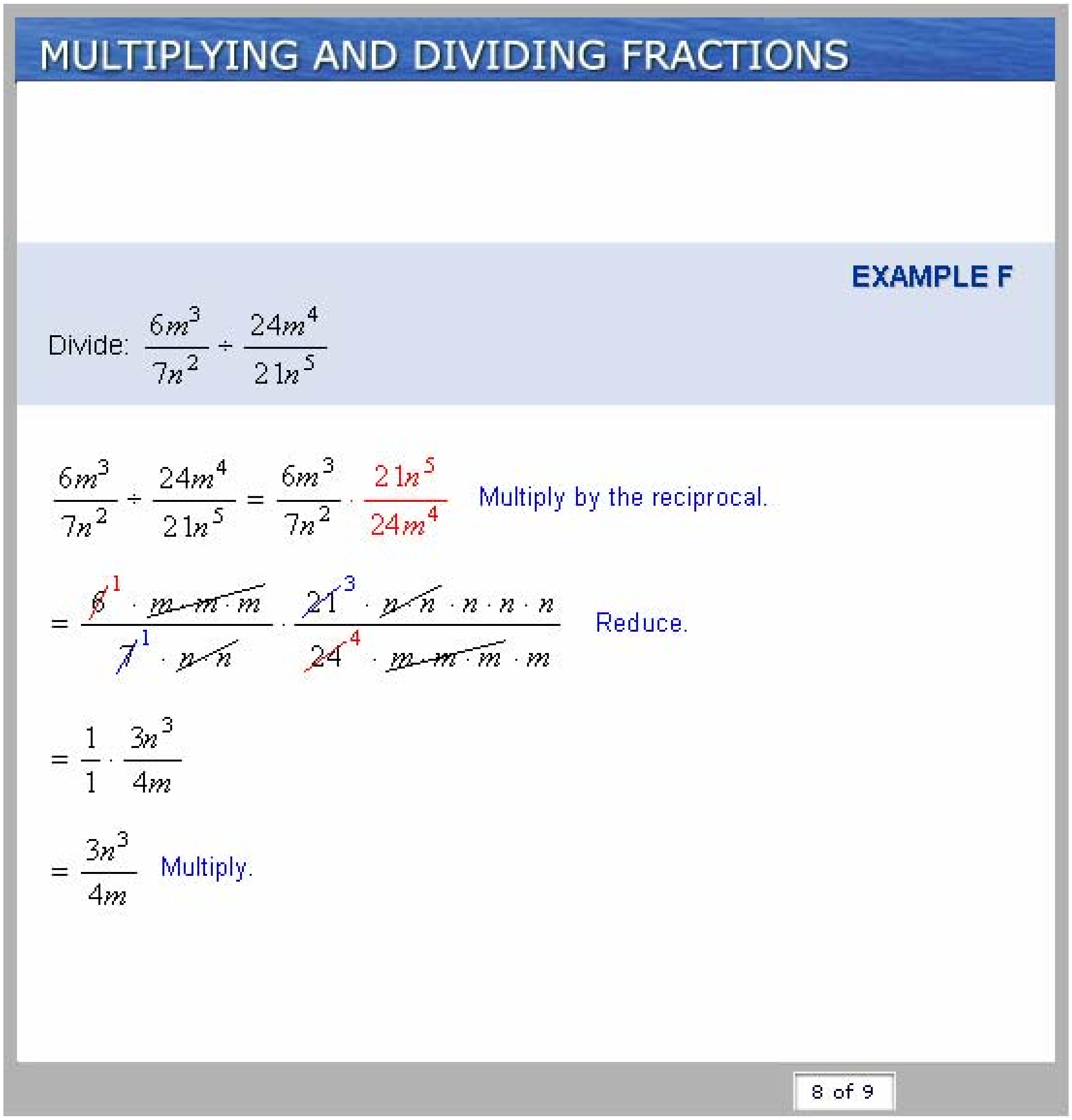
EXAMPLE E

Divide: $\frac{5}{6a^3} \div \frac{15}{24a^5}$

$\frac{5}{6a^3} \div \frac{15}{24a^5} = \frac{5}{6a^3} \cdot \frac{24a^5}{15}$ Change the division problem into multiplication by the reciprocal of the second fraction.

$= \frac{\cancel{5}^{1}}{\cancel{6}^{1} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}} \cdot \frac{\cancel{24}^{4} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a \cdot a}{\cancel{15}^{3}}$ Reduce 5 and 15 by the common factor, 5, and 24 and 6 by the common factor, 6. Cancel a's as possible.

$= \frac{1 \cdot 4 \cdot a \cdot a}{1 \cdot 3} = \frac{4a^2}{3}$ Multiply.



MULTIPLYING AND DIVIDING FRACTIONS

EXAMPLE F

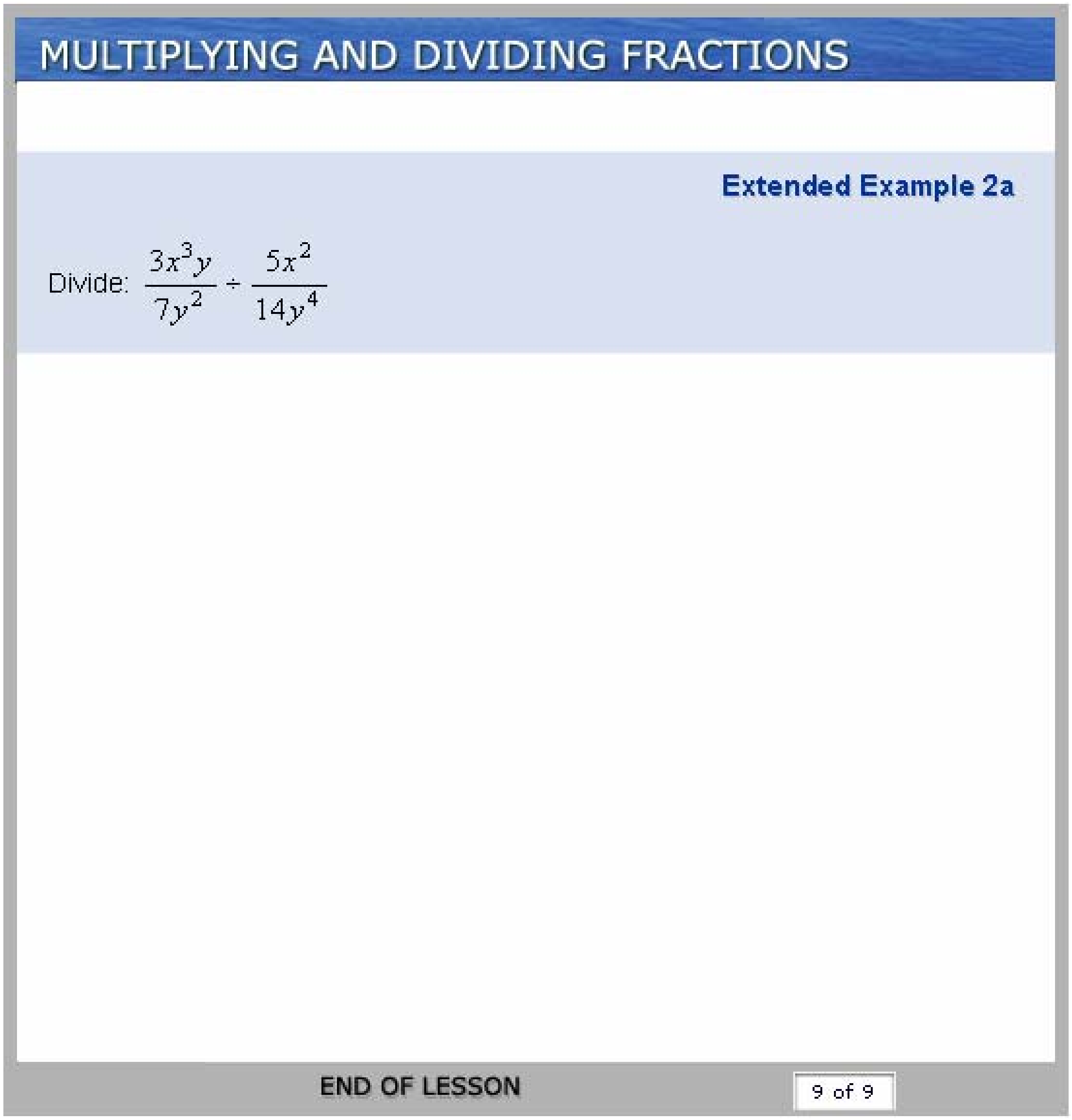
Divide: $\frac{6m^3}{7n^2} \div \frac{24m^4}{21n^5}$

$$\frac{6m^3}{7n^2} \div \frac{24m^4}{21n^5} = \frac{6m^3}{7n^2} \cdot \frac{21n^5}{24m^4}$$ Multiply by the reciprocal.

$$= \frac{\cancel{6}^{1} \cdot \cancel{m \cdot m \cdot m}}{\cancel{7}^{1} \cdot \cancel{n \cdot n}} \cdot \frac{\cancel{21}^{3} \cdot \cancel{n \cdot n} \cdot n \cdot n \cdot n}{\cancel{24}^{4} \cdot \cancel{m \cdot m \cdot m} \cdot m}$$ Reduce.

$$= \frac{1}{1} \cdot \frac{3n^3}{4m}$$

$$= \frac{3n^3}{4m}$$ Multiply.



MULTIPLYING AND DIVIDING FRACTIONS

Extended Example 2a

Divide: $\frac{3x^3y}{7y^2} \div \frac{5x^2}{14y^4}$

Multiply.

$$\frac{4m^3}{9n^5} \cdot \frac{27}{3m^5} =$$

Multiply.

$$\frac{2r}{8s^8} \cdot \frac{16r^3s^4}{4r^6s} =$$

Divide.

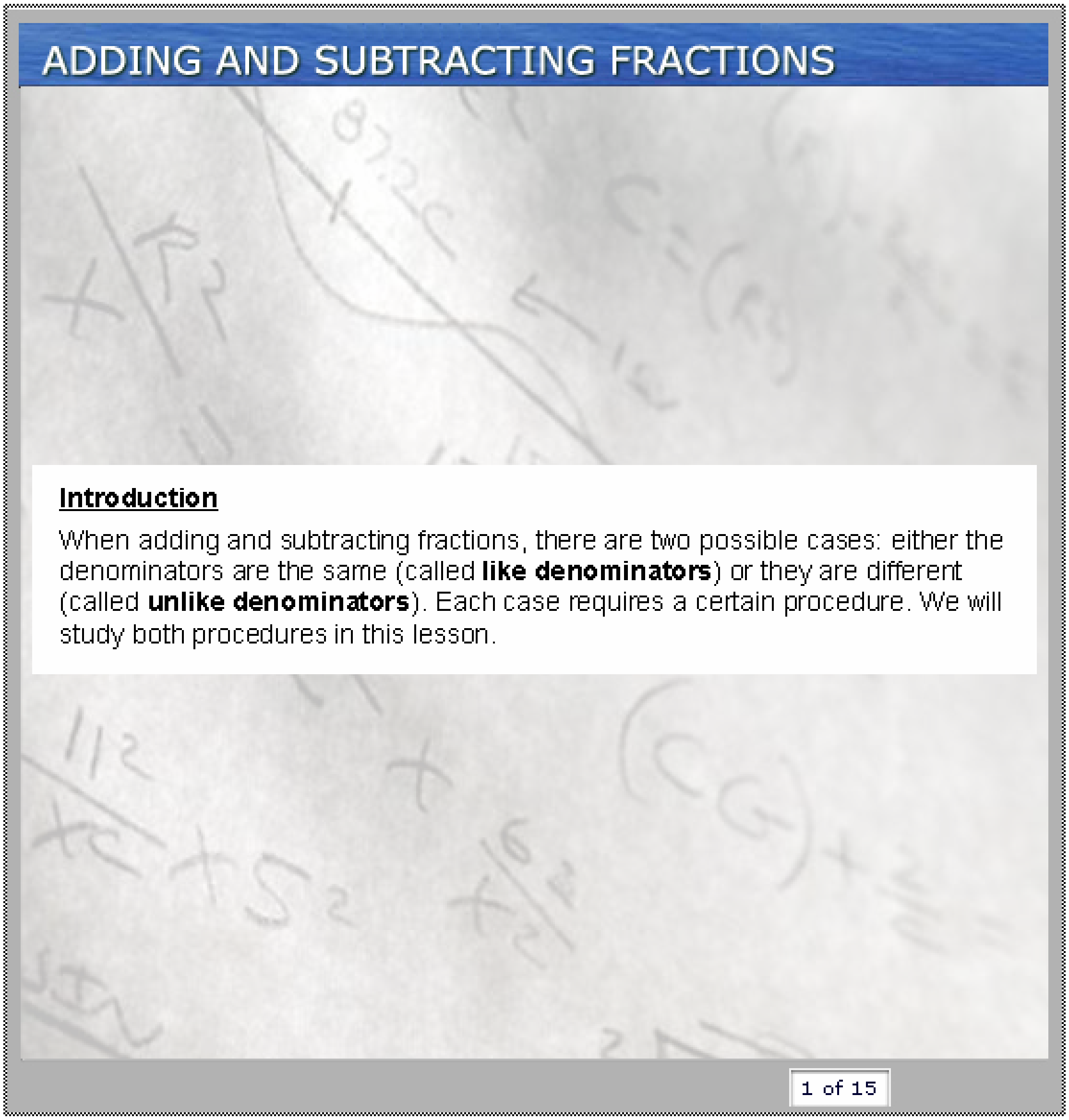
$$\frac{7m}{15} \div \frac{14}{-5n} =$$

Divide.

$$\frac{5x^4}{7y^3} \div \frac{25x^3}{35y} =$$

Divide.

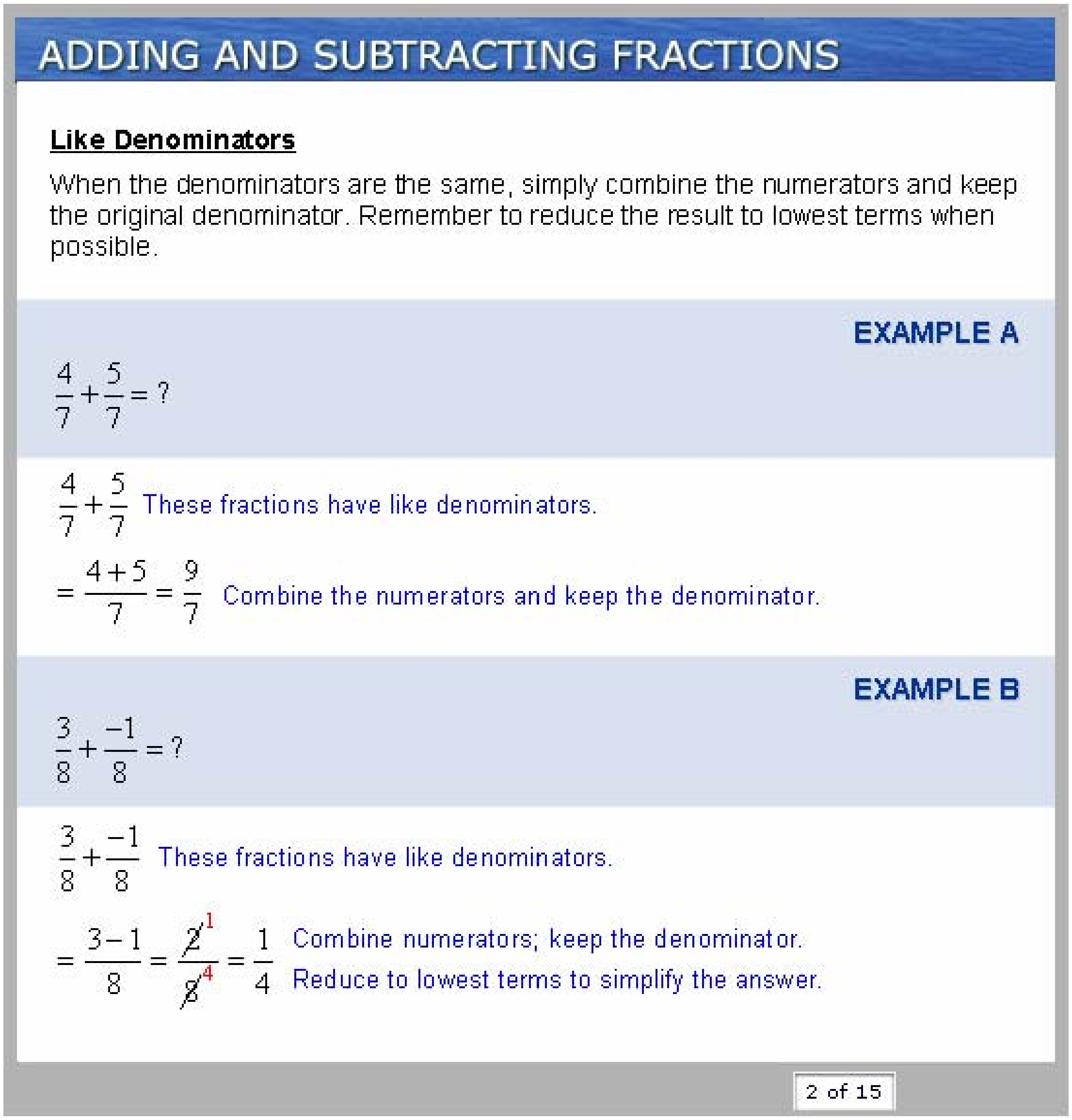
$$\frac{p^3q^5}{10} \div \frac{p^5q^3r^3}{-4r^4} =$$



ADDING AND SUBTRACTING FRACTIONS

Introduction

When adding and subtracting fractions, there are two possible cases: either the denominators are the same (called **like denominators**) or they are different (called **unlike denominators**). Each case requires a certain procedure. We will study both procedures in this lesson.



ADDING AND SUBTRACTING FRACTIONS

Like Denominators

When the denominators are the same, simply combine the numerators and keep the original denominator. Remember to reduce the result to lowest terms when possible.

EXAMPLE A

$\frac{4}{7} + \frac{5}{7} = ?$

$\frac{4}{7} + \frac{5}{7}$ These fractions have like denominators.

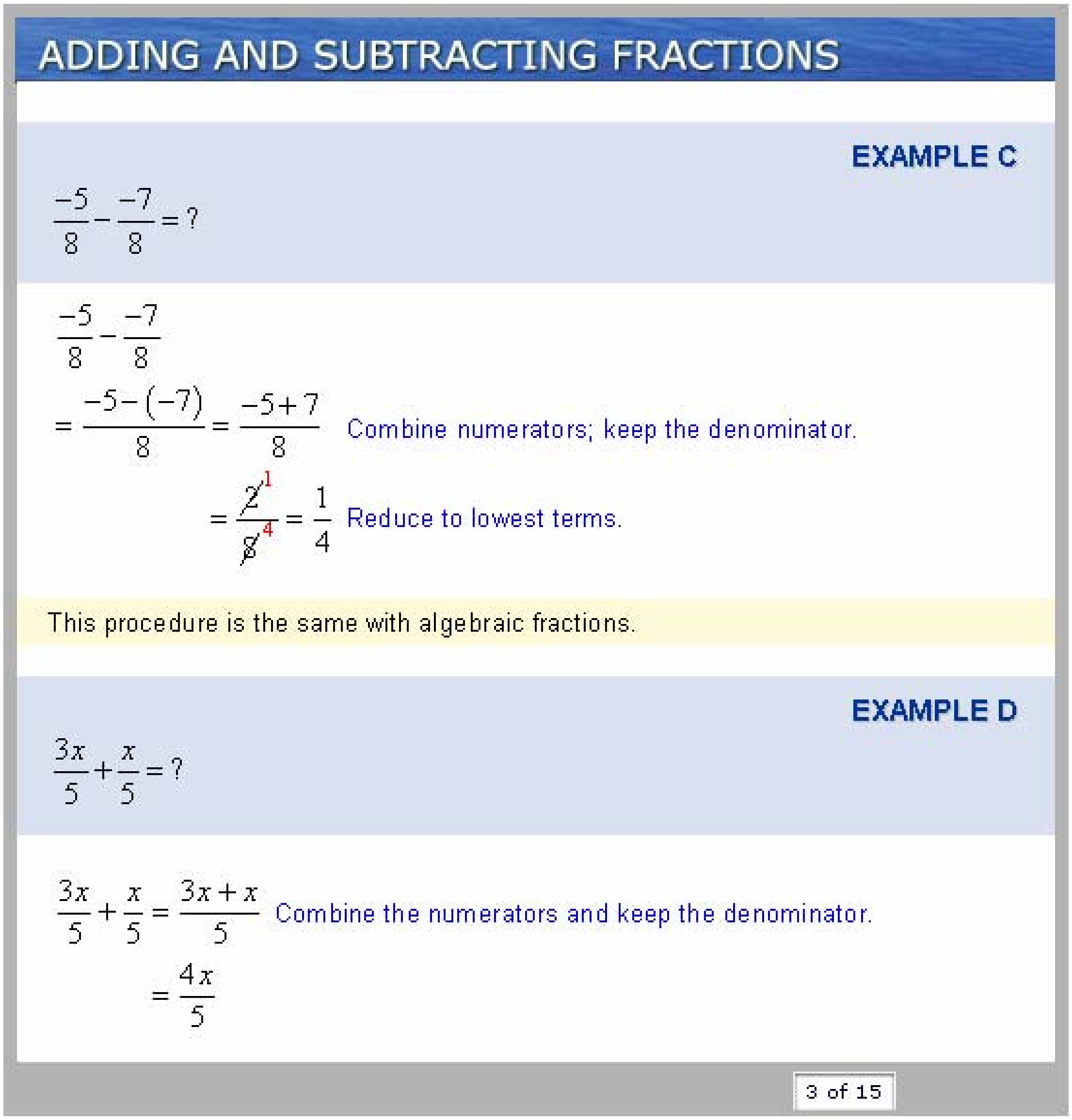
$= \frac{4+5}{7} = \frac{9}{7}$ Combine the numerators and keep the denominator.

EXAMPLE B

$\frac{3}{8} + \frac{-1}{8} = ?$

$\frac{3}{8} + \frac{-1}{8}$ These fractions have like denominators.

$= \frac{3-1}{8} = \frac{\cancel{2}^{1}}{\cancel{8}^{4}} = \frac{1}{4}$ Combine numerators; keep the denominator.
Reduce to lowest terms to simplify the answer.



ADDING AND SUBTRACTING FRACTIONS

EXAMPLE C

$$\frac{-5}{8}-\frac{-7}{8}=?$$

$$\frac{-5}{8}-\frac{-7}{8}$$

$$=\frac{-5-(-7)}{8}=\frac{-5+7}{8}$$ Combine numerators; keep the denominator.

$$=\frac{\cancel{2}^{1}}{\cancel{8}^{4}}=\frac{1}{4}$$ Reduce to lowest terms.

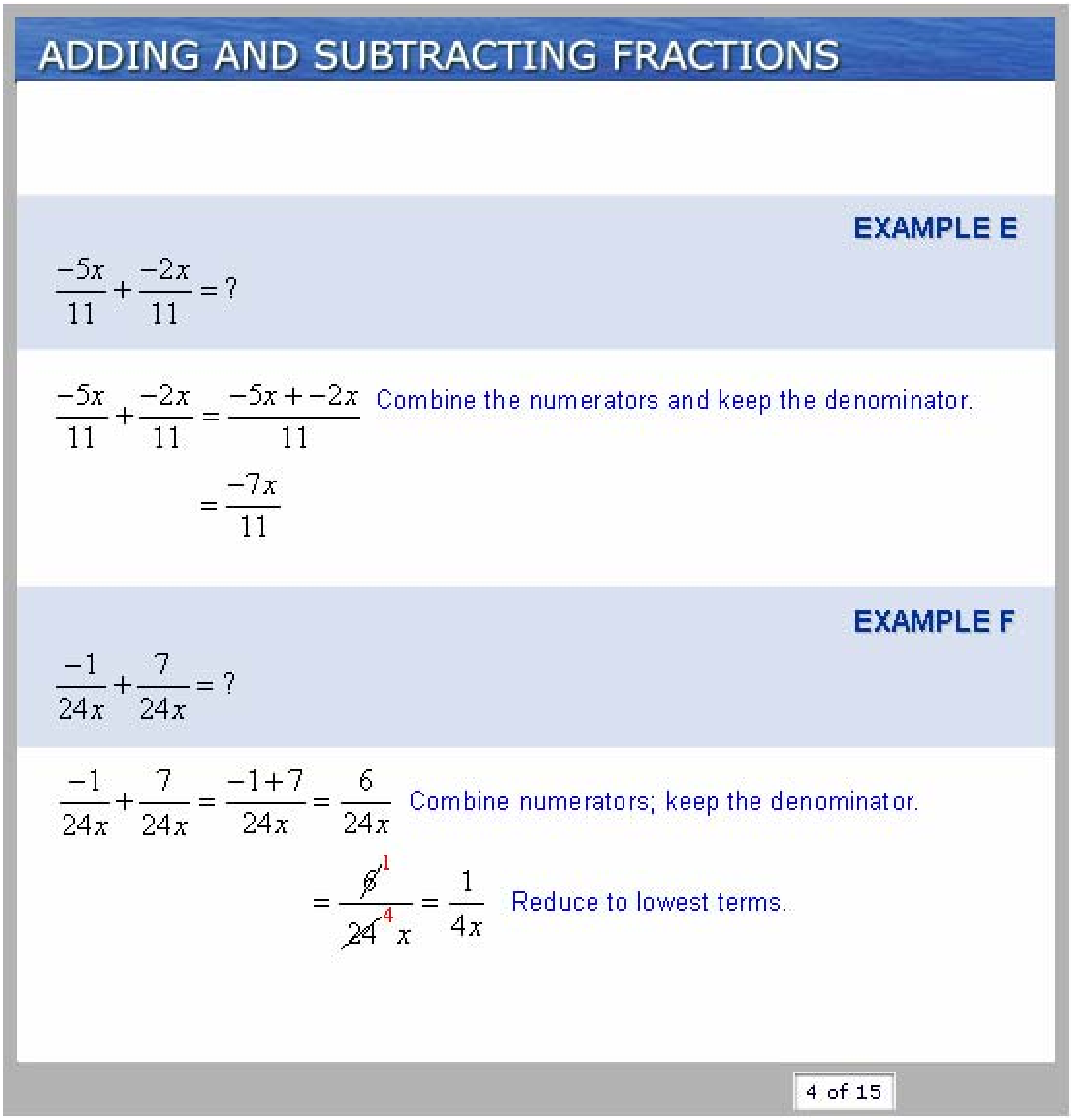
This procedure is the same with algebraic fractions.

EXAMPLE D

$$\frac{3x}{5}+\frac{x}{5}=?$$

$$\frac{3x}{5}+\frac{x}{5}=\frac{3x+x}{5}$$ Combine the numerators and keep the denominator.

$$=\frac{4x}{5}$$



ADDING AND SUBTRACTING FRACTIONS

EXAMPLE E

$$\frac{-5x}{11}+\frac{-2x}{11}=?$$

$$\frac{-5x}{11}+\frac{-2x}{11}=\frac{-5x+-2x}{11}$$ Combine the numerators and keep the denominator.

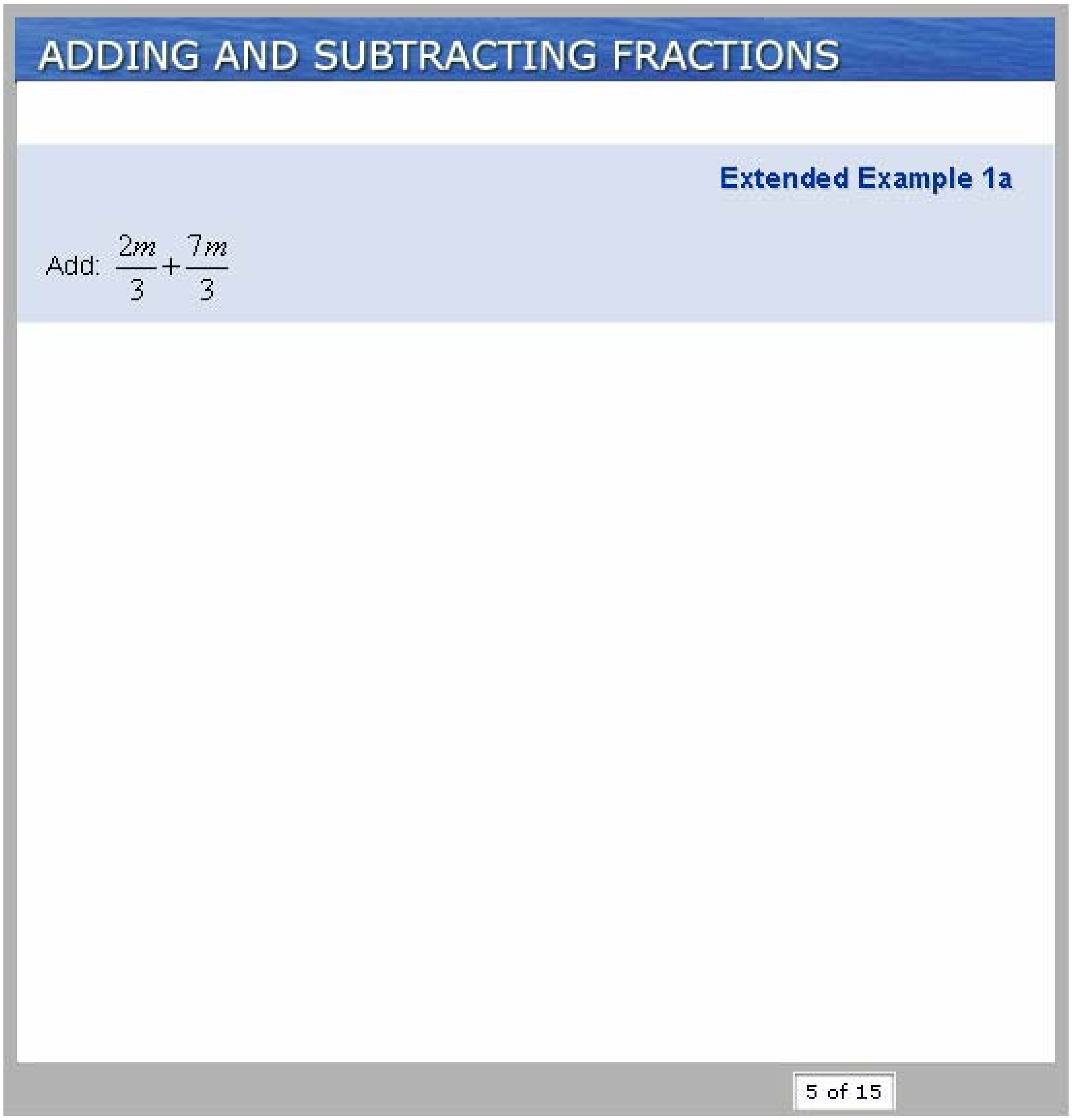
$$=\frac{-7x}{11}$$

EXAMPLE F

$$\frac{-1}{24x}+\frac{7}{24x}=?$$

$$\frac{-1}{24x}+\frac{7}{24x}=\frac{-1+7}{24x}=\frac{6}{24x}$$ Combine numerators; keep the denominator.

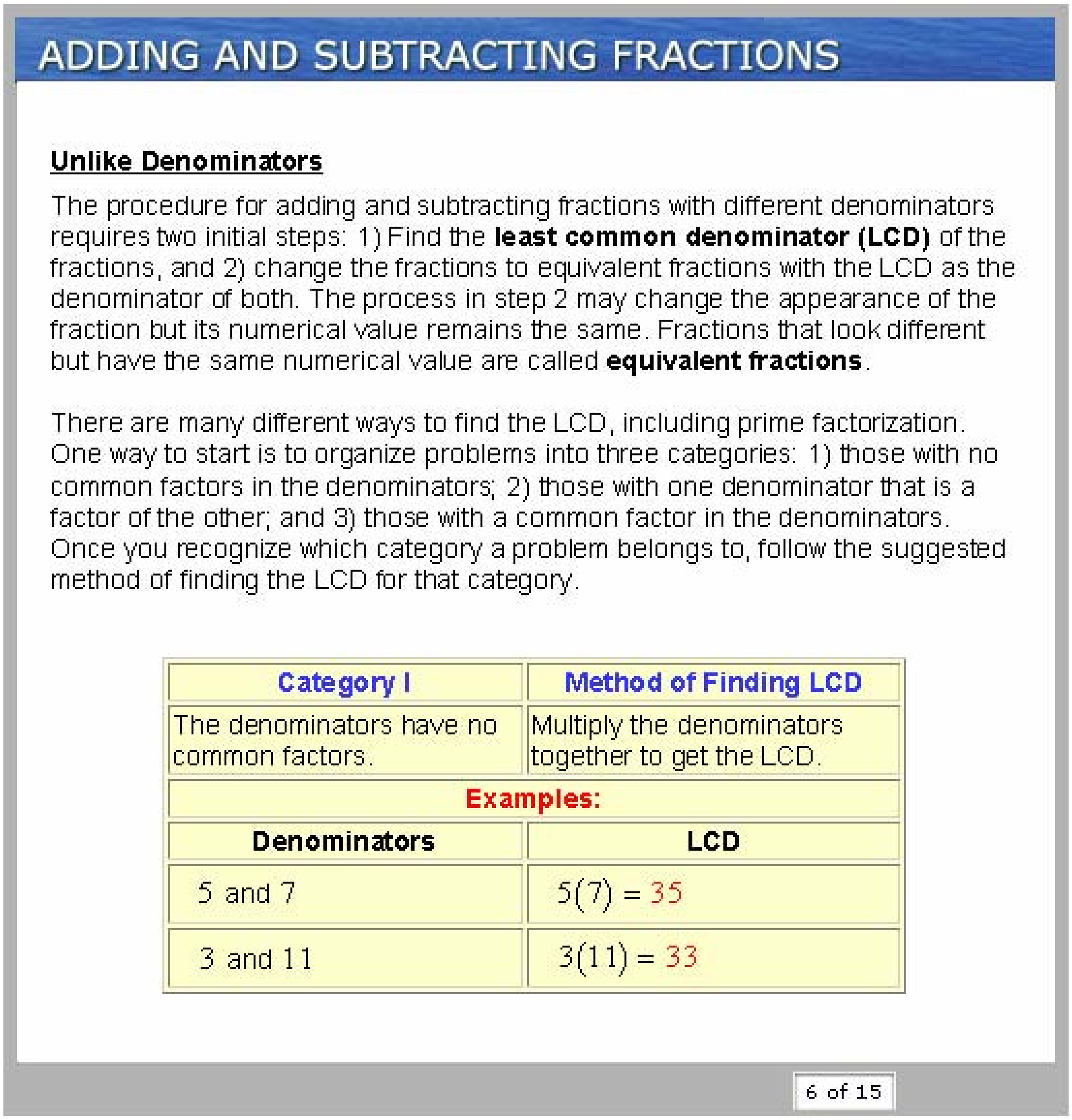
$$=\frac{\cancel{6}^{1}}{\cancel{24}^{4}x}=\frac{1}{4x}$$ Reduce to lowest terms.



ADDING AND SUBTRACTING FRACTIONS

Extended Example 1a

Add: $\frac{2m}{3}+\frac{7m}{3}$



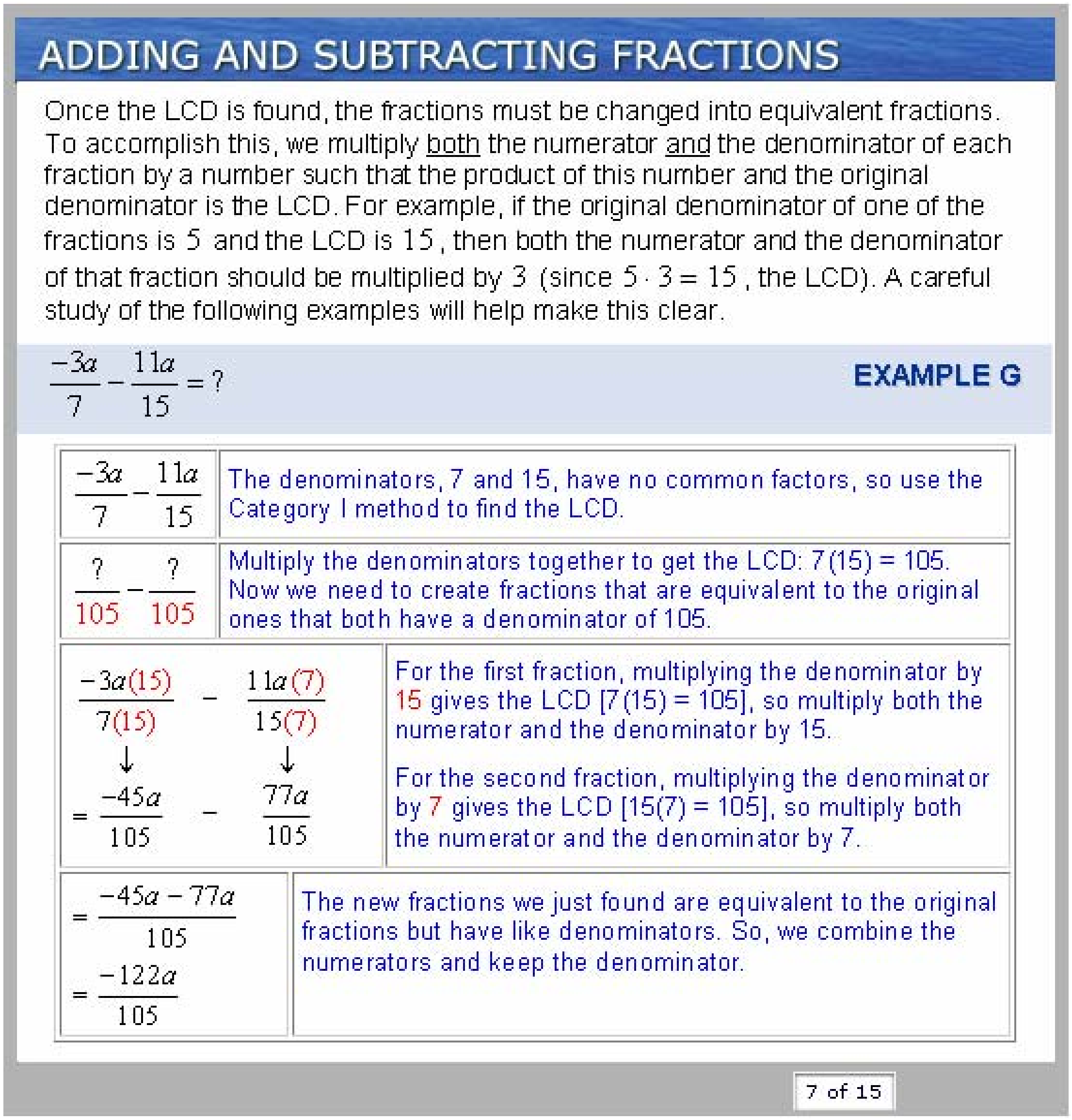
ADDING AND SUBTRACTING FRACTIONS

Unlike Denominators

The procedure for adding and subtracting fractions with different denominators requires two initial steps: 1) Find the **least common denominator (LCD)** of the fractions, and 2) change the fractions to equivalent fractions with the LCD as the denominator of both. The process in step 2 may change the appearance of the fraction but its numerical value remains the same. Fractions that look different but have the same numerical value are called **equivalent fractions**.

There are many different ways to find the LCD, including prime factorization. One way to start is to organize problems into three categories: 1) those with no common factors in the denominators; 2) those with one denominator that is a factor of the other; and 3) those with a common factor in the denominators. Once you recognize which category a problem belongs to, follow the suggested method of finding the LCD for that category.

Category I	**Method of Finding LCD**
The denominators have no common factors.	Multiply the denominators together to get the LCD.
Examples:	
Denominators	**LCD**
5 and 7	$5(7) = 35$
3 and 11	$3(11) = 33$

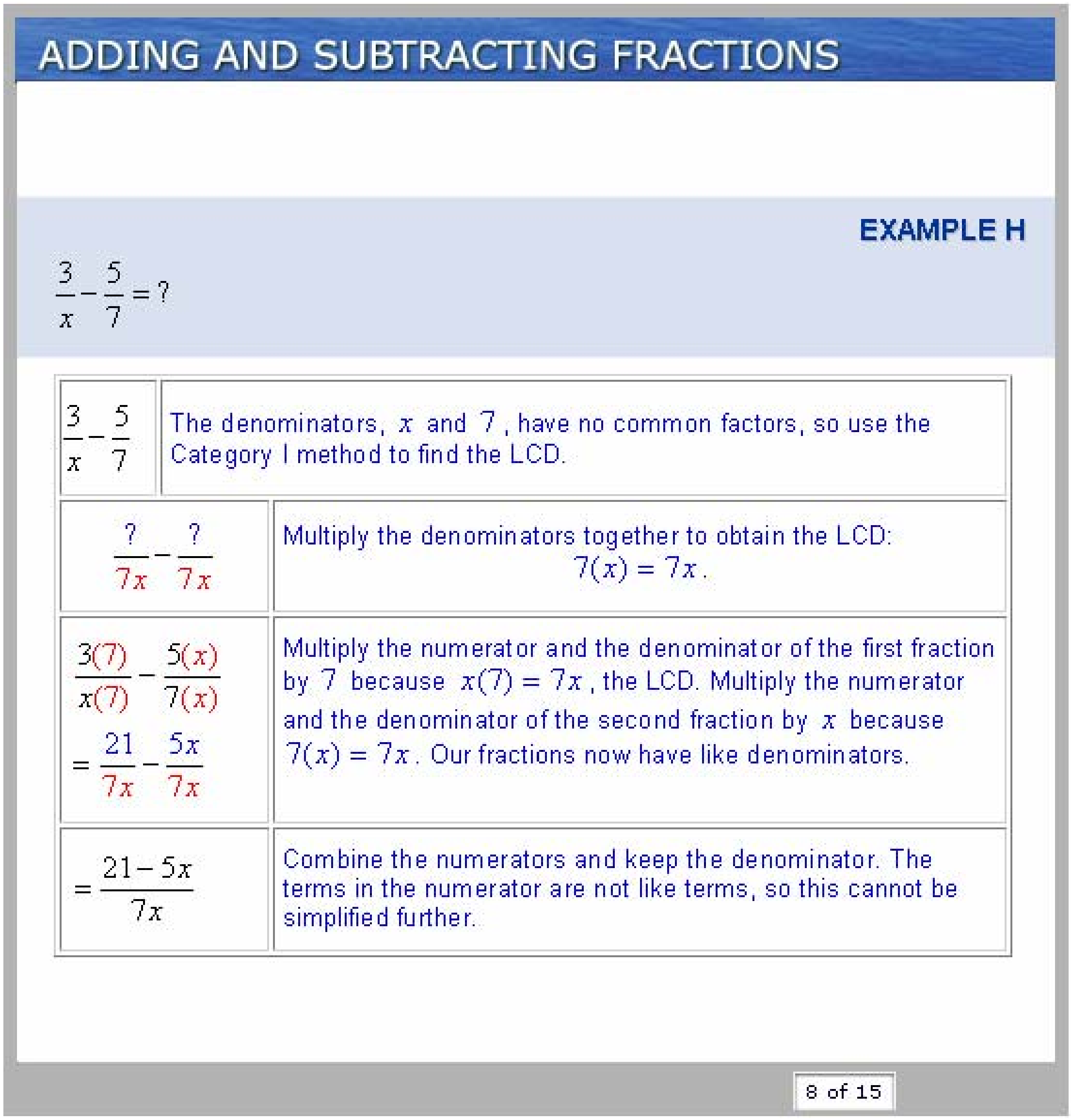


ADDING AND SUBTRACTING FRACTIONS

Once the LCD is found, the fractions must be changed into equivalent fractions. To accomplish this, we multiply both the numerator and the denominator of each fraction by a number such that the product of this number and the original denominator is the LCD. For example, if the original denominator of one of the fractions is 5 and the LCD is 15, then both the numerator and the denominator of that fraction should be multiplied by 3 (since $5 \cdot 3 = 15$, the LCD). A careful study of the following examples will help make this clear.

$\frac{-3a}{7} - \frac{11a}{15} = ?$ **EXAMPLE G**

$\frac{-3a}{7} - \frac{11a}{15}$	The denominators, 7 and 15, have no common factors, so use the Category I method to find the LCD.
$\frac{?}{105} - \frac{?}{105}$	Multiply the denominators together to get the LCD: 7(15) = 105. Now we need to create fractions that are equivalent to the original ones that both have a denominator of 105.
$\frac{-3a(15)}{7(15)} - \frac{11a(7)}{15(7)}$ $\downarrow \quad \downarrow$ $= \frac{-45a}{105} - \frac{77a}{105}$	For the first fraction, multiplying the denominator by 15 gives the LCD [7(15) = 105], so multiply both the numerator and the denominator by 15. For the second fraction, multiplying the denominator by 7 gives the LCD [15(7) = 105], so multiply both the numerator and the denominator by 7.
$= \frac{-45a - 77a}{105}$ $= \frac{-122a}{105}$	The new fractions we just found are equivalent to the original fractions but have like denominators. So, we combine the numerators and keep the denominator.

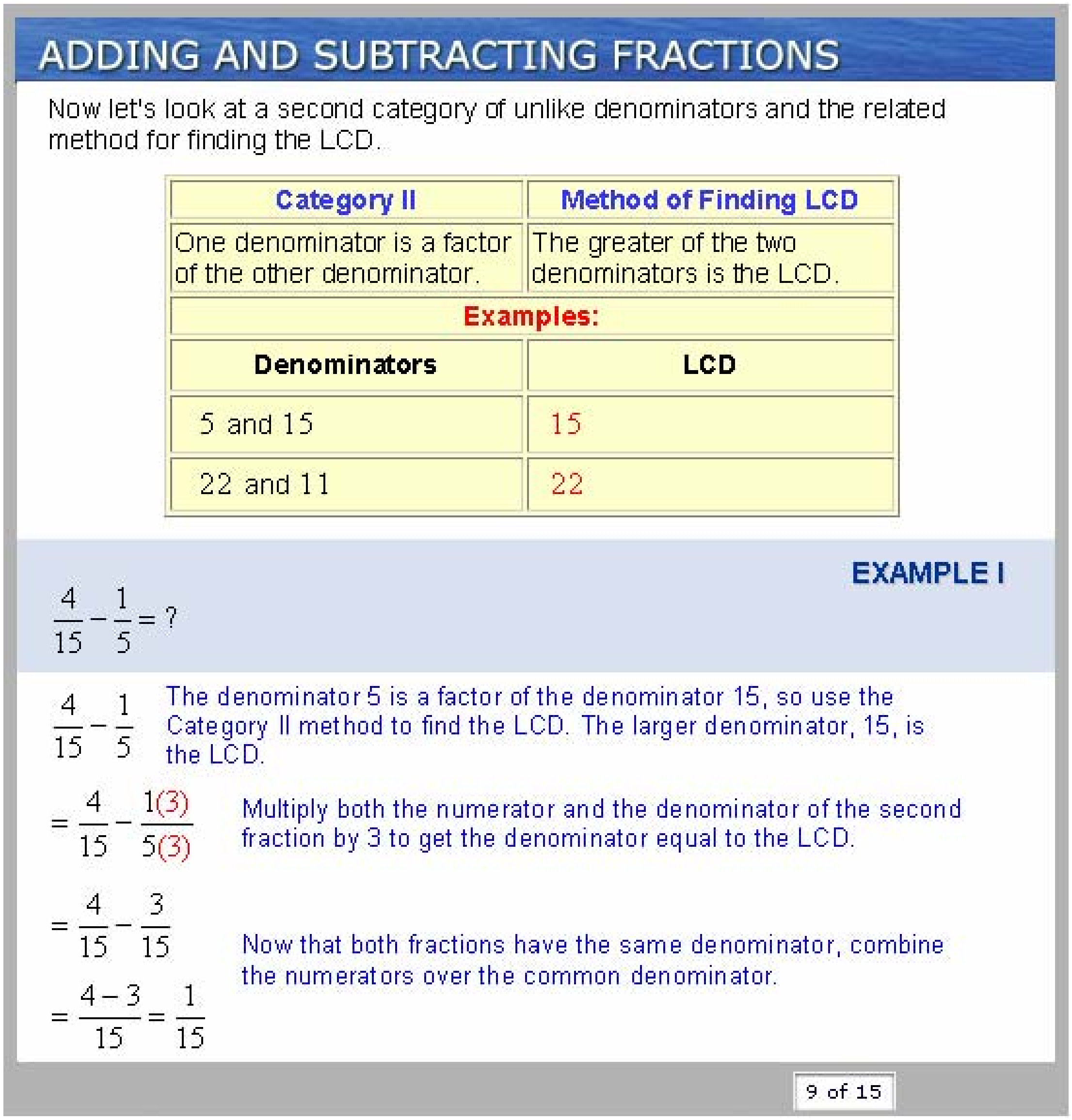


ADDING AND SUBTRACTING FRACTIONS

EXAMPLE H

$$\frac{3}{x}-\frac{5}{7}=?$$

$\frac{3}{x}-\frac{5}{7}$	The denominators, x and 7, have no common factors, so use the Category I method to find the LCD.
$\frac{?}{7x}-\frac{?}{7x}$	Multiply the denominators together to obtain the LCD: $7(x)=7x$.
$\frac{3(7)}{x(7)}-\frac{5(x)}{7(x)}$ $=\frac{21}{7x}-\frac{5x}{7x}$	Multiply the numerator and the denominator of the first fraction by 7 because $x(7)=7x$, the LCD. Multiply the numerator and the denominator of the second fraction by x because $7(x)=7x$. Our fractions now have like denominators.
$=\frac{21-5x}{7x}$	Combine the numerators and keep the denominator. The terms in the numerator are not like terms, so this cannot be simplified further.



ADDING AND SUBTRACTING FRACTIONS

Now let's look at a second category of unlike denominators and the related method for finding the LCD.

Category II	**Method of Finding LCD**
One denominator is a factor of the other denominator.	The greater of the two denominators is the LCD.
Examples:	
Denominators	**LCD**
5 and 15	15
22 and 11	22

EXAMPLE I

$$\frac{4}{15}-\frac{1}{5}=?$$

$$\frac{4}{15}-\frac{1}{5}$$

The denominator 5 is a factor of the denominator 15, so use the Category II method to find the LCD. The larger denominator, 15, is the LCD.

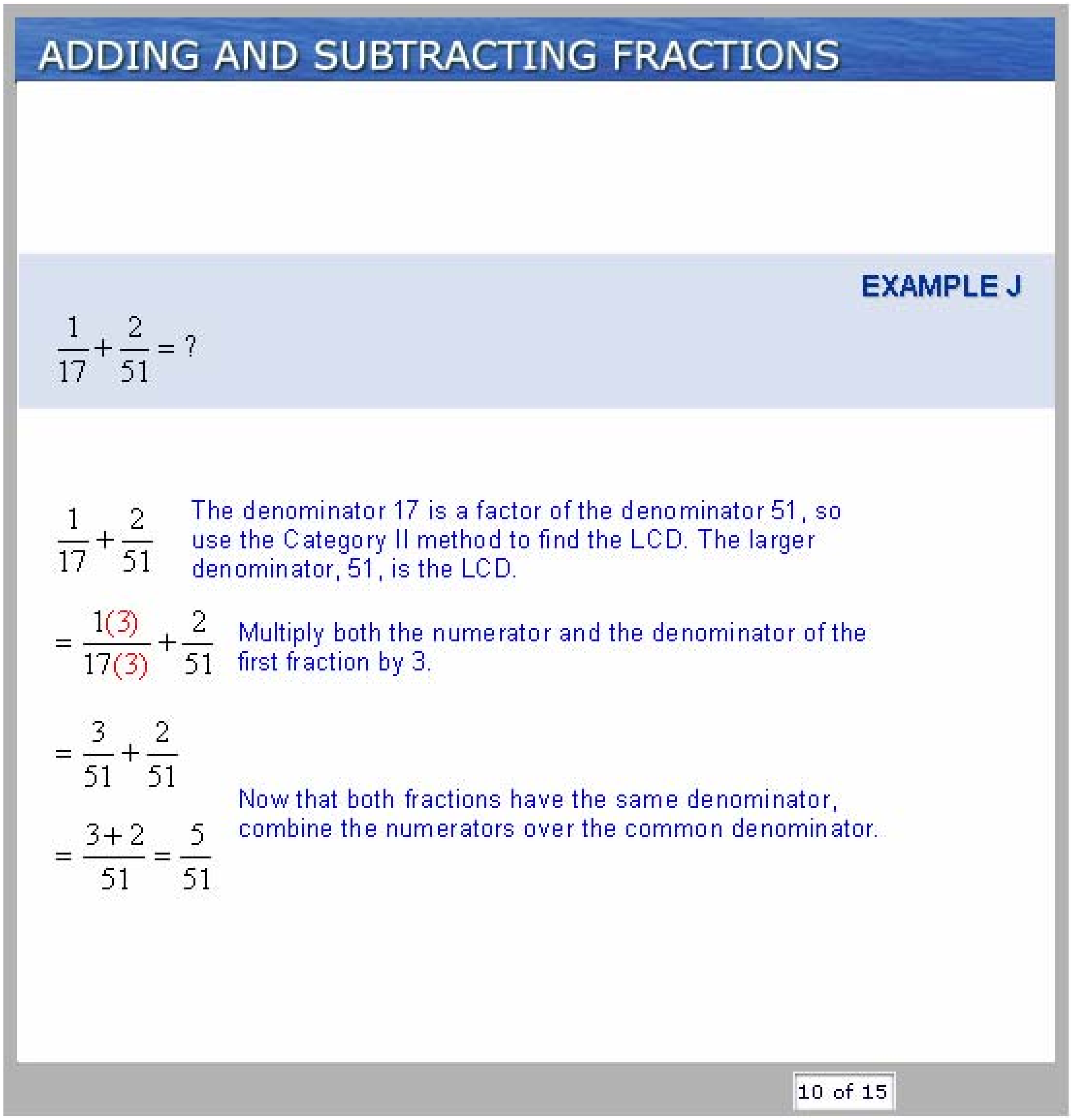
$$=\frac{4}{15}-\frac{1(3)}{5(3)}$$

Multiply both the numerator and the denominator of the second fraction by 3 to get the denominator equal to the LCD.

$$=\frac{4}{15}-\frac{3}{15}$$

Now that both fractions have the same denominator, combine the numerators over the common denominator.

$$=\frac{4-3}{15}=\frac{1}{15}$$



ADDING AND SUBTRACTING FRACTIONS

EXAMPLE J

$$\frac{1}{17}+\frac{2}{51}=?$$

$$\frac{1}{17}+\frac{2}{51}$$

The denominator 17 is a factor of the denominator 51, so use the Category II method to find the LCD. The larger denominator, 51, is the LCD.

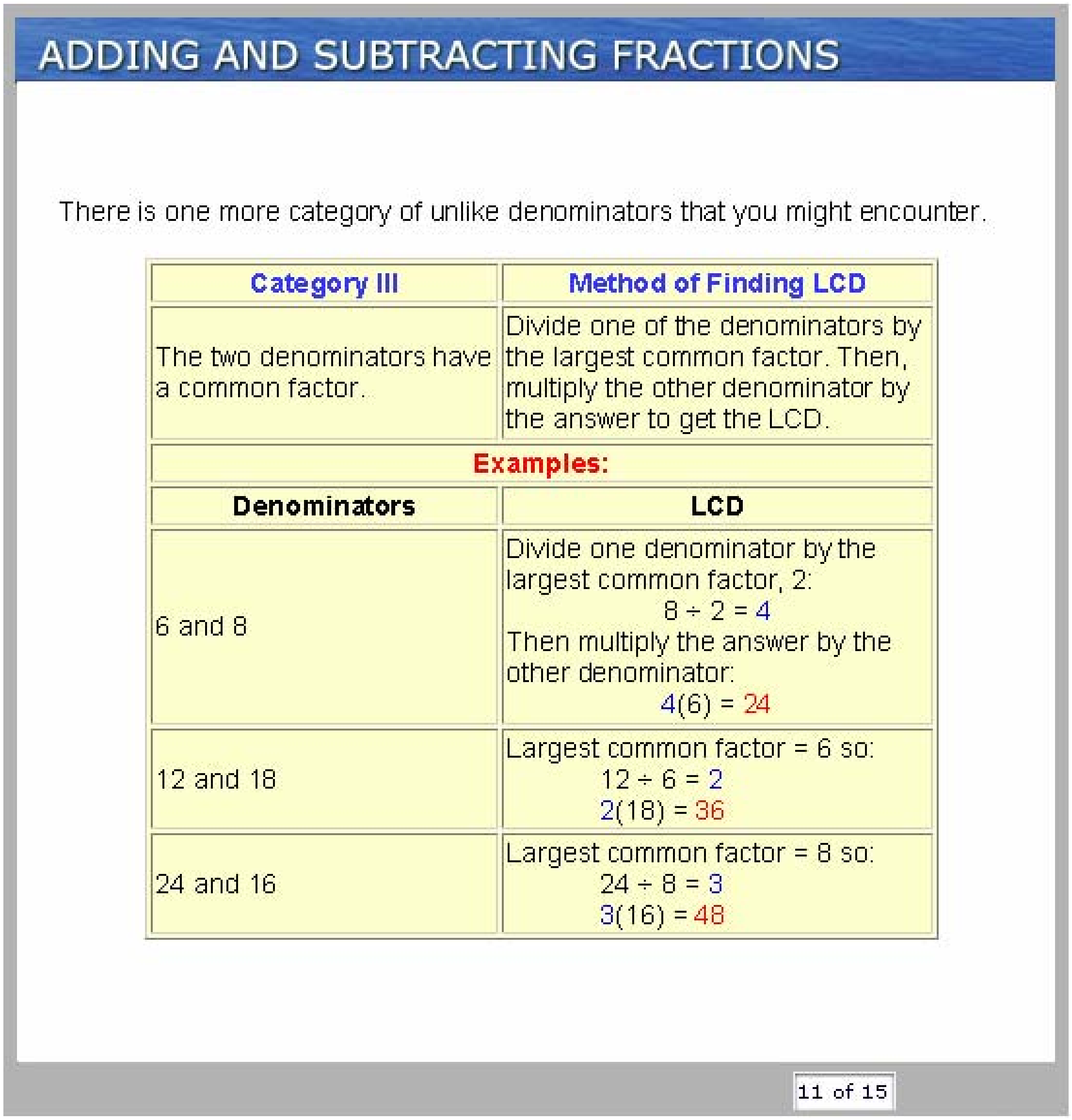
$$=\frac{1(3)}{17(3)}+\frac{2}{51}$$

Multiply both the numerator and the denominator of the first fraction by 3.

$$=\frac{3}{51}+\frac{2}{51}$$

Now that both fractions have the same denominator, combine the numerators over the common denominator.

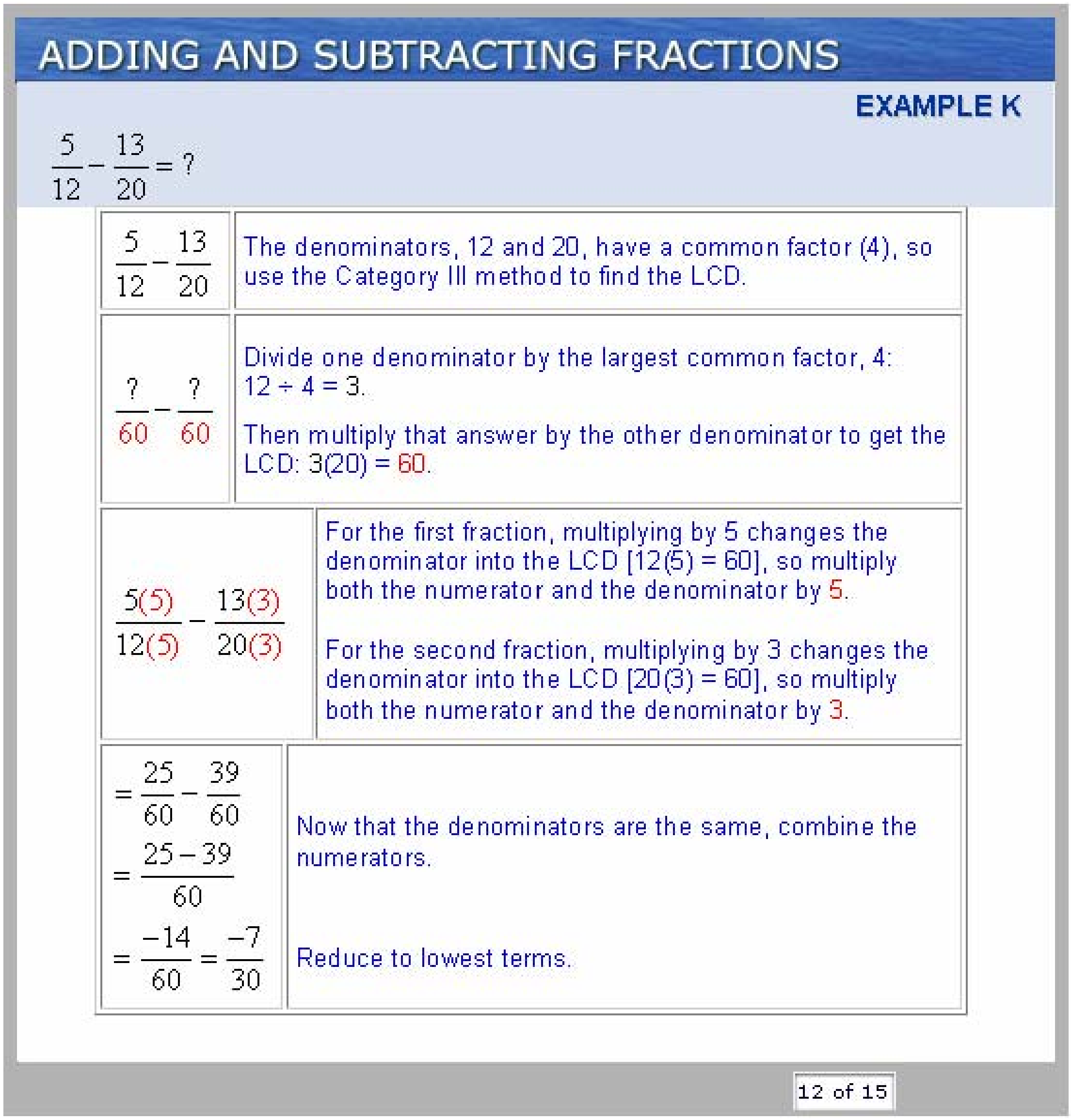
$$=\frac{3+2}{51}=\frac{5}{51}$$



ADDING AND SUBTRACTING FRACTIONS

There is one more category of unlike denominators that you might encounter.

Category III	**Method of Finding LCD**
The two denominators have a common factor.	Divide one of the denominators by the largest common factor. Then, multiply the other denominator by the answer to get the LCD.
Examples:	
Denominators	**LCD**
6 and 8	Divide one denominator by the largest common factor, 2: 8 ÷ 2 = 4 Then multiply the answer by the other denominator: 4(6) = 24
12 and 18	Largest common factor = 6 so: 12 ÷ 6 = 2 2(18) = 36
24 and 16	Largest common factor = 8 so: 24 ÷ 8 = 3 3(16) = 48

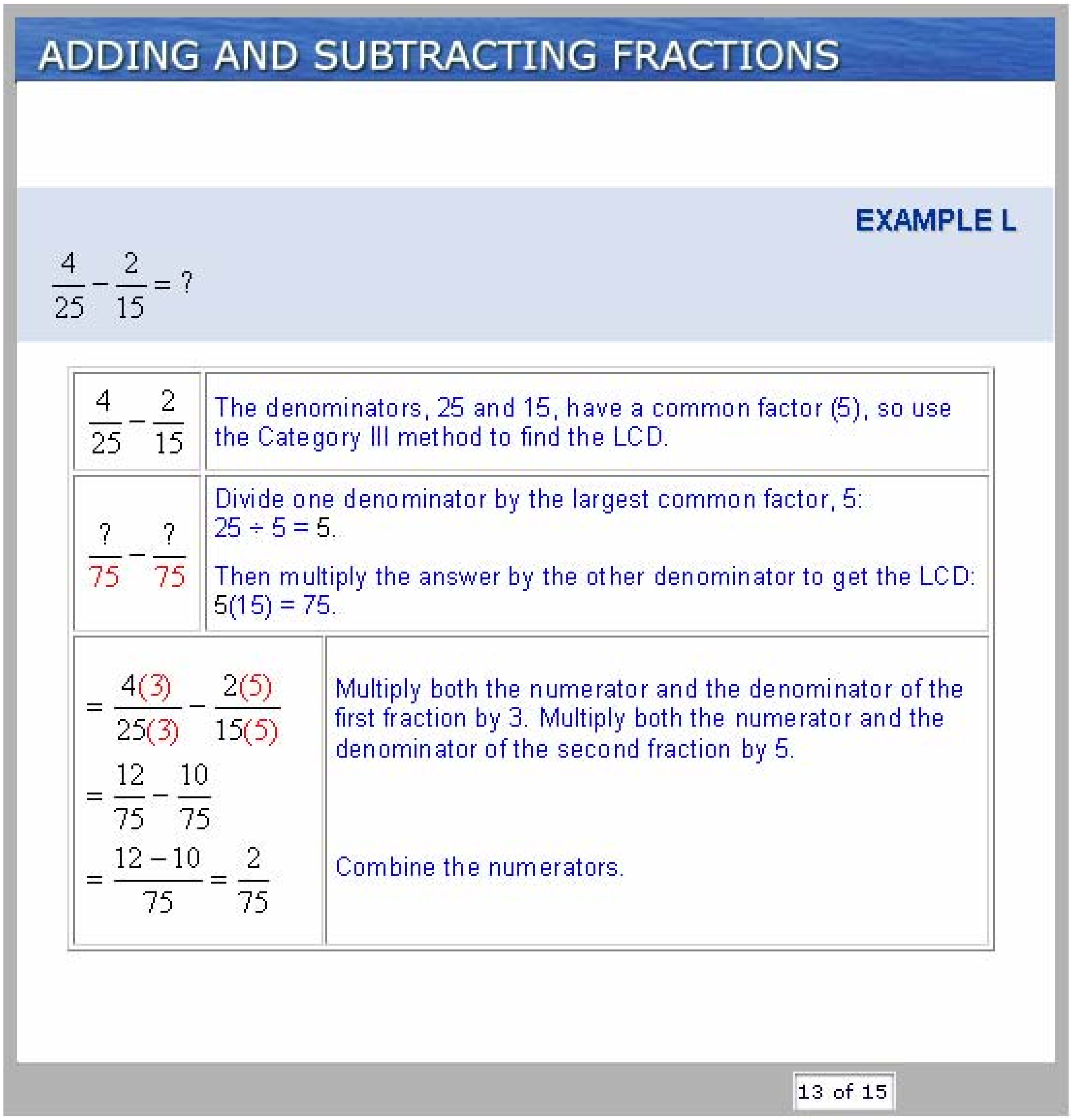


ADDING AND SUBTRACTING FRACTIONS

EXAMPLE K

$\frac{5}{12} - \frac{13}{20} = ?$

$\frac{5}{12} - \frac{13}{20}$	The denominators, 12 and 20, have a common factor (4), so use the Category III method to find the LCD.
$\frac{?}{60} - \frac{?}{60}$	Divide one denominator by the largest common factor, 4: 12 ÷ 4 = 3. Then multiply that answer by the other denominator to get the LCD: 3(20) = 60.
$\frac{5(5)}{12(5)} - \frac{13(3)}{20(3)}$	For the first fraction, multiplying by 5 changes the denominator into the LCD [12(5) = 60], so multiply both the numerator and the denominator by 5. For the second fraction, multiplying by 3 changes the denominator into the LCD [20(3) = 60], so multiply both the numerator and the denominator by 3.
$= \frac{25}{60} - \frac{39}{60}$ $= \frac{25-39}{60}$ $= \frac{-14}{60} = \frac{-7}{30}$	Now that the denominators are the same, combine the numerators. Reduce to lowest terms.

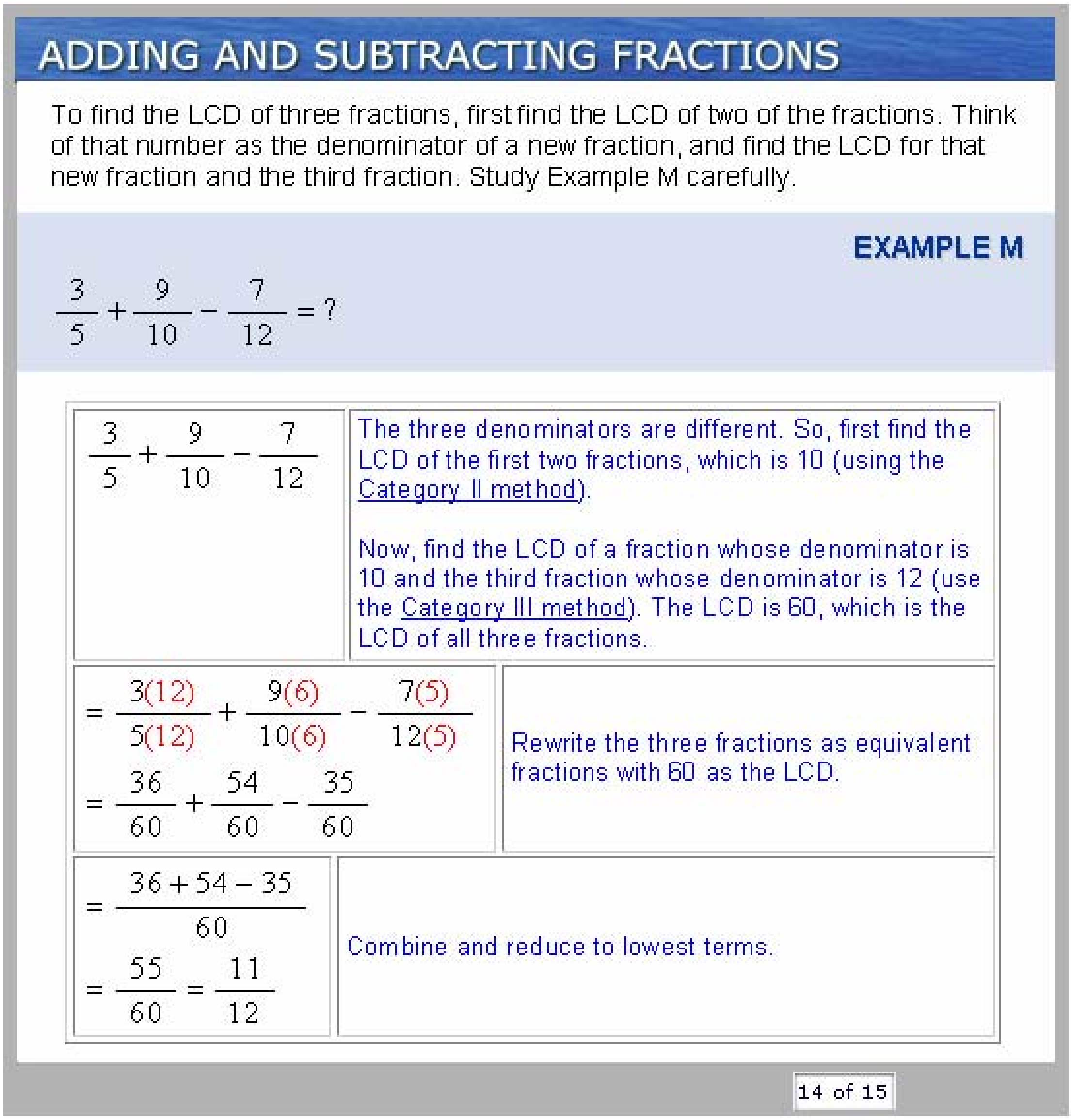


ADDING AND SUBTRACTING FRACTIONS

EXAMPLE L

$$\frac{4}{25}-\frac{2}{15}=?$$

$\frac{4}{25}-\frac{2}{15}$	The denominators, 25 and 15, have a common factor (5), so use the Category III method to find the LCD.
$\frac{?}{75}-\frac{?}{75}$	Divide one denominator by the largest common factor, 5: $25 \div 5 = 5$. Then multiply the answer by the other denominator to get the LCD: $5(15) = 75$.
$=\frac{4(3)}{25(3)}-\frac{2(5)}{15(5)}$ $=\frac{12}{75}-\frac{10}{75}$	Multiply both the numerator and the denominator of the first fraction by 3. Multiply both the numerator and the denominator of the second fraction by 5.
$=\frac{12-10}{75}=\frac{2}{75}$	Combine the numerators.



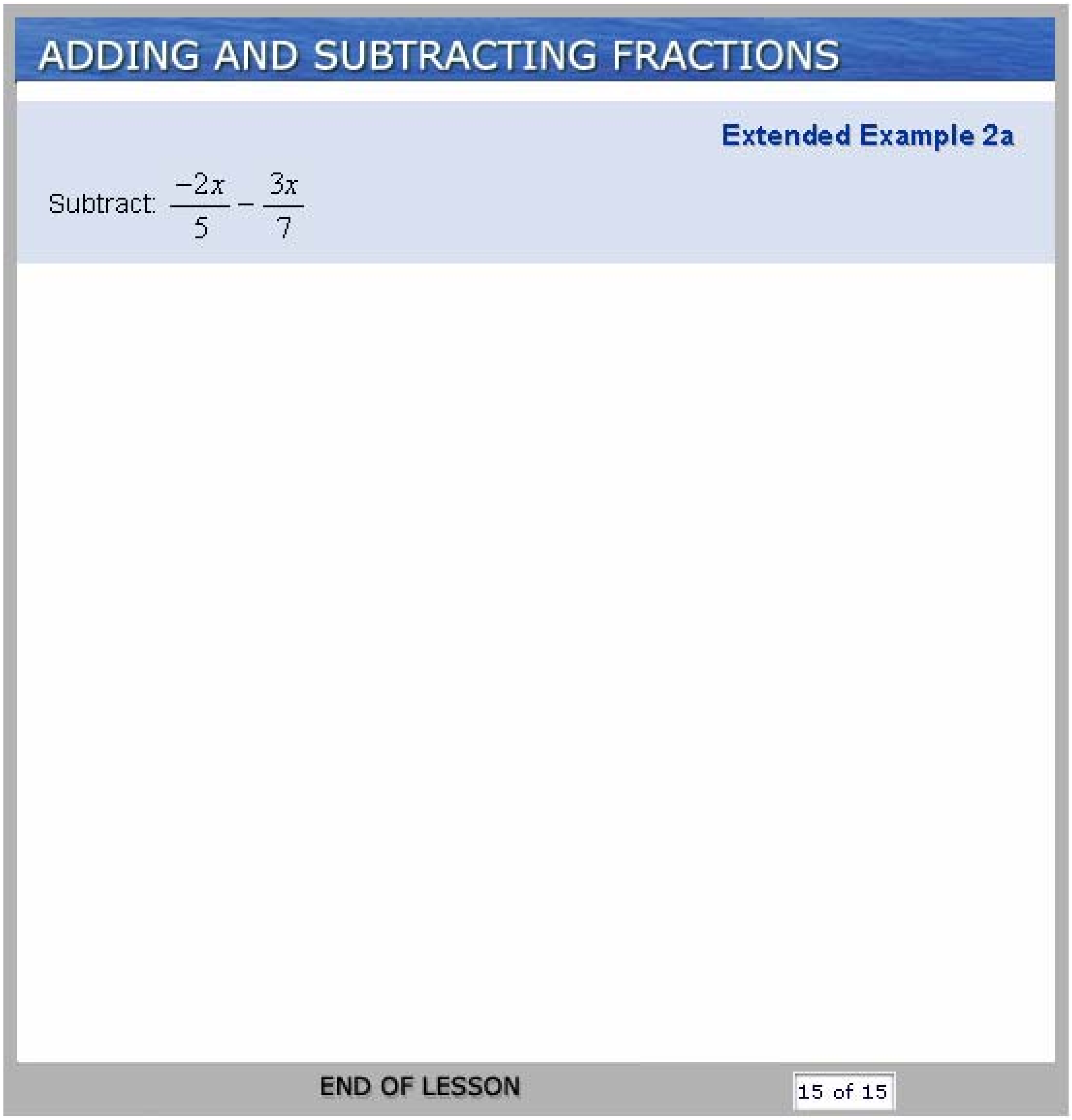
ADDING AND SUBTRACTING FRACTIONS

To find the LCD of three fractions, first find the LCD of two of the fractions. Think of that number as the denominator of a new fraction, and find the LCD for that new fraction and the third fraction. Study Example M carefully.

EXAMPLE M

$$\frac{3}{5} + \frac{9}{10} - \frac{7}{12} = ?$$

$\frac{3}{5} + \frac{9}{10} - \frac{7}{12}$	The three denominators are different. So, first find the LCD of the first two fractions, which is 10 (using the Category II method). Now, find the LCD of a fraction whose denominator is 10 and the third fraction whose denominator is 12 (use the Category III method). The LCD is 60, which is the LCD of all three fractions.
$= \frac{3(12)}{5(12)} + \frac{9(6)}{10(6)} - \frac{7(5)}{12(5)}$ $= \frac{36}{60} + \frac{54}{60} - \frac{35}{60}$	Rewrite the three fractions as equivalent fractions with 60 as the LCD.
$= \frac{36 + 54 - 35}{60}$ $= \frac{55}{60} = \frac{11}{12}$	Combine and reduce to lowest terms.



ADDING AND SUBTRACTING FRACTIONS

Extended Example 2a

Subtract: $\frac{-2x}{5}-\frac{3x}{7}$

For the set of denominators given, find the least common denominator.

19 and 76

For the set of denominators given, find the least common denominator.

17 and 13

Add or subtract.

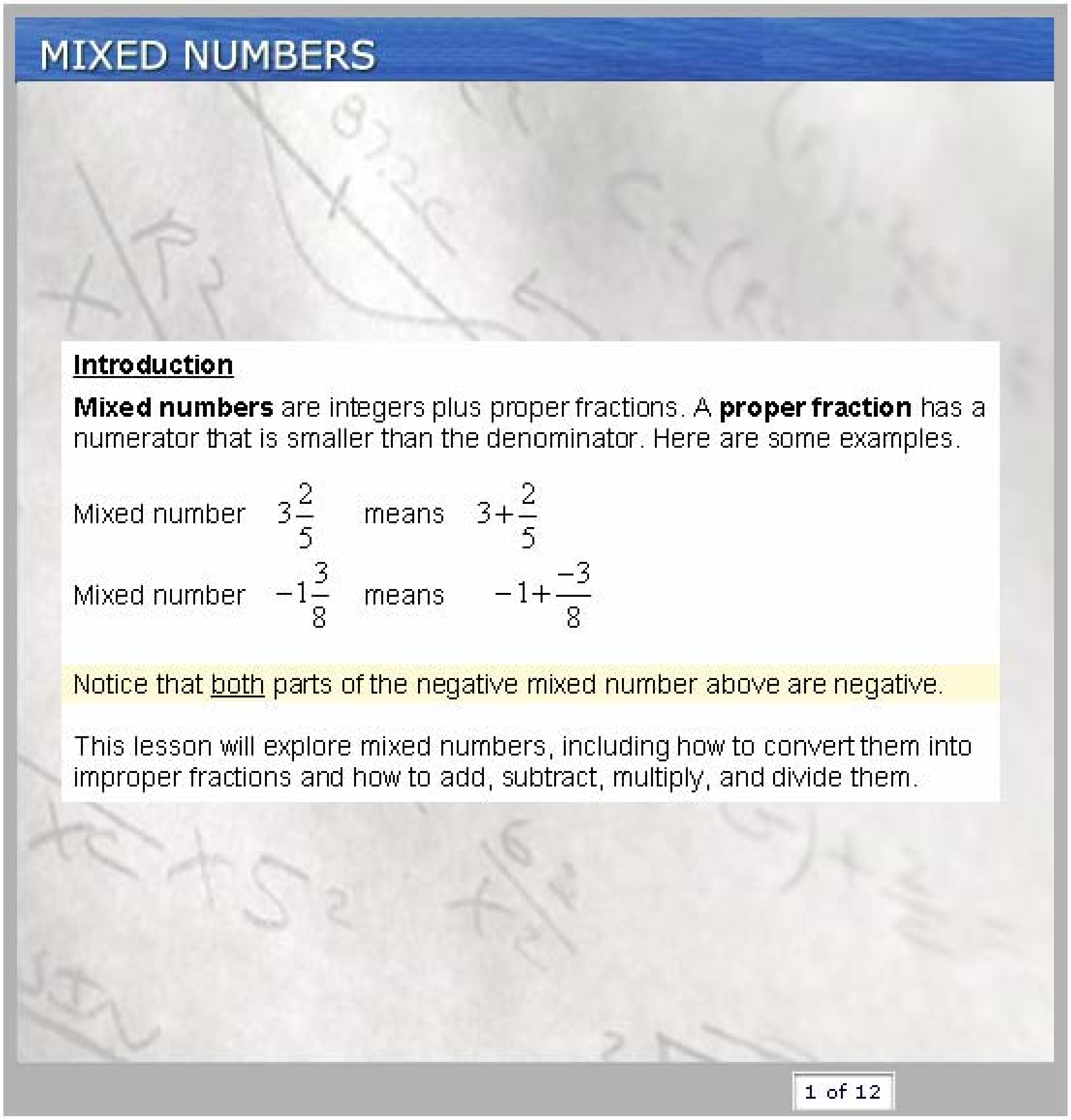
$$\frac{6y}{32}-\frac{7y}{8}=$$

Add or subtract.

$$\frac{3}{x}-\frac{4}{5}=$$

Add or subtract.

$$\frac{-1n}{2m}+\frac{-2}{5}=$$



MIXED NUMBERS

Introduction

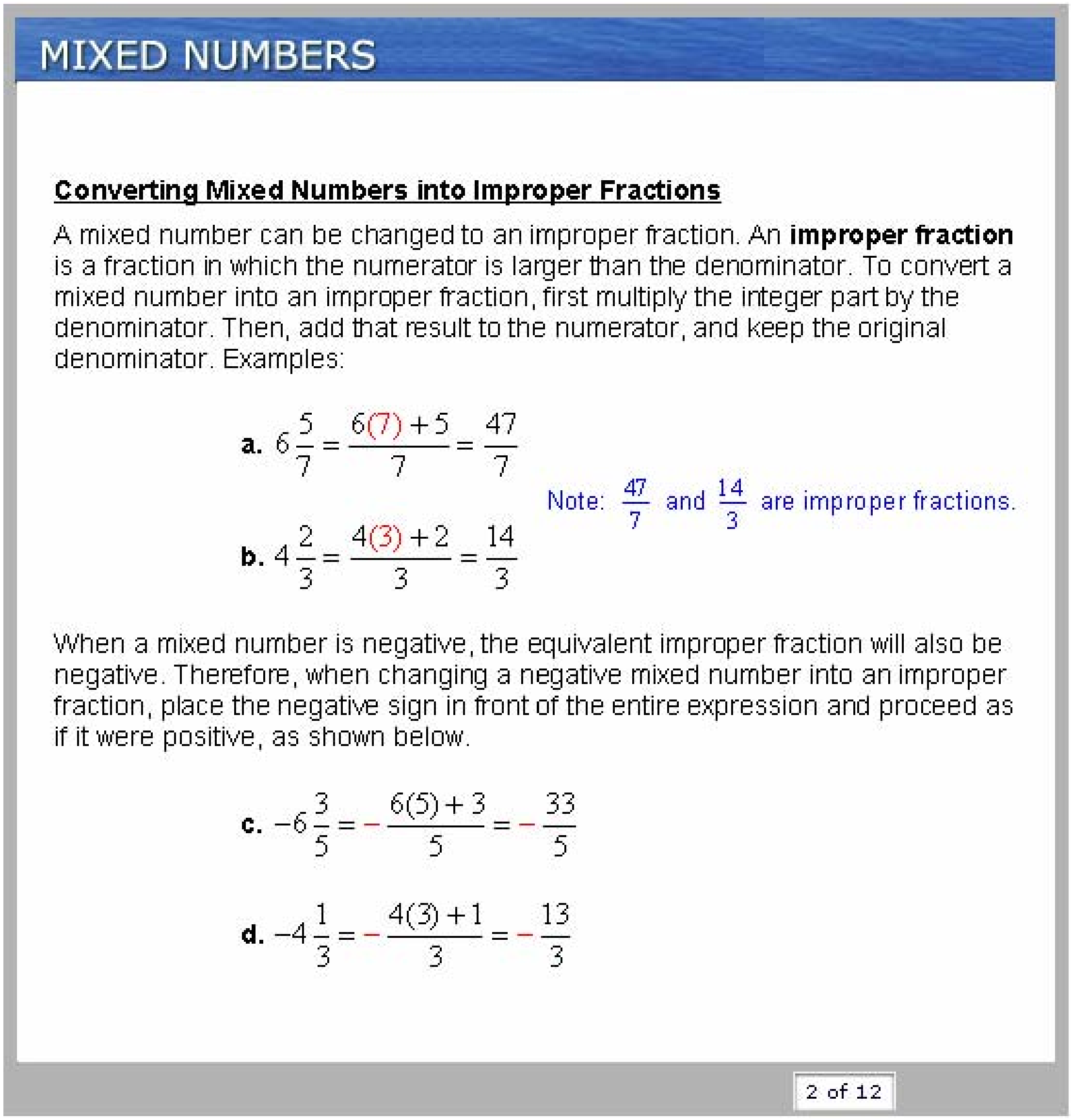
Mixed numbers are integers plus proper fractions. A **proper fraction** has a numerator that is smaller than the denominator. Here are some examples.

Mixed number $3\frac{2}{5}$ means $3+\frac{2}{5}$

Mixed number $-1\frac{3}{8}$ means $-1+\frac{-3}{8}$

Notice that both parts of the negative mixed number above are negative.

This lesson will explore mixed numbers, including how to convert them into improper fractions and how to add, subtract, multiply, and divide them.



MIXED NUMBERS

Converting Mixed Numbers into Improper Fractions

A mixed number can be changed to an improper fraction. An **improper fraction** is a fraction in which the numerator is larger than the denominator. To convert a mixed number into an improper fraction, first multiply the integer part by the denominator. Then, add that result to the numerator, and keep the original denominator. Examples:

a. $6\frac{5}{7}=\frac{6(7)+5}{7}=\frac{47}{7}$

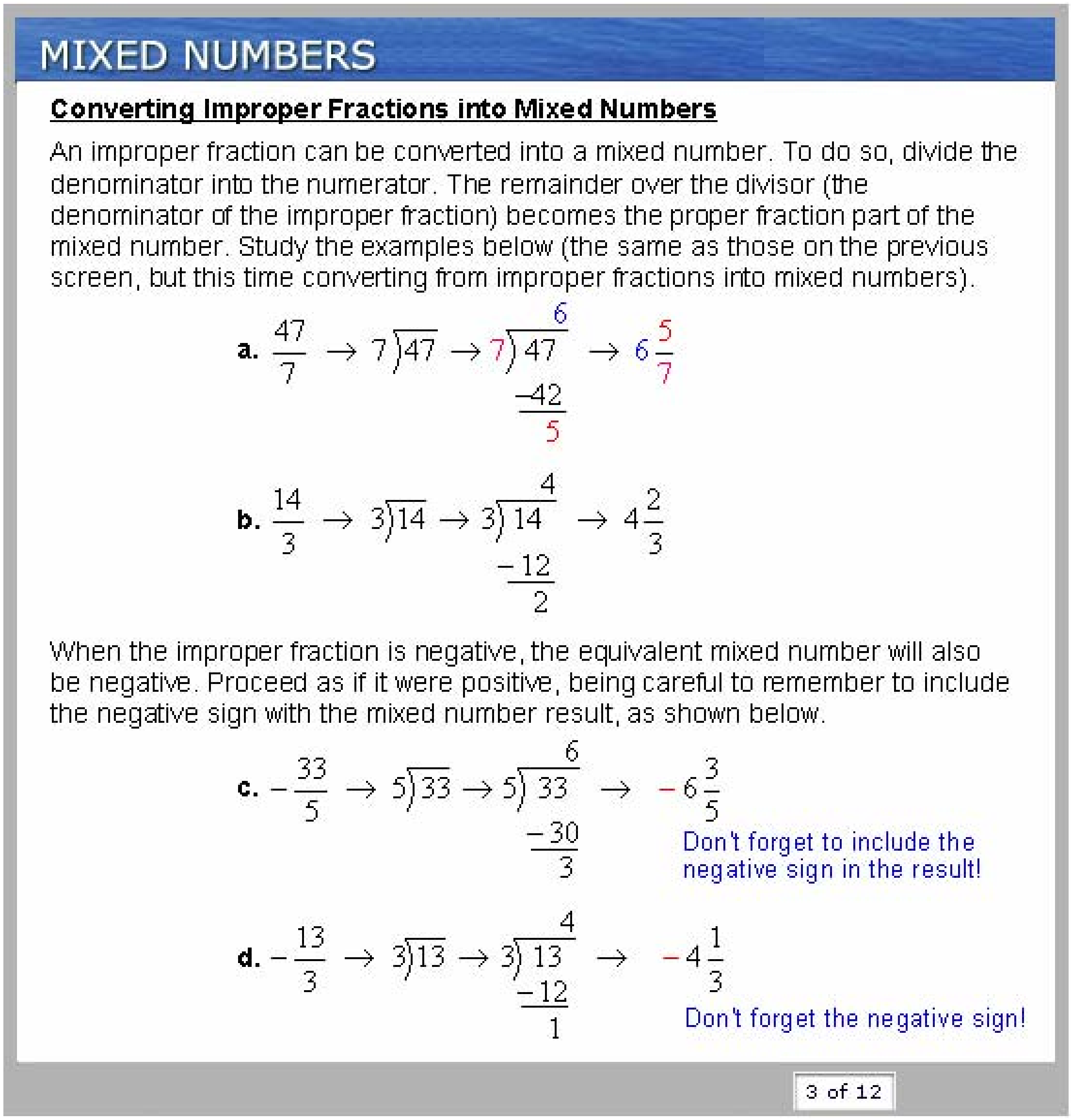
b. $4\frac{2}{3}=\frac{4(3)+2}{3}=\frac{14}{3}$

Note: $\frac{47}{7}$ and $\frac{14}{3}$ are improper fractions.

When a mixed number is negative, the equivalent improper fraction will also be negative. Therefore, when changing a negative mixed number into an improper fraction, place the negative sign in front of the entire expression and proceed as if it were positive, as shown below.

c. $-6\frac{3}{5}=-\frac{6(5)+3}{5}=-\frac{33}{5}$

d. $-4\frac{1}{3}=-\frac{4(3)+1}{3}=-\frac{13}{3}$



MIXED NUMBERS

Converting Improper Fractions into Mixed Numbers

An improper fraction can be converted into a mixed number. To do so, divide the denominator into the numerator. The remainder over the divisor (the denominator of the improper fraction) becomes the proper fraction part of the mixed number. Study the examples below (the same as those on the previous screen, but this time converting from improper fractions into mixed numbers).

a. $$\frac{47}{7} \rightarrow 7\overline{)47} \rightarrow \begin{array}{r} 6 \\ 7\overline{)\,47} \\ -42 \\ \hline 5 \end{array} \rightarrow 6\frac{5}{7}$$

b. $$\frac{14}{3} \rightarrow 3\overline{)14} \rightarrow \begin{array}{r} 4 \\ 3\overline{)\,14} \\ -12 \\ \hline 2 \end{array} \rightarrow 4\frac{2}{3}$$

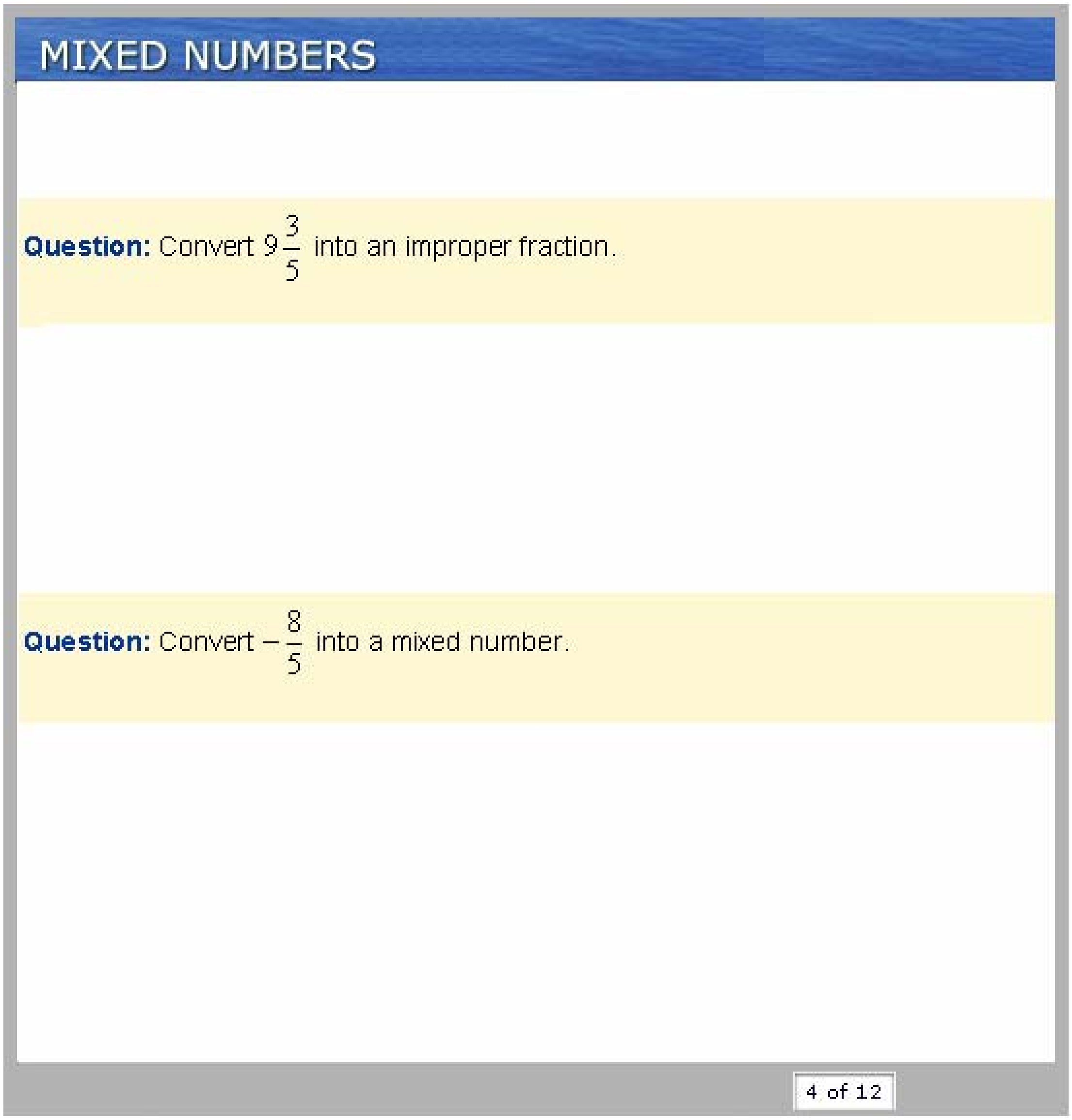
When the improper fraction is negative, the equivalent mixed number will also be negative. Proceed as if it were positive, being careful to remember to include the negative sign with the mixed number result, as shown below.

c. $$-\frac{33}{5} \rightarrow 5\overline{)33} \rightarrow \begin{array}{r} 6 \\ 5\overline{)\,33} \\ -30 \\ \hline 3 \end{array} \rightarrow -6\frac{3}{5}$$

Don't forget to include the negative sign in the result!

d. $$-\frac{13}{3} \rightarrow 3\overline{)13} \rightarrow \begin{array}{r} 4 \\ 3\overline{)\,13} \\ -12 \\ \hline 1 \end{array} \rightarrow -4\frac{1}{3}$$

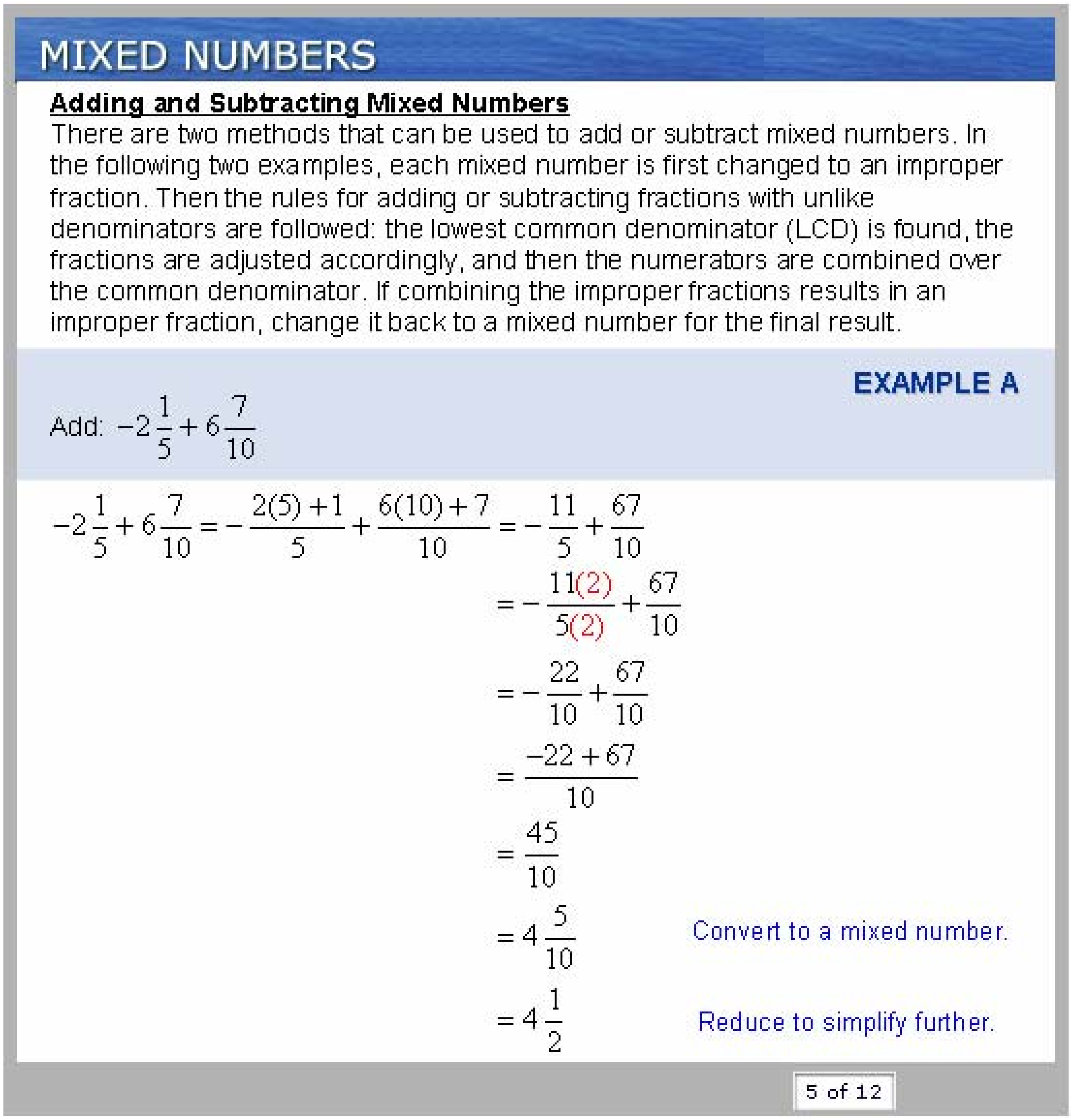
Don't forget the negative sign!



MIXED NUMBERS

Question: Convert $9\frac{3}{5}$ into an improper fraction.

Question: Convert $-\frac{8}{5}$ into a mixed number.



MIXED NUMBERS

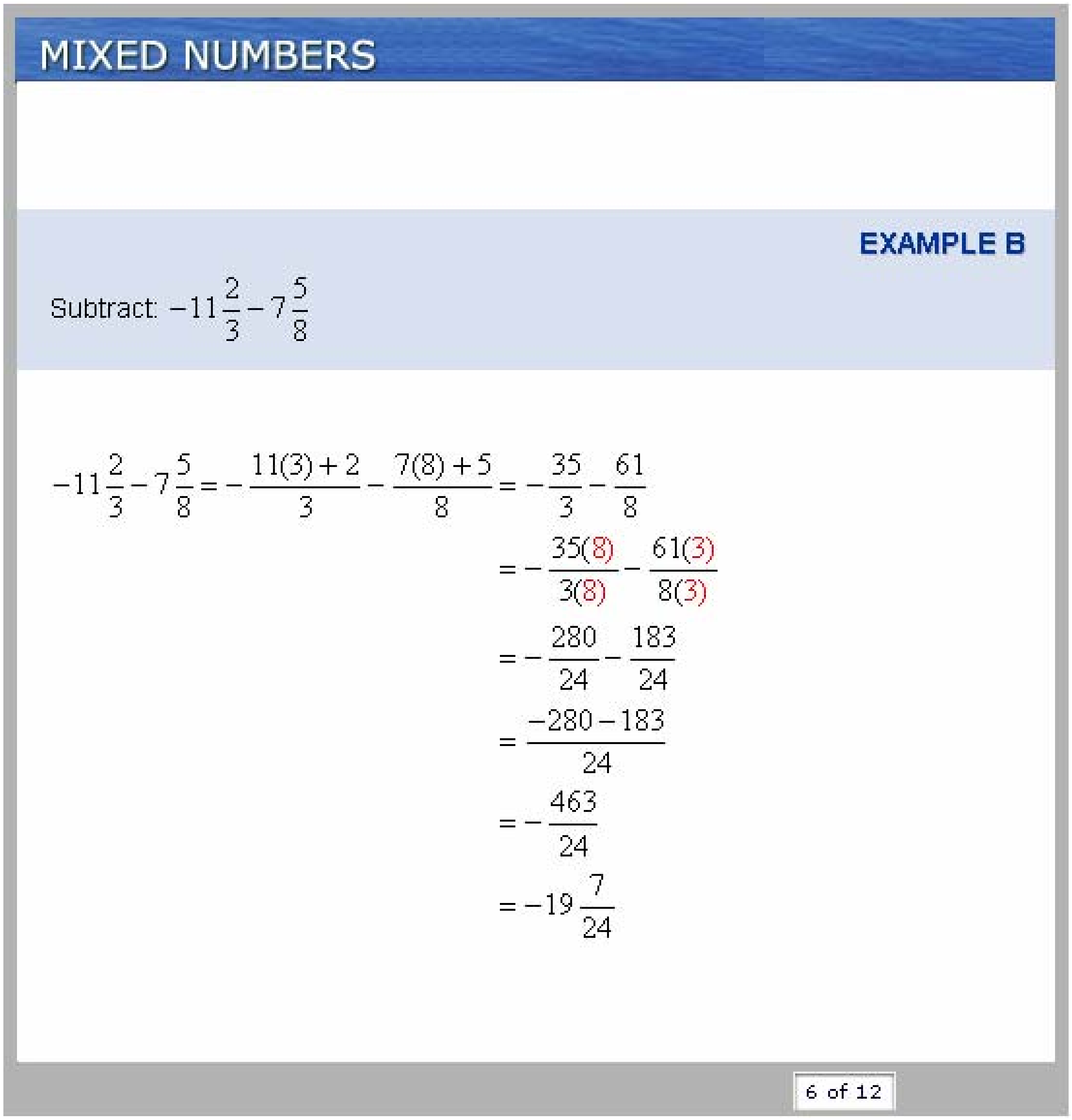
Adding and Subtracting Mixed Numbers

There are two methods that can be used to add or subtract mixed numbers. In the following two examples, each mixed number is first changed to an improper fraction. Then the rules for adding or subtracting fractions with unlike denominators are followed: the lowest common denominator (LCD) is found, the fractions are adjusted accordingly, and then the numerators are combined over the common denominator. If combining the improper fractions results in an improper fraction, change it back to a mixed number for the final result.

EXAMPLE A

Add: $-2\frac{1}{5}+6\frac{7}{10}$

$$
\begin{aligned}
-2\frac{1}{5}+6\frac{7}{10} = -\frac{2(5)+1}{5}+\frac{6(10)+7}{10} &= -\frac{11}{5}+\frac{67}{10} \\
&= -\frac{11(2)}{5(2)}+\frac{67}{10} \\
&= -\frac{22}{10}+\frac{67}{10} \\
&= \frac{-22+67}{10} \\
&= \frac{45}{10} \\
&= 4\frac{5}{10} \quad \text{Convert to a mixed number.} \\
&= 4\frac{1}{2} \quad \text{Reduce to simplify further.}
\end{aligned}
$$

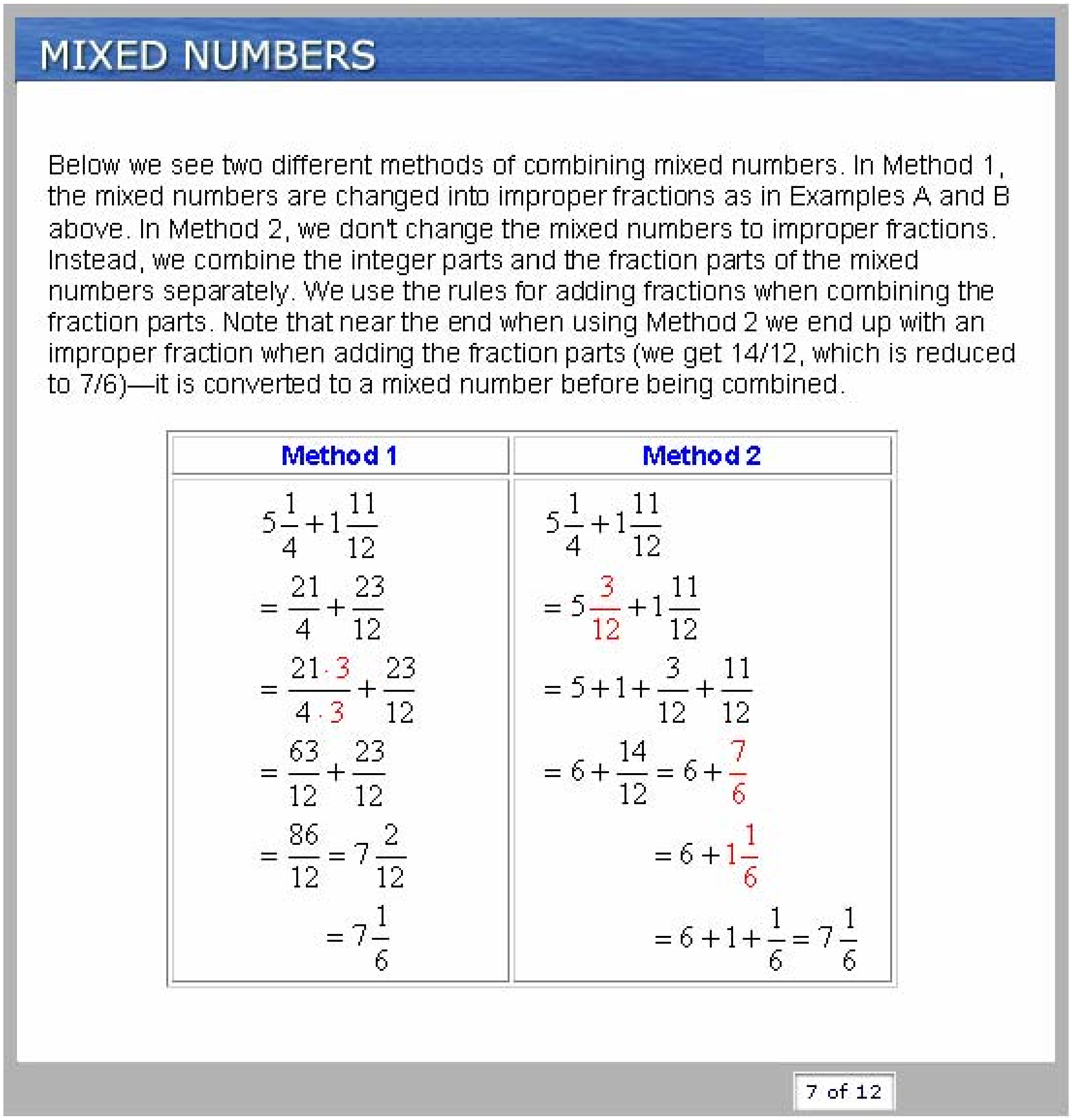


MIXED NUMBERS

EXAMPLE B

Subtract: $-11\frac{2}{3}-7\frac{5}{8}$

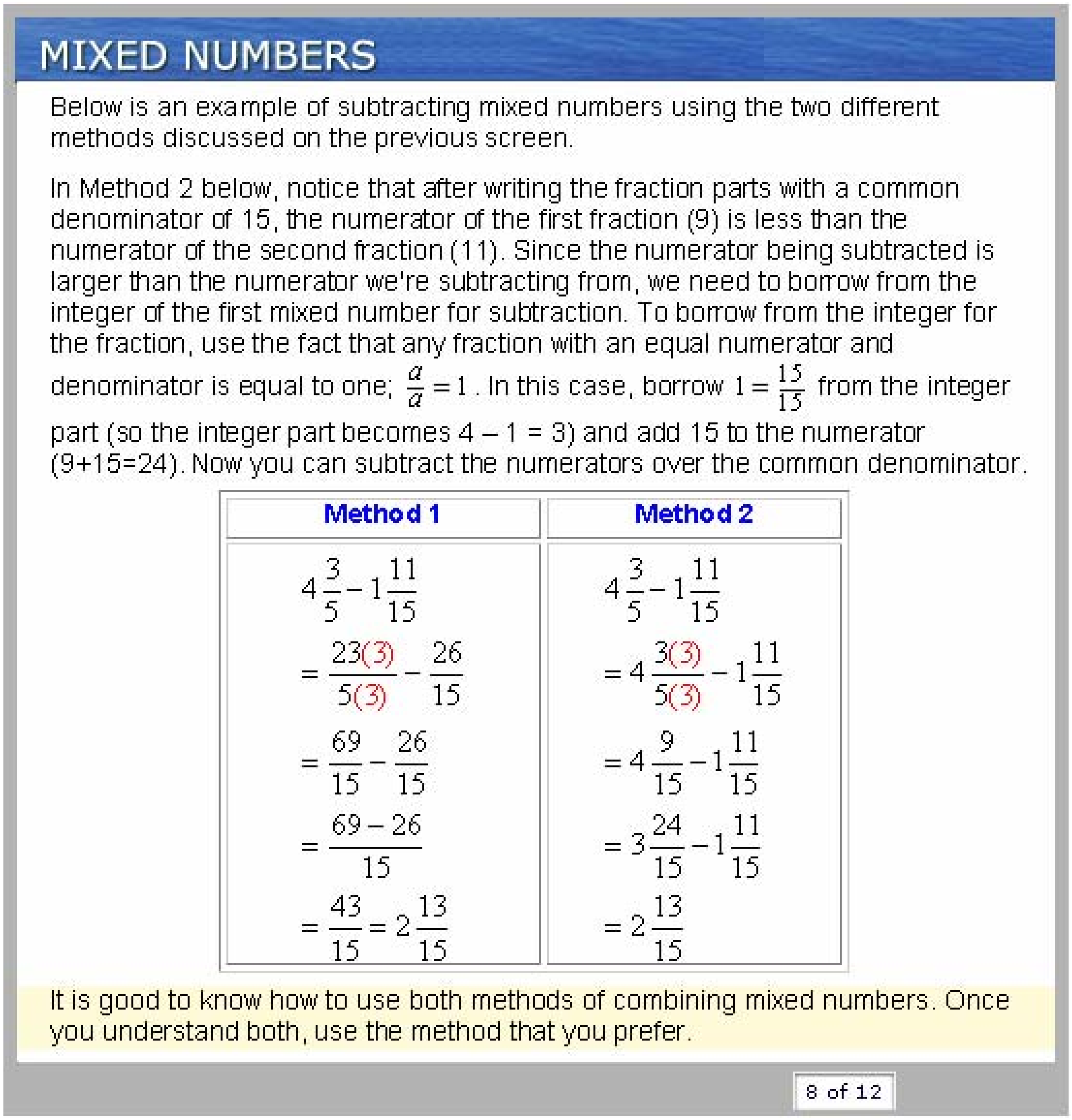
$$\begin{aligned}
-11\frac{2}{3}-7\frac{5}{8} &= -\frac{11(3)+2}{3}-\frac{7(8)+5}{8} = -\frac{35}{3}-\frac{61}{8} \\
&= -\frac{35(8)}{3(8)}-\frac{61(3)}{8(3)} \\
&= -\frac{280}{24}-\frac{183}{24} \\
&= \frac{-280-183}{24} \\
&= -\frac{463}{24} \\
&= -19\frac{7}{24}
\end{aligned}$$



MIXED NUMBERS

Below we see two different methods of combining mixed numbers. In Method 1, the mixed numbers are changed into improper fractions as in Examples A and B above. In Method 2, we don't change the mixed numbers to improper fractions. Instead, we combine the integer parts and the fraction parts of the mixed numbers separately. We use the rules for adding fractions when combining the fraction parts. Note that near the end when using Method 2 we end up with an improper fraction when adding the fraction parts (we get 14/12, which is reduced to 7/6)—it is converted to a mixed number before being combined.

Method 1	**Method 2**
$5\frac{1}{4}+1\frac{11}{12}$ $=\frac{21}{4}+\frac{23}{12}$ $=\frac{21\cdot 3}{4\cdot 3}+\frac{23}{12}$ $=\frac{63}{12}+\frac{23}{12}$ $=\frac{86}{12}=7\frac{2}{12}$ $=7\frac{1}{6}$	$5\frac{1}{4}+1\frac{11}{12}$ $=5\frac{3}{12}+1\frac{11}{12}$ $=5+1+\frac{3}{12}+\frac{11}{12}$ $=6+\frac{14}{12}=6+\frac{7}{6}$ $=6+1\frac{1}{6}$ $=6+1+\frac{1}{6}=7\frac{1}{6}$



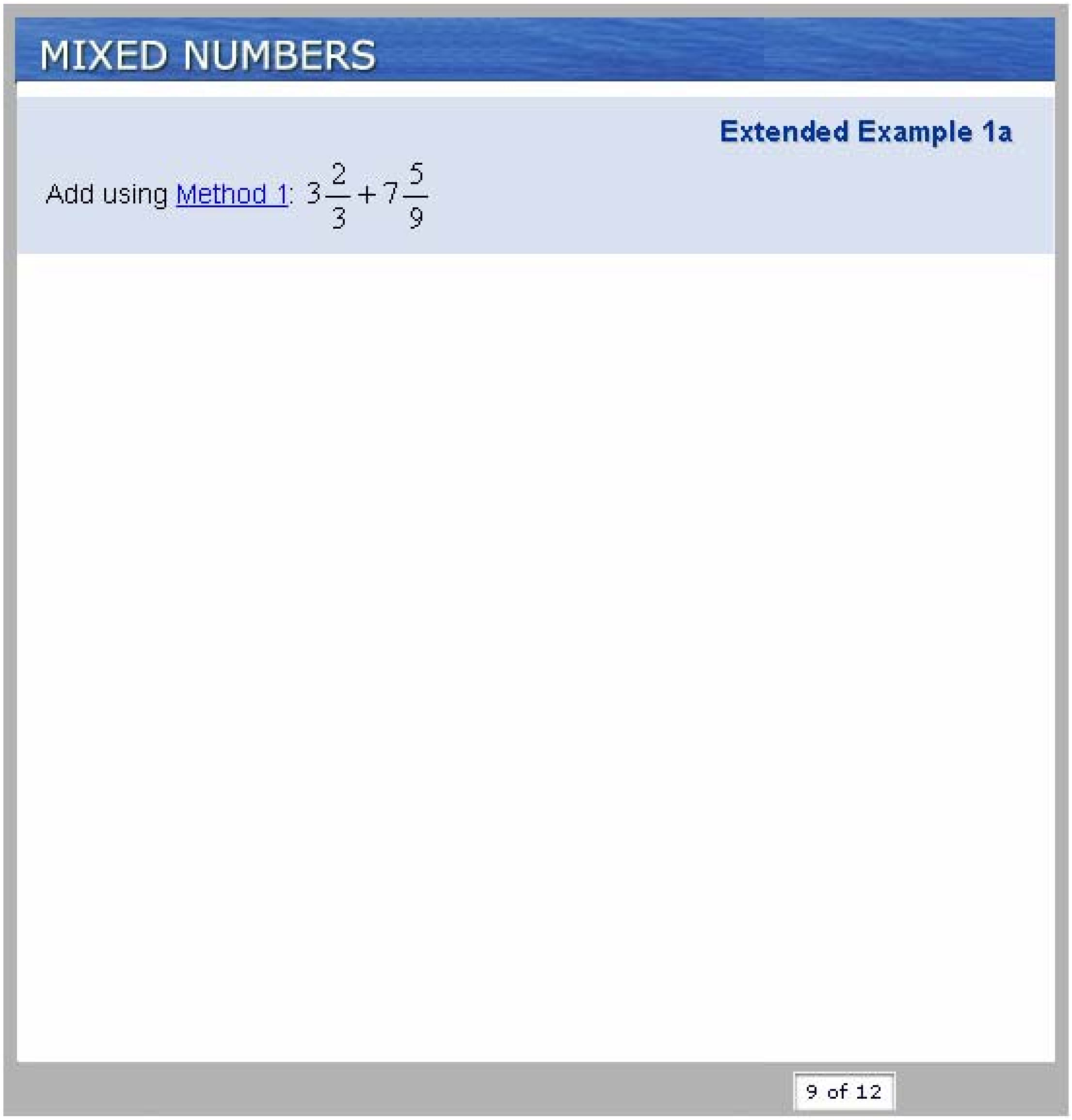
MIXED NUMBERS

Below is an example of subtracting mixed numbers using the two different methods discussed on the previous screen.

In Method 2 below, notice that after writing the fraction parts with a common denominator of 15, the numerator of the first fraction (9) is less than the numerator of the second fraction (11). Since the numerator being subtracted is larger than the numerator we're subtracting from, we need to borrow from the integer of the first mixed number for subtraction. To borrow from the integer for the fraction, use the fact that any fraction with an equal numerator and denominator is equal to one; $\frac{a}{a}=1$. In this case, borrow $1=\frac{15}{15}$ from the integer part (so the integer part becomes 4 – 1 = 3) and add 15 to the numerator (9+15=24). Now you can subtract the numerators over the common denominator.

Method 1	**Method 2**
$4\frac{3}{5}-1\frac{11}{15}$	$4\frac{3}{5}-1\frac{11}{15}$
$=\frac{23(3)}{5(3)}-\frac{26}{15}$	$=4\frac{3(3)}{5(3)}-1\frac{11}{15}$
$=\frac{69}{15}-\frac{26}{15}$	$=4\frac{9}{15}-1\frac{11}{15}$
$=\frac{69-26}{15}$	$=3\frac{24}{15}-1\frac{11}{15}$
$=\frac{43}{15}=2\frac{13}{15}$	$=2\frac{13}{15}$

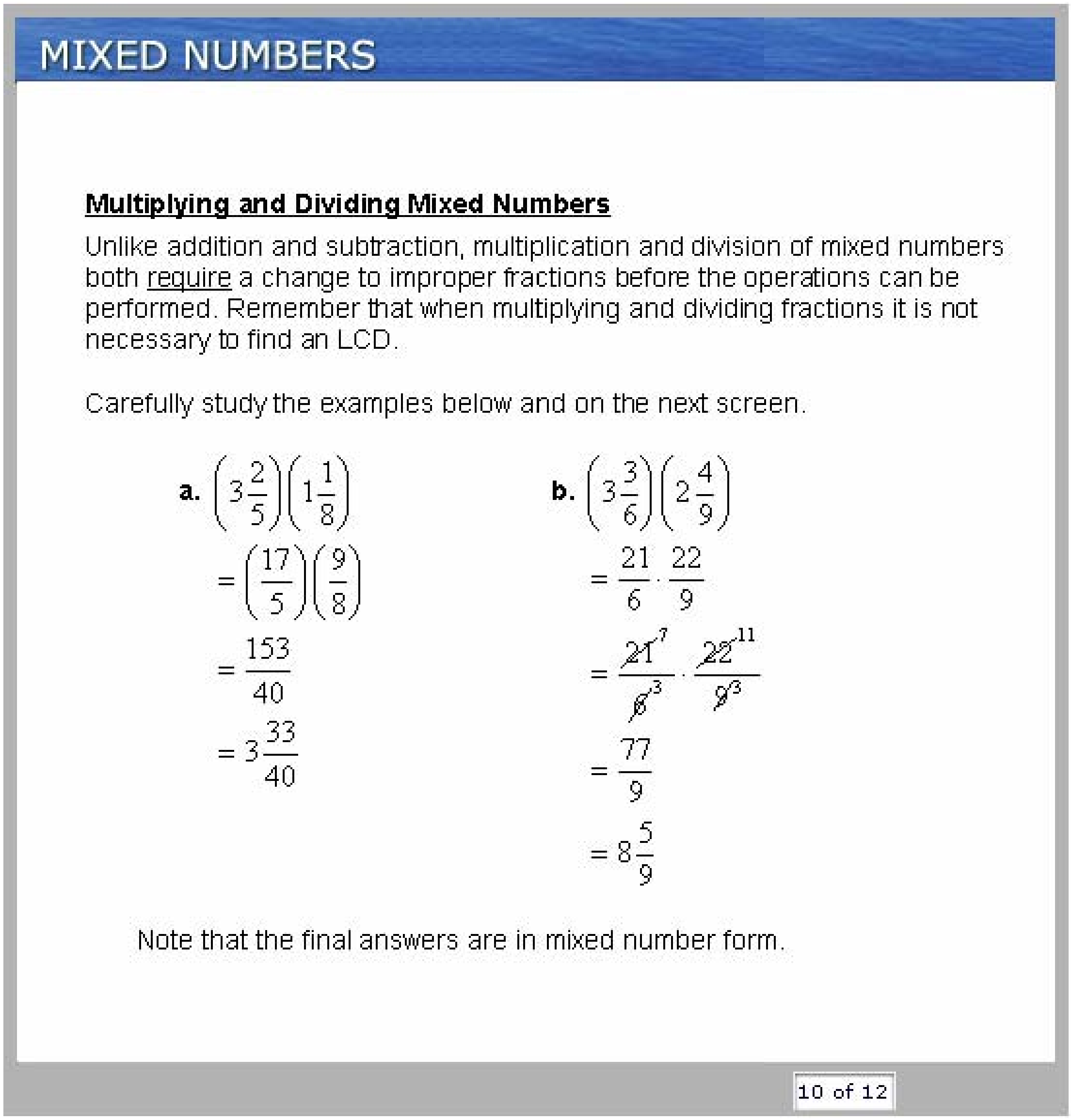
It is good to know how to use both methods of combining mixed numbers. Once you understand both, use the method that you prefer.



MIXED NUMBERS

Extended Example 1a

Add using Method 1: $3\frac{2}{3}+7\frac{5}{9}$



MIXED NUMBERS

Multiplying and Dividing Mixed Numbers

Unlike addition and subtraction, multiplication and division of mixed numbers both require a change to improper fractions before the operations can be performed. Remember that when multiplying and dividing fractions it is not necessary to find an LCD.

Carefully study the examples below and on the next screen.

a. $\left(3\frac{2}{5}\right)\left(1\frac{1}{8}\right)$

$= \left(\frac{17}{5}\right)\left(\frac{9}{8}\right)$

$= \frac{153}{40}$

$= 3\frac{33}{40}$

b. $\left(3\frac{3}{6}\right)\left(2\frac{4}{9}\right)$

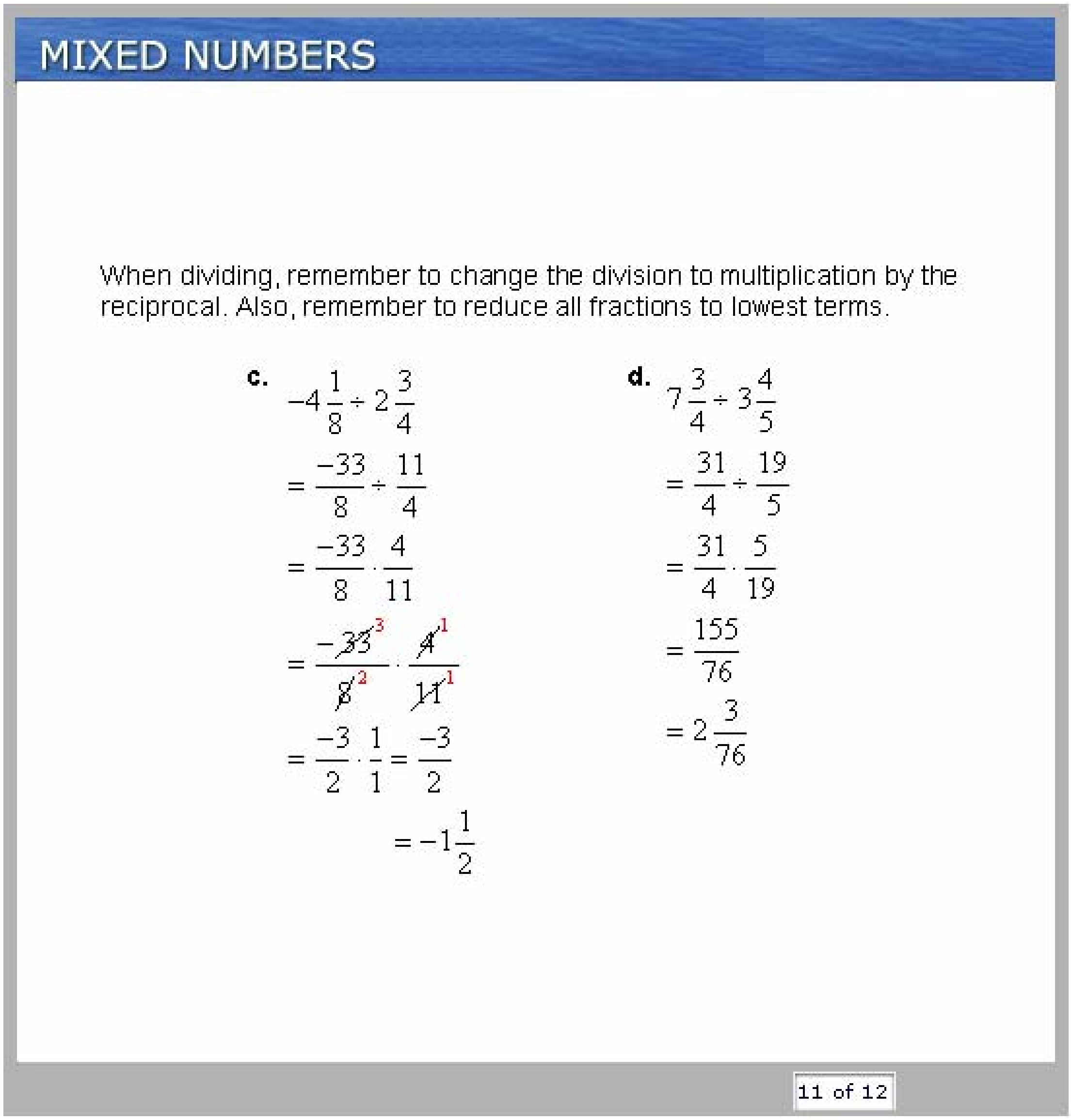
$= \frac{21}{6} \cdot \frac{22}{9}$

$= \frac{\cancel{21}^{7}}{\cancel{6}^{3}} \cdot \frac{\cancel{22}^{11}}{\cancel{9}^{3}}$

$= \frac{77}{9}$

$= 8\frac{5}{9}$

Note that the final answers are in mixed number form.



MIXED NUMBERS

When dividing, remember to change the division to multiplication by the reciprocal. Also, remember to reduce all fractions to lowest terms.

c.

$$-4\frac{1}{8} \div 2\frac{3}{4}$$

$$= \frac{-33}{8} \div \frac{11}{4}$$

$$= \frac{-33}{8} \cdot \frac{4}{11}$$

$$= \frac{-\cancel{33}^{3}}{\cancel{8}^{2}} \cdot \frac{\cancel{4}^{1}}{\cancel{11}^{1}}$$

$$= \frac{-3}{2} \cdot \frac{1}{1} = \frac{-3}{2}$$

$$= -1\frac{1}{2}$$

d.

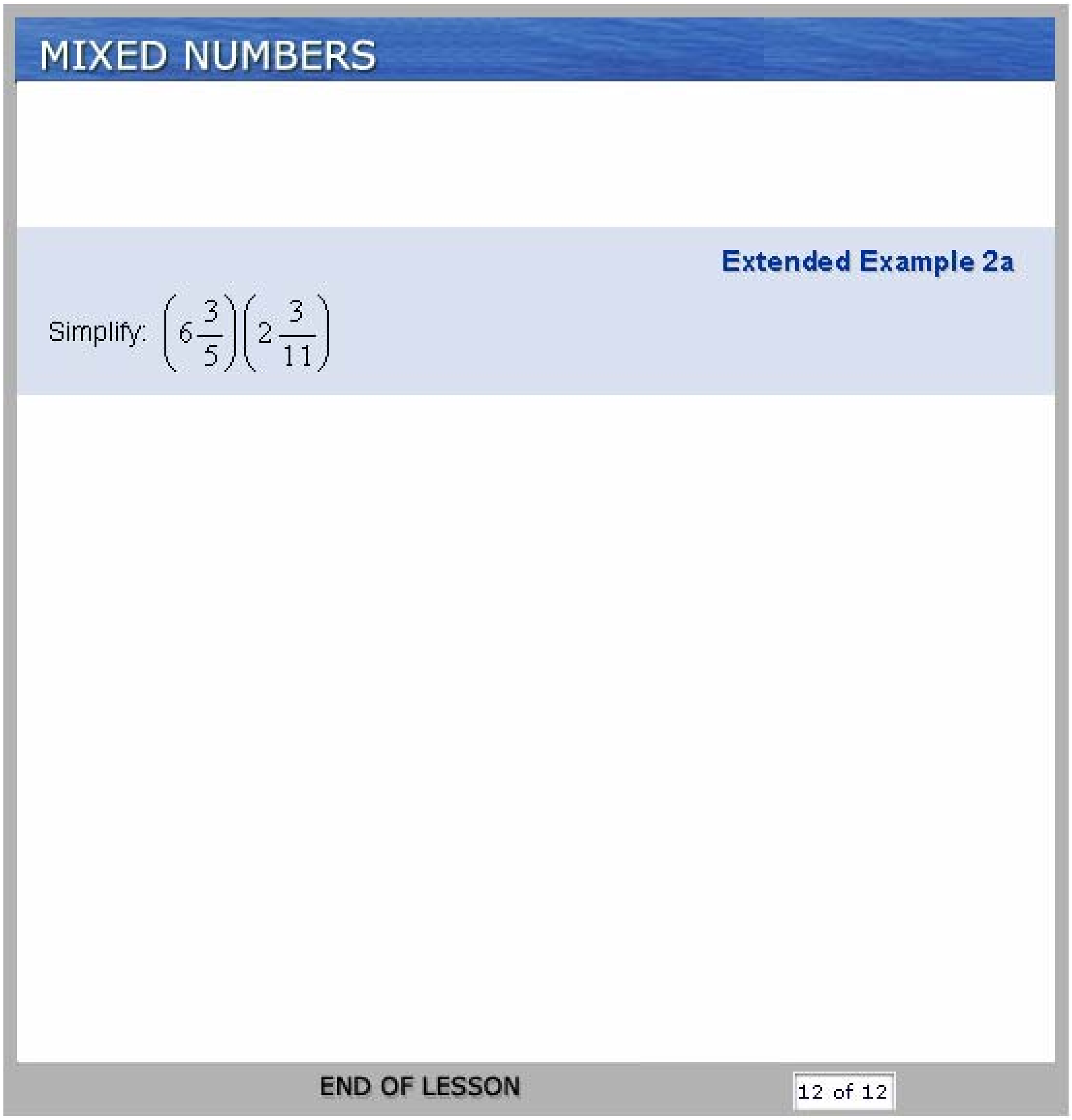
$$7\frac{3}{4} \div 3\frac{4}{5}$$

$$= \frac{31}{4} \div \frac{19}{5}$$

$$= \frac{31}{4} \cdot \frac{5}{19}$$

$$= \frac{155}{76}$$

$$= 2\frac{3}{76}$$



MIXED NUMBERS

Extended Example 2a

Simplify: $\left(6\frac{3}{5}\right)\left(2\frac{3}{11}\right)$

Convert the mixed number into an improper fraction.

$$8\frac{2}{3} =$$

Add or subtract.

$$6 - \frac{4}{9} =$$

Add or subtract.

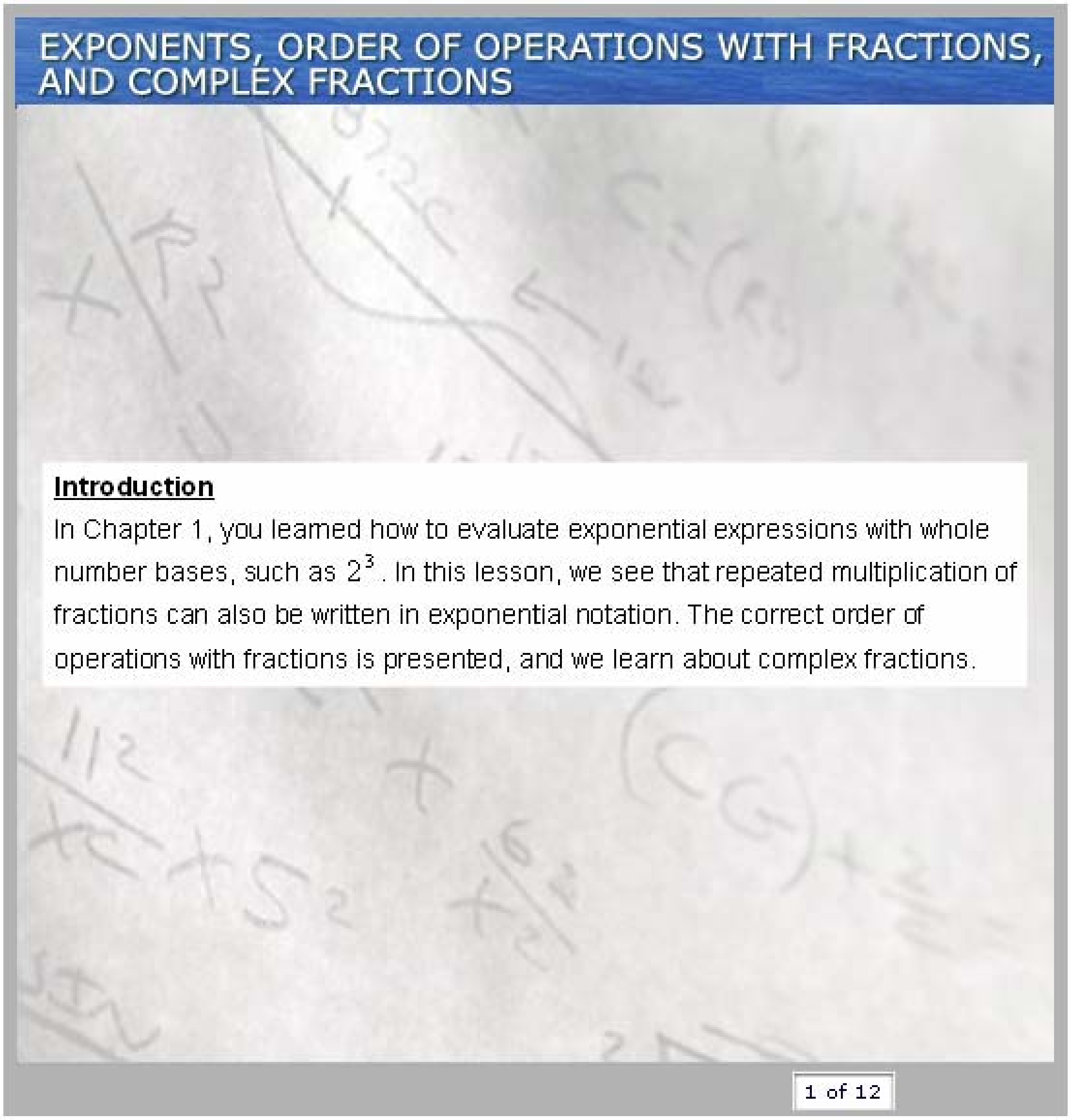
$$15\frac{4}{5} - 5\frac{3}{4} =$$

Multiply or divide.

$$\left(10\frac{8}{9}\right)\left(\frac{3}{4}\right)=$$

Multiply or divide.

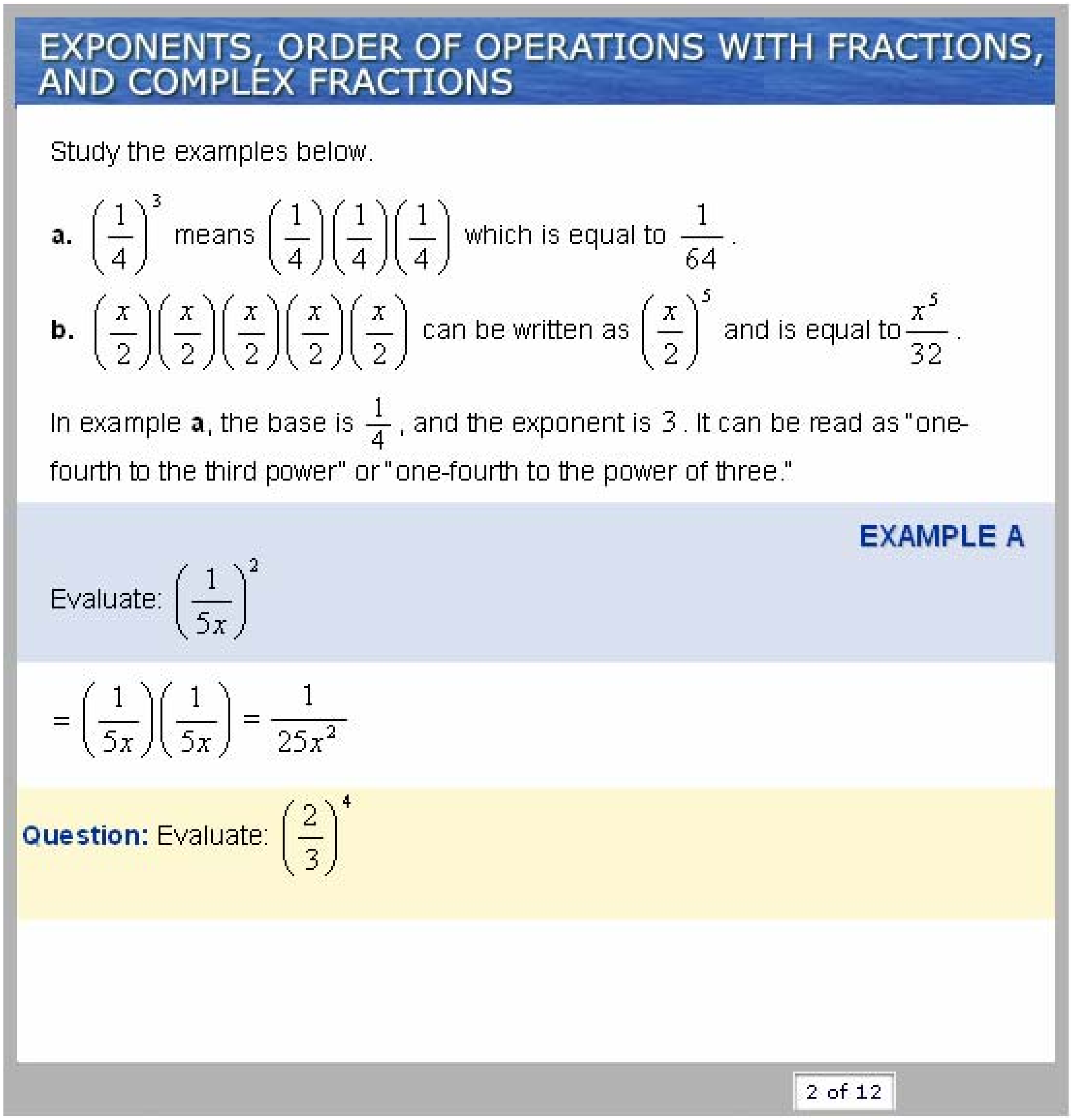
$$4\frac{3}{5}\div 3\frac{1}{2}=$$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

Introduction

In Chapter 1, you learned how to evaluate exponential expressions with whole number bases, such as 2^3. In this lesson, we see that repeated multiplication of fractions can also be written in exponential notation. The correct order of operations with fractions is presented, and we learn about complex fractions.



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

Study the examples below.

a. $\left(\frac{1}{4}\right)^3$ means $\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$ which is equal to $\frac{1}{64}$.

b. $\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)\left(\frac{x}{2}\right)$ can be written as $\left(\frac{x}{2}\right)^5$ and is equal to $\frac{x^5}{32}$.

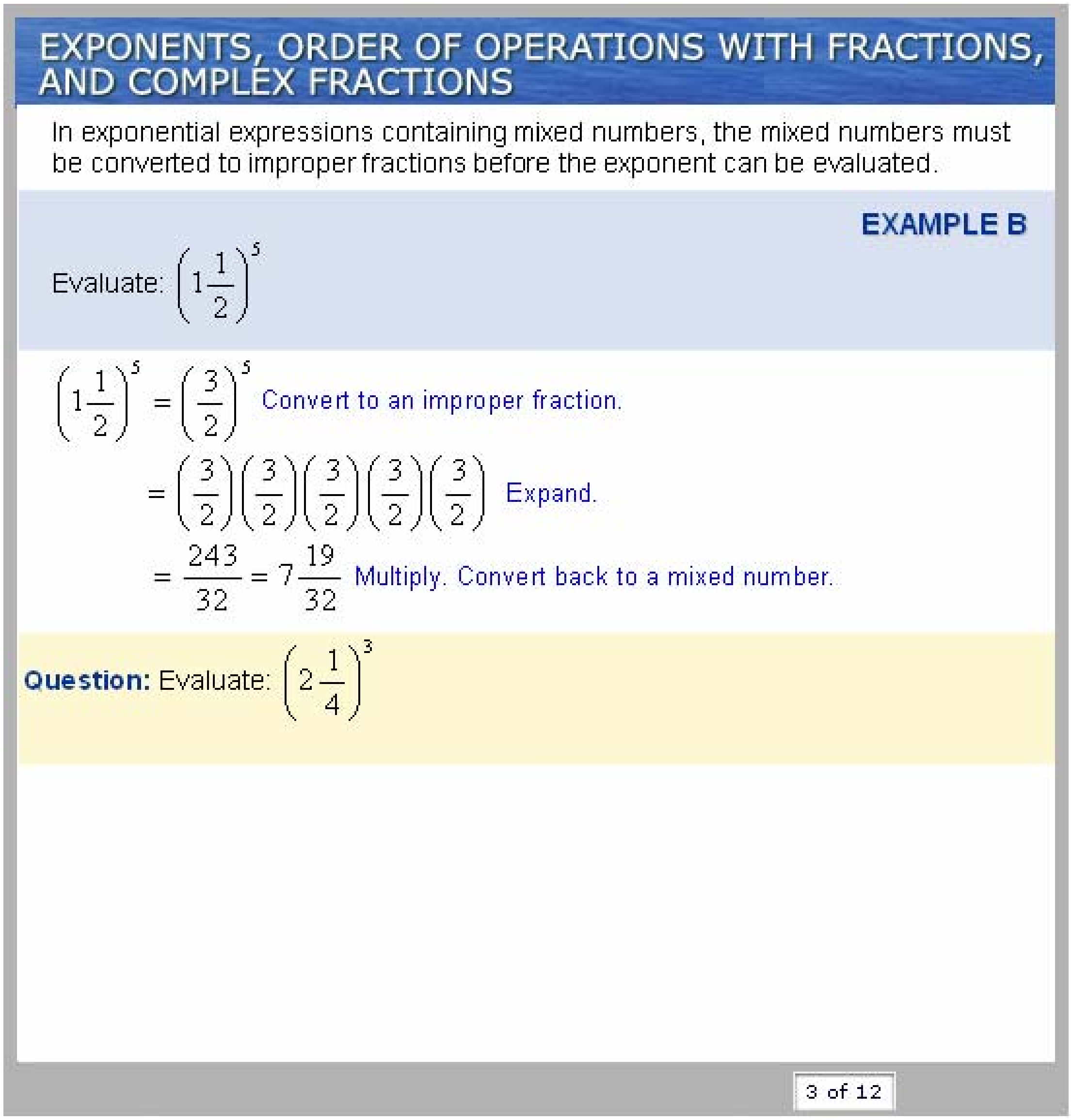
In example **a**, the base is $\frac{1}{4}$, and the exponent is 3. It can be read as "one-fourth to the third power" or "one-fourth to the power of three."

EXAMPLE A

Evaluate: $\left(\frac{1}{5x}\right)^2$

$$= \left(\frac{1}{5x}\right)\left(\frac{1}{5x}\right) = \frac{1}{25x^2}$$

Question: Evaluate: $\left(\frac{2}{3}\right)^4$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

In exponential expressions containing mixed numbers, the mixed numbers must be converted to improper fractions before the exponent can be evaluated.

EXAMPLE B

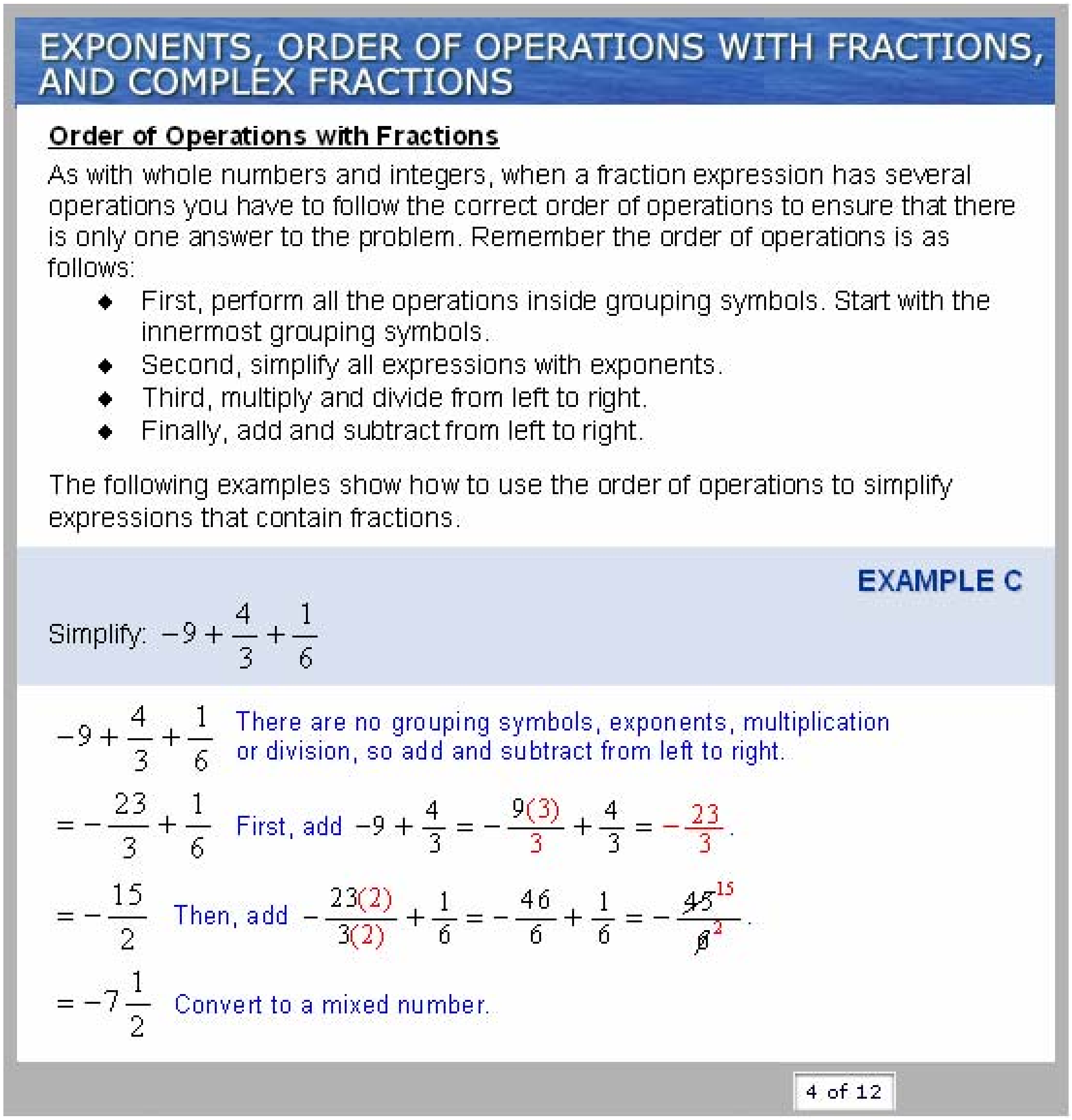
Evaluate: $\left(1\frac{1}{2}\right)^5$

$\left(1\frac{1}{2}\right)^5 = \left(\frac{3}{2}\right)^5$ Convert to an improper fraction.

$= \left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)$ Expand.

$= \frac{243}{32} = 7\frac{19}{32}$ Multiply. Convert back to a mixed number.

Question: Evaluate: $\left(2\frac{1}{4}\right)^3$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

Order of Operations with Fractions

As with whole numbers and integers, when a fraction expression has several operations you have to follow the correct order of operations to ensure that there is only one answer to the problem. Remember the order of operations is as follows:

- First, perform all the operations inside grouping symbols. Start with the innermost grouping symbols.
- Second, simplify all expressions with exponents.
- Third, multiply and divide from left to right.
- Finally, add and subtract from left to right.

The following examples show how to use the order of operations to simplify expressions that contain fractions.

EXAMPLE C

Simplify: $-9+\frac{4}{3}+\frac{1}{6}$

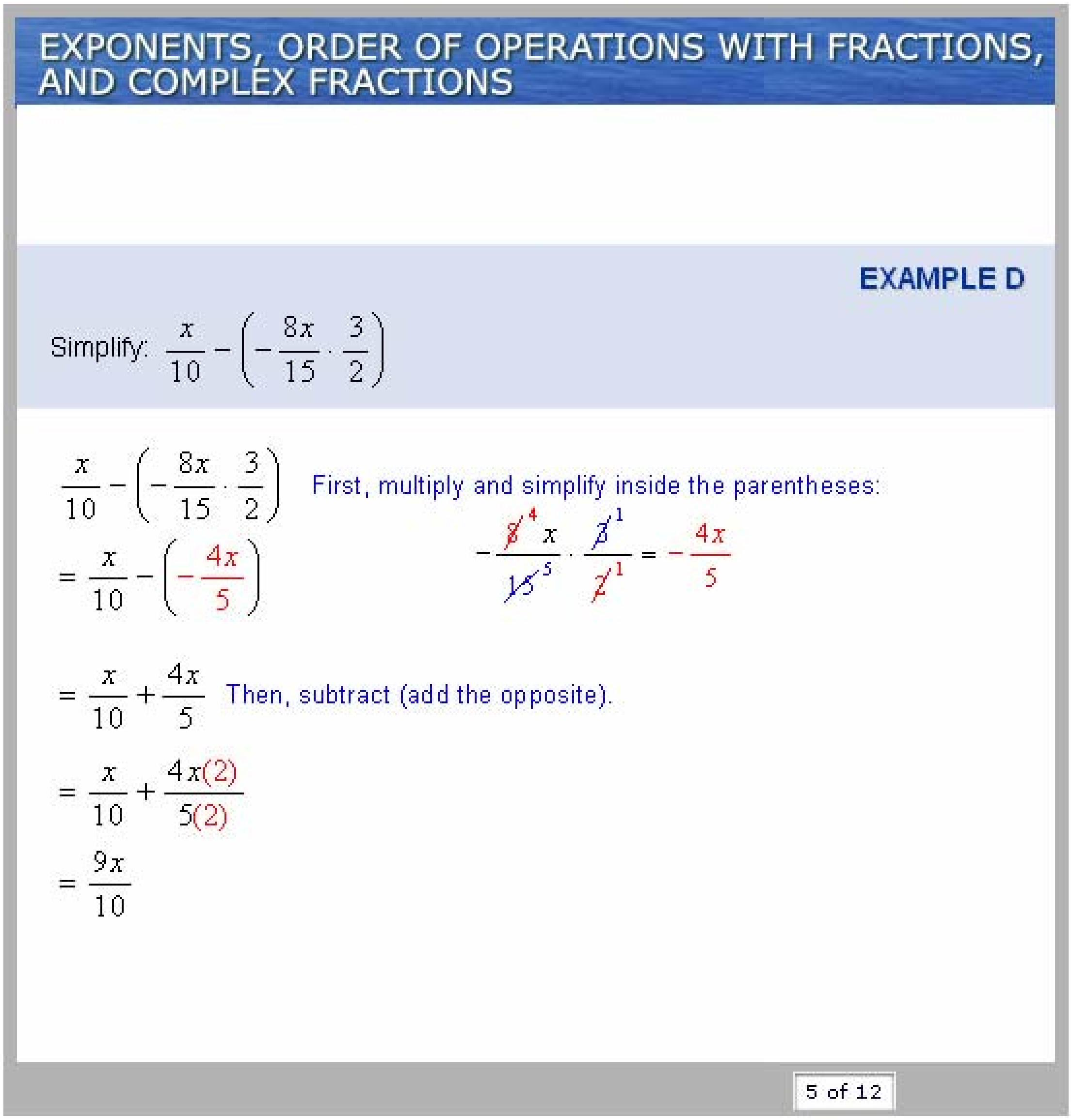
$-9+\frac{4}{3}+\frac{1}{6}$ There are no grouping symbols, exponents, multiplication or division, so add and subtract from left to right.

$=-\frac{23}{3}+\frac{1}{6}$ First, add $-9+\frac{4}{3}=-\frac{9(3)}{3}+\frac{4}{3}=-\frac{23}{3}$.

$=-\frac{15}{2}$ Then, add $-\frac{23(2)}{3(2)}+\frac{1}{6}=-\frac{46}{6}+\frac{1}{6}=-\frac{\cancel{45}^{15}}{\cancel{6}^{2}}$.

$=-7\frac{1}{2}$ Convert to a mixed number.

4 of 12



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

EXAMPLE D

Simplify: $\frac{x}{10} - \left(-\frac{8x}{15} \cdot \frac{3}{2}\right)$

$\frac{x}{10} - \left(-\frac{8x}{15} \cdot \frac{3}{2}\right)$ First, multiply and simplify inside the parentheses:

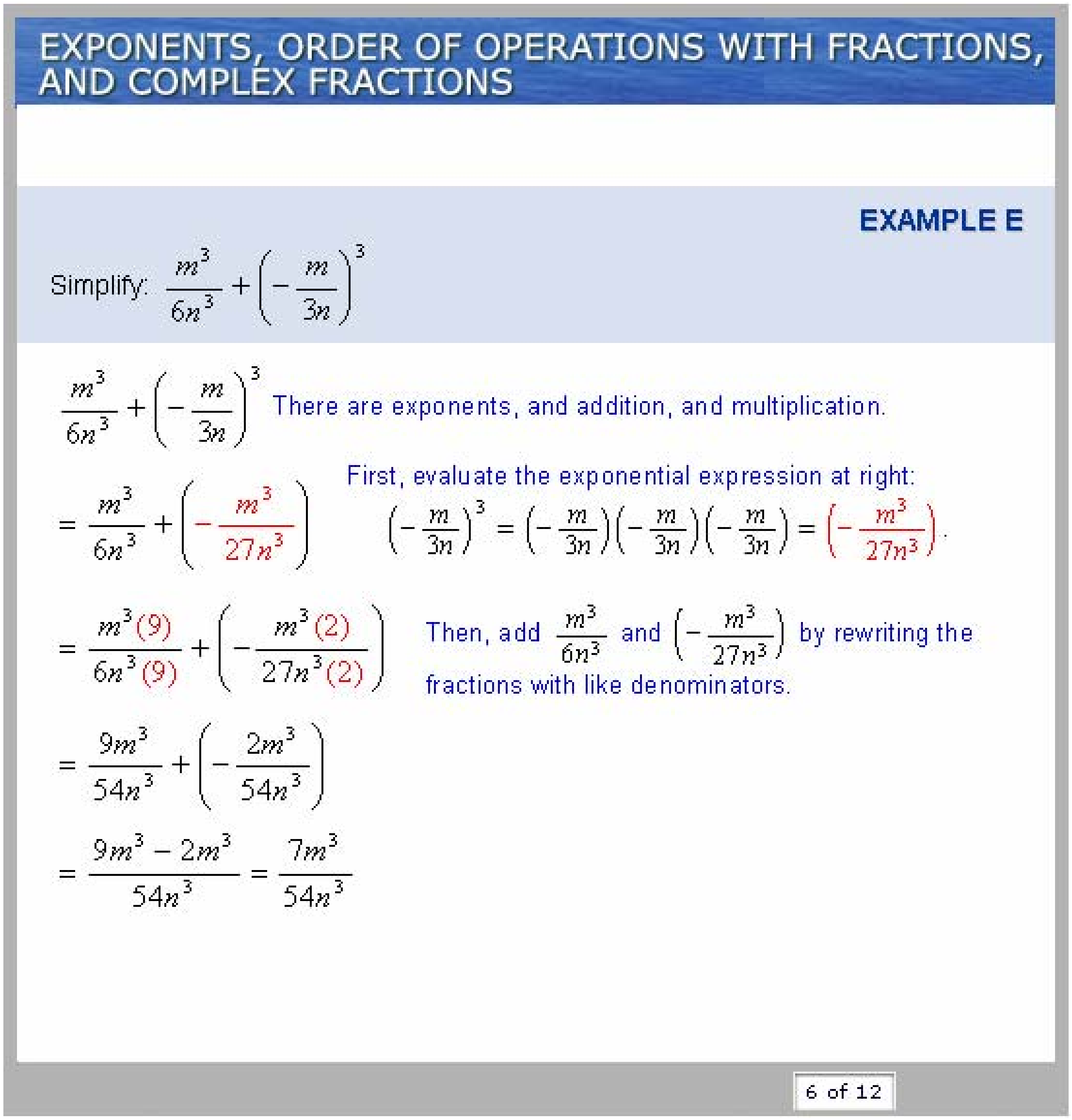
$$-\frac{\cancel{8}^{4} x}{\cancel{15}^{5}} \cdot \frac{\cancel{3}^{1}}{\cancel{2}^{1}} = -\frac{4x}{5}$$

$= \frac{x}{10} - \left(-\frac{4x}{5}\right)$

$= \frac{x}{10} + \frac{4x}{5}$ Then, subtract (add the opposite).

$= \frac{x}{10} + \frac{4x(2)}{5(2)}$

$= \frac{9x}{10}$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

EXAMPLE E

Simplify: $\frac{m^3}{6n^3}+\left(-\frac{m}{3n}\right)^3$

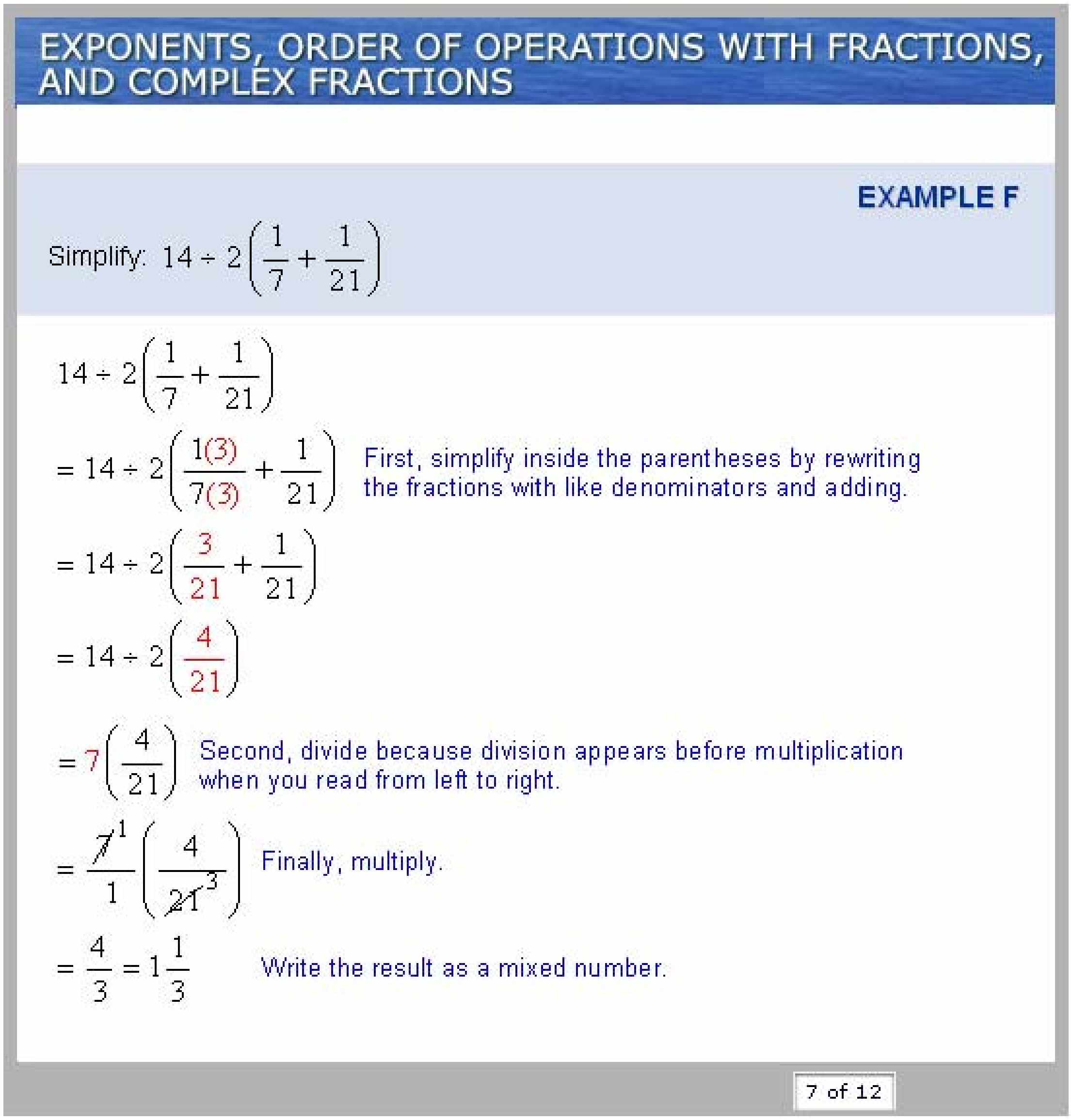
$\frac{m^3}{6n^3}+\left(-\frac{m}{3n}\right)^3$ There are exponents, and addition, and multiplication.

$=\frac{m^3}{6n^3}+\left(-\frac{m^3}{27n^3}\right)$ First, evaluate the exponential expression at right: $\left(-\frac{m}{3n}\right)^3=\left(-\frac{m}{3n}\right)\left(-\frac{m}{3n}\right)\left(-\frac{m}{3n}\right)=\left(-\frac{m^3}{27n^3}\right)$.

$=\frac{m^3(9)}{6n^3(9)}+\left(-\frac{m^3(2)}{27n^3(2)}\right)$ Then, add $\frac{m^3}{6n^3}$ and $\left(-\frac{m^3}{27n^3}\right)$ by rewriting the fractions with like denominators.

$=\frac{9m^3}{54n^3}+\left(-\frac{2m^3}{54n^3}\right)$

$=\frac{9m^3-2m^3}{54n^3}=\frac{7m^3}{54n^3}$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

EXAMPLE F

Simplify: $14 \div 2\left(\frac{1}{7} + \frac{1}{21}\right)$

$$14 \div 2\left(\frac{1}{7} + \frac{1}{21}\right)$$

$$= 14 \div 2\left(\frac{1(3)}{7(3)} + \frac{1}{21}\right)$$

First, simplify inside the parentheses by rewriting the fractions with like denominators and adding.

$$= 14 \div 2\left(\frac{3}{21} + \frac{1}{21}\right)$$

$$= 14 \div 2\left(\frac{4}{21}\right)$$

$$= 7\left(\frac{4}{21}\right)$$

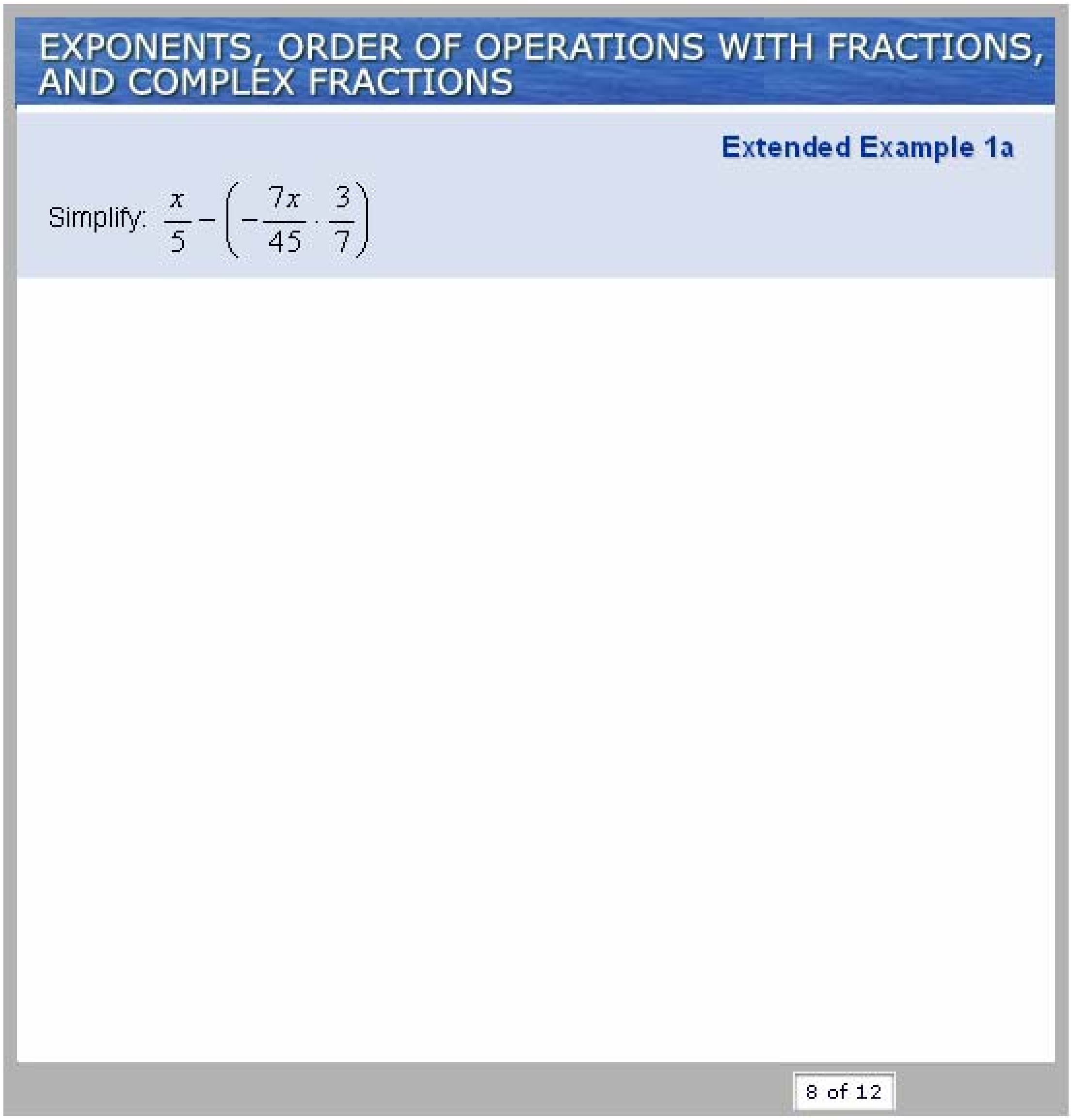
Second, divide because division appears before multiplication when you read from left to right.

$$= \frac{\cancel{7}^{1}}{1}\left(\frac{4}{\cancel{21}^{3}}\right)$$

Finally, multiply.

$$= \frac{4}{3} = 1\frac{1}{3}$$

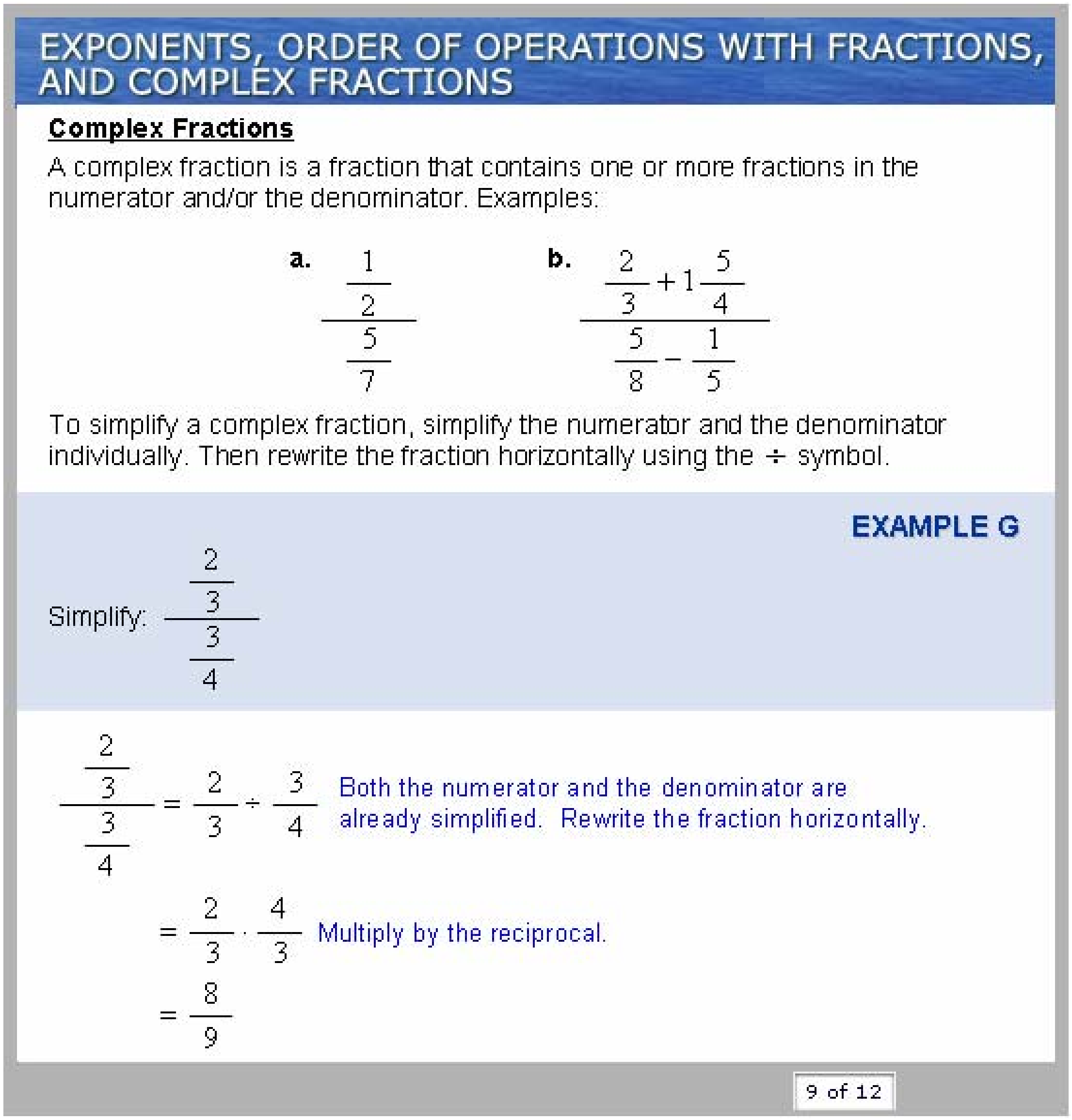
Write the result as a mixed number.



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

Extended Example 1a

Simplify: $\frac{x}{5}-\left(-\frac{7x}{45}\cdot\frac{3}{7}\right)$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

Complex Fractions

A complex fraction is a fraction that contains one or more fractions in the numerator and/or the denominator. Examples:

a. $\dfrac{\frac{1}{2}}{\frac{5}{7}}$ **b.** $\dfrac{\frac{2}{3}+1\frac{5}{4}}{\frac{5}{8}-\frac{1}{5}}$

To simplify a complex fraction, simplify the numerator and the denominator individually. Then rewrite the fraction horizontally using the ÷ symbol.

EXAMPLE G

Simplify: $\dfrac{\frac{2}{3}}{\frac{3}{4}}$

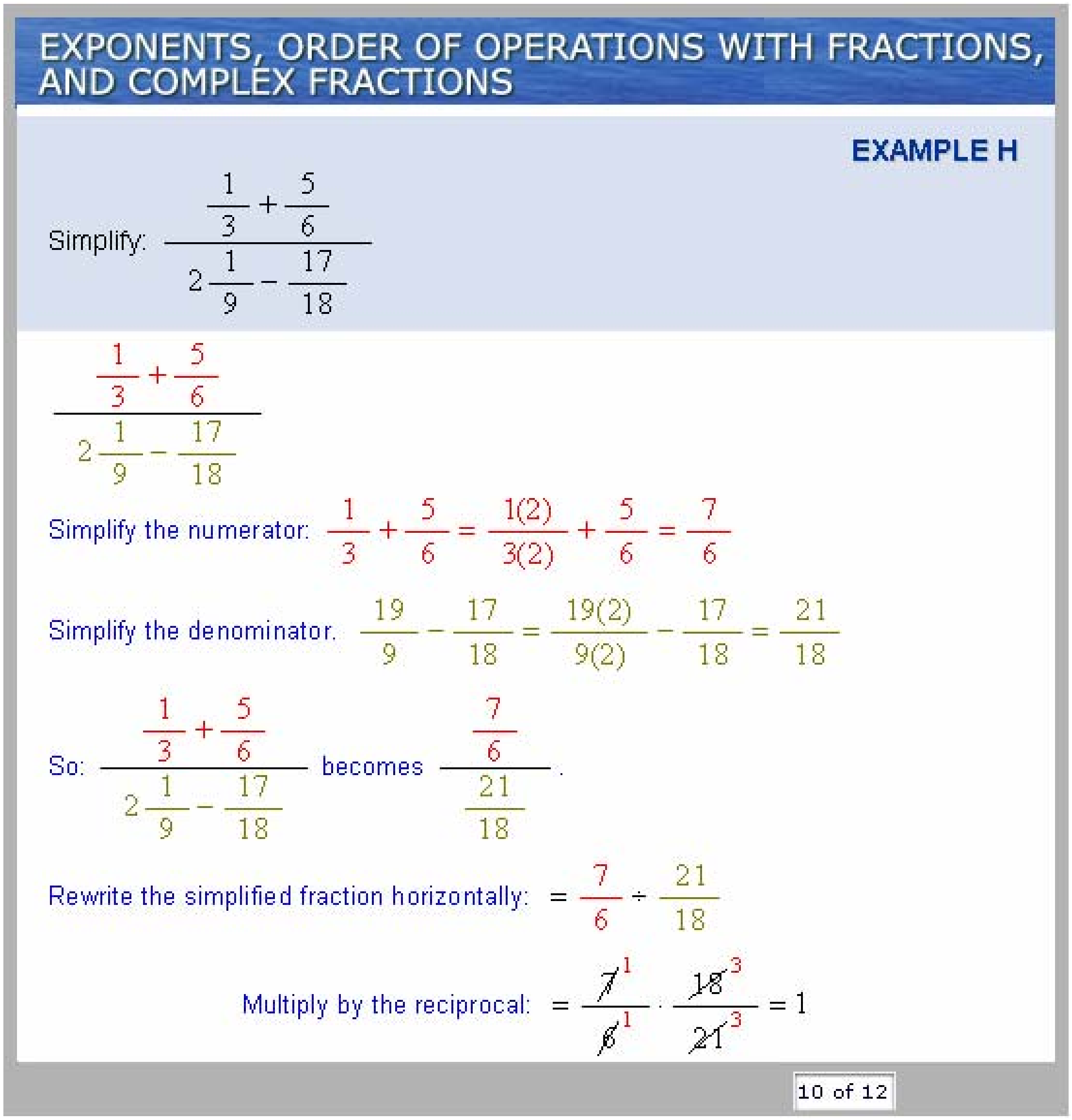
$$\frac{\frac{2}{3}}{\frac{3}{4}} = \frac{2}{3} \div \frac{3}{4}$$

Both the numerator and the denominator are already simplified. Rewrite the fraction horizontally.

$$= \frac{2}{3} \cdot \frac{4}{3}$$

Multiply by the reciprocal.

$$= \frac{8}{9}$$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

EXAMPLE H

Simplify: $$\frac{\frac{1}{3}+\frac{5}{6}}{2\frac{1}{9}-\frac{17}{18}}$$

$$\frac{\frac{1}{3}+\frac{5}{6}}{2\frac{1}{9}-\frac{17}{18}}$$

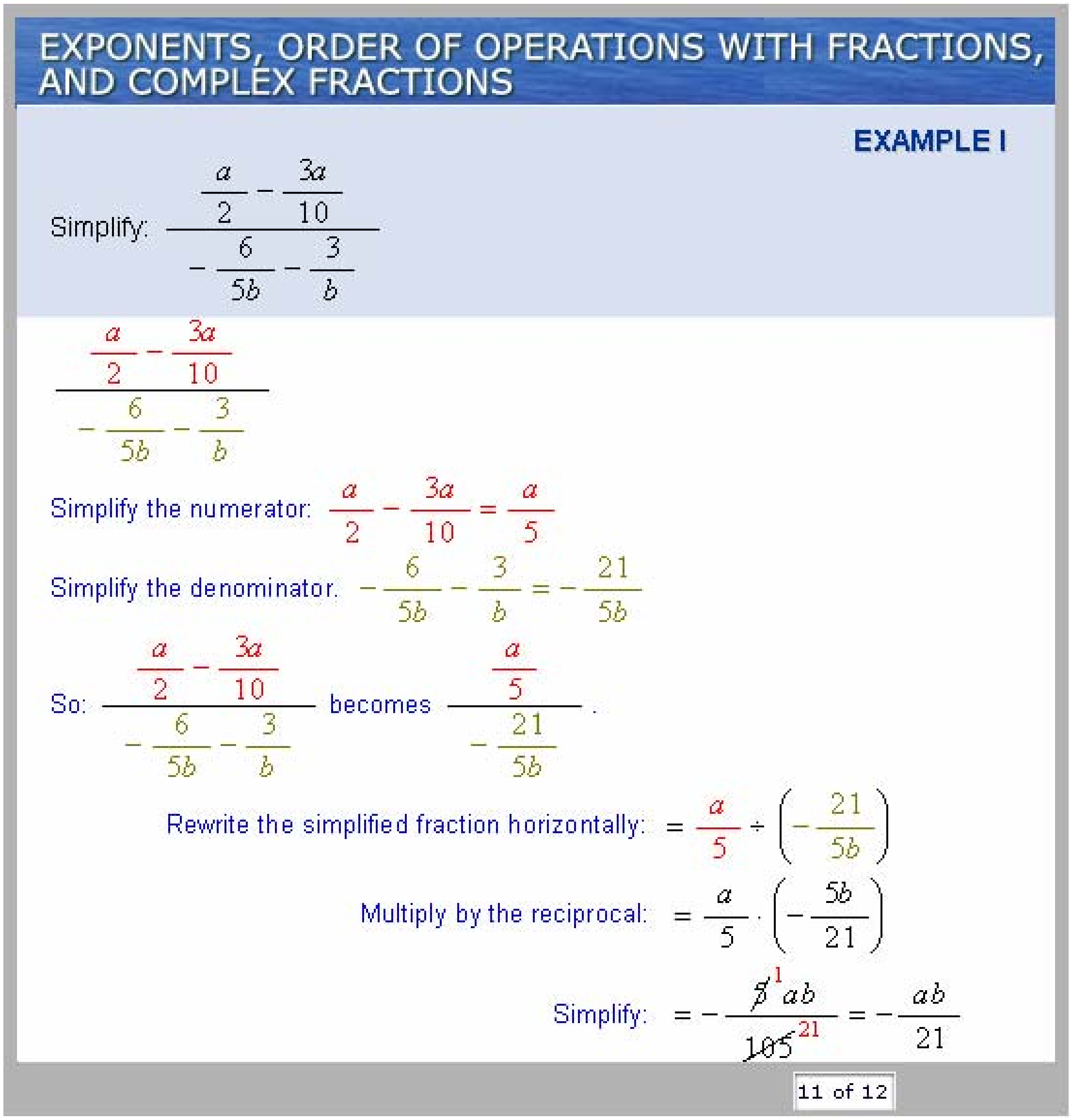
Simplify the numerator: $\frac{1}{3}+\frac{5}{6}=\frac{1(2)}{3(2)}+\frac{5}{6}=\frac{7}{6}$

Simplify the denominator. $\frac{19}{9}-\frac{17}{18}=\frac{19(2)}{9(2)}-\frac{17}{18}=\frac{21}{18}$

So: $\frac{\frac{1}{3}+\frac{5}{6}}{2\frac{1}{9}-\frac{17}{18}}$ becomes $\frac{\frac{7}{6}}{\frac{21}{18}}$.

Rewrite the simplified fraction horizontally: $=\frac{7}{6}\div\frac{21}{18}$

Multiply by the reciprocal: $=\frac{\cancel{7}^{1}}{\cancel{6}^{1}}\cdot\frac{\cancel{18}^{3}}{\cancel{21}^{3}}=1$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

EXAMPLE I

Simplify: $$\frac{\frac{a}{2}-\frac{3a}{10}}{-\frac{6}{5b}-\frac{3}{b}}$$

$$\frac{\frac{a}{2}-\frac{3a}{10}}{-\frac{6}{5b}-\frac{3}{b}}$$

Simplify the numerator: $\frac{a}{2}-\frac{3a}{10}=\frac{a}{5}$

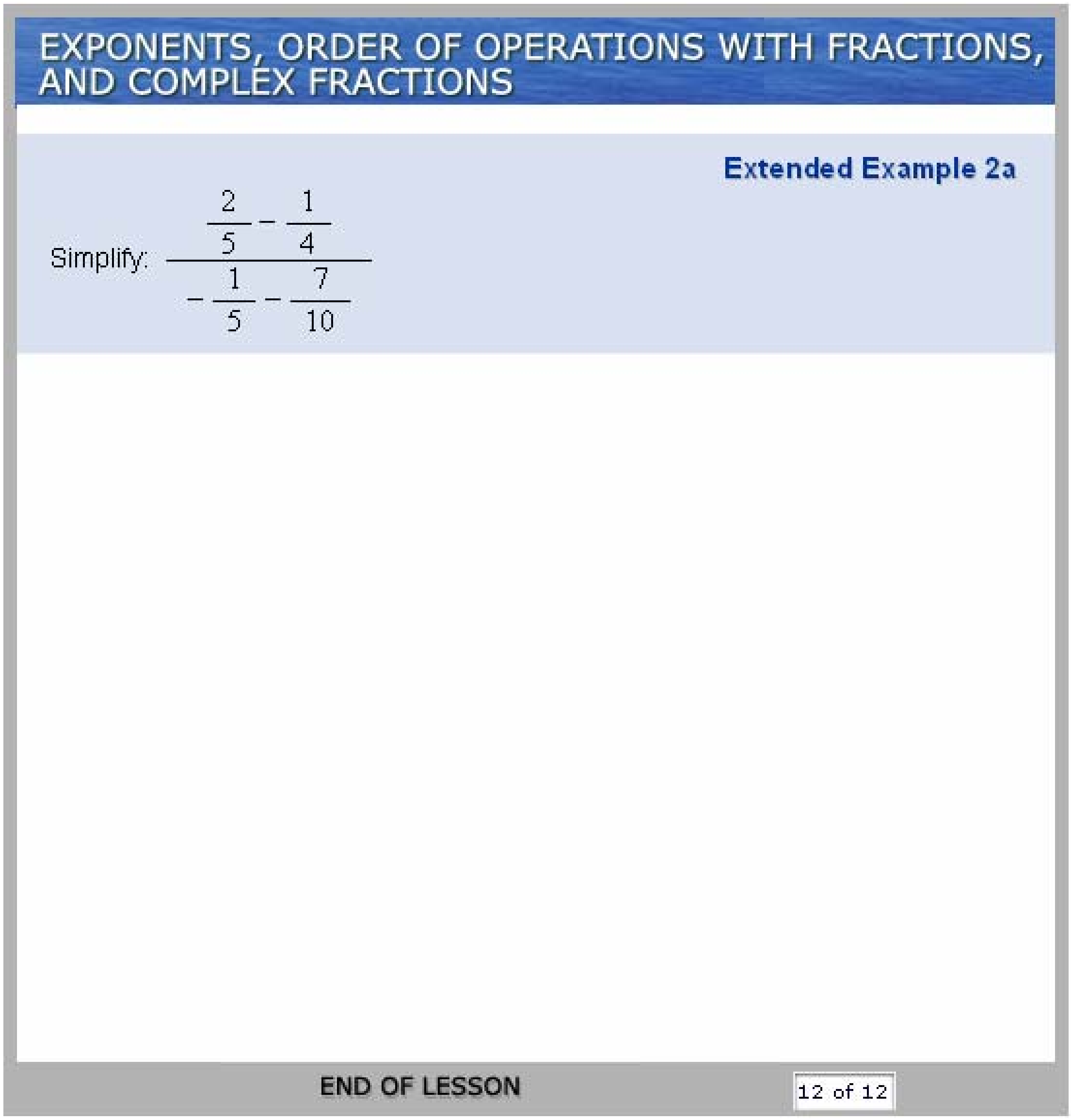
Simplify the denominator: $-\frac{6}{5b}-\frac{3}{b}=-\frac{21}{5b}$

So: $\frac{\frac{a}{2}-\frac{3a}{10}}{-\frac{6}{5b}-\frac{3}{b}}$ becomes $\frac{\frac{a}{5}}{-\frac{21}{5b}}$.

Rewrite the simplified fraction horizontally: $=\frac{a}{5}\div\left(-\frac{21}{5b}\right)$

Multiply by the reciprocal: $=\frac{a}{5}\cdot\left(-\frac{5b}{21}\right)$

Simplify: $=-\frac{\cancel{5}^{1}ab}{\cancel{105}^{21}}=-\frac{ab}{21}$



EXPONENTS, ORDER OF OPERATIONS WITH FRACTIONS, AND COMPLEX FRACTIONS

Extended Example 2a

Simplify: $$\frac{\frac{2}{5}-\frac{1}{4}}{-\frac{1}{5}-\frac{7}{10}}$$

Evaluate.

$$\left(-2\frac{1}{4}\right)^3 =$$

Use the Order of Operations to simplify.

$$\frac{4v^3}{7}+\left(\frac{2v}{3}\right)^2 \div \frac{28}{6v}=$$

Use the Order of Operations to simplify.

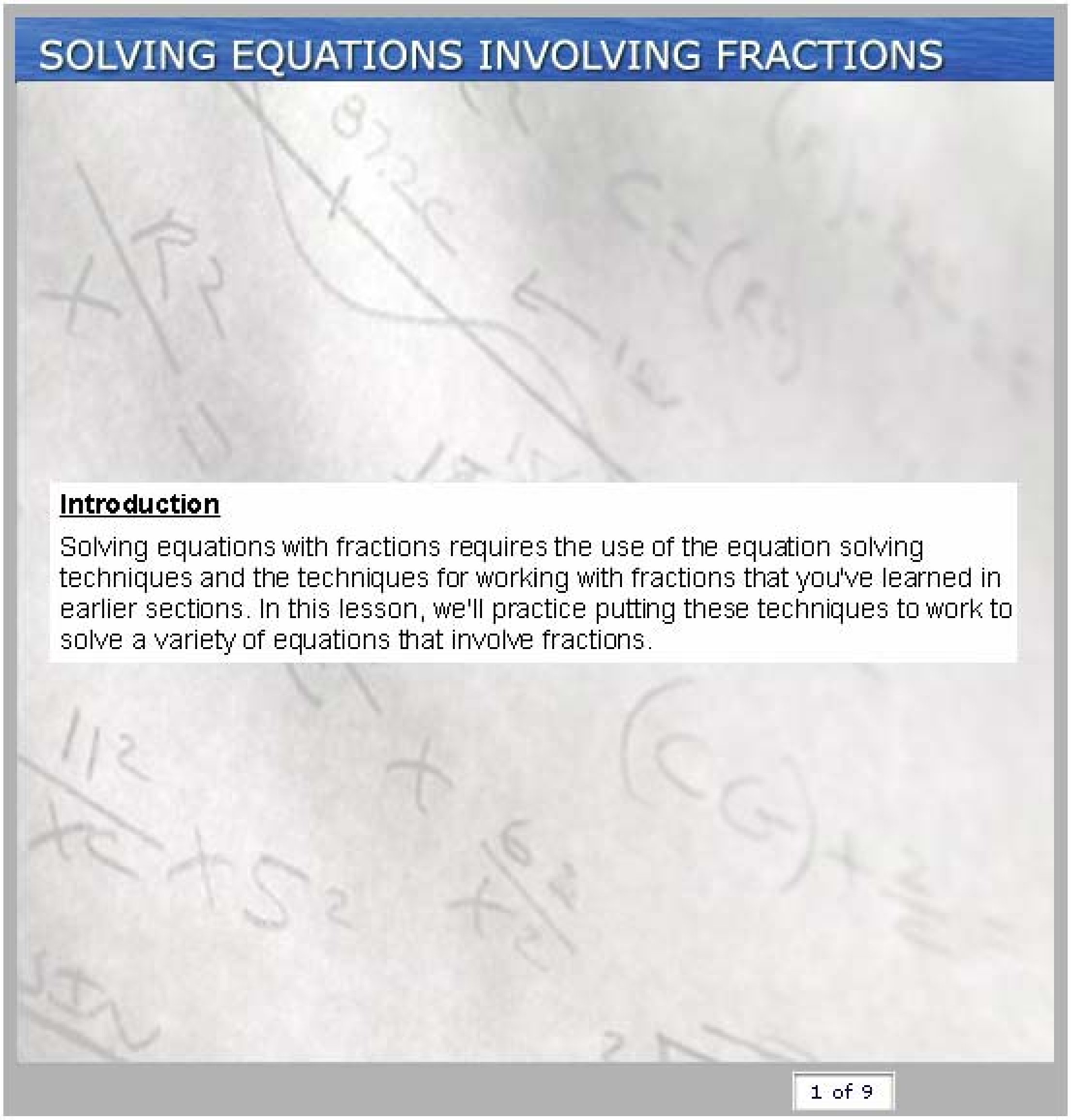
$$\frac{3}{4r}+\left(\frac{r}{2}\right)^5 \div \frac{r^6}{2}-\frac{1}{r}=$$

Simplify.

$$\frac{\frac{5}{6}-\frac{7}{12}}{6}=$$

Simplify.

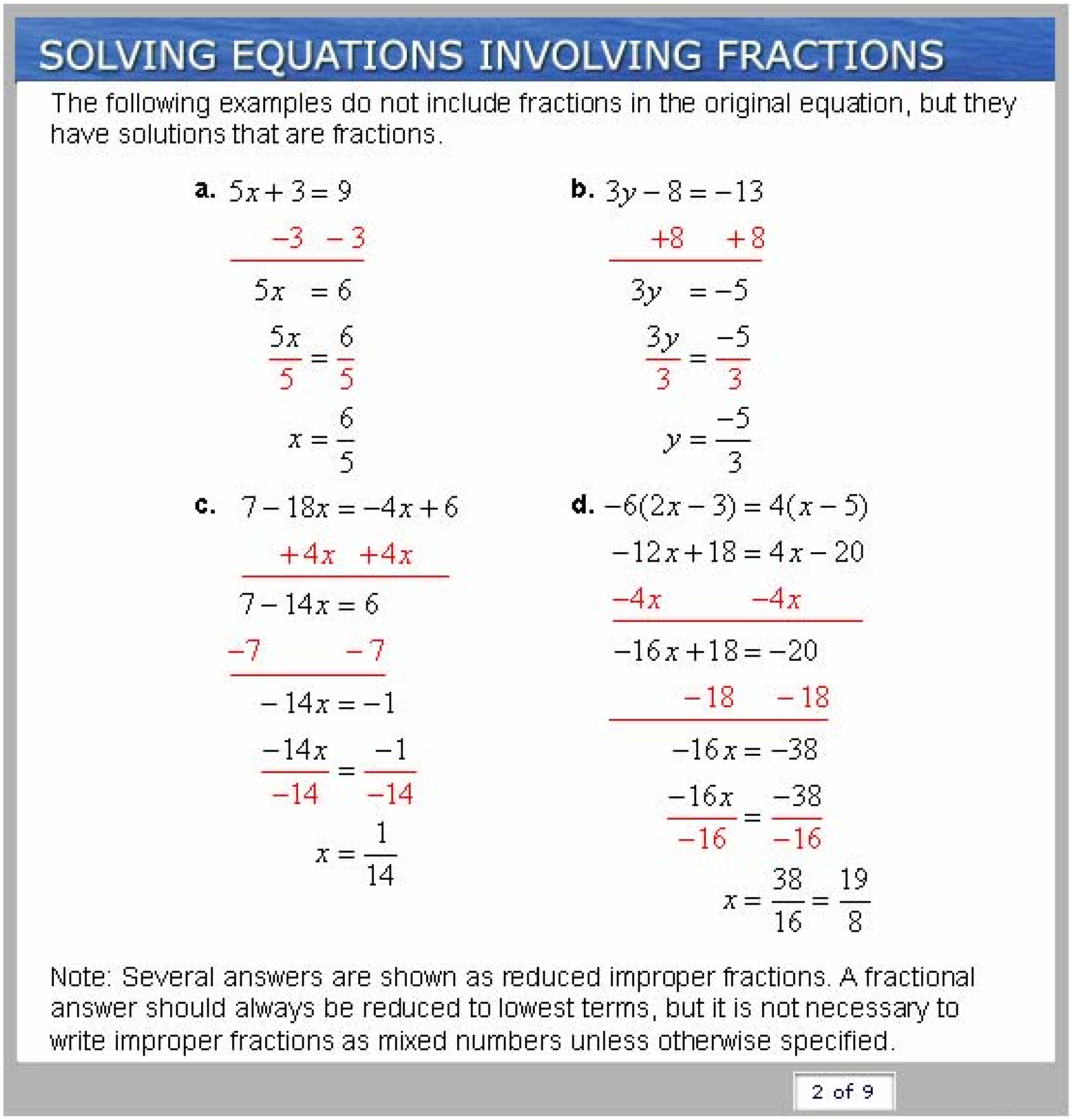
$$\frac{\frac{2x}{7}-\frac{11x}{14}}{\frac{x^2}{2}+\frac{4x^2}{7}}=$$



SOLVING EQUATIONS INVOLVING FRACTIONS

Introduction

Solving equations with fractions requires the use of the equation solving techniques and the techniques for working with fractions that you've learned in earlier sections. In this lesson, we'll practice putting these techniques to work to solve a variety of equations that involve fractions.



SOLVING EQUATIONS INVOLVING FRACTIONS

The following examples do not include fractions in the original equation, but they have solutions that are fractions.

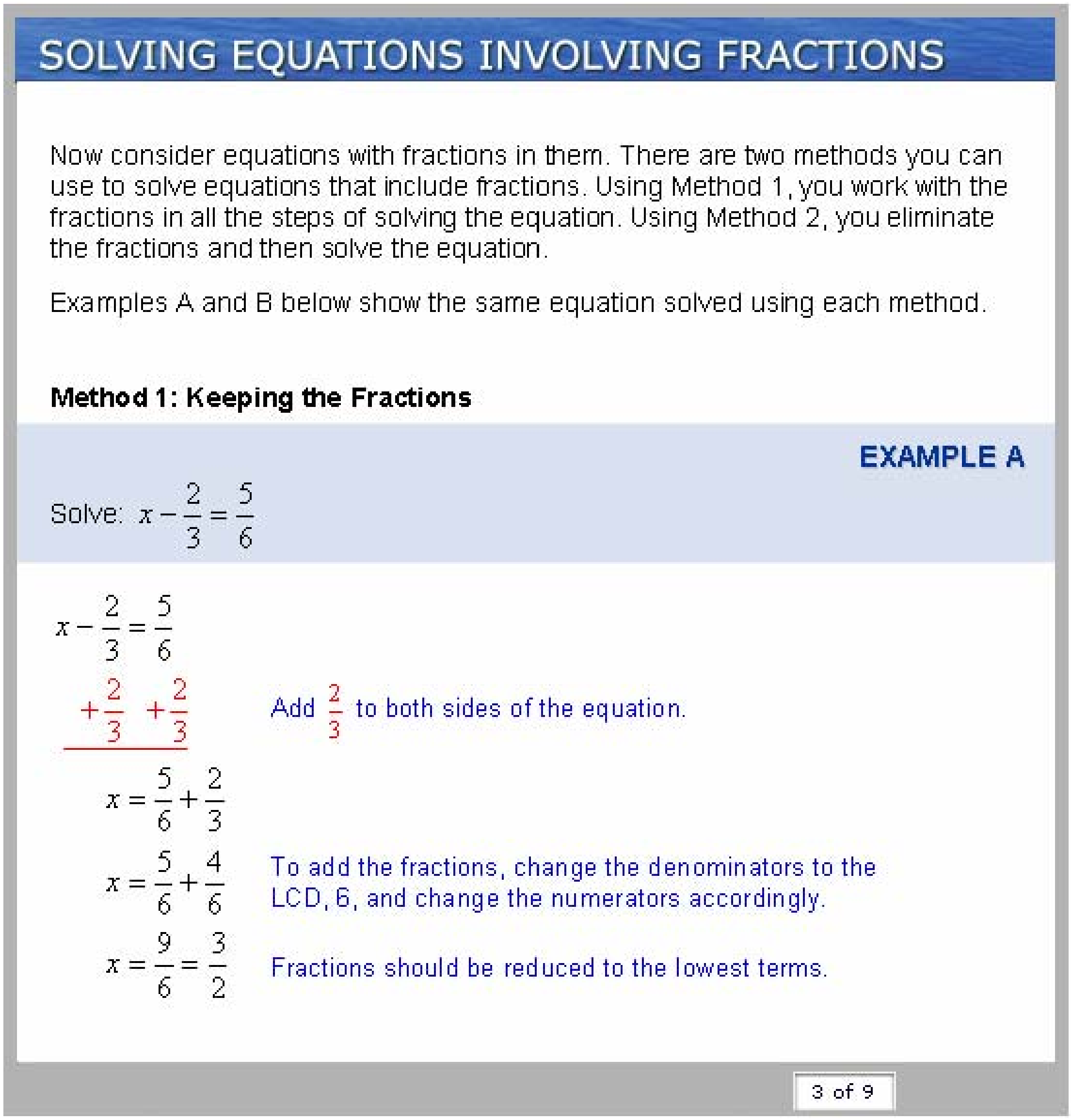
a.
$$\begin{aligned} 5x+3&=9\\ \underline{\quad -3\;}&\underline{\;\;-3\quad}\\ 5x\quad&=6\\ \frac{5x}{5}&=\frac{6}{5}\\ x&=\frac{6}{5} \end{aligned}$$

b.
$$\begin{aligned} 3y-8&=-13\\ \underline{\quad +8\;}&\underline{\;\;+8\quad}\\ 3y\quad&=-5\\ \frac{3y}{3}&=\frac{-5}{3}\\ y&=\frac{-5}{3} \end{aligned}$$

c.
$$\begin{aligned} 7-18x&=-4x+6\\ \underline{\quad +4x\;}&\underline{\;\;+4x\quad}\\ 7-14x&=6\\ \underline{-7\qquad\;}&\underline{\;\;-7\;}\\ -14x&=-1\\ \frac{-14x}{-14}&=\frac{-1}{-14}\\ x&=\frac{1}{14} \end{aligned}$$

d.
$$\begin{aligned} -6(2x-3)&=4(x-5)\\ -12x+18&=4x-20\\ \underline{-4x\qquad\;}&\underline{\;\;-4x\qquad}\\ -16x+18&=-20\\ \underline{\quad -18\;}&\underline{\;\;-18\;}\\ -16x&=-38\\ \frac{-16x}{-16}&=\frac{-38}{-16}\\ x&=\frac{38}{16}=\frac{19}{8} \end{aligned}$$

Note: Several answers are shown as reduced improper fractions. A fractional answer should always be reduced to lowest terms, but it is not necessary to write improper fractions as mixed numbers unless otherwise specified.



SOLVING EQUATIONS INVOLVING FRACTIONS

Now consider equations with fractions in them. There are two methods you can use to solve equations that include fractions. Using Method 1, you work with the fractions in all the steps of solving the equation. Using Method 2, you eliminate the fractions and then solve the equation.

Examples A and B below show the same equation solved using each method.

Method 1: Keeping the Fractions

EXAMPLE A

Solve: $x - \frac{2}{3} = \frac{5}{6}$

$$x - \frac{2}{3} = \frac{5}{6}$$

$$\underline{+\frac{2}{3} \quad +\frac{2}{3}}$$

Add $\frac{2}{3}$ to both sides of the equation.

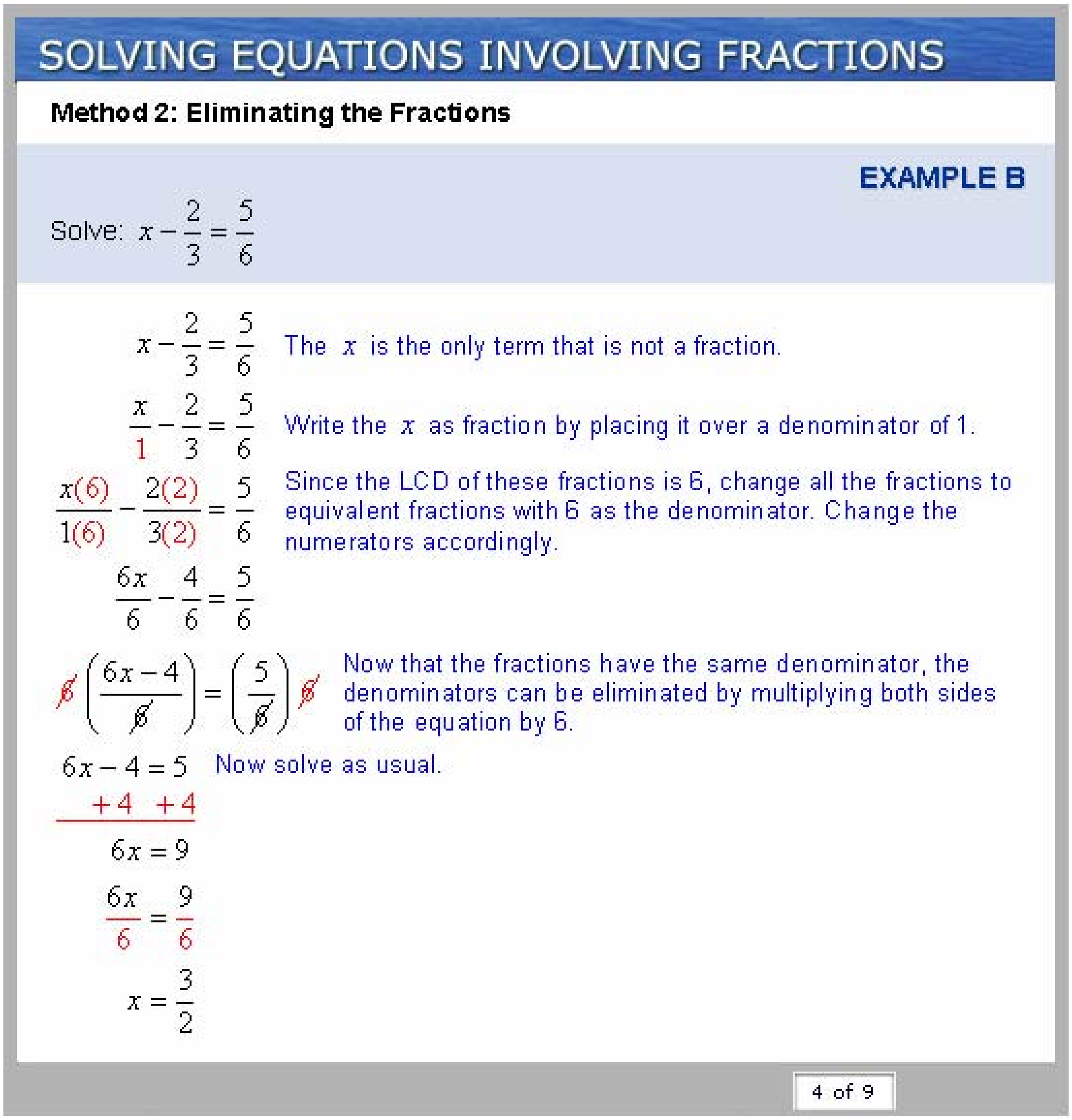
$$x = \frac{5}{6} + \frac{2}{3}$$

$$x = \frac{5}{6} + \frac{4}{6}$$

To add the fractions, change the denominators to the LCD, 6, and change the numerators accordingly.

$$x = \frac{9}{6} = \frac{3}{2}$$

Fractions should be reduced to the lowest terms.



SOLVING EQUATIONS INVOLVING FRACTIONS

Method 2: Eliminating the Fractions

EXAMPLE B

Solve: $x-\frac{2}{3}=\frac{5}{6}$

$$x-\frac{2}{3}=\frac{5}{6}$$

The x is the only term that is not a fraction.

$$\frac{x}{1}-\frac{2}{3}=\frac{5}{6}$$

Write the x as fraction by placing it over a denominator of 1.

$$\frac{x(6)}{1(6)}-\frac{2(2)}{3(2)}=\frac{5}{6}$$

Since the LCD of these fractions is 6, change all the fractions to equivalent fractions with 6 as the denominator. Change the numerators accordingly.

$$\frac{6x}{6}-\frac{4}{6}=\frac{5}{6}$$

$$\cancel{6}\left(\frac{6x-4}{\cancel{6}}\right)=\left(\frac{5}{\cancel{6}}\right)\cancel{6}$$

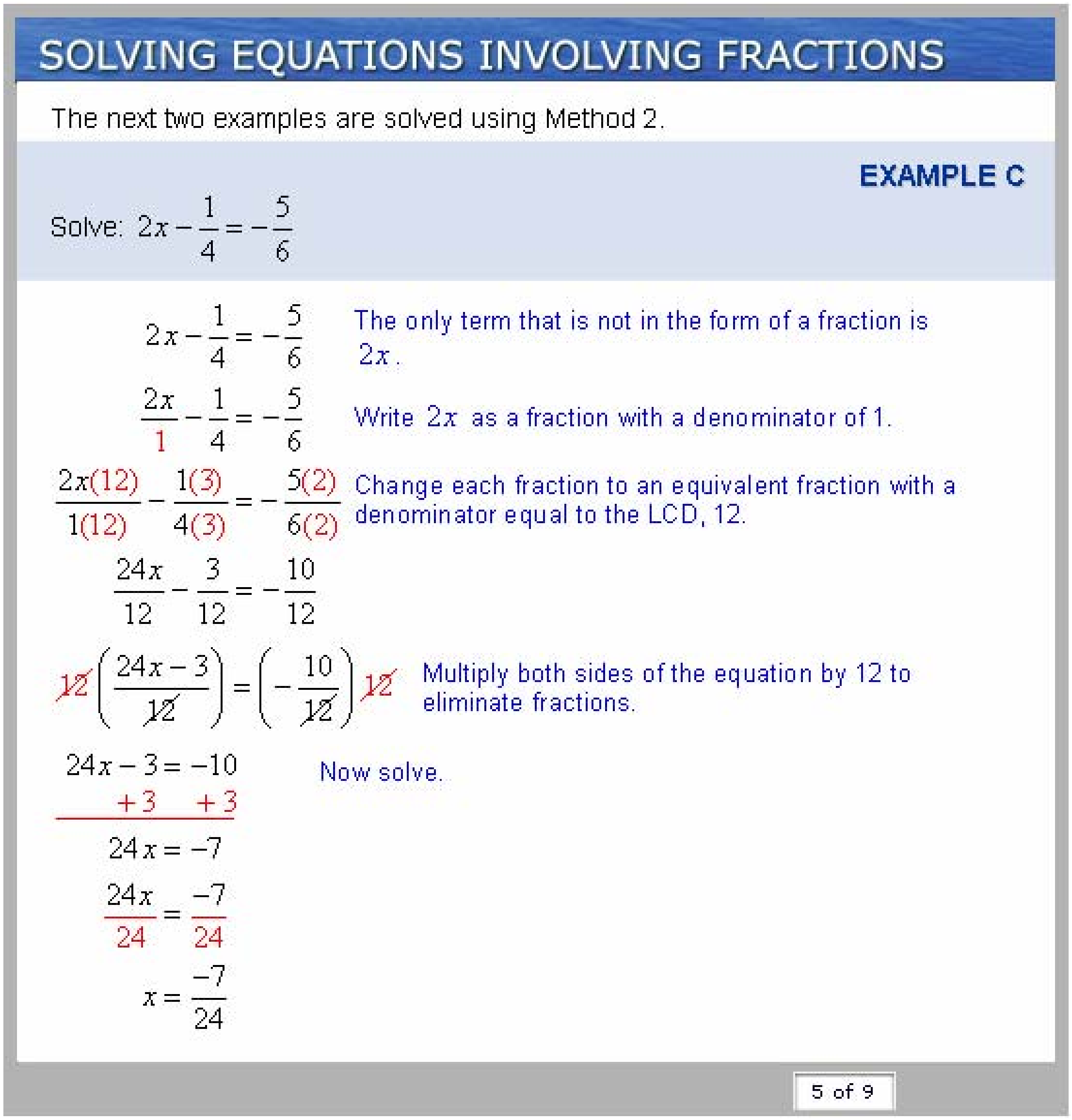
Now that the fractions have the same denominator, the denominators can be eliminated by multiplying both sides of the equation by 6.

$$\begin{array}{r} 6x-4=5 \\ \underline{+4\quad +4} \\ 6x=9 \end{array}$$

Now solve as usual.

$$\frac{6x}{6}=\frac{9}{6}$$

$$x=\frac{3}{2}$$



SOLVING EQUATIONS INVOLVING FRACTIONS

The next two examples are solved using Method 2.

EXAMPLE C

Solve: $2x - \frac{1}{4} = -\frac{5}{6}$

$$2x - \frac{1}{4} = -\frac{5}{6}$$

The only term that is not in the form of a fraction is $2x$.

$$\frac{2x}{1} - \frac{1}{4} = -\frac{5}{6}$$

Write $2x$ as a fraction with a denominator of 1.

$$\frac{2x(12)}{1(12)} - \frac{1(3)}{4(3)} = -\frac{5(2)}{6(2)}$$

Change each fraction to an equivalent fraction with a denominator equal to the LCD, 12.

$$\frac{24x}{12} - \frac{3}{12} = -\frac{10}{12}$$

$$\cancel{12}\left(\frac{24x-3}{\cancel{12}}\right) = \left(-\frac{10}{\cancel{12}}\right)\cancel{12}$$

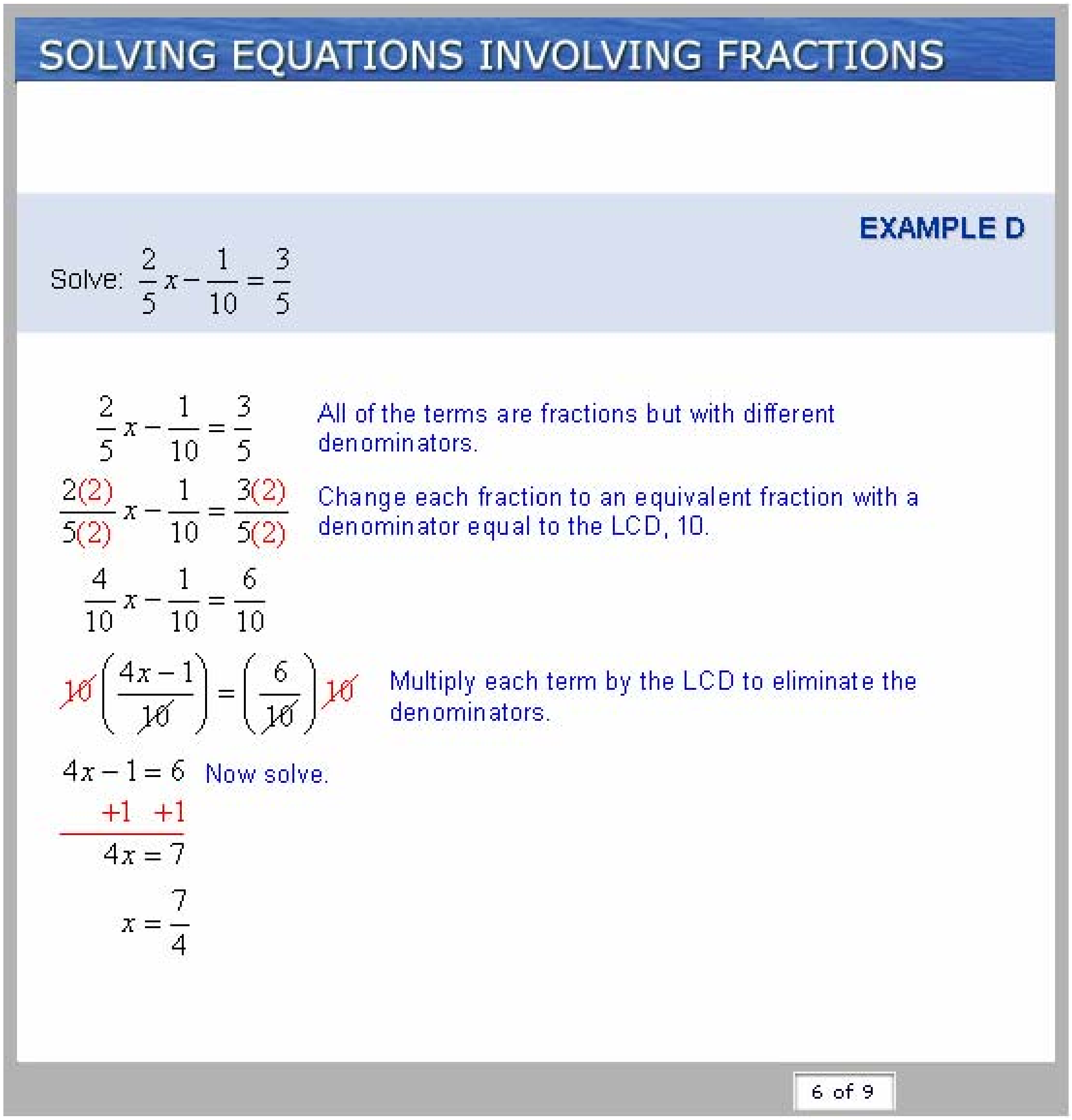
Multiply both sides of the equation by 12 to eliminate fractions.

$$\begin{aligned} 24x - 3 &= -10 \\ +3 \quad &\quad +3 \\ \hline 24x &= -7 \end{aligned}$$

Now solve.

$$\frac{24x}{24} = \frac{-7}{24}$$

$$x = \frac{-7}{24}$$



SOLVING EQUATIONS INVOLVING FRACTIONS

EXAMPLE D

Solve: $\frac{2}{5}x-\frac{1}{10}=\frac{3}{5}$

$$\frac{2}{5}x-\frac{1}{10}=\frac{3}{5}$$

All of the terms are fractions but with different denominators.

$$\frac{2(2)}{5(2)}x-\frac{1}{10}=\frac{3(2)}{5(2)}$$

Change each fraction to an equivalent fraction with a denominator equal to the LCD, 10.

$$\frac{4}{10}x-\frac{1}{10}=\frac{6}{10}$$

$$\cancel{10}\left(\frac{4x-1}{\cancel{10}}\right)=\left(\frac{6}{\cancel{10}}\right)\cancel{10}$$

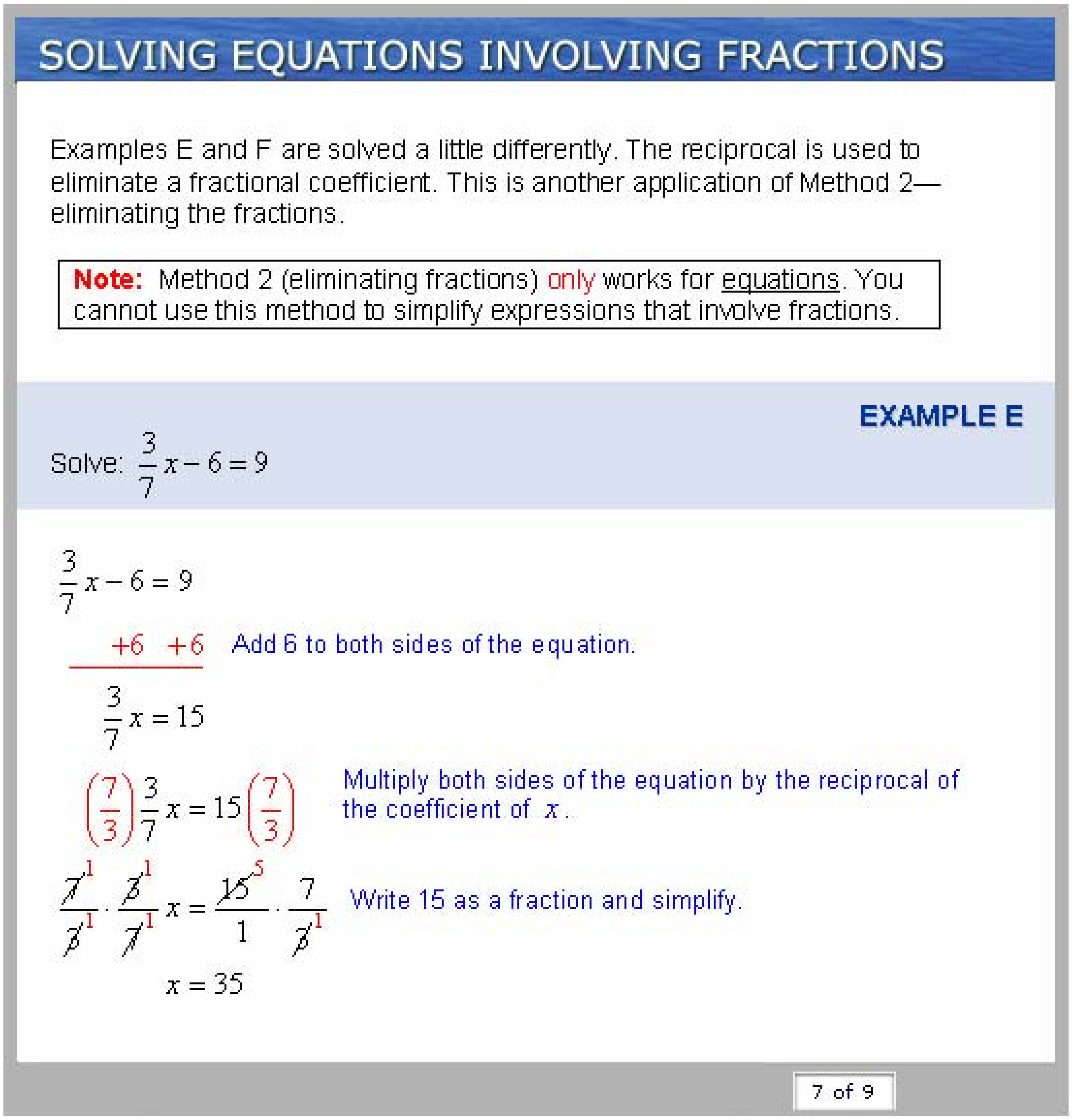
Multiply each term by the LCD to eliminate the denominators.

$$4x-1=6$$

Now solve.

$$\begin{array}{r} 4x-1=6 \\ +1\;\;+1 \\ \hline 4x=7 \end{array}$$

$$x=\frac{7}{4}$$



SOLVING EQUATIONS INVOLVING FRACTIONS

Examples E and F are solved a little differently. The reciprocal is used to eliminate a fractional coefficient. This is another application of Method 2—eliminating the fractions.

Note: Method 2 (eliminating fractions) only works for equations. You cannot use this method to simplify expressions that involve fractions.

EXAMPLE E

Solve: $\frac{3}{7}x-6=9$

$$\frac{3}{7}x-6=9$$

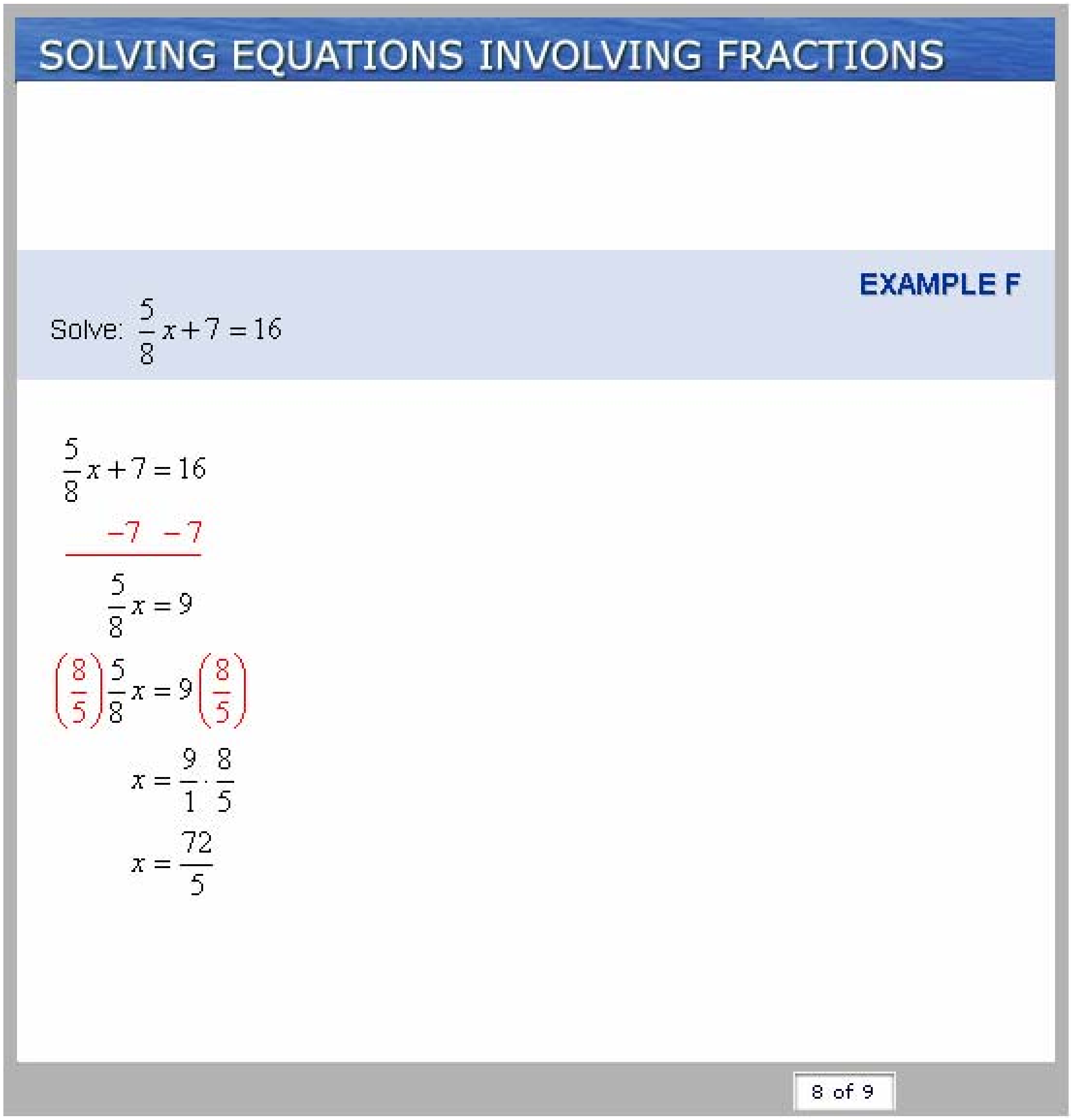
$$\underline{\quad +6 \quad +6}$$ Add 6 to both sides of the equation.

$$\frac{3}{7}x=15$$

$$\left(\frac{7}{3}\right)\frac{3}{7}x=15\left(\frac{7}{3}\right)$$ Multiply both sides of the equation by the reciprocal of the coefficient of x.

$$\frac{\cancel{7}^{1}}{\cancel{3}^{1}}\cdot\frac{\cancel{3}^{1}}{\cancel{7}^{1}}x=\frac{\cancel{15}^{5}}{1}\cdot\frac{7}{\cancel{3}^{1}}$$ Write 15 as a fraction and simplify.

$$x=35$$



SOLVING EQUATIONS INVOLVING FRACTIONS

EXAMPLE F

Solve: $\frac{5}{8}x+7=16$

$$\frac{5}{8}x+7=16$$

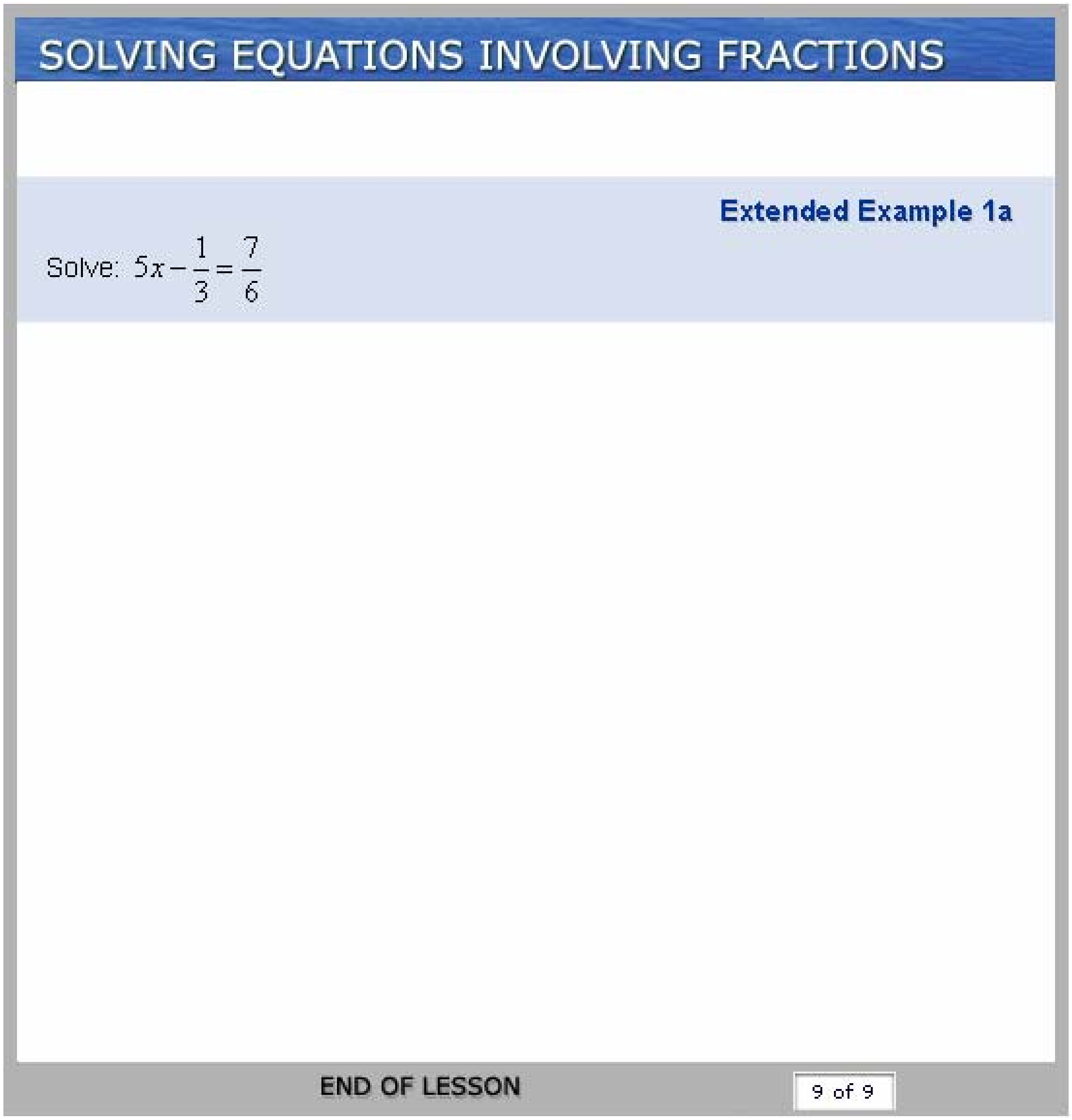
$$\underline{\quad -7 \quad -7}$$

$$\frac{5}{8}x=9$$

$$\left(\frac{8}{5}\right)\frac{5}{8}x=9\left(\frac{8}{5}\right)$$

$$x=\frac{9}{1}\cdot\frac{8}{5}$$

$$x=\frac{72}{5}$$



SOLVING EQUATIONS INVOLVING FRACTIONS

Extended Example 1a

Solve: $5x-\frac{1}{3}=\frac{7}{6}$

Solve.

$$8x - 3x = 12x - 17$$

Solve.

$$\frac{3g}{14} = -4$$

Solve.

$$-\frac{2}{9}y - 1 = 7$$

Solve.

$$-\frac{3}{7}x+\frac{2}{14}=\frac{1}{2}$$

Solve.

$$\frac{x}{5}+x=-\frac{24}{5}$$