TRANSLATING WORDS INTO ALGEBRA

Introduction

To solve "word" or "story" problems, it is necessary to translate the words in the problem into numbers and mathematical symbols in order to write an equation or expression. That equation or expression can then be solved or evaluated in order to answer the question posed in the word problem. In this lesson, we define specific words in terms of mathematical symbols and operations, and we practice translating words into mathematical expressions using these definitions.

TRANSLATING WORDS INTO ALGEBRA

Review the definitions in the table below.

The answer to an addition problem is called the **sum**.
The answer to a subtraction problem is called the **difference**.
The answer to a multiplication problem is called the **product**.
The answer to a division problem is called the **quotient**.

Besides the list above, there are other words or short statements that suggest which basic operation needs to be used to solve a problem. The following table lists some common words or phrases that indicate each operation.

Addition	Subtraction	Multiplication	Division
sum	difference	product	quotient
increased by	decreased by	times	divided by
added	less than	twice	
more than	subtracted from	triple	
added to		of	
total			

TRANSLATING WORDS INTO ALGEBRA

The missing item in a word problem (such as a number, an amount of money, a length, etc.) is called the **unknown**. The unknown in a mathematical expression or an equation is represented by a **variable** (for example, x, n, t). When translating words into algebra, you first have to recognize the unknown and assign it a variable. Then set up an expression or an equation using the definitions from the tables above to determine the appropriate operation. Carefully study the examples below and on the next screen.

Phrase	Expression	Notes
a. $\underbrace{\text{Five}}_{5}\ \underbrace{\text{more than}}_{+}\ \underbrace{\text{a number}}_{n}$	$5+n$ or $n+5$	Addition is commutative; the order of the terms makes no difference.
b. $\underbrace{\text{Seven}}_{7}\ \underbrace{\text{times}}_{\cdot}\ \underbrace{\text{a number}}_{x}$	$7x$	
c. $\underbrace{\text{A number}}_{n}\ \underbrace{\text{decreased by}}_{-}\ \underbrace{\text{six}}_{6}$	$n-6$ <u>not</u> $6-n$	Subtraction is <u>not</u> commutative; you need to be careful about the order of the terms.
d. $\underbrace{\text{The difference}}_{-}\ \text{of}\ \underbrace{\text{a number}}_{x}\ \text{and}\ \underbrace{2}_{2}$	$x-2$ <u>not</u> $2-x$	Subtraction is <u>not</u> commutative; you need to be careful about the order of the terms.
e. $\underbrace{\text{Triple}}_{3\cdot}\ \underbrace{\text{a number}}_{n}$	$3n$	

TRANSLATING WORDS INTO ALGEBRA

Phrase	Expression	Notes
f. $\underbrace{10}_{10}\ \underbrace{\text{decreased by}}_{-}\ \underbrace{\text{a number}}_{x}$	$10-x$ not $x-10$	Subtraction is not commutative; you need to be careful about the order of the terms.
g. $\underbrace{15}_{15}\ \underbrace{\text{less than}}_{-}\ \underbrace{\text{twice}}_{2\cdot}\ \underbrace{\text{a number}}_{n}$	$2n-15$	With "less than," the second term must be written first in the expression.
h. $\underbrace{\text{The quotient}}_{\div}\ \text{of}\ \underbrace{\text{a number}}_{n}\ \text{and}\ \underbrace{5}_{5}$	$n \div 5$ or $\frac{n}{5}$ not $5 \div n$ not $\frac{5}{n}$	Division is not commutative; you need to be careful about the order of the terms.
i. $\underbrace{\text{The product}}_{\cdot}\ \text{of}\ \underbrace{\text{two numbers}}_{x \text{ and } y}$	xy	In this case you need two variables.
j. $\underbrace{\text{Twice}}_{2\cdot}\ \underbrace{\text{the total}}_{+}\ \text{of}\ \underbrace{x}_{x}\ \text{and}\ \underbrace{2y}_{2y}$	$2(x+2y)$	Note that the x and $2y$ are inside parentheses because we need to multiply by a sum: the total of $x+2y$. The order of operations says that we simplify within grouping symbols first, so the parentheses give us the sum that we multiply by 2.

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TRANSLATING WORDS INTO ALGEBRA

Translation Practice

On your own, determine the mathematical expression for each phrase below. Remember that to write equivalent mathematical expressions for the following phrases, you have to first recognize the part that is unknown and assign it a variable. Then, use the vocabulary learned above to write the mathematical expression that represents the words. After you are finished, click the question marks to check your answers.

Phrase	Mathematical Expression
a. Twice the price of a desk	
b. Triple the sum of a number and 3	
c. Six less than the product of a number and 4	
d. The lottery money was divided by 5 people	
e. The difference between Jim's age and 5 years old	

TRANSLATING WORDS INTO ALGEBRA

Extended Example 1a

Write an expression for "five less than twice a number."

END OF LESSON

Write an equivalent mathematical expression for the statement.

A number and 4 decreased by 8.

Write an equivalent mathematical expression for the statement.

The difference of a number and 23.

Write an equivalent mathematical expression for the statement.

Jolanta's new job pays her $15,000 more a year.

Write an equivalent mathematical expression for the statement.

The quotient of 12 and a number increased by 8.

Write an equivalent mathematical expression for the statement.

The difference of a number and the sum of 15 and 5.

WORD PROBLEMS WITH ONE UNKNOWN

Introduction

In the previous section you learned how to translate verbal statements into equivalent mathematical expressions. In this section, you will apply those skills to translating word problems into equations that can then be solved.

WORD PROBLEMS WITH ONE UNKNOWN

Equations have equal signs. Below are some of the words and phrases that indicate that an equal sign should be used.

Results in	**Equal to**	**Is**	**The result is**	**Is the same as**

The following word problems have only one unknown. To solve them, start by assigning a variable to the unknown. Then set up an equation and solve it. The numerical value that you find for the variable is the answer to the problem.

EXAMPLE A

The sum of a number and 5 is 12. Find the number.

A number = x

The $\underbrace{\text{sum of a number and 5}}_{x+5}$ $\underbrace{\text{is}}_{=}$ $\underbrace{12}_{12}$.

$x + 5 = 12$

$\underline{\;-5 \quad -5\;}$

$x = 7$ So, the number is 7.

WORD PROBLEMS WITH ONE UNKNOWN

EXAMPLE B

The sum of twice a number and 6 is equal to 22. What is the number?

The number = x .

$$\text{The sum of } \underbrace{\text{twice a number}}_{2x} \ \underbrace{\text{and}}_{+} \ \underbrace{6}_{6} \ \underbrace{\text{is equal to}}_{=} \ \underbrace{22}_{22}.$$

$$\begin{aligned} 2x + 6 &= 22 \\ -6 \quad & \quad -6 \\ \hline 2x &= 16 \\ x &= 8 \end{aligned}$$

So, the number is 8.

EXAMPLE C

Sherry paid $78 for a pair of shoes after receiving a $14 discount. What was the original price of the shoes?

Price of the shoes = p .

$$\begin{aligned} p - 14 &= 78 \\ +14 \quad & \quad +14 \\ \hline p &= 92 \end{aligned}$$

The original price of the shoes was $92.

WORD PROBLEMS WITH ONE UNKNOWN

EXAMPLE D

Each month, Amy spends $450 on food and other expenses, which is $220 less than what she pays in rent. How much does she pay in rent?

Amount of rent = r .

$\underbrace{\text{\$450 for food and other}}_{450}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{\$220 less than rent}}_{r-220}.$

$$r - 220 = 450$$

$$\underline{\quad +220 \qquad +220}$$

$$r = 670$$

The rent is $670.

WORD PROBLEMS WITH ONE UNKNOWN

Extended Example 1a

The quotient of number and 9 is -5. Find the number.

END OF LESSON

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Write an equation for the problem and solve for the unknown.

A number increased by –16 is 9.

Write an equation for the problem and solve for the unknown.

Twice a number and 5 is equal to 33.

Write an equation for the problem and solve for the unknown.

Three times the sum of twice a number and 4 is 36.

Write an equation for the problem and solve for the unknown.

A 38 foot log is cut into two pieces. If one piece is 29 feet long, what is the length of the other piece?

Write an equation for the problem and solve for the unknown.

Mary spent $78 on two books. She received a $22 discount for buying two books. How much was the price of each book originally?

WORD PROBLEMS WITH TWO UNKNOWNS

Introduction

In the previous section, you learned how to write equations for word problems with one unknown. In this section, you will learn how to write expressions and equations for word problems with two unknowns.

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WORD PROBLEMS WITH TWO UNKNOWNS

When two pieces of information are missing in a word problem, first assign a variable to one of the unknowns. Then, write an expression for the second unknown in terms of the first unknown that you defined. Translate the information in the problem into an equation. The solution to the equation is the value of the first unknown. Substitute this value into the expression for the second unknown.

How to Solve a Word Problem with Two Unknowns

1. Read the problem carefully (reread it if you have to).
2. Identify the two unknowns.
3. Assign one of them a variable such as x.
4. Write an expression for the second unknown in terms of x based on the description provided in the problem.
5. Write an equation based on the phrase or sentence provided in the problem.
6. Solve the equation for x; this is the value of the first unknown.
7. Substitute the answer to x (from step 6) into the expression you wrote in step 4 to find the value of the second unknown.

This might seem complicated at first, but it will make more sense as you study the examples in this lesson.

WORD PROBLEMS WITH TWO UNKNOWNS

Study the following examples carefully.

EXAMPLE A

One number is twice the other number. Their sum is 21. What are the two numbers?

Translate "one number":	First number = x
Translate "twice the other number":	Second number = $2x$
Translate "Their sum is 21":	$x+2x=21$
Combine like terms and solve the equation for x:	$3x=21$ $x=7$
First number = x:	First number = 7
Second number = $2x$:	Second number = $2(7)=14$
Check:	$7+14=21$ ✓

EXAMPLE B

The distance from Las Vegas to San Francisco is 30 miles more than twice the distance from Las Vegas to Los Angeles. If the total of the distances is 855 miles, find the distance from Las Vegas to Los Angeles and the distance from Las Vegas to San Francisco.

WORD PROBLEMS WITH TWO UNKNOWNS

EXAMPLE C

James paid \$325 for 3 books and a briefcase. Each book was the same price and the briefcase was \$130 more than the total price of the three books. Find the price of each book and the price of the briefcase.

Translate "price of each book":	Price of each book = x
Translate "The briefcase was \$130 more than the total price of the three books":	Price of all 3 books = $3x$ Price of the briefcase = $3x+130$
Translate "James paid \$325 for 3 books and a briefcase": Combine like terms:	$\underbrace{\text{3 books}}_{3x}\ \underbrace{\text{and}}_{+}\ \underbrace{\text{1 briefcase}}_{3x+130} = 325$ $3x + 3x+130 = 325$ $6x + 130 = 325$
Solve the equation for x:	$6x+130=325$ $6x = 325-130$ $6x=195$ $x=32.5$
Price of one book = x:	Price of one book = \$32.50
Price of the briefcase = $3x+130$:	Price of the briefcase = $3(32.5)+130 = 97.5+130$ $= 227.5$

Each book cost \$32.50 and the briefcase cost \$227.50.

WORD PROBLEMS WITH TWO UNKNOWNS

EXAMPLE D

Twice the sum of Mary's age and her mom's age is 160 years. If her mom's age is 4 years less than twice Mary's age, how old is each woman?

The mom's age is given in terms of Mary's age, so let x represent Mary's age.

Mary's age = x

Translate "mom's age is 4 years less than twice Mary's age":

Mom's age = $2x-4$

Translate "Twice the sum of Mary's age and her mom's age is 160 years":

$$\underbrace{\text{Twice the}}_{2}\ \text{sum of}\ \underbrace{\underbrace{\text{Mary's age}}_{x}\ \text{and her}\ \underbrace{\text{mom's age}}_{2x-4}}_{(x+2x-4)}\ \underbrace{\text{is 160}}_{=160}.$$

$$2(x+2x-4)=160$$

Combine like terms and solve the equation for x:

$$2(x+2x-4)=160$$
$$2(3x-4)=160$$
$$6x-8=160$$
$$6x=168$$
$$x=28$$

Mary's age = x: Mary's age = 28

Mom's age = $2x-4$: Mom's age = $2(28)-4=56-4=52$

Mary is 28 and her mother is 52 years old.

WORD PROBLEMS WITH TWO UNKNOWNS

Extended Example 1a

Ali paid $125 for a sweater and a blouse. The price of the sweater was $25 less than twice the price of the blouse. How much did Ali paid for each item?

END OF LESSON

Write an equation for the problem and solve for the two unknowns.

A number is 9 less than three times another number. Their sum is 55.

Write an equation for the problem and solve for the two unknowns.

One number is twice the sum of another number and 12. The sum of the two numbers is 3.

Solve each word problem.

The price of an mp3 player is $120 more than three memory sticks. Sherry paid $864 for one mp3 player and three memory sticks. What was the cost of each?

Solve each word problem.

The number of fiction books Lacey owns is seven less than four times the number of non-fiction books. She has a total of 83 books. How many of each type of book does she have?

Solve each word problem.

The length of a swimming pool is 4 feet more than twice its width. If the sum of the length and width is 34 feet, find the length and width of the pool.

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

Introduction

Earlier, you learned to solve equations that contain a variable term on one side. This lesson introduces solving equations that have variable terms on both sides. You will put what you've already learned to work, as well as learn a few new tricks, in order to solve such equations.

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

Recall the concept of opposite numbers: The sum of a number and its opposite is always equal to zero ($-3+3=0$). The same concept can be extended to variable terms: the sum of a term and its opposite is always equal to zero. For example:

Term	Opposite of the Term	Property of Opposite Numbers
x	$-x$	$x+(-x)=0$
$-4y$	$4y$	$-4y+4y=0$
$\frac{1}{3}x^2$	$-\frac{1}{3}x^2$	$\frac{1}{3}x^2+\left(-\frac{1}{3}x^2\right)=0$

To solve equations of the type $ax+b=cx+d$, with variable terms on both sides, follow these steps:

1. Bring all variable terms to one side of the equation. First decide which variable term you want to move, and then add the opposite of that term to both sides of the equation. (Remember that the sum of opposite terms is zero).

2. Bring all non-variable terms (constants) to the other side of the equation. To move a constant term to the other side of the equation, add its opposite to both sides of the equation.

3. Completely isolate the variable. Finish solving the equation by performing the operations necessary to isolate the variable.

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

EXAMPLE A

Solve: $2x - 5 = x + 12$

$$\begin{array}{r} 2x - 5 = x + 12 \\ -x \qquad -x \\ \hline x - 5 = 12 \\ +5 \quad +5 \\ \hline x = 17 \end{array}$$

Choose which variable term to move: x. To eliminate x from the right side of the equation, add $-x$ to both sides.

To eliminate the –5 from the left side of the equation, add 5 to both sides.

Note: You can choose to move either term; we used x to end up with a positive x-value on the left side.

EXAMPLE B

Solve: $7y + 11 = 4y - 34$

$$\begin{array}{r} 7y + 11 = 4y - 34 \\ -4y \qquad -4y \\ \hline 3y + 11 = 0 - 34 \\ 3y + 11 = -34 \\ -11 \quad -11 \\ \hline 3y + 0 = -45 \\ \dfrac{3y}{3} = \dfrac{-45}{3} \\ y = -15 \end{array}$$

Choose which variable term to move: $4y$. Add $-4y$ to both sides.

To move 11 to right side of the equation, add –11 to both sides.

To completely isolate y, divide both sides by the coefficient of y, which is 3.

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SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

EXAMPLE C

Solve: $-3m - 24 = -m + 18$

$$-3m - 24 = -m + 18$$

$$+m \qquad\qquad +m$$

$$-2m - 24 = 0 + 18$$

$$-2m - 24 = 18$$

$$+24 \quad +24$$

$$-2m + 0 = 42$$

$$\frac{-2m}{-2} = \frac{42}{-2}$$

$$m = -21$$

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

To solve some equations, you may have to first simplify each side of the equation by combining like terms.

EXAMPLE D

Solve: $3x + 12 - 6x = -5x - 4$

$3x + 12 - 6x = -5x - 4$ — First combine like terms on the left.

$-3x + 12 = -5x - 4$ — Choose which variable term to move: $-3x$.

$+3x \qquad +3x$ — Add $3x$ to both sides.

$0 + 12 = -2x - 4$

$12 = -2x - 4$

$-4 \qquad -4$ — Move 4 to the left to isolate the variable term by adding –4 to both sides.

$8 = -2x$

$\frac{8}{-2} = \frac{-2x}{-2}$ — To completely isolate x, divide both sides by –2.

$-4 = x$

EXAMPLE E

Solve: $7t + 13 - t + 16 = 11t + 9 + 3t + 12$

SOLVING EQUATIONS WITH VARIABLES ON BOTH SIDES

Extended Example 1a

Solve: $6x - 8 = x + 12$

END OF LESSON

Solve.

$-7m - 16 = -3m - 24$

Solve.

$x + 6 = -6 - 4x - 8$

Solve.

$a + 6 - 17 = -5a - 10 - 19$

Solve.

$2x - 80 + 17x + 9 = -21 + x + 5x + 2$

Solve.

$r - 25 - 32 + 9 = 18r - 6 - 19r + 5r$

SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

Introduction

In this section, you will learn how to solve equations that first have to be simplified using the distributive property to remove parentheses. After simplifying, apply the techniques for solving equations that you learned earlier.

SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

EXAMPLE A

Solve: $8(2y + 5) = -24$

$$8(2y+5) = -24$$
$$16y + 40 = -24$$
$$-40 \quad -40$$
$$16y = -64$$
$$\frac{16y}{16} = \frac{-64}{16}$$
$$y = -4$$

EXAMPLE B

Solve: $-3(m - 6) = 60$

$$-3(m-6) = 60$$
$$-3m + 18 = 60$$
$$-18 \quad -18$$
$$-3m = 42$$
$$\frac{-3m}{-3} = \frac{42}{-3}$$
$$m = -14$$

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SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

EXAMPLE C

Solve: $-3(-m+9)=72$

$$-3(-m+9)=72$$
$$3m-27=72$$
$$\underline{\quad +27 \quad +27}$$
$$3m=99$$
$$\frac{\cancel{3}m}{\cancel{3}}=\frac{99}{3}$$
$$m=33$$

EXAMPLE D

Solve: $2(7n-8)=5n-52$

$$2(7n-8)=5n-52$$
$$14n-16=5n-52$$
$$\underline{-5n \quad\quad -5n}$$
$$9n-16=-52$$
$$\underline{\quad +16 \quad +16}$$
$$9n=-36$$
$$\frac{\cancel{9}n}{\cancel{9}}=\frac{-36}{9}$$
$$n=-4$$

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SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

EXAMPLE E

Solve: $-(x+5) = 6(-2x+1)$

$$-(x+5) = 6(-2x+1)$$
$$-x-5 = -12x+6$$
$$\underline{+12x \qquad +12x}$$
$$11x-5 = 0+6$$
$$11x-5 = 6$$
$$\underline{\quad +5 \quad +5}$$
$$11x = 11$$
$$\frac{\cancel{11}x}{\cancel{11}} = \frac{11}{11}$$
$$x = 1$$

SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

Extended Example 1a

Solve: $-2(3x-7)=20$

END OF LESSON

Solve.

$-(10m - 7) = -33$

Solve.

$4n = 5(n - 16) + 3n$

Solve.

$4(f + 3) - 5(2f - 7) = 29$

Solve.

$$3(2x - 5) - (4x + 6) = -71$$

Solve.

$$3(5 - 6y) - 2(2y + 3) = -7(y + 18)$$