INTEGERS, ABSOLUTE VALUE, AND INTEGER COMPARISON

Introduction

The set of real numbers called **integers** is discussed in this lesson, as are the related concepts of **absolute value** and **integer comparison**. You will see how to simplify expressions involving absolute value, and how to compare integers.

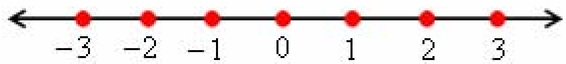
INTEGERS, ABSOLUTE VALUE, AND INTEGER COMPARISON

Integers

The set of **integers** is the entire set of whole numbers and their opposites:

$$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

The three dots at the beginning and at the end of the list of numbers indicate that the numbers continue in both directions.



On the number line, integers become greater as we move from left to right. All of the numbers to the right of zero are **positive** numbers and all of the numbers to the left of zero are **negative** numbers. The arrows at each end of the number line indicate that the numbers continue in both directions.

To better understand the concept of negative numbers, think of negative numbers as opposites of positive numbers. Some real-life concepts also have opposites. For example:

> The opposite of gaining 10 pounds is losing 10 pounds.
>
> The opposite of depositing $65 into your bank account is withdrawing $65.
>
> The opposite of being 20 minutes late is being 20 minutes early.

In algebra the **opposite** of a number is written as the negative of that number. For example, the opposite of 15 is negative 15, which is written as -15. The opposite of -7 is 7.

INTEGERS, ABSOLUTE VALUE, AND INTEGER COMPARISON

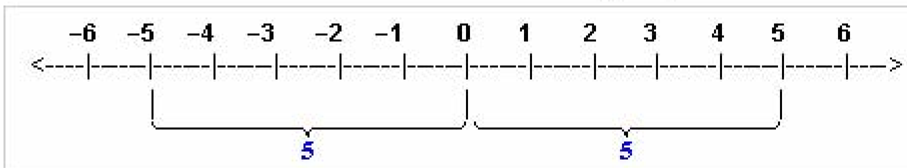
In general, $-(a) = -a$ and $-(-a) = a$.

How we write it in math	**What it means in words**
$-(18) = -18$	The opposite of 18 is negative 18.
$-(-5) = 5$	The opposite of -5 is positive 5.
$-(37) = -37$	The opposite of 37 is negative 37.
$-(-11) = 11$	The opposite of -11 is positive 11.

Removing the parentheses in the examples above simplifies the expressions.

Absolute Value

The **absolute value** of a number is its distance from zero on the number line. For example, on the number line below, notice that the distance from 5 to zero is 5 units. What is the distance from -5 to zero? Again, the answer is 5 units.

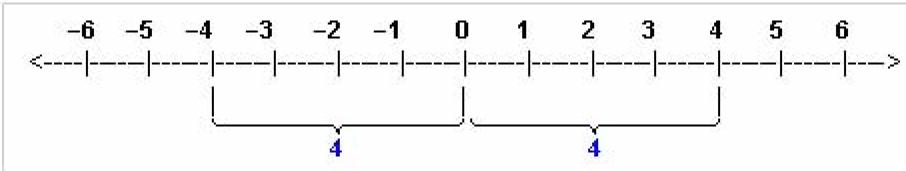


Because distance is always positive, the absolute value of a number is always positive. The absolute value of a is written as $|a|$. Since the distance from 5 or -5 to zero is five units, the absolute value of 5 or -5 is 5. So:

$$|5| = 5 \quad \text{and} \quad |-5| = 5.$$

INTEGERS, ABSOLUTE VALUE, AND INTEGER COMPARISON

Look at the number line again.



The distance from 4 to zero is the same as the distance from −4 to zero. Both of these distances are equal to 4 units. You can show this as:

$$|4| = 4 \quad \text{and} \quad |-4| = 4 .$$

To simplify absolute value expressions such as $|8-5|$, first simplify the expression inside the absolute value symbols, and then find the absolute value, as shown in the examples below.

EXAMPLE A

Simplify: $|8-5|$

$|8-5| = |3|$

$= 3$

EXAMPLE B

Simplify: $|6-16|$

$|6-16| = |-10|$

$= 10$

Extended Example 1a

Simplify: $|9-4|$

INTEGERS, ABSOLUTE VALUE, AND INTEGER COMPARISON

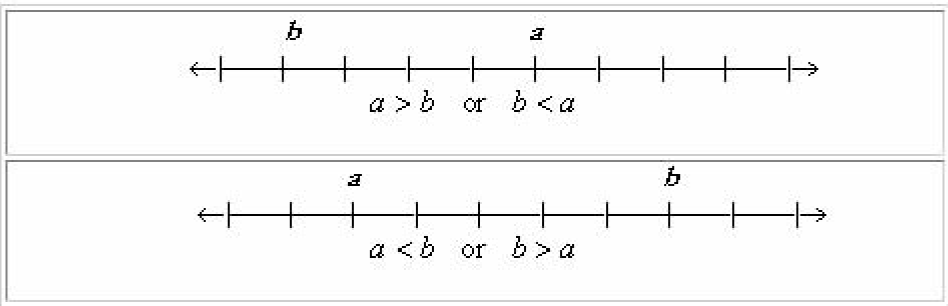
Comparing Integers

If two numbers are not equal to each other, then one number is either **less than** or **greater than** the other number. To compare unequal numbers, use the symbol < for "less than" and > for "greater than."

Larger number > Smaller number

Smaller number < Larger number

Therefore, $a > b$ means "a is greater than b," or "a is to the right of b on the number line." Also, $a < b$ means "a is less than b," or "a is to the left of b on the number line."



For example, 5 is less than 13 because 5 is to the left of 13 on the number line. You could express this as $5 < 13$. You could also say that 13 is greater than 5, or $13 > 5$. To compare –6 and 4, you can write $4 > -6$ because 4 is greater than –6. Or, since –6 is less than 4, you can write $-6 < 4$.

INTEGERS, ABSOLUTE VALUE, AND INTEGER COMPARISON

EXAMPLE C

Which number is larger: $|-5|$ or $|-14|$? Use < or > to show the comparison.

$|-5| = 5$ and $|-14| = 14$

$5 < 14$

So, $|-5|$ is less than $|-14|$ or $|-5| < |-14|$.

EXAMPLE D

Which number is larger: $|13|$ or $|-4|$? Use < or > to show the comparison.

$|13| = 13$ and $|-4| = 4$

$13 > 4$

So, $|13|$ is greater than $|-4|$ or $|13| > |-4|$.

Extended Example 2a

Use < or > to compare $|-16|$ and $|8|$.

END OF LESSON

Simplify

$$-(-13) =$$

Use > or < to compare.

$$-|3| \ \_\_ \ -(-3)$$

Simplify.

$$|8-7|+|-15|$$

Use > or < to compare.

$$|-6| \underline{\ <\ } |-37|$$

Simplify.

$$|-8| - |3|$$

ADDING INTEGERS

Introduction

In this section and the next section, you will learn how to add and subtract (or "combine") integers. You have already used the "+" symbol to represent addition and the "–" symbol to represent subtraction. Because these symbols are also used to show positive and negative numbers, adding and subtracting integers can be confusing.

Here are some real-life examples that illustrate the concepts of positive and negative numbers.

Gaining 4 pounds	+4 pounds or 4 pounds Note: It is not necessary to write the + sign for positive numbers
Losing 3 pounds	–3 pounds
Winning 33 dollars	+33 dollars or 33 dollars
Losing 12 dollars	–12 dollars
Dropping 5 degrees in temperature	–5 degrees
Withdrawing $20 from the bank	–20 dollars

ADDING INTEGERS

The following rules should make adding integers easier to understand. Each rule will be illustrated with a real-life example and a number line demonstration.

Adding Integers

When adding two integers there are three possible cases.

Case #1: Both integers are positive.
In this case, the rule is to simply add the numbers and the sum will be positive.

EXAMPLE A

Simplify: $8+11$

$8+11=19$ Both 8 and 11 are positive, so the sum is positive.

Question: Simplify: $24+13$

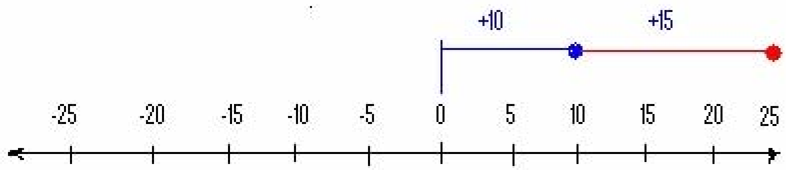
ADDING INTEGERS

EXAMPLE B

Suppose you deposit $10 in your bank account on Monday and then deposit another $15 on Friday. How much has the balance of your bank account increased due to these deposits?

Deposit of $10 on Monday: +$10
Deposit of $15 on Monday: +$15

Total increase: $10 + $15 = $25. The balance was increased by $25.



Case #2: Both integers are negative.
In this case, add the absolute values of the numbers and the sum will be negative.

EXAMPLE C

Simplify: $-12+(-18)$

$-12+(-18)=-30$ Both –12 and –18 are negative. Find the absolute value of each: |–12| = 12 and |–18| = 18. Add 12 and 18; the sum is negative.

3 of 11

ADDING INTEGERS

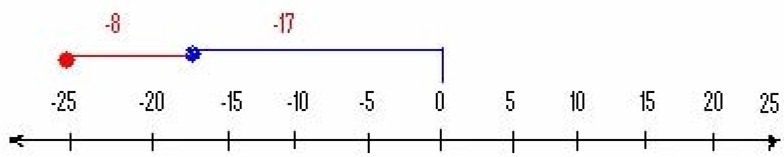
Question: Simplify: $-9+(-15)$

EXAMPLE D

Suppose you withdraw \$17 from your bank account on Tuesday and then withdraw another \$8 on Thursday. How much has the balance in your account decreased due to these withdrawals?

Withdrawal of \$17: $-\$17$
Withdrawal of \$8: $-\$8$

Total decrease: $-\$17+(-\$8)=-\$25$. The balance decreased by \$25.



You may have noticed from the examples so far that when you add a positive number, you move toward the right along the number line and when you add a negative number, you move toward the left along the number line.

ADDING INTEGERS

Case #3: One integer is positive and the other is negative.
In this case, find the difference between the absolute values of the numbers. The answer has the sign of the integer with the greater absolute value.

EXAMPLE E

Simplify: $25+(-10)$

$25+(-10)=15$

25 is positive and –10 is negative, so find the absolute value of each: $|25| = 25$ and $|-10| = 10$. Subtract 10 from 25. Since $|25| > |-10|$, the answer is positive.

EXAMPLE F

Simplify: $-4+14$

$-4+14=10$

14 is positive and 4 is negative, so find the absolute value of each: $|-4| = 4$ and $|14| = 14$. Subtract: $14 - 4=10$. Since $|14| > |-4|$, the answer is positive.

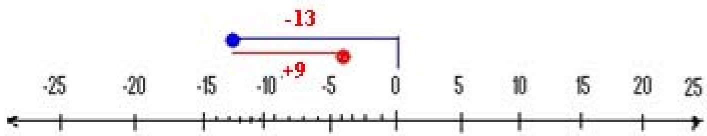
Question: Simplify: $-17+5$

ADDING INTEGERS

EXAMPLE G

Suppose you withdraw \$13 from your bank account and then deposit \$9. How much did your balance increase or decrease due to these transactions?

Withdraw \$13: $-\$13$
Deposit \$9: $+\$9$
$-\$13 + \$9 = -\$4$, so your account balance decreased by \$4.



The following table summarizes the rules for adding integers:

Rules for Adding Integers
If the signs of the numbers are the same: Find the SUM and keep the sign.
If the signs of the numbers are different: Find the DIFFERENCE of absolute values and keep the sign of the number with the greater absolute value.

The same rules apply when adding more than two numbers.

EXAMPLE H

Simplify: $-12+5+(-8)+1+(-3)$

$-12+5+(-8)+1+(-3)$
$= -12+(-8)+(-3)+5+1$
$= -23+6=-17$

Group numbers with like signs together. The sum of all the negative numbers is –23. The sum of the positive numbers is 6. Finally, find the difference of the absolute values of each number: $|-23| - |6| = 23 - 6 = 17$. Since $|-23| > |6|$, the sum is negative: –17.

ADDING INTEGERS

Extended Example 1a

Simplify: $-8+9+(-11)+4+(-5)$

ADDING INTEGERS

Combining Integers

In algebra, we call both addition and subtraction of signed numbers **combining integers**. For example, observe and compare problems **a** and **b** below. Although both problems are essentially the same, we think of problem **a** as an addition problem and problem **b** as a subtraction problem.

a. $7+(-3)$ $=4$ **b.** $7-3$ $=4$

The two signs in problem **a** have been combined into a single sign in problem **b**. Notice that both expressions equal 4; therefore, $7-3$ and $7+(-3)$ are equivalent expressions:

$$7-3 \Leftrightarrow 7+(-3) .$$

This suggests that we can actually think of the operation in problem **b** as addition, where 7 is positive and 3 is negative. Since the signs are different, we subtract the absolute values and use the sign of the number with greater absolute value. This discussion also suggests that subtracting 3 is the same as adding its opposite, –3. In general, when you subtract a number it's the same as adding it's opposite.

ADDING INTEGERS

When adding and subtracting integers, parentheses are often placed around negative numbers to keep the signs clear. From the discussion about examples **a** and **b** above, we know we can write an equivalent expression without parentheses. For example:

$$-6+(-7)=-6-7 \text{ and } 9+(-3)=9-3.$$

The addition problems were simplified to their equivalent subtraction form without parentheses by combining integers. Note that in either form you should be able to recognize the sign of each number involved.

EXAMPLE I

Simplify: $-6-7$

$-6-7=-13$ Both –6 and –7 are negative; same sign. Thinking of $-6-7$ as addition, use the rule for adding integers with the same sign: add the numbers, keep the sign.

EXAMPLE J

Simplify: $9-3$

$9-3=6$ Think of this as addition. In this case, 9 is positive and –3 is negative; different signs. We need to subtract the absolute values and use the sign of the number with greater absolute value: $|9|-|-3|=9-3=6$. Since $|9|>|-3|$, our answer is positive.

9 of 11

ADDING INTEGERS

Extended Example 2a

Simplify: $-8+9+(-11)$

Additive Inverse

Two numbers that are the same distance from zero on the number line are called opposites of each other. The sum of a number and its opposite is always equal to zero. Such numbers are called the **additive inverses** of each other.

$8+(-8)=0$	8 and –8 are additive inverses.
$12+(-12)=0$	12 and –12 are additive inverses.
$-9+9=0$	–9 and 9 are additive inverses.
$-a+a=a+(-a)=0$ **Any number and its opposite are additive inverses of each other.**	

EXAMPLE K

Simplify: $-10+7+10-7$

$-10+10+7-7$ Rearrange the numbers so that additive inverses (opposite numbers) are next to each other.

$\underbrace{-10+10}_{0}+\underbrace{7-7}_{0}$ Simplify the additive inverses.

$0 + 0 = 0$

Indicate whether the following is True *or* False.

$$5 + (-8) = 5 + 8$$

Simplify.

$$7 + (-10)$$

Simplify.

$$-78 + 39$$

Simplify.

$-51 + 22 + (-8)$

Use what you know about additive inverses to simplify.

$-49 + 49 + 5$

SUBTRACTING INTEGERS

Introduction

You have already learned how to add integers. You can use the same rules to subtract integers. Think of subtraction as adding the opposite, as shown in the example below.

$8-(7)=8+(-7)$ $8+(-7)=1$	The opposite of 7 is -7, so subtracting $8-(7)$ is the same as adding $8+(-7)$. The addends have different signs, so we find the difference of absolute values and keep the sign of the number with the greater absolute value.

SUBTRACTING INTEGERS

Subtracting Integers

Just as with addition of integers, there are three possible cases when subtracting integers:

1) Both numbers are positive.

2) Both numbers are negative.

3) One number is positive and the other is negative.

In all three cases, first remove the parentheses.

Recall that $-(a) = -a$ and $-(-a) = a$.

So, $5-(3) = 5-3$ and $5-(-3) = 5+3$.

Once you have removed the parentheses and written the expression in equivalent form, combine the integers by assuming that the operation is addition (see Chapter 2, Section 2). Then apply the appropriate rule for adding integers, which you also learned in the previous section.

SUBTRACTING INTEGERS

Case #1: Both numbers are positive.

EXAMPLE A

Simplify: 17 – 5

$$17 - 5 = 17 + (-5)$$
$$= 12$$

Rewrite the problem as addition. 17 is positive and –5 is negative, so find the absolute value of each: |17| = 17 and |–5| = 5. Subtract 5 from 17 = 12. Since |17| > |–5|, the answer is positive.

EXAMPLE B

Simplify: 20 – 27

$$20 - 27 = 20 + (-27)$$
$$= -7$$

Rewrite the problem as addition. 20 is positive and –27 is negative, so find the absolute value of each: |20| = 20 and |–27| = 27. Subtract 20 from 27 = 7. Since |–27| > |20| the answer is negative.

SUBTRACTING INTEGERS

Case #2: Both numbers are negative.

EXAMPLE C

Simplify: $-10-(-17)$

$$-10-(-17)=-10+17$$
$$=7$$

Remove ( ) to find the equivalent addition expression. Remember that $-(-a) = a$.

–10 is negative and 17 is positive; so find the absolute value of each: |–10| = 10 and |17| = 17. Subtract 10 from 17 = 7. Since |17| > |–10|, the answer is positive.

EXAMPLE D

Simplify: $-36-(-15)$

$$-36-(-15)=-36+15$$
$$=-21$$

Remove ( ) to find the equivalent addition expression. Remember that $-(-a) = a$.

15 is positive and –36 is negative, so find the absolute value of each: |–36| = 36 and |15| = 15. Subtract 15 from 36 = 21. Since |–36| > |15|, the answer is negative.

4 of 11

SUBTRACTING INTEGERS

Case #3: One number is positive and the other negative

EXAMPLE E

Simplify: $-10-(17)$

$$-10-(17) = -10+(-17)$$
$$= -27$$

Find the equivalent addition expression. Since both –10 and –17 are negative, add the absolute values and keep the negative sign.

EXAMPLE F

Simplify: $10-(-17)$

$$10-(-17) = 10+17$$
$$= 27$$

Remove ( ) to find the equivalent addition expression. Since both 10 and 17 are positive, add the numbers and keep the positive sign.

SUBTRACTING INTEGERS

The next two boxes summarize the rules described in this section.

Rules for Subtracting Integers
Rewrite the problem as an equivalent addition problem (add the opposite).
When numbers have the same sign: Find the SUM and keep the sign.
When numbers have different signs: Find the DIFFERENCE and keep the sign of the number with the greater absolute value.

Removing Parentheses by Combining Signs
$-(-a) = a$
$-(a) = -a$
$+(a) = a$
$+(-a) = -a$

Study the following additional examples of combining integers using the information provided in the two boxes above.

a. $9 - 12 = 9 + (-12) = -3$

b. $-8 - 14 = -8 + (-14) = -22$

c. $-2 - (-5) = -2 + 5 = 3$

d. $-24 - (-20) = -24 + 20 = -4$

e. $-8 - (-8) = -8 + 8 = 0$

f. $8 - (-8) = 8 + 8 = 16$

SUBTRACTING INTEGERS

Extended Example 1a

Simplify: $-19-(-4)$

SUBTRACTING INTEGERS

To simplify an expression involving adding and subtracting several integers, follow the same steps:

- Remove the parentheses, recalling that $-(a) = -a$, $-(-a) = a$, and $+(-a) = -a$.
- Rearrange the numbers based on their signs.
- Combine all of the numbers with positive signs, and all of the numbers with negative signs.
- Combine the integers, following the rule for adding numbers with different signs: subtract the absolute values of the numbers and keep the sign of the number with the greater absolute value.

EXAMPLE G

Simplify: $29+(-32)-24+58-90-(-14)+(-12)-18+7-(-25)$

$$
\begin{aligned}
&29+(-32)-24+58-90-(-14)+(-12)-18+7-(-25)\\
&=29-32-24+58-90+14-12-18+7+25\\
&=\underline{-32-24-90-12-18}+\underline{\underline{29+58+14+7+25}}\\
&=\underline{-176}+\underline{\underline{133}}\\
&=-43
\end{aligned}
$$

8 of 11

SUBTRACTING INTEGERS

Extended Example 2a

Simplify: $4-7+9-(-8)+1-(6)-3$

9 of 11

SUBTRACTING INTEGERS

Simplifying Absolute Values

To simplify expressions with absolute values, first evaluate the absolute values and then add or subtract terms. If there is an operation within the absolute value symbols, simplify the operation before evaluating the absolute value sign. In the order of operations, absolute value is treated as a grouping symbol. Study the examples below.

a. $|-5| = 5$

b. $-|4| + |-3| = -4 + 3$

$= -1$

c. $-|-8| - |-6| = -8 - 6$

$= -14$

d. $|-8 - 6| = |-14|$

$= 14$

e. $-|5-9| + |-11-5| = -|5-9| + |-11-5|$

$= -|-4| + |-16|$

$= -4 + 16$

$= 12$

Notice in problems **b**, **d**, and **e** there is a negative sign before the absolute value symbol (shown in red). Since these negative signs are <u>outside</u> of the absolute value symbols, they do not get "cancelled out" by the absolute value symbol.

Extended Example 3a

Simplify: $|14-6|-|-15-3|$

Simplify.

$-27 - 14$

Simplify.

$-(-6) - 7 - (13) + (-16) - (-25)$

Simplify.

$-|5 - 11| + |18 - (-14)|$

Indicate whether the following is True *or* False.

$$(-3) - (7) = -3 + 7$$

Simplify.

$$-22 + 27 - 15 - 11 + 9$$

MULTIPLYING INTEGERS

Introduction

In this lesson, you will learn the rules for multiplying integers. Also, you will see how to simplify a negative number raised to an exponent.

1 of 10

MULTIPLYING INTEGERS

Multiplying Integers

Recall that multiplication is a shortcut for adding the same number several times. Here is an example.

$5+5+5=15$ This can be written as $3(5)=15$.

Similarly, $3+3+3+3+3=15$ can be written as $5(3)=15$.

So, $3(5)=5(3)=15$.

The same concept can be applied to negative numbers.

$-5+(-5)+(-5)=-15$, and this can be written as $3(-5)=-15$.

Similarly, $-3+(-3)+(-3)+(-3)+(-3)=-15$ can be written as $5(-3)=-15$.

So, $3(-5)=5(-3)=-15$.

Notice that in the expression $3(-5)=-15$, the 3 is positive, the -5 is negative, and the product is negative -15. (Recall that a "product" is the answer to a multiplication problem.) In the expression $5(-3)=-15$, multiplying 5 by -3 also gives a product of -15.

MULTIPLYING INTEGERS

From the discussion on the previous screen, we see that the following is true:

A <u>positive</u> times a <u>negative</u> results in a NEGATIVE.
A <u>negative</u> times a <u>positive</u> results in a NEGATIVE.

Here are additional examples:

a. $10(-4) = -40$ **b.** $25(-11) = -275$

c. $-4(10) = -40$ **d.** $-11(25) = -275$

Next, note that the product of two negative numbers is positive. One way to show this is to observe the following example of an interesting pattern.

$$4(-3) = -12$$
$$3(-3) = -9$$
$$2(-3) = -6$$
$$1(-3) = -3$$
$$0(-3) = 0$$
$$-1(-3) = 3$$
$$-2(-3) = 6$$
$$-3(-3) = 9$$

Notice that the first factor in each problem is one less than in the problem above it (4, 3, 2, ... ,−2, −3).

Also notice that the product increases by three each time (−12, −9, −6, ... , 6, 9).

The pattern example above shows the following.

A <u>negative</u> times a <u>negative</u> results in a POSITIVE.

MULTIPLYING INTEGERS

Study the following table carefully. Notice the pattern.

Signs of Factors	Sign of Product
Positive times positive: $(+)(+)$	Positive: $(+)$
Negative times negative: $(-)(-)$	Positive: $(+)$
Positive times negative: $(+)(-)$	Negative: $(-)$
Negative times positive: $(-)(+)$	Negative: $(-)$

The rules for multiplying integers can be simplified:

The product of two numbers with like signs is always POSITIVE.
The product of two numbers with different signs is always NEGATIVE.

Note: Multiplication of signed numbers can easily be mistaken for subtraction of integers. Study the following problems, and learn to identify the operations correctly.

Subtraction		Multiplication
$4-(6)$		$4(-6)$
$4-(6)=4-6=-2$		$4(-6)=-24$

MULTIPLYING INTEGERS

Multiplying Multiple Signed Numbers

In multiplication of more than two signed numbers, you need to count the number of negative signs in the string of factors. If there is an even number of negative signs, then the answer will be positive. But if there is an odd number of negative signs, then the answer will be negative. Study the following problems.

a. $-7(5)(2)$ One negative (odd)
$= -35(2)$
$= -70$

b. $-5(-4)(8)$ Two negatives (even)
$= 20(8)$
$= 160$

c. $6(-1)(-3)(-5)$ Three negatives (odd)
$= -6(-3)(-5)$
$= 18(-5)$
$= -90$

d. $-6(-4)(-2)(-1)$ Four negatives (even)
$= 24(-2)(-1)$
$= -48(-1)$
$= 48$

To summarize:

When there is an even number of negative factors, the product is POSITIVE.
When there is an odd number of negative factors, the product is NEGATIVE.

Knowing these rules, you can figure out the sign of the answer separately when you are multiplying a string of signed numbers. Then, just multiply the numbers, without paying attention to their signs. For instance, in example **d** above, $-6(-4)(-2)(-1)$, there is an even number of negatives, so the result has a positive sign. After recognizing the fact that the product will be positive, you can ignore the signs and just multiply 6 by 4 by 2 by 1 to find the answer, 48.

MULTIPLYING INTEGERS

EXAMPLE A

Simplify: $-4(5)(-11)(-1)$

$-4(5)(-11)(-1)$

$= -220$

The multiplication of three negative numbers gives a negative product (odd number of negative factors) .

Multiplying $4 \times 5 \times 11 \times 1$ results in 220. The final answer is negative: -220.

EXAMPLE B

Simplify: $10(-20)(-2)$

$10(-20)(-2)$

$= 400$

The multiplication of two negative numbers gives a positive product (even number of negative factors).

Multiplying $10 \times 20 \times 2$ results in 400. The final answer is positive: 400.

MULTIPLYING INTEGERS

Extended Example 1a

Simplify: $-9(-7)(6)(-1)5$

MULTIPLYING INTEGERS

Exponents

Recall the definition of an exponent. An exponent is a shorthand notation for multiplication of a number by itself several times. Earlier, you learned how to simplify an expression with an exponent when the base is a positive number. Here you will learn how to simplify an exponent problem when the base is negative. One thing that you need to keep in mind is that if the base of an exponent problem is negative, then the base has to be written inside parentheses.

In calculating the answer to an exponent problem, apply the same rule used for multiplication of signed numbers. If the exponent is odd, there will be an odd number of negative factors so the answer will be negative (as in examples **b** and **d** below). If the exponent is even, there will be an even number of negative factors so the answer will be positive (as in example **c** below).

a. 3^3

$= 3(3)(3)$

$= 27$

b. $(-2)^3$

$= (-2)(-2)(-2)$

$= -8$

Negative base must be inside parentheses.

c. $(-4)^4$

$= (-4)(-4)(-4)(-4)$

$= 256$

d. $(-1)^5$

$= (-1)(-1)(-1)(-1)(-1)$

$= -1$

MULTIPLYING INTEGERS

EXAMPLE C

Simplify: $(-5)^4$

$(-5)^4$ The base is negative and written inside parentheses. Therefore, you have to multiply four factors of –5.

$= (-5)(-5)(-5)(-5)$

EXAMPLE D

Simplify: -5^4

$-5^4 = -(5^4)$ The base is positive. The negative sign is not part of the base.

$= -(5 \times 5 \times 5 \times 5)$

$= -(625)$

EXAMPLE E

Simplify: -7^5

$-7^5 = -(7^5)$ The base is positive. The negative sign is not part of the base.

$= -(7 \times 7 \times 7 \times 7 \times 7)$

$= -(16{,}807)$

$= -16{,}807$

Note: It is important to distinguish between two types of problems: in Example C the base is negative, but in Examples D and E the bases are positive. Remember, a negative base is always written inside parentheses!

9 of 10

MULTIPLYING INTEGERS

Extended Example 2a

Simplify: -4^5

END OF LESSON

Simplify.

$-5(-8)$

Simplify.

$-4(-4)(3)(-2)(-7)$

Simplify.

$3(4^2)(-1)^5$

Simplify.

$-6(-11)(9)$

Simplify.

$2(-3)^4$

DIVIDING INTEGERS

Introduction

In the previous section, you learned how to multiply integers. This lesson focuses on division of integers.

DIVIDING INTEGERS

Multiplying Integers

Recall that when multiplying integers, if there is an odd number of negative factors, then the product is negative. If there is an even number of negative factors, then the product is positive.

The product of two numbers with like signs is positive.	$(+)(+) = +$ and $(-)(-) = +$
The product of two numbers with different signs is negative.	$(+)(-) = -$ and $(-)(+) = -$

Dividing Integers

Because division is a form of multiplication, the rule for multiplication of integers also applies to the division of integers. (Remember that a "quotient" is the answer to a division problem).

The quotient of two numbers with like signs is positive.	$^{+}/_{+} = +$ and $^{-}/_{-} = +$
The quotient of two numbers with different signs is negative.	$^{+}/_{-} = -$ and $^{-}/_{+} = -$

DIVIDING INTEGERS

There are two different ways of writing division problems. In algebra a fraction bar is usually used to indicate division, but a division sign (÷) can also be used.

In each example below, study the signs of the integers. Pay close attention to the sign of the quotient!

a. $12 \div 3 = 4$ or $\frac{12}{3} = 4$

b. $-12 \div (-3) = 4$ or $\frac{-12}{-3} = 4$

c. $-12 \div 3 = -4$ or $\frac{-12}{3} = -4$

d. $12 \div (-3) = -4$ or $\frac{12}{-3} = -4$

e. $\frac{-1}{-1} = 1$

f. $\frac{-6}{-3} = 2$

g. $\frac{-12}{4} = -3$

h. $\frac{10}{-5} = -2$

i. $\frac{-18}{-3} = 6$

j. $\frac{-15}{-5} = 3$

k. $\frac{-7}{1} = -7$

l. $\frac{8}{-4} = -2$

DIVIDING INTEGERS

When you are writing division problems in the form of a fraction, you need to know that the negative sign in a fraction such as $-\frac{2}{3}$ can be placed before the fraction (as shown) or in either the numerator or denominator.

$$-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}.$$

However, it is customary to write $\frac{2}{-3}$ as $-\frac{2}{3}$ or $\frac{-2}{3}$.

The fact that the negative sign can be repositioned is useful when you want to eliminate the negative sign in the denominator of a fraction. To do so, simply move the negative sign from the denominator to the numerator or place it before the fraction, as in the following examples.

a. $-36 \div 6 = -6 \quad \rightarrow \quad \frac{-36}{6} = -6$ or $-\frac{36}{6} = -6$
b. $49 \div (-7) = -7 \quad \rightarrow \quad \frac{49}{-7} = -7$ or $-\frac{49}{7} = -7$

4 of 5

DIVIDING INTEGERS

Extended Example 1a

Simplify: $\frac{-81}{27}$

END OF LESSON

Simplify.

$-30 \div (-5)$

Simplify.

$$\frac{-189}{-21}$$

Simplify.

$$-\frac{56}{7}$$

ORDER OF OPERATIONS WITH INTEGERS

Introduction

In Chapter 1, we saw that when an expression has several operations we must perform the operations in an accepted order to get the correct answer to the problem. In this lesson, we review this order and apply it to simplifying expressions that involve integers.

ORDER OF OPERATIONS WITH INTEGERS

Order of Operations with Integers

Recall that the correct **order of operations** is as follows:

1. First, perform all the operations inside grouping symbols, if there are any. Grouping symbols are parentheses ( ), brackets [ ], braces { }, and fraction bars. Start with the innermost grouping symbols.

   Note: Recall that in an expression such as $7(5)$, the parentheses are not considered grouping symbols. To be considered grouping symbols, the ( ), [ ], or { } must have an operation inside them. For example, in $7(5+2)$, the parentheses are considered grouping symbols.

2. Second, simplify all expressions with exponents, if there are any.
3. Third, multiply and divide from left to right.
4. Finally, add and subtract from left to right.

The following examples show how to use the order of operations to simplify expressions with integers.

EXAMPLE A

Simplify: $-9+4+3$

$-9+4+3$ There is no grouping symbol, exponent, or multiplication or division, so add and subtract from left to right.

$= -5+3$

$= -2$

ORDER OF OPERATIONS WITH INTEGERS

EXAMPLE B

Simplify: $12 \div 2(-3)$

$12 \div 2(-3)$ There are two operations here.

$= 6(-3)$ First, divide 12 by 2 because division comes first from left to right.

$= -18$ Then, multiply 6 by −3.

EXAMPLE C

Simplify: $8 - 4(-9 + 3)$

$8 - 4(-9 + 3)$ Note the operations and grouping symbols involved.

$= 8 - 4(-6)$ First simplify inside the parentheses.

$= 8 + 24$ Then multiply: $-4(-6) = 24$.

$= 32$ Finally, add.

EXAMPLE D

Simplify: $25 - 3(5 + 2)$

$25 - 3(5 + 2)$ Note the operations and grouping symbols involved.

$= 25 - 3(7)$ First, simplify within the parentheses.

$= 25 - 21$ Second, multiply.

$= 4$ Finally, subtract.

3 of 6

ORDER OF OPERATIONS WITH INTEGERS

EXAMPLE E

Simplify: $(5+4)^2 - 2^2$

$(5+4)^2 - 2^2$ Decide which operation you need to do first.

$= (9)^2 - 2^2$ First, simplify inside the parentheses.

$= 81 - 4$ Second, simplify the exponents. Notice that the negative sign before the 2 is not in parentheses, so it is not part of the base.

$= 77$ Finally, subtract.

Even within grouping symbols, you must follow the order of operations.

EXAMPLE F

Simplify: $4\left[(-3) - 2(-1)^2\right]$

$4\left[(-3) - 2(-1)^2\right]$

$= 4[(-3) - 2(1)]$ First, simplify inside the brackets. Within the brackets simplify the exponent.

$= 4[(-3) - 2]$ Multiply 2 by 1.

$= 4[-5]$ Rewrite (–3)–2 as addition: (–3)+(–2) and add.

$= -20$ Multiply.

4 of 6

ORDER OF OPERATIONS WITH INTEGERS

In a fraction, simplify the numerator and the denominator separately, following the order of operations.

EXAMPLE G

Simplify: $\dfrac{8-2(3+8)}{3(4)-5}$

$$\frac{8-2(3+8)}{3(4)-5}$$
$$=\frac{8-2(11)}{12-5}$$
$$=\frac{8-22}{7}$$
$$=\frac{-14}{7}=-2$$

Simplify the numerator following the order of operations, and simplify the denominator following the order of operations.

EXAMPLE H

Simplify: $\dfrac{4^3-8(2)}{2-2^3}$

$$\frac{4^3-8(2)}{2-2^3}$$
$$=\frac{64-8(2)}{2-8}$$
$$=\frac{64-16}{-6}$$
$$=\frac{48}{-6}=-8$$

Simplify the numerator following the order of operations, and simplify the denominator following the order of operations.

ORDER OF OPERATIONS WITH INTEGERS

Extended Example 1a

Simplify: $-36 \div 2(3) - 3(1 - 6)$

END OF LESSON

6 of 6

Simplify.

$$21 - 15 \div 5$$

Simplify.

$$4 + 3(8 - 6)^2$$

Simplify.

$$2(6 + 2 \cdot 2)^2 - (-5)$$

Simplify.

$$\frac{4[8-2(5)]}{-(5)+(-7)+4}$$

Simplify.

$$\frac{2(5)(4^3-38)}{15^2 \div 45}$$