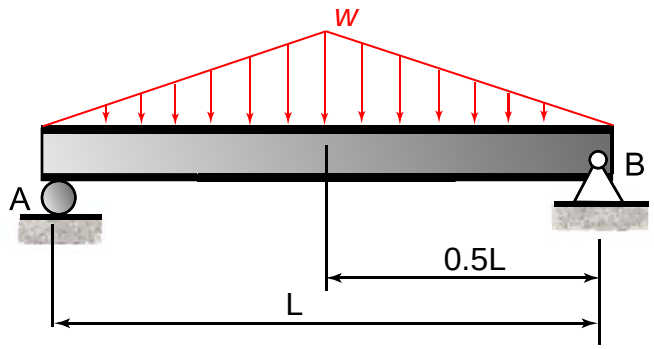
Chapter 9
Deflection of Beams

INTRODUCTION

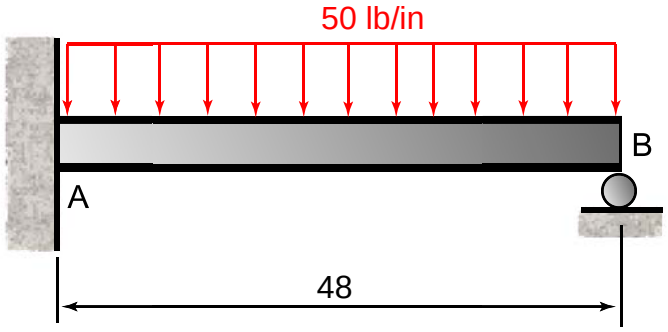
Deflection of Beams using Integration

Deflection of Beams using Superposition

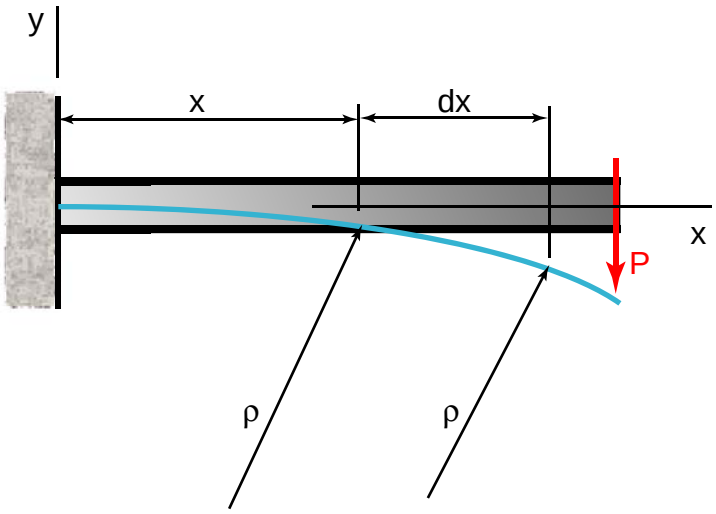


Statically Indeterminate Beams using Integration

Statically Indeterminate Beams using Superposition



EQUATION OF THE ELASTIC CURVE



Review,

$$\frac{dM}{dx} = V \qquad \frac{dV}{dx} = -w$$

$$\frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx} = \frac{M(x)}{EI}$$

Noting,

$$TAN\theta = \frac{dy}{dx} \cong \theta$$

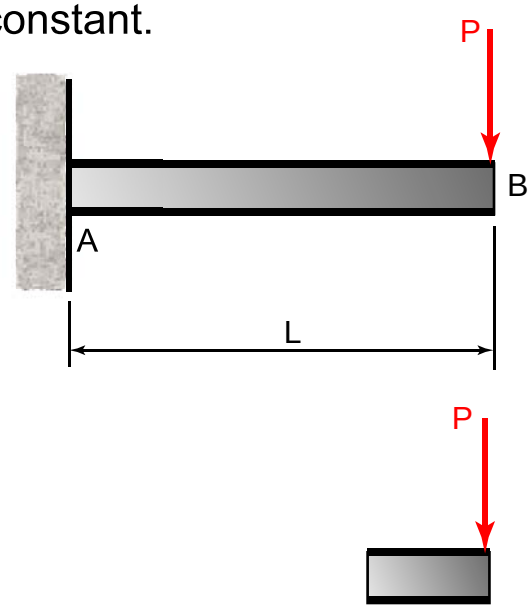
$$\therefore \frac{d\theta}{dx} = \boxed{\frac{d^2y}{dx^2} = \frac{M(x)}{EI}}$$

$$\frac{d^3y}{dx^3} = \frac{dM}{EIdx} = \frac{V(x)}{EI}$$

$$\frac{d^4y}{dx^4} = \frac{dV}{dxEI} = -\frac{w(x)}{EI} \qquad \boxed{\frac{d^4y}{dx^4} = -\frac{w(x)}{EI}}$$

Example

a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the second order differential equation to solve. EI is constant.



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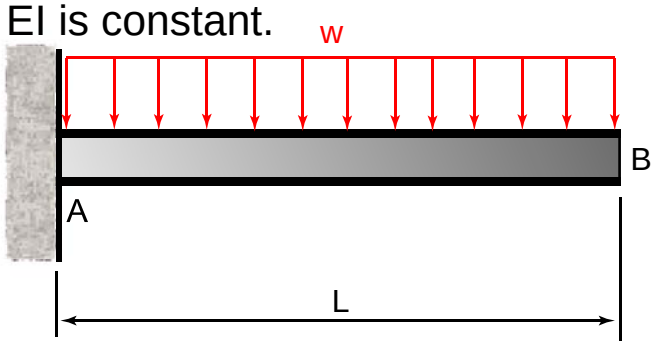

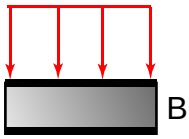

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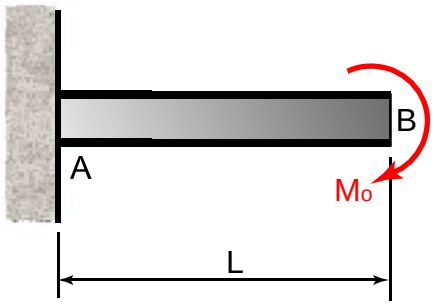
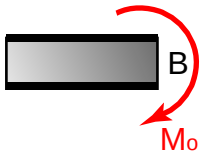
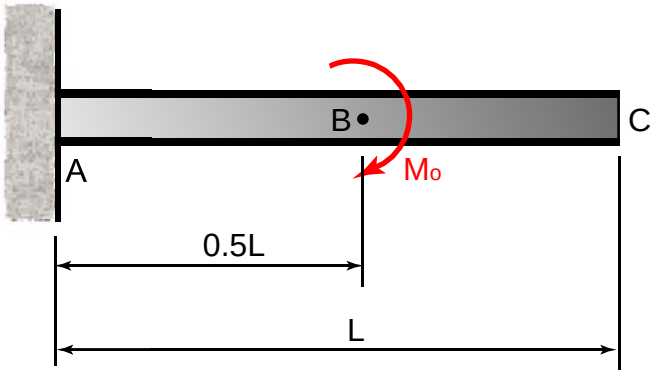
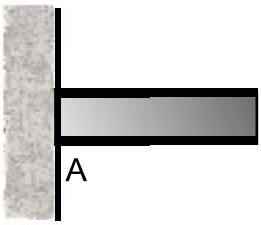


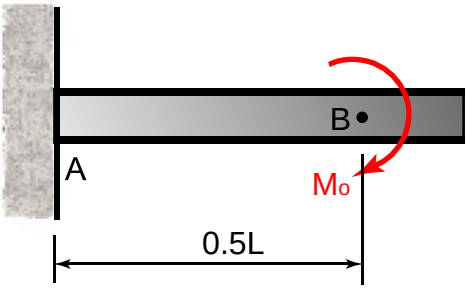


Example

Determine the vertical displacement and slope at point c. Use the second order differential equation to solve. EI is constant.

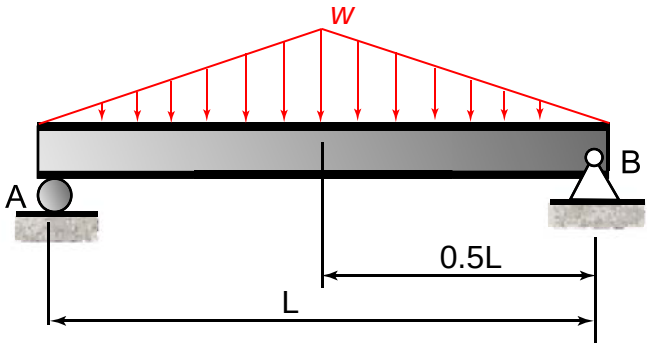


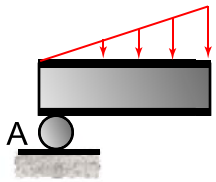




Example

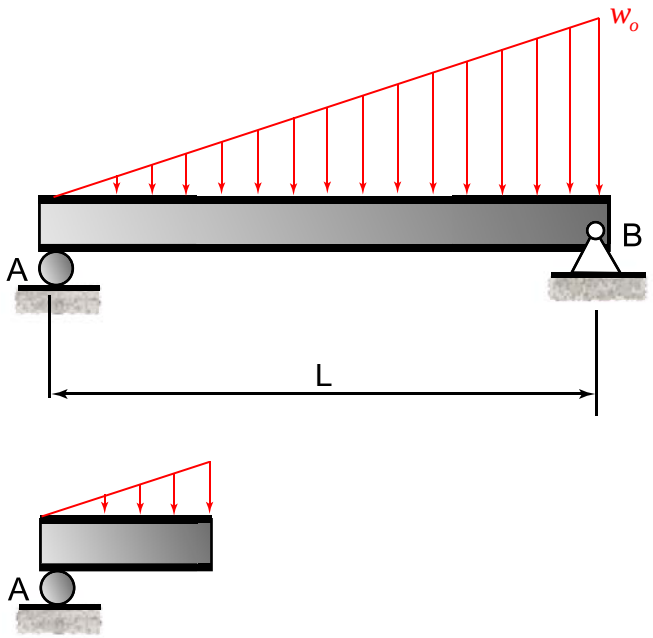
Determine the vertical displacement at the center of the beam. Use the second order differential equation to solve. EI is constant.





Example

Determine the maximum vertical displacement of the beam. Use the second order differential equation to solve. EI is constant.

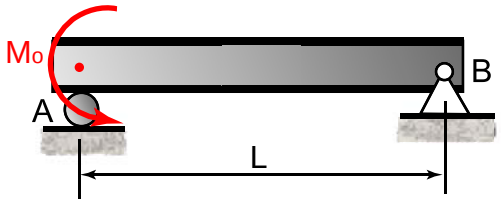


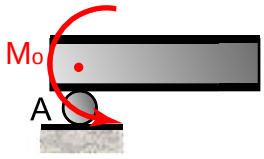
Example

Determine the maximum vertical displacement of the beam. Use the second order differential equation to solve. EI is constant.
Units: kN, m.

$$E = 200GPa$$

$$I = 22.2x10^{6mm^4}$$



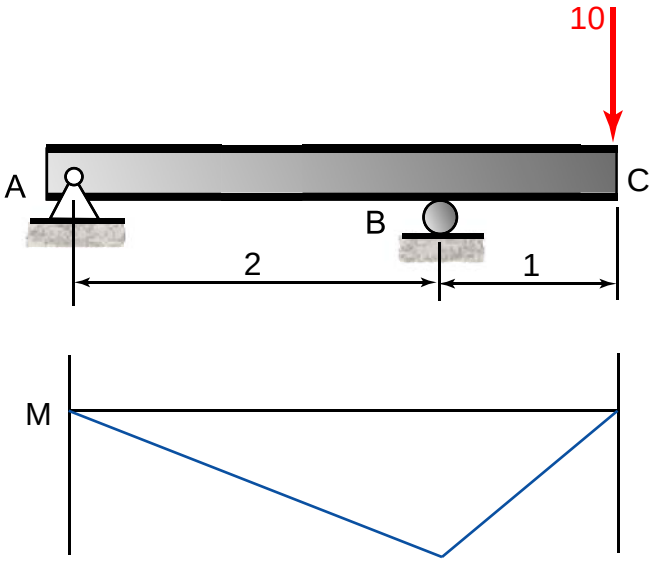


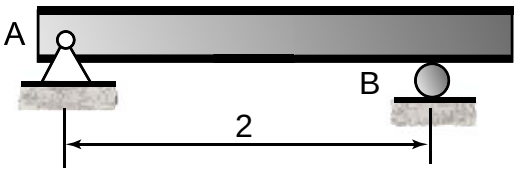
Example

Determine the vertical displacement at C. Use the second order differential equation to solve. EI is constant.
Units: kN, m.

$$E = 200GPa$$

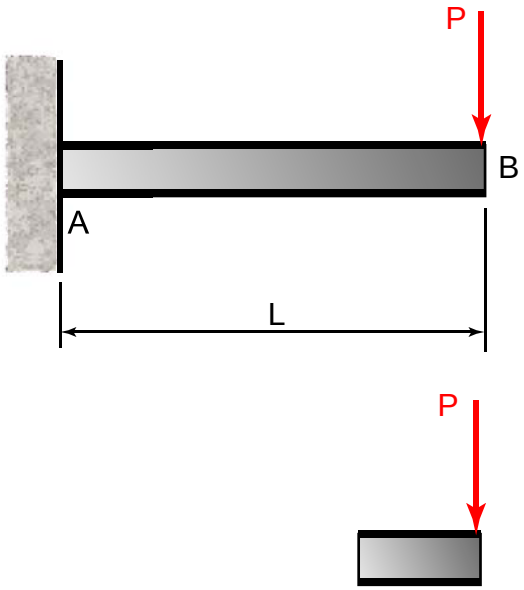
$$I = 22.2x10^{6mm^4}$$





Example

a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the fourth order differential equation to solve. EI is constant.



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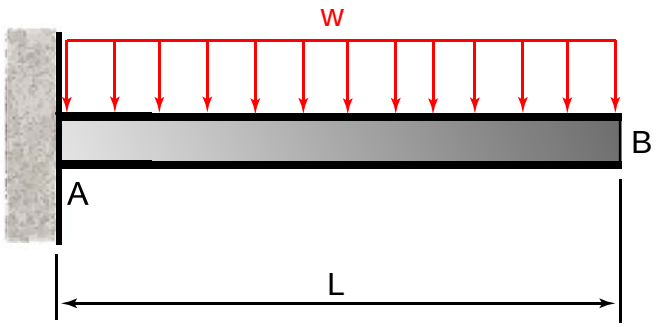

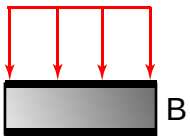

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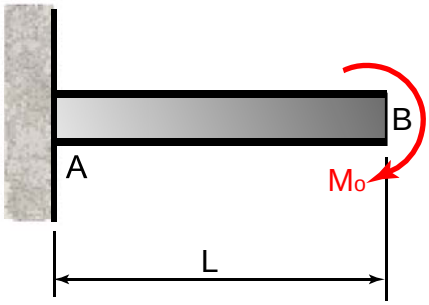
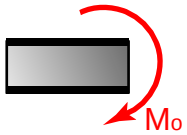
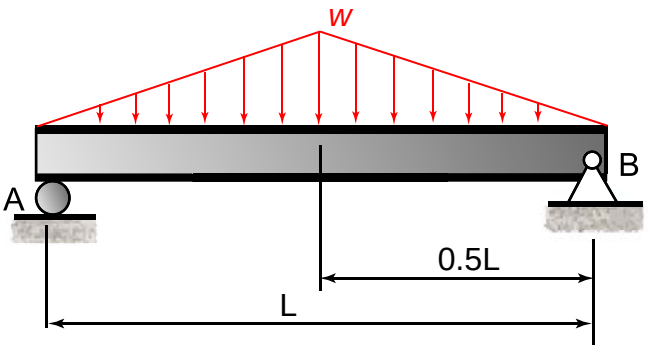
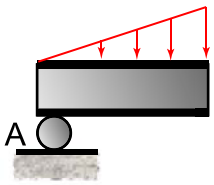




Example

a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement at L/2 and the slope at A. Use the fourth order differential equation to solve. EI is constant.

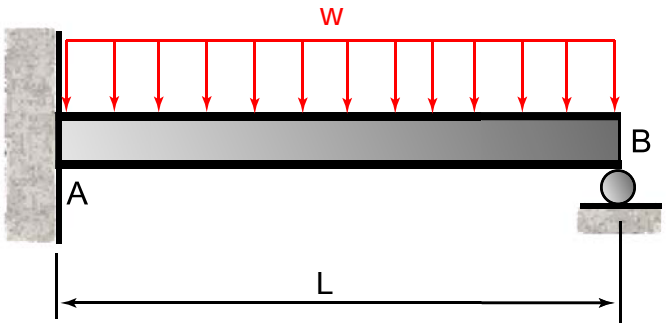




STATICALLY INDETERMINATE BEAMS USING INTEGRATION

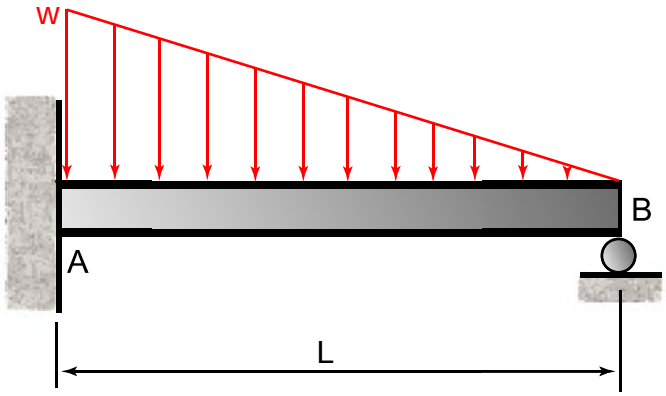
Example

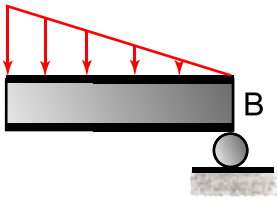
Determine the reactions at A and B. Use the fourth order differential equation to solve. EI is constant. Units: lb, in.



Example

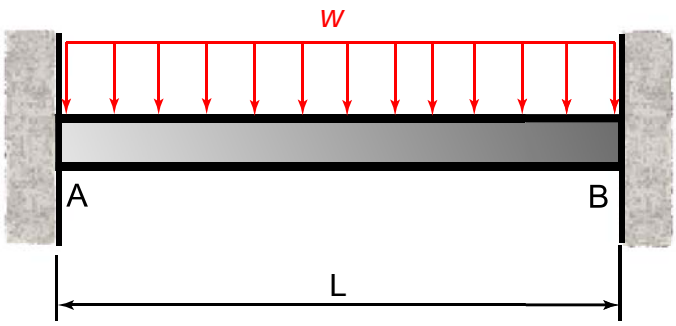
Determine the reactions at A and B. Use the second order differential equation to solve. EI is constant. Units: lb, in.

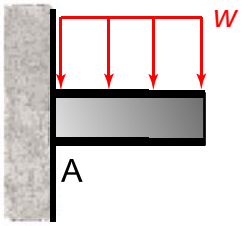




Example

Determine the reactions at A and B. Use the second order differential equation to solve. Neglect the effect of any axial reactions. EI is constant. Units: lb, in.





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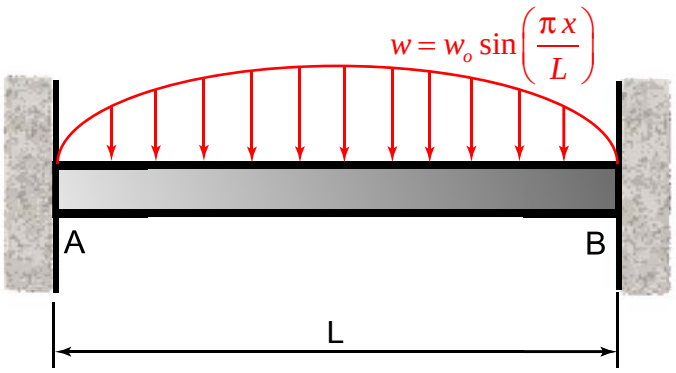


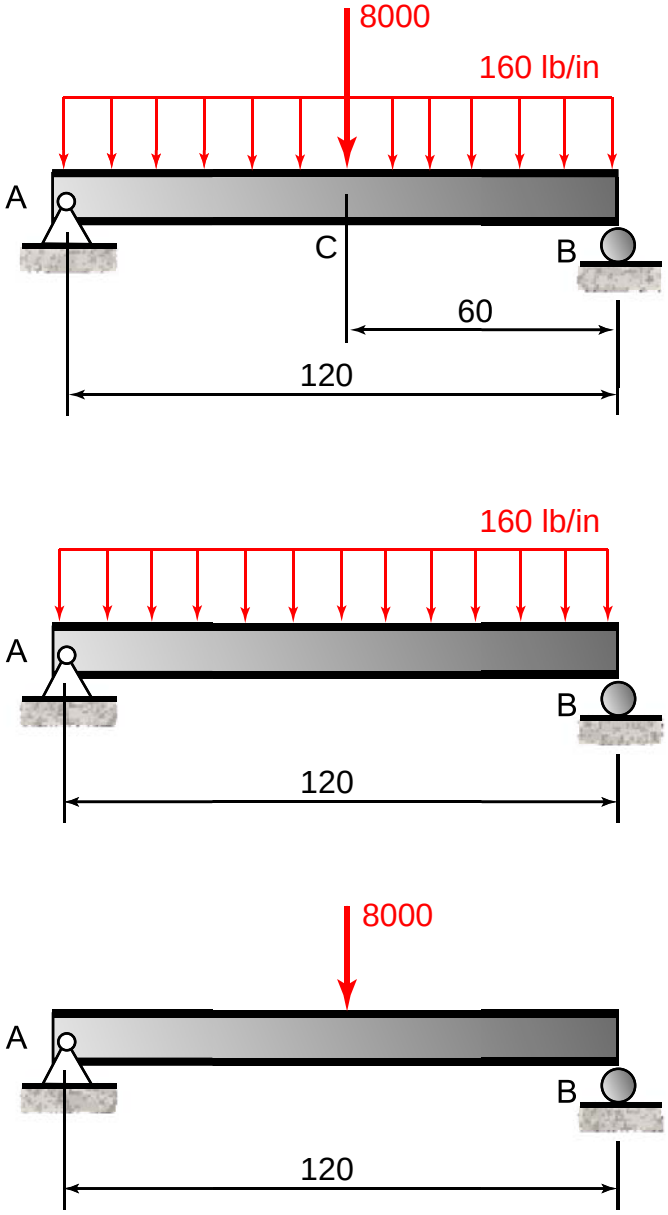
DEFLECTION OF BEAMS USING SUPERPOSITION

Example

Using superposition, determine the displacement at C. EI is constant. Units: lb, in.

$$E = 29x10^{6\,psi}$$

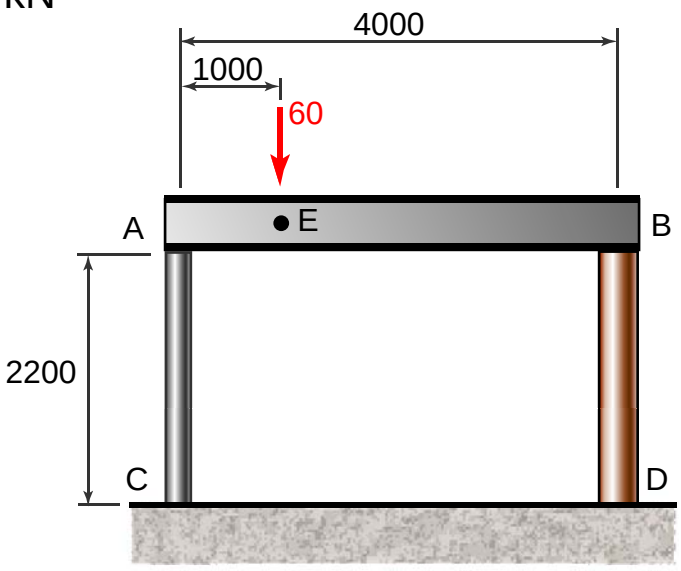
$$I = 53.4^{in^4}$$

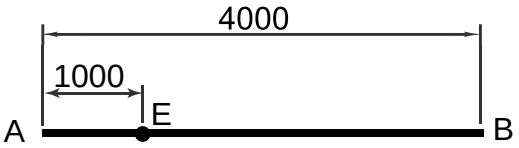


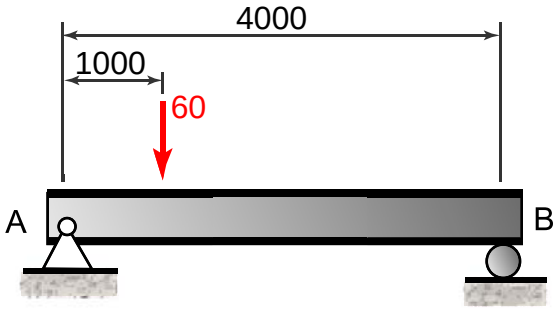
Example

Post AC is made of steel and has a diameter of 18 mm, and BD is made of copper and has a diameter of 42 mm. Determine the displacement of point E on the steel beam AB. E(steel)= 200 GPa, E(copper)= 120 GPa. Units: mm, kN

$I = 45.5x10^{6mm^4}$

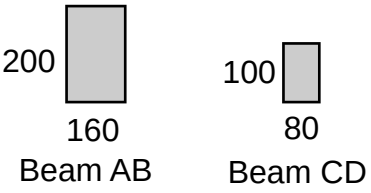


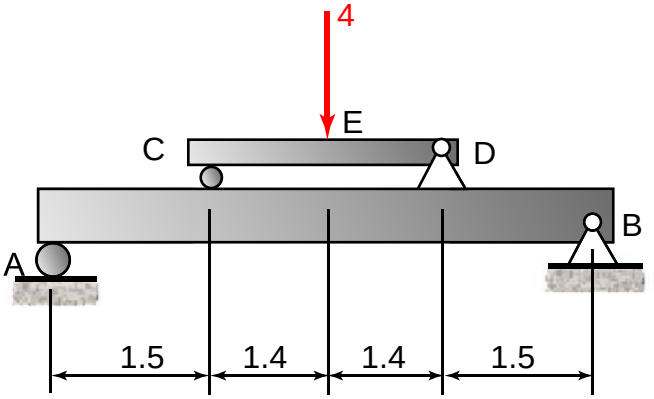


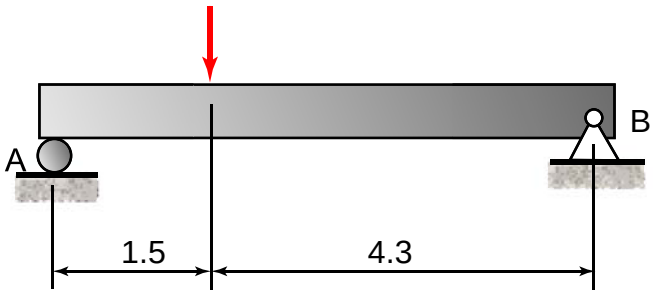


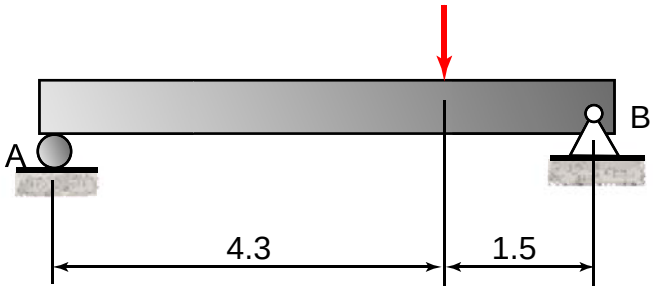
Example

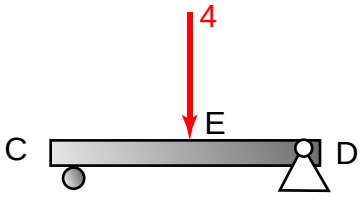
Knowing that each beam has a rectangular cross section as shown, determine the displacement at E. E= 200GPa. EI is constant. Units: kN, m.







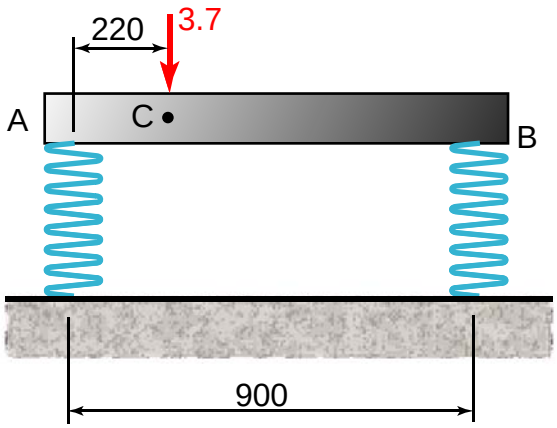


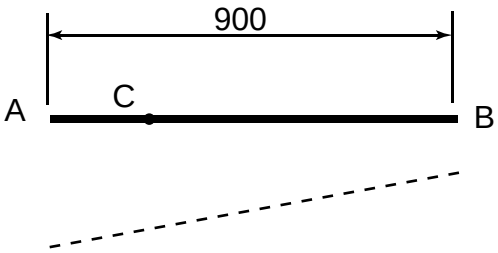


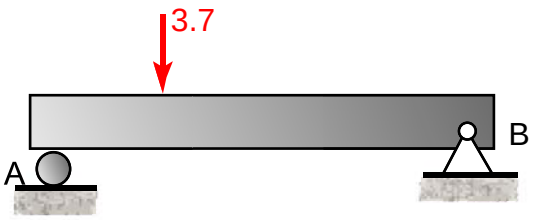
Example

The horizontal beam AB rests on the two short springs with the same length. The spring at A has stiffness of 250 kN/m and the spring at B has a stiffness of 150 kN/m. Determine the displacement under the load. Units: kN, mm.

$EI = 15x10^{3m^2}$



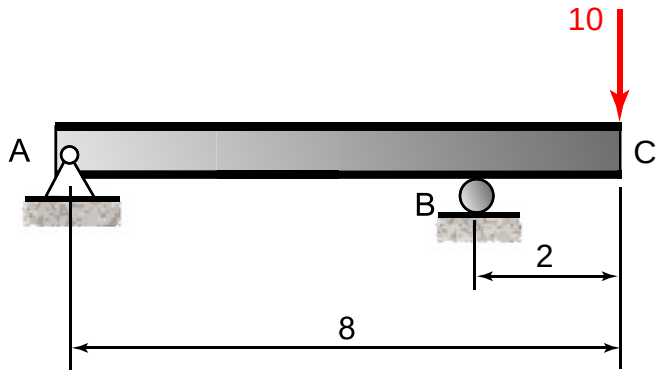


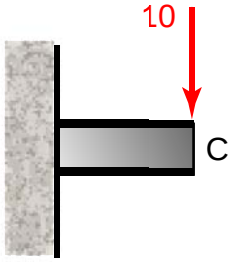


Example

Using superposition, determine the displacement at C. EI is constant. Units: kN, m.

$EI = 21.4x10^{6m^2}$



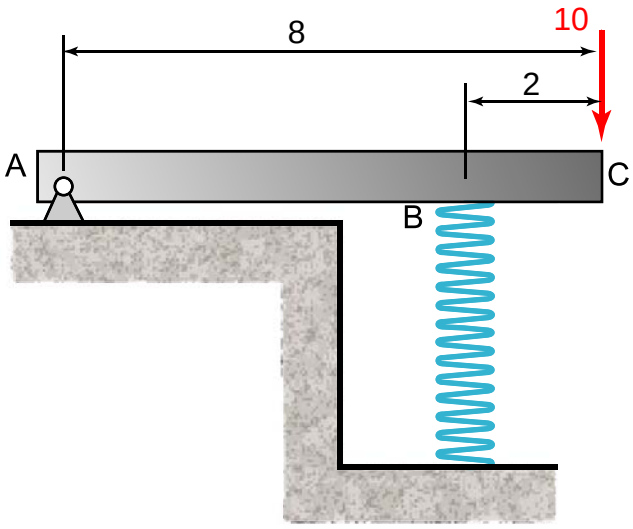


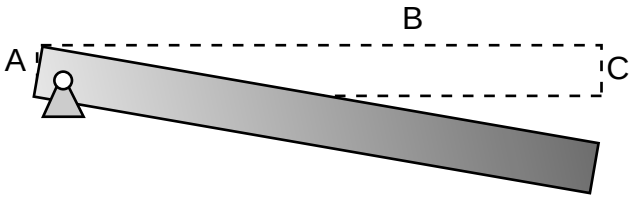
Example

The 160x200 mm rectangular beam ABC rests on a spring at B. The spring at B has stiffness of 2500 kN/m. Determine the displacement at C. Units: kN, m.

$$EI = 21.4x10^{6m^2}$$

From a previous solution with point B being a rigid roller:
y_C= 4.99 mm

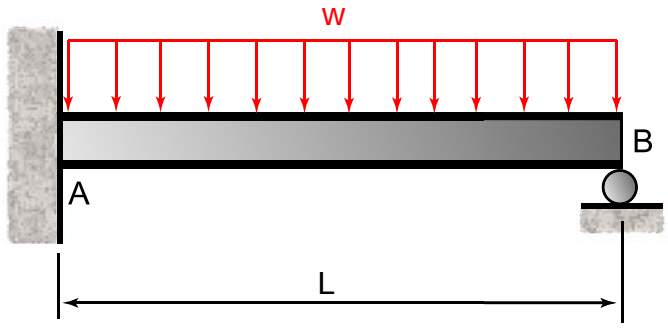


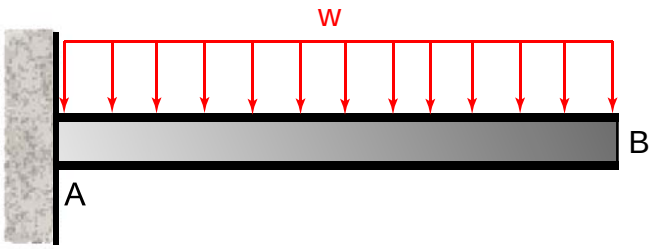


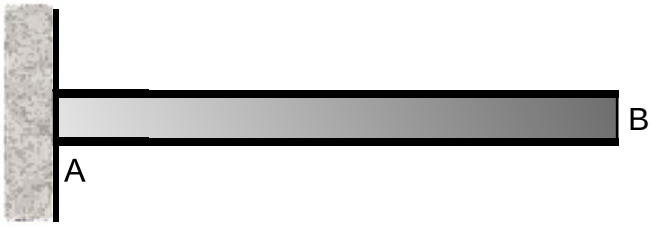
STATICALLY INDETERMINATE BEAMS USING SUPERPOSITION

Example

Using superposition, determine the reactions at A and B.
EI is constant. Units: lb, in.





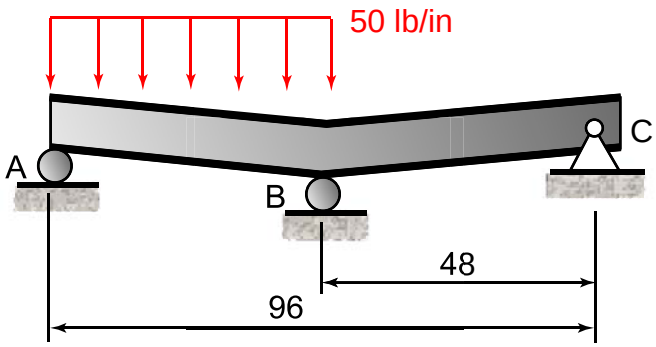


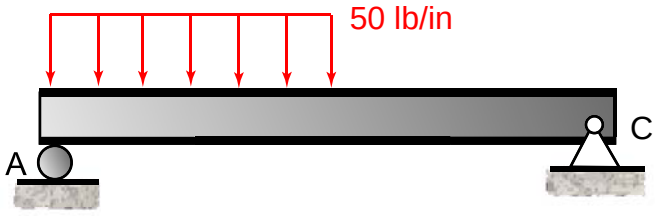
Example

Due to the loading and poor construction, support B settles 1/16".
Using superposition, determine the reactions at A, B, and C.
EI is constant. Units: lb, in.

$$E = 29x10^{6\,psi}$$

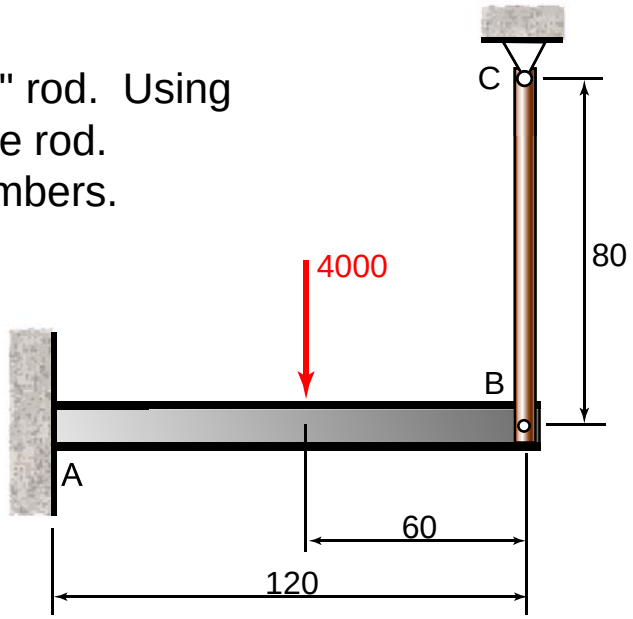
$$I = 11.3^{in^4}$$

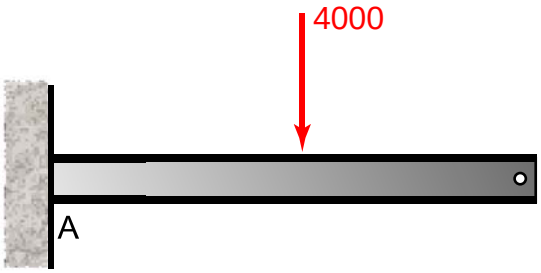




Example

The W6x20 is supported at B by a 0.25" rod. Using superposition, determine the force in the rod.
EI is constant. E=29E6 psi for both members.
Units: lb, in.

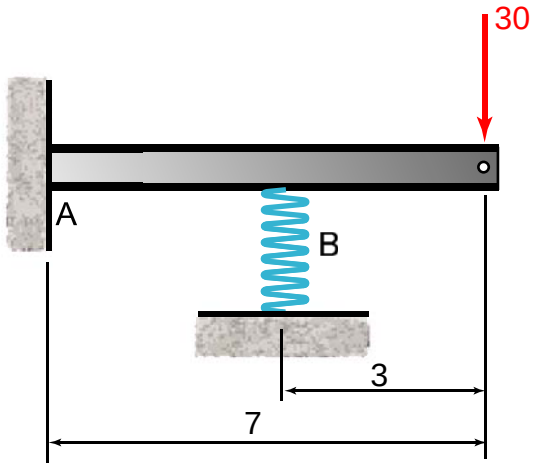


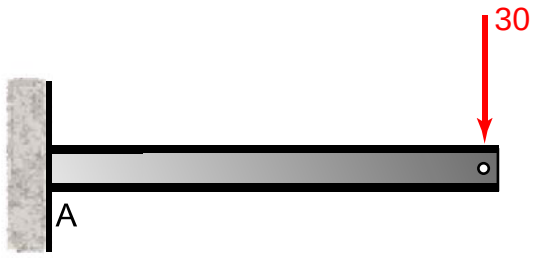


Example

Using superposition, determine the reactions at A and the force in the spring at B. The spring constant is 1 kN/mm. EI is constant.
Units: kN, m.

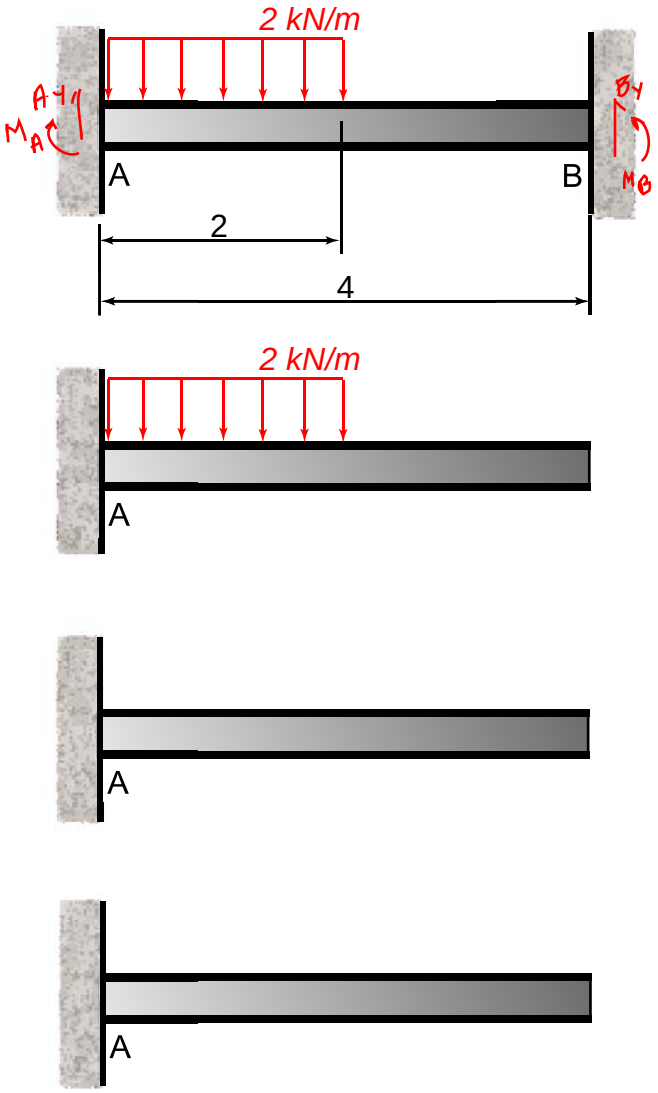
$$EI = 100x10^{6m^2}$$





Example

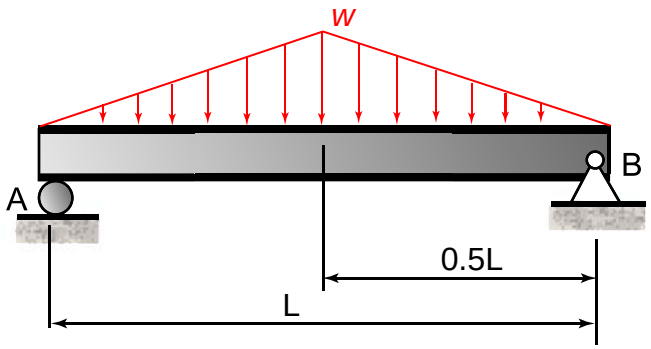
Using superposition, determine the reactions at A and B. Neglect the effect of any axial reactions. EI is constant. Units: kN, m.



SUMMARY

Deflection of Beams using Integration

Deflection of Beams using Superposition



Statically Indeterminate Beams using Integration

Statically Indeterminate Beams using Superposition