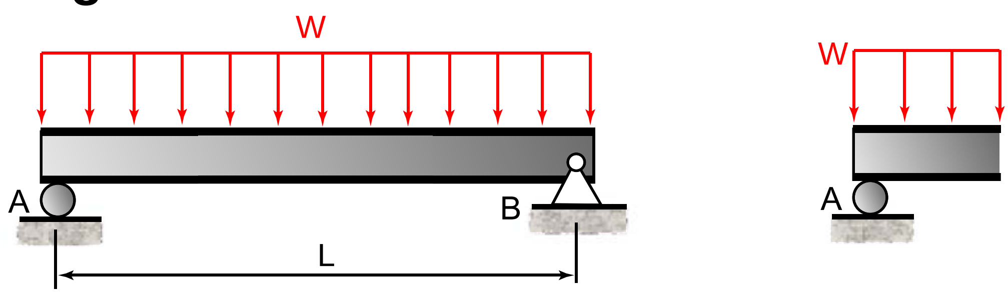
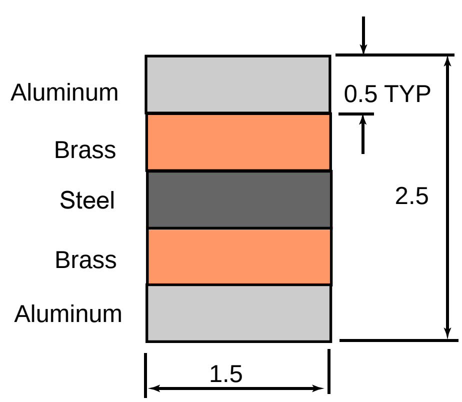
Chapter 4
Pure Bending

INTRODUCTION

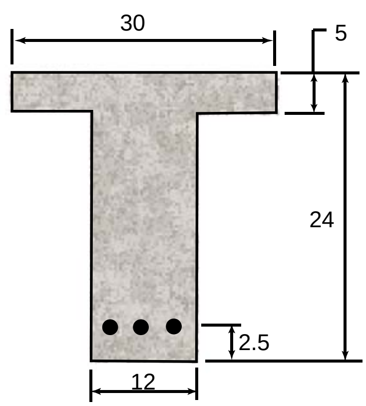
Bending Stress



Bending of Members made of Several Materials

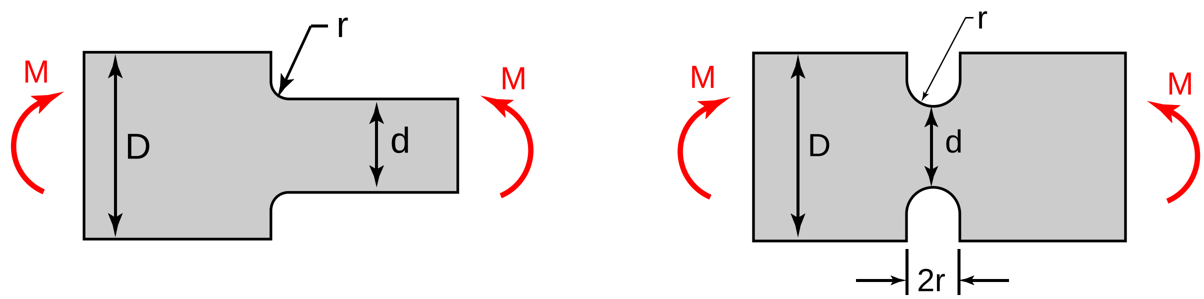


Cross-section, in

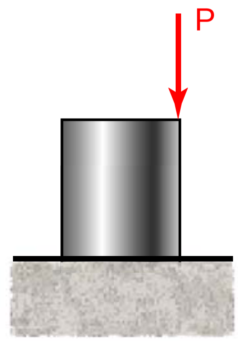


Cross-section, in

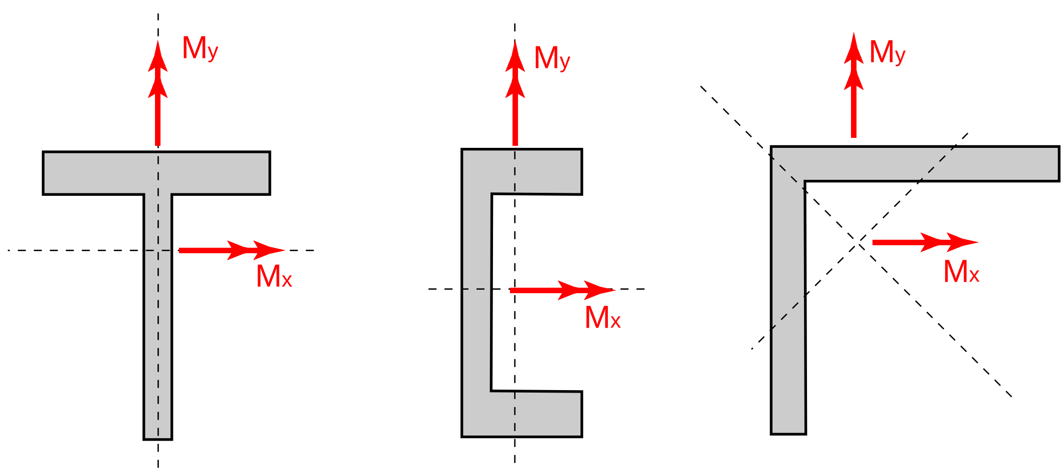
Stress Concentrations



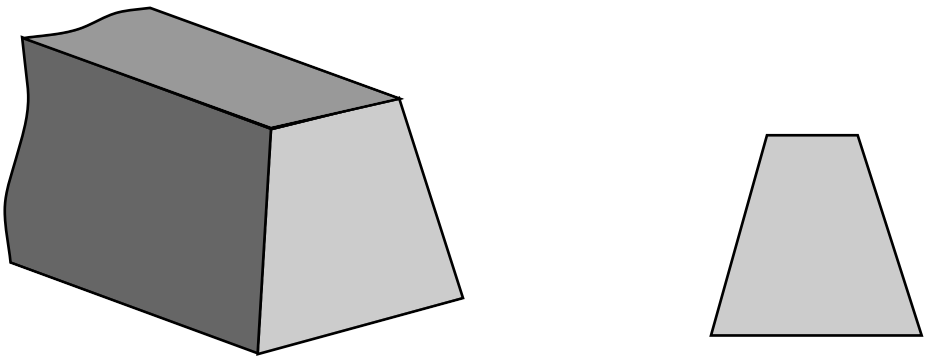
Eccentric Axial Loading in a Plane of Symmetry

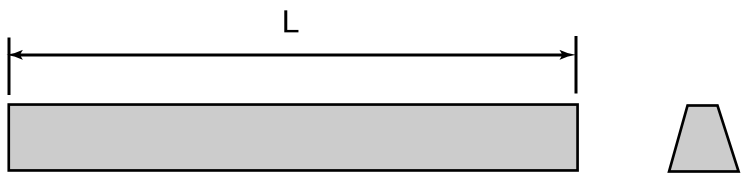


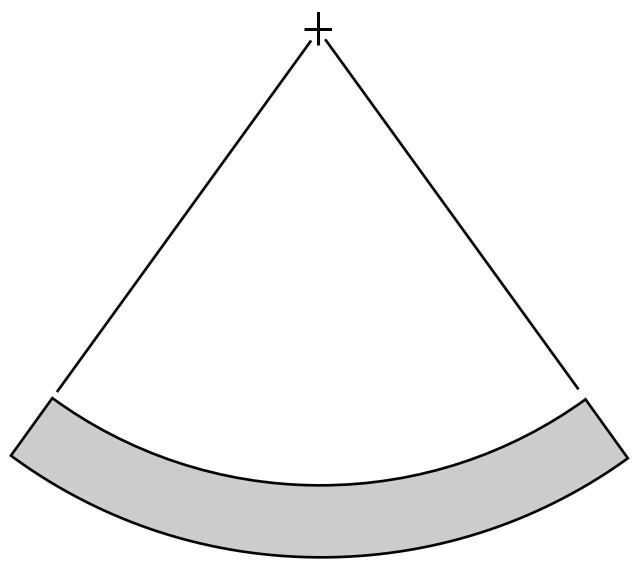
Unsymmetric Bending



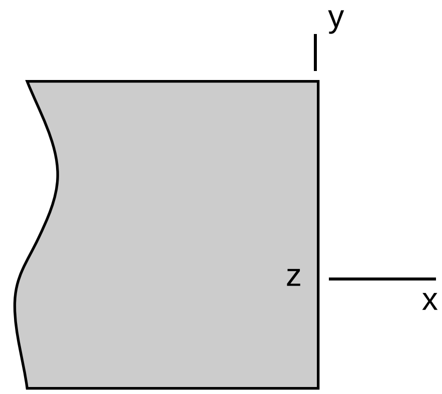
BENDING STRESS

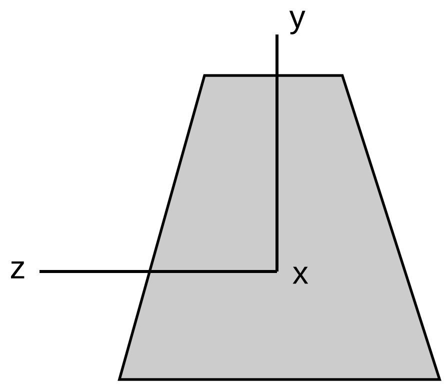






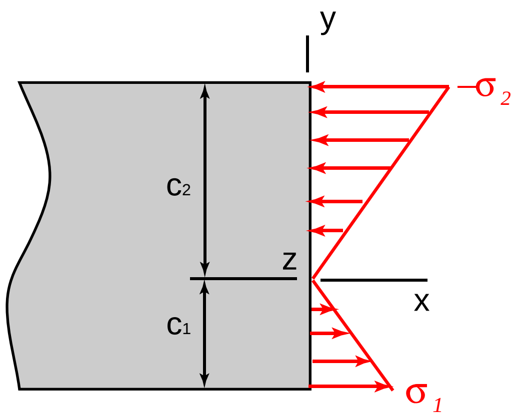
BENDING STRESS- continued

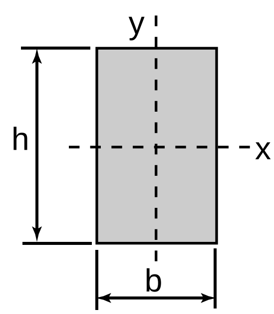




$$\sigma_x = -\frac{My}{I}$$

SECTION MODULUS

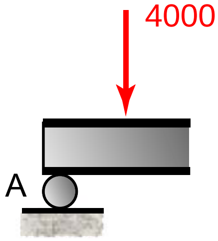


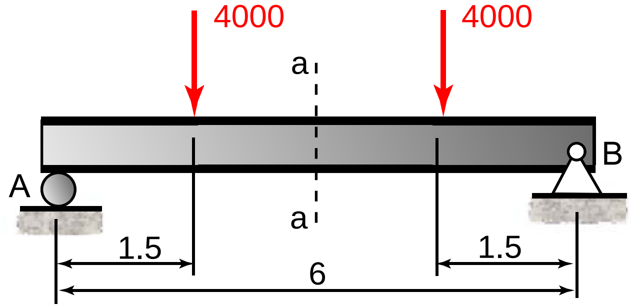


$$\sigma_x = -\frac{My}{I} = -\frac{M}{S}$$

Example

Find the maximum bending stress at section a-a (3 m from A) of the W150x29.8 beam. Units: N, m.





W150x29.8

Area, $A = 3790^{mm^2}$

Depth, $d = 157^{mm}$

Flange Width, $b_f = 153^{mm}$

Flange Thickness, $t_f = 9.3^{mm}$

Web Thickness, $t_w = 6.6^{mm}$

$I_x = 17.2x10^{6mm^4}$

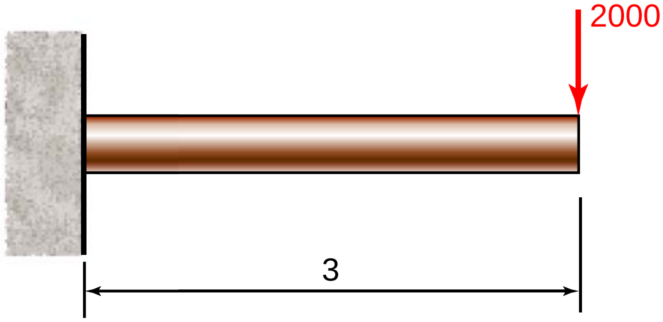
$I_y = 5.56x10^{6mm^4}$

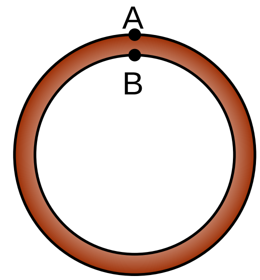
$S_x = 219x10^{3mm^3}$

$S_y = 72.7x10^{3mm^3}$

Example

Find the bending stresses at the wall at points A and B for a 6" pipe with a wall thickness of 0.125". Units: lb, ft.

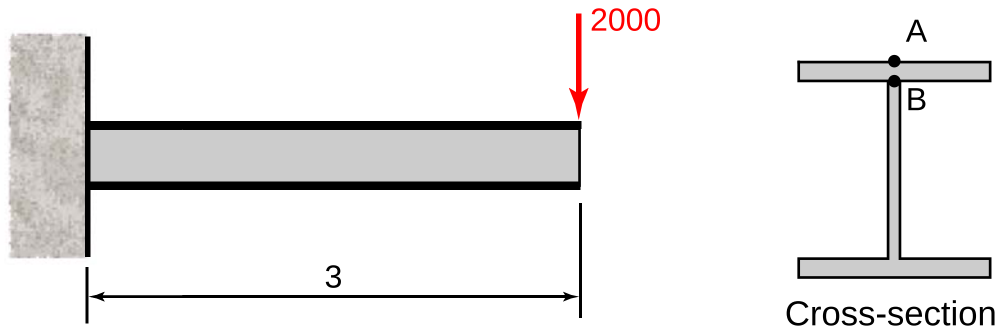




Cross-section

Example

Find the bending stresses at the wall at points A and B for the W6x20 beam. Units: lb, ft.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

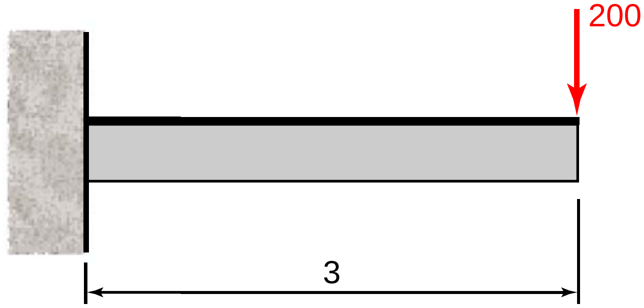
$I_y = 13.3^{in^4}$

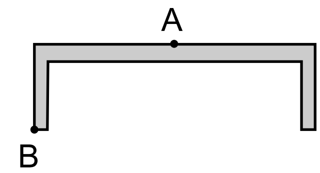
$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

Find the bending stresses at the wall at points A and B for the C6x13 beam. Units: lb, ft.





Cross-section

<u>C6x13</u>

Area, A $= 3.83^{in^2}$

Depth, d $= 6.00^{in}$

Flange Width, $b_f = 2.16^{in}$

Flange Thickness, $t_f = 0.343^{in}$

Web Thickness, $t_w = 0.437^{in}$

$I_x = 17.4^{in^4}$

$I_y = 1.05^{in^4}$

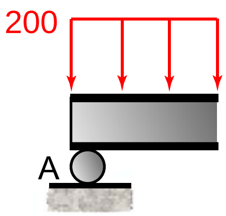
$S_x = 5.80^{in^3}$

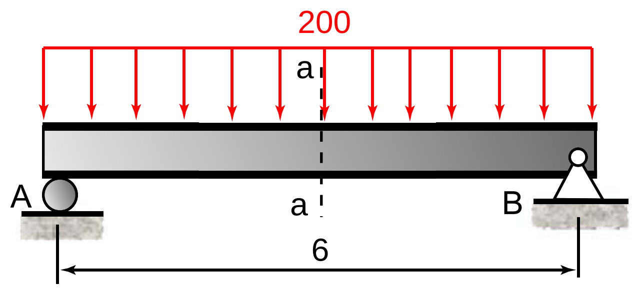
$S_y = 0.642^{in^3}$

$\overline{x} = 0.514^{in}$

Example

Find the maximum bending stress at section a-a (3 m from A) of the W150x29.8 beam. Units: N/m, m.



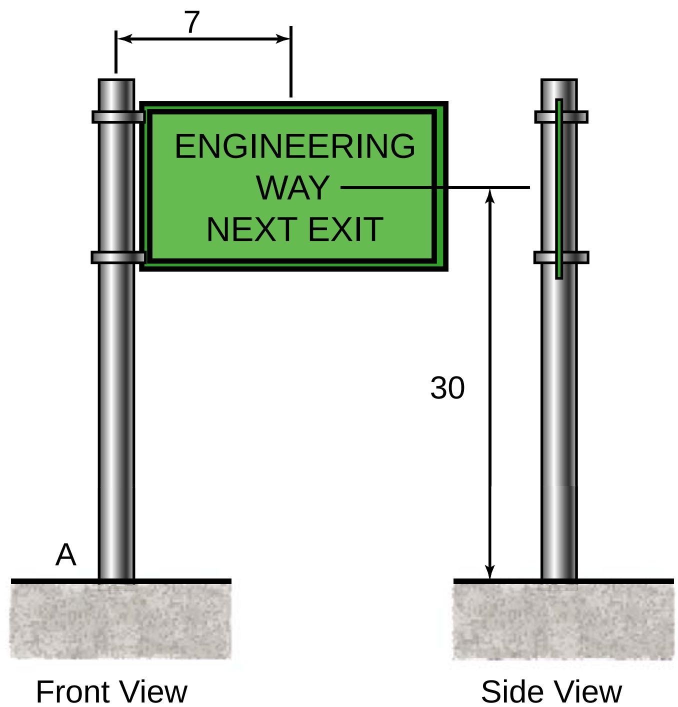


W150x29.8

$$\text{Area, A} = 3790^{mm^2}$$
$$\text{Depth, d} = 157^{mm}$$
$$\text{Flange Width, } b_f = 153^{mm}$$
$$\text{Flange Thickness, } t_f = 9.3^{mm}$$
$$\text{Web Thickness, } t_w = 6.6^{mm}$$
$$I_x = 17.2x10^{6 mm^4}$$
$$I_y = 5.56x10^{6 mm^4}$$
$$S_x = 219x10^{3 mm^3}$$
$$S_y = 72.7x10^{3 mm^3}$$

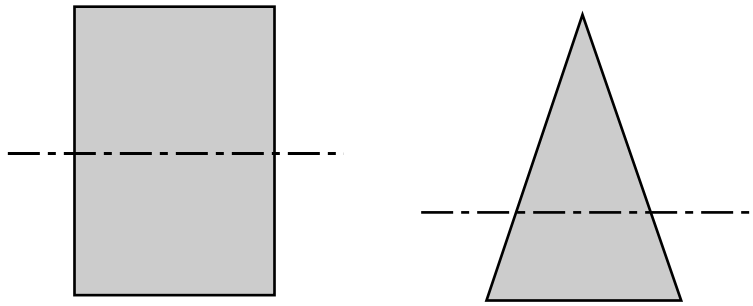
Example

The sign is subjected to a wind force of 250 lb at it’s centroid 7' from the center of the column. The column has an outside diameter of 10" and a wall thickness of 0.25". Considering only this force, determine the maximum bending stress. Units: ft.

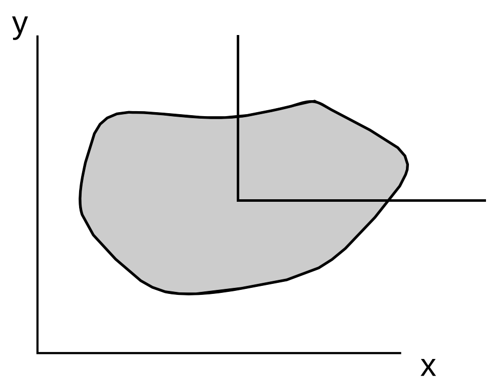


PARALLEL-AXIS THEOREM

MOMENTS OF INERTIA

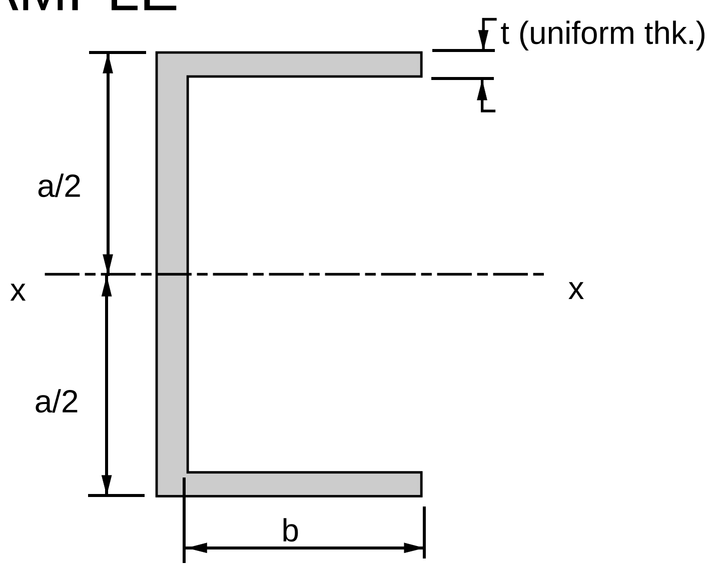


PARALLEL-AXIS THEOREM



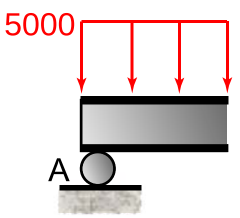
$$I_x = \sum (I_{x'} + Ad_y^2)$$
$$I_y = \sum (I_{y'} + Ad_x^2)$$

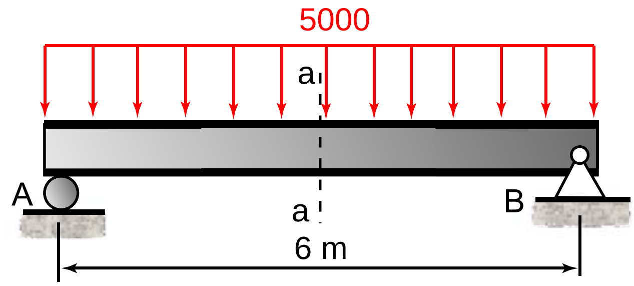
EXAMPLE

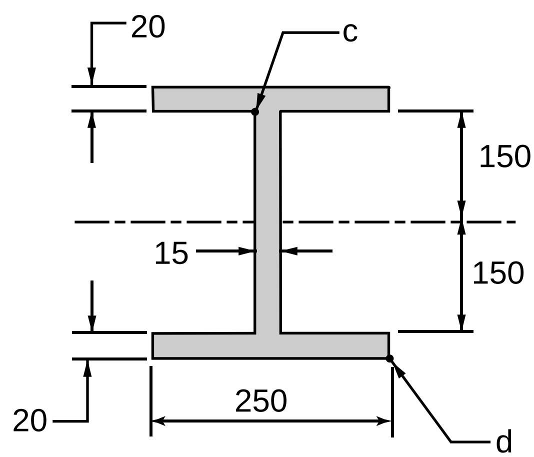


Example

The simply-supported beam below has a cross-sectional area as shown. Determine the bending stress that acts at points c and d, located at section a-a (3 m from A). Units: N/m, mm (uno).

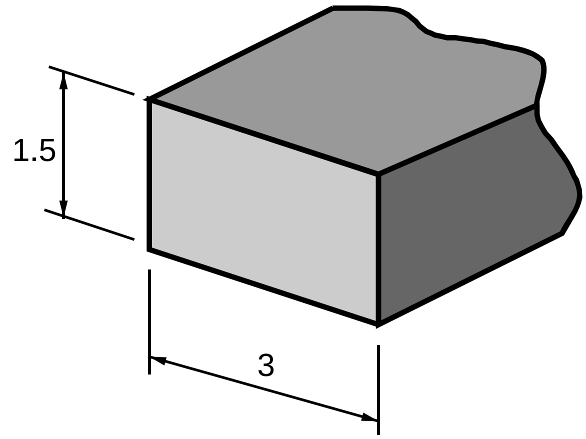


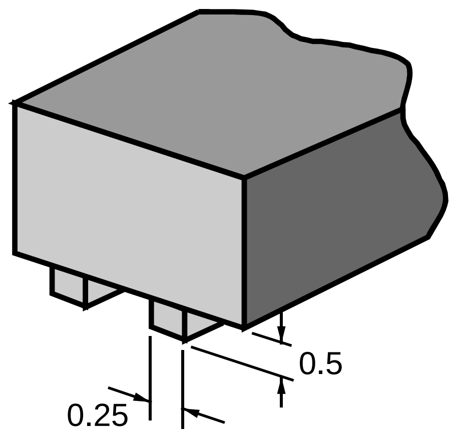


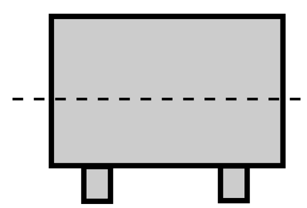


Example

The member is designed to resist a moment of 5 kip•in about the horizontal axis. Determine the maximum normal stress in the member for the two similar cross-sections. Units: in.

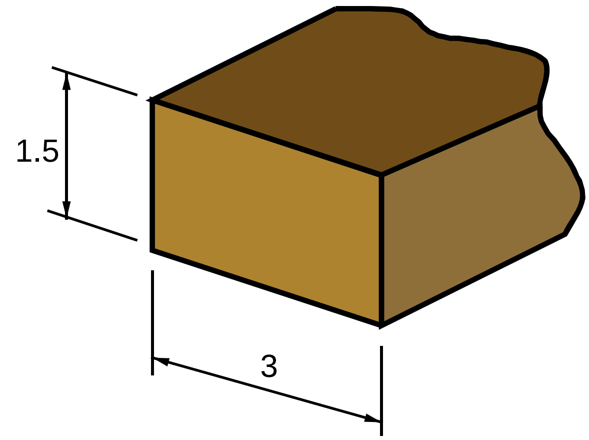




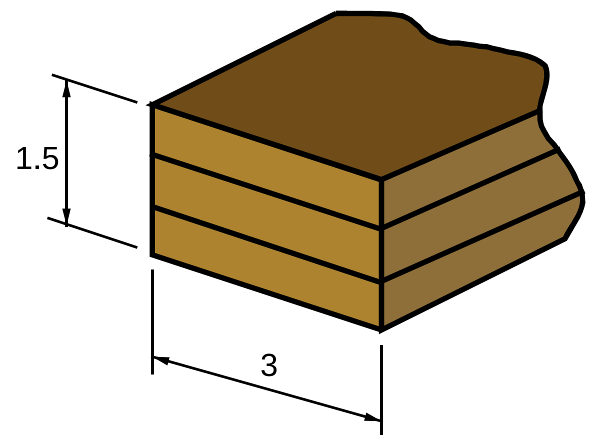


Example

Compare the bending stresses between the two cases for a moment about the horizontal axis. Case 1 is a simple solid cross-section, whereas case 2 is made up of 3 identical boards. The 3 boards aren't connected together and simply rest on one another. Units: in.



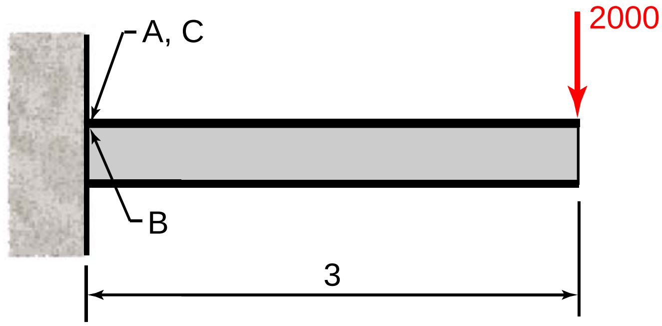
Case 1

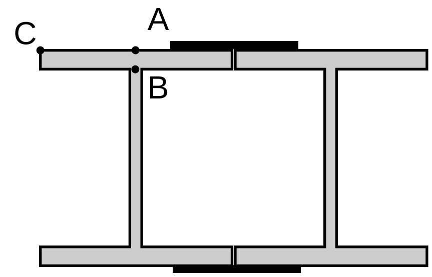


Case 2

Example

The two beams are connected by a thin rigid plate on the top and bottom side of the flanges. Find the bending stresses at the wall at points A, B and C for the W6x20 beam. Units: lb, ft





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

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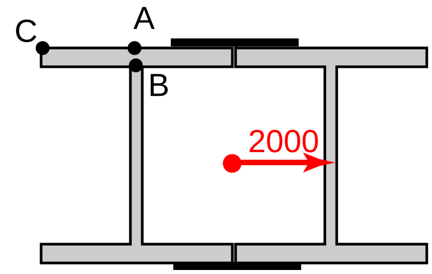

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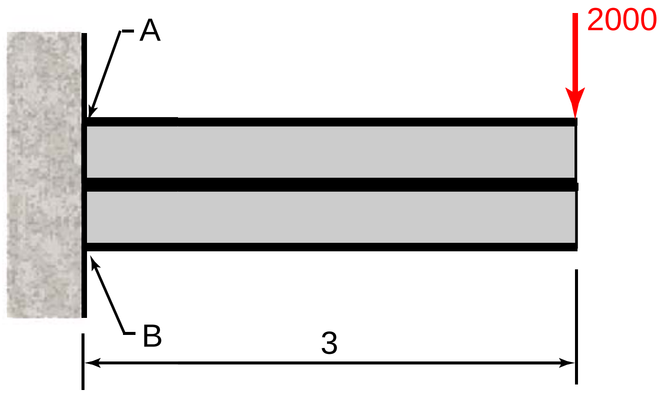
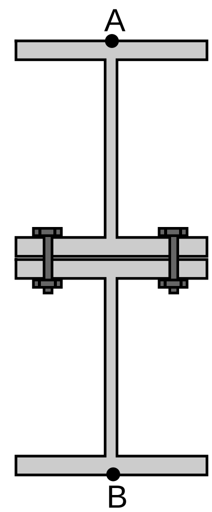
Example

The two beams are connected by bolts through the flanges. Find the bending stresses at the wall at points A and B for the W6x20 beam.
Units: lb, ft





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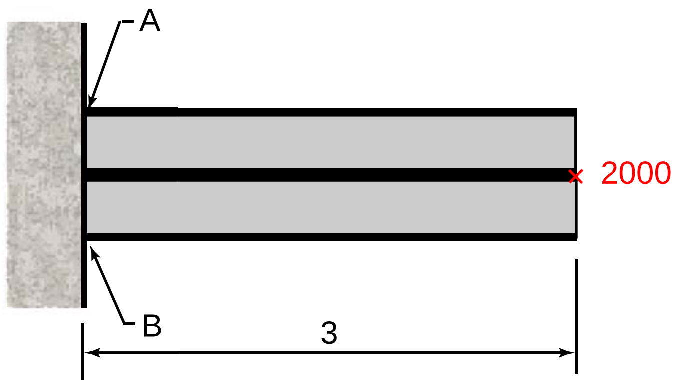

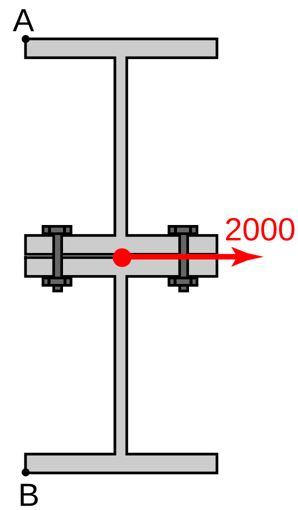

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Example

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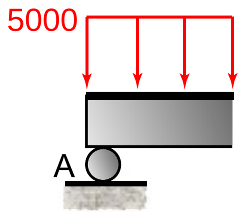
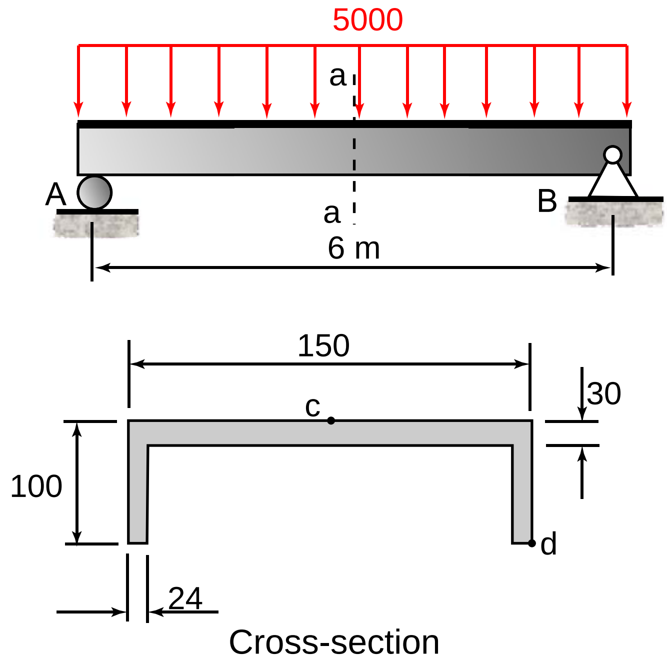
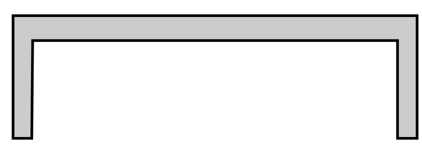
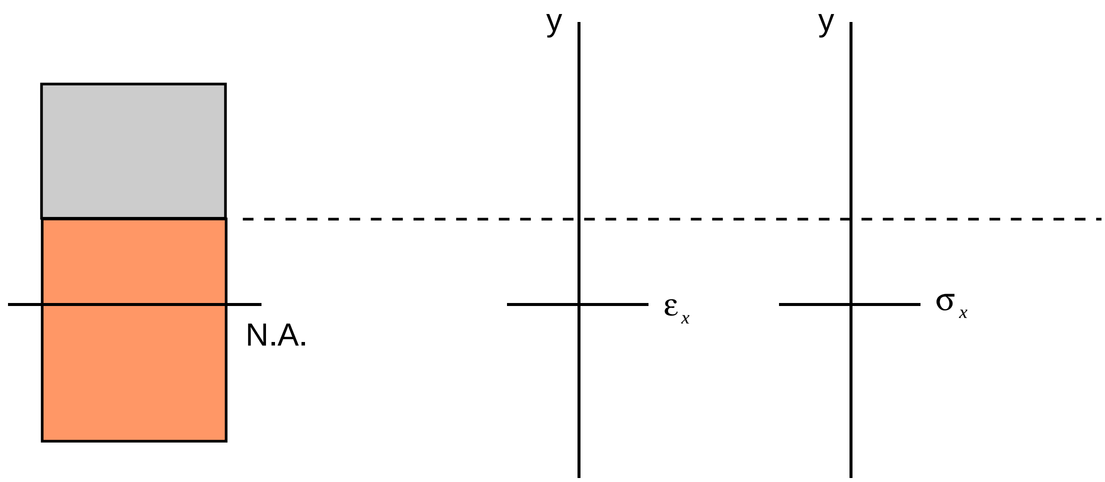
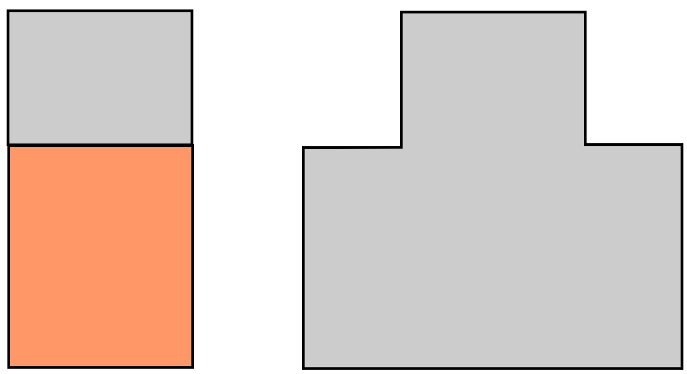






BENDING OF MEMBERS MADE OF SEVERAL MATERIALS



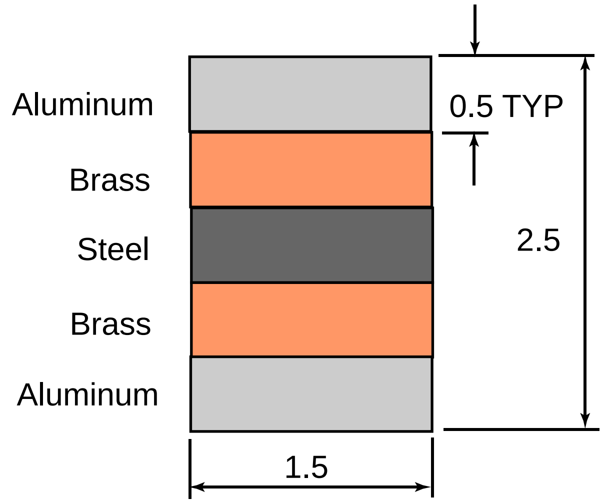


$$\sigma_x = -n\frac{My}{I}$$

Example

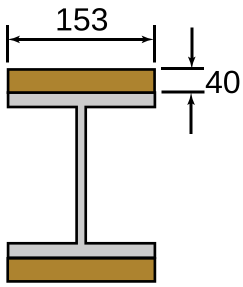
Find the stress in each of the three metals if a moment of 12 k-in is applied about the horizontal axis.

E (aluminum)= 10E6, E(steel)= 30E6, E (brass)= 15E6 psi. Units: in.

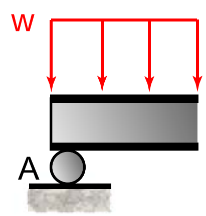


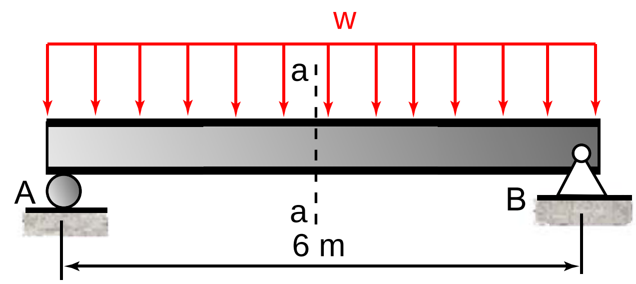
Example

A W150x29.8 wide flange beam is reinforced with wood planks that are securely connected to the flanges. E_{steel}/E_{wood}= 20. If the allowable stresses in the wood and steel are 4.5 MPa and 52 MPa, respectively, determine the allowable distributed load w based on section a-a (3 m from A). Units: N/m, mm (UNO).



Cross-section





W150x29.8

$$\text{Area, A} = 3790^{mm^2}$$
$$\text{Depth, d} = 157^{mm}$$
$$\text{Flange Width, } b_f = 153^{mm}$$
$$\text{Flange Thickness, } t_f = 9.3^{mm}$$
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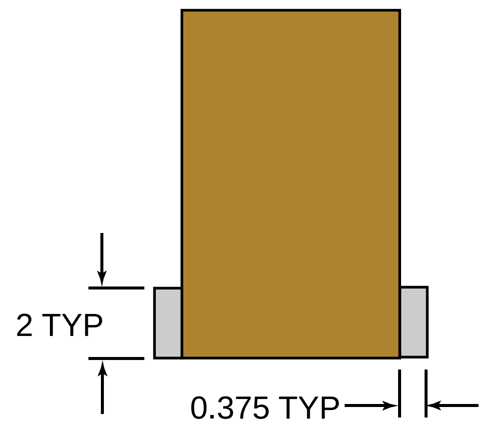
$$I_x = 17.2x10^{6 mm^4}$$
$$I_y = 5.56x10^{6 mm^4}$$
$$S_x = 219x10^{3 mm^3}$$
$$S_y = 72.7x10^{3 mm^3}$$

Example

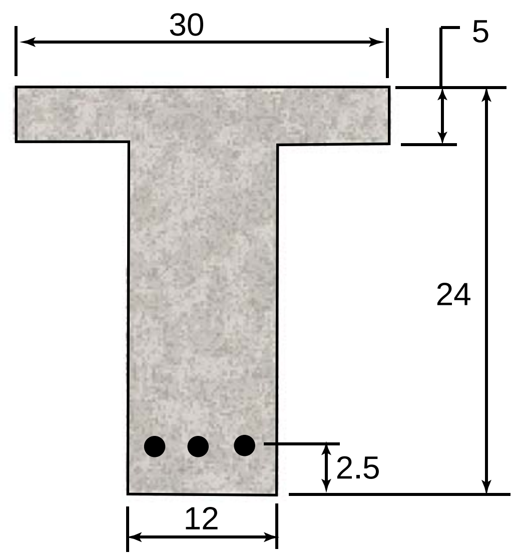
Two steel plates are securely fastened to a 6"x10" wood beam. $E_{steel}/E_{wood} = 20$. Knowing that the beam is bent about the horizontal axis by a 125 kip-in moment, determine the maximum stress in (a) the wood, (b) the steel. Units: in.



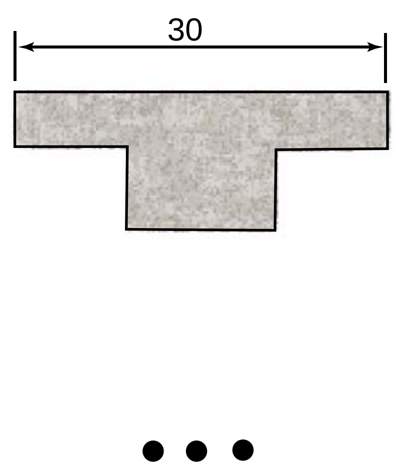
Cross-section

Example

Determine the stress in the concrete and steel if a moment of 1500 kip-in is applied about the horizontal axis. Area of steel= 3.14 sq. in. E (steel)= 30E6 psi, E (concrete)= 3.75E6 psi. Units: in.



Cross-section

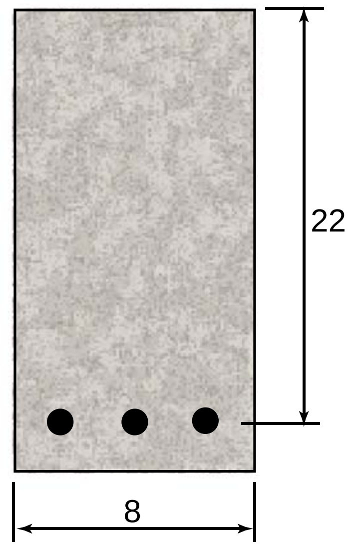


Cross-section

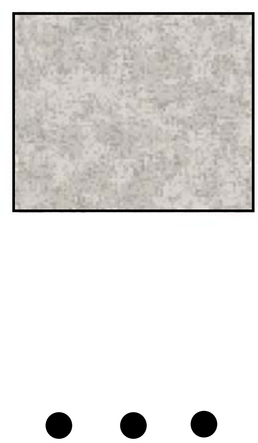
Example

Determine the required steel area for the beam to be balanced. Allowable stress in the steel and concrete are 33,000 and 3,000 psi respectively.

E (steel)= 29E6 psi, E (concrete)= 3.5E6 psi. Units: in.

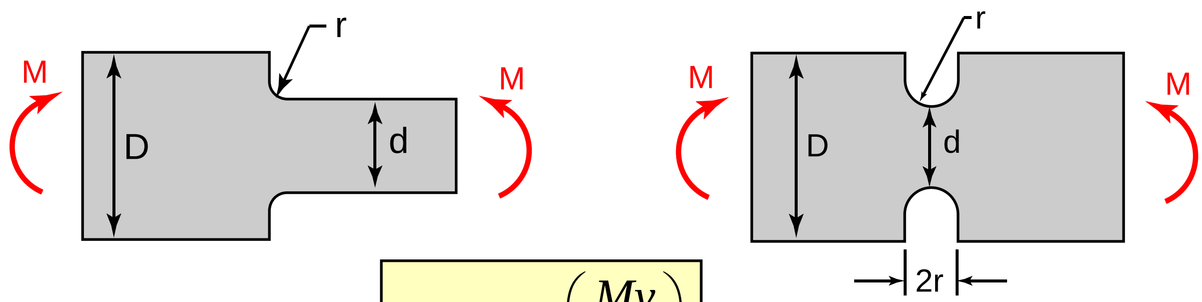


Cross-section

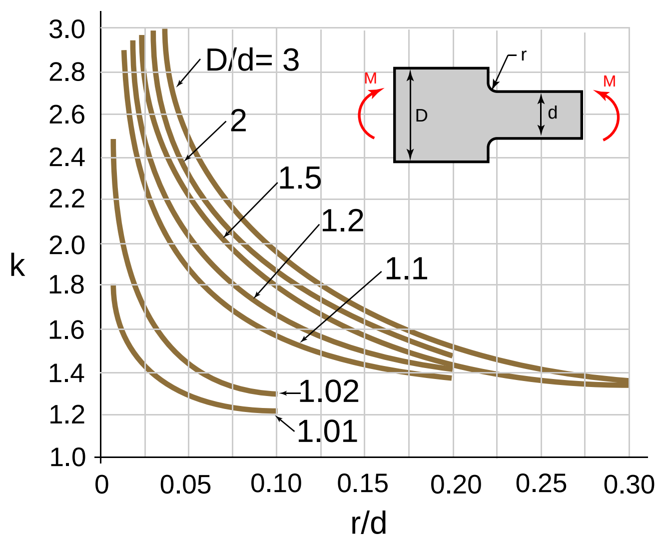


Cross-section

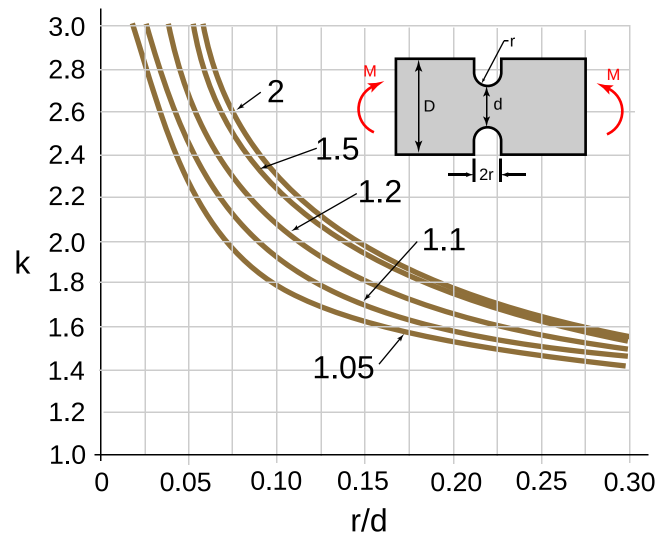
STRESS CONCENTRATIONS



$$\sigma_{max} = k\left(\frac{My}{I}\right)$$



Stress-concentration factors for flat bars with fillets under pure bending

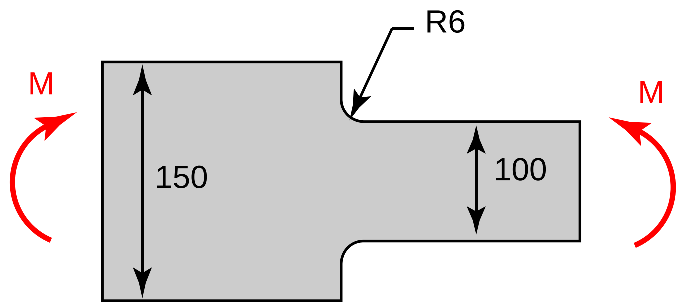


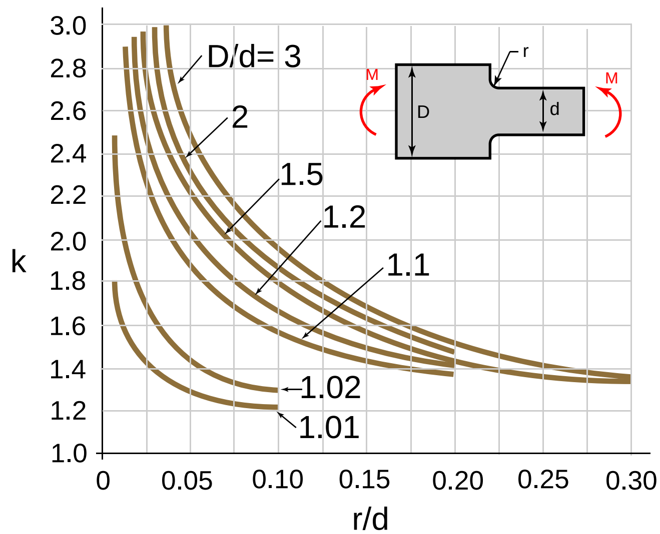
Stress-concentration factors for flat bars with groves under pure bending

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

For the 13 mm thick plate, determine the largest bending moment that can be applied if the allowable bending stress is 90 MPa. Units: mm.



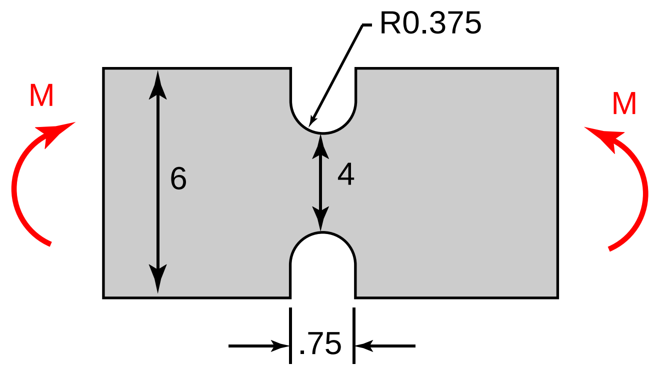


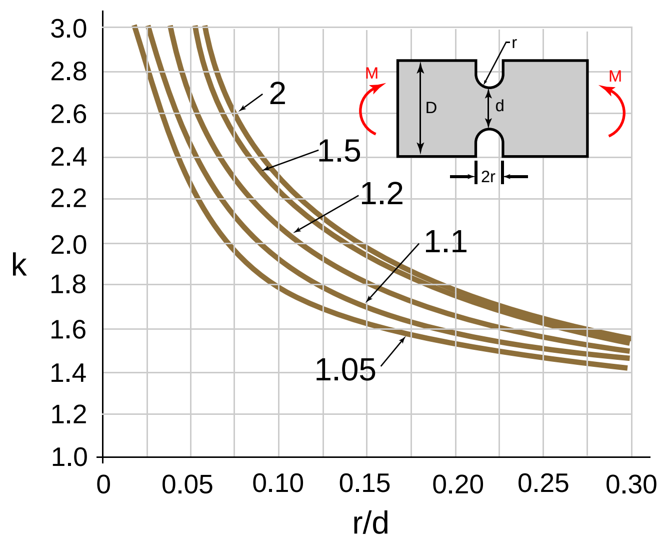
Stress-concentration factors for flat bars with fillets under pure bending

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

For the 7/8" thick plate, determine the largest bending moment that can be applied if the allowable bending stress is 24,000 psi. Units: in.

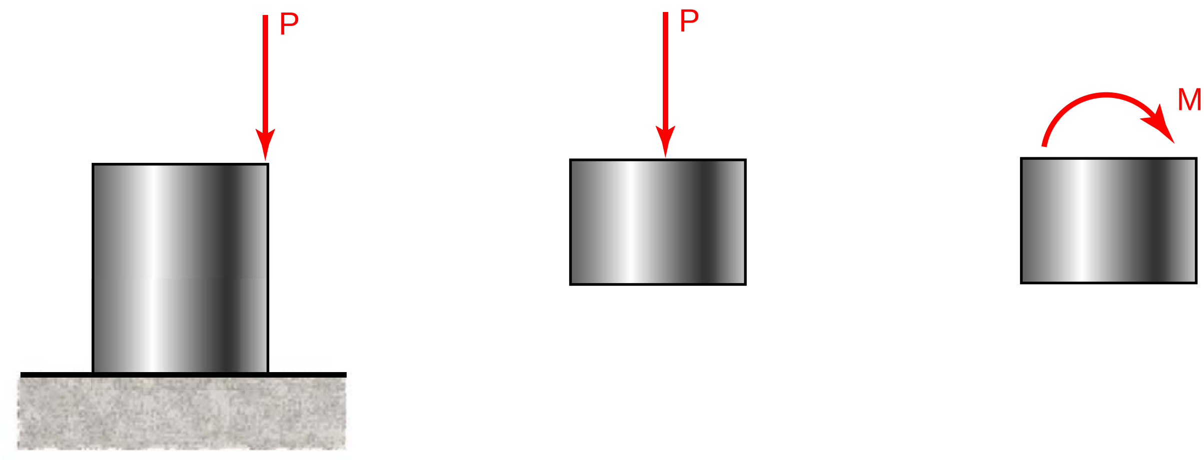




Stress-concentration factors for flat bars with groves under pure bending

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

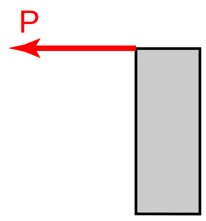


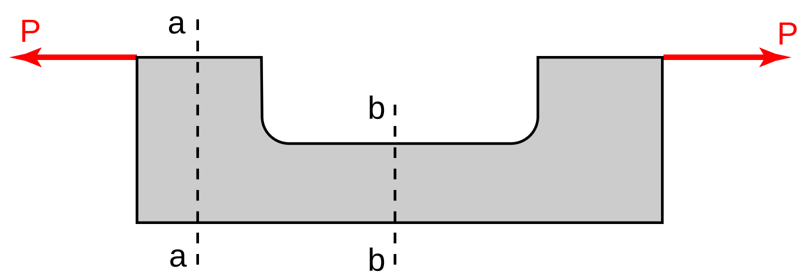
In general,

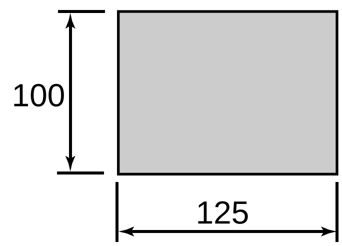
$$\sigma_x = \frac{P}{A} + \frac{My}{I}$$

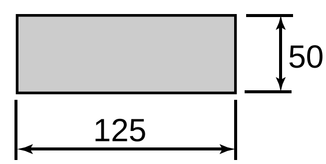
Example

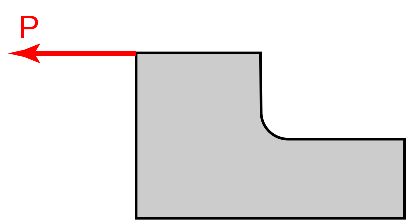
For the solid rectangular bar, determine the largest load P that can be applied based on a maximum normal stress of 130 MPa. Ignore any stress concentrations. Units: mm.





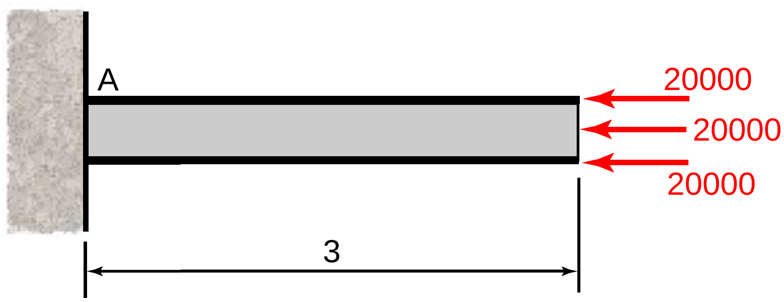


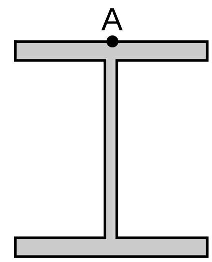




Example

The three loads are applied at the end of the W6x20 beam. Find the normal stress at the wall at point A for the beam, (a) if all three loads are applied, (b) the bottom load is removed. Units: lb, ft.





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

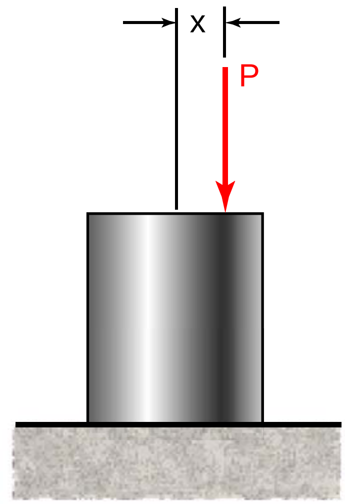
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

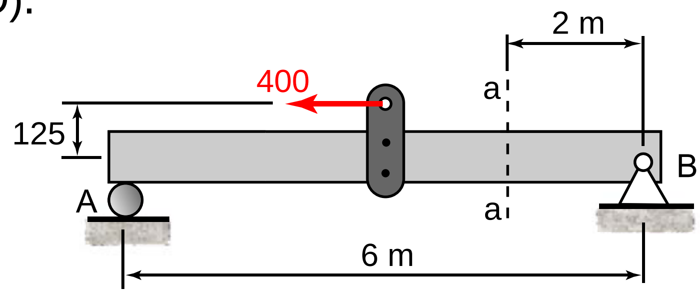
Example

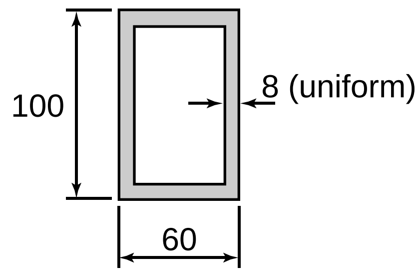
The 100 mm diameter solid circular bar has an eccentric load P applied. Determine the maximum location x that the load can be placed without inducing any tensile stresses. Units: mm.



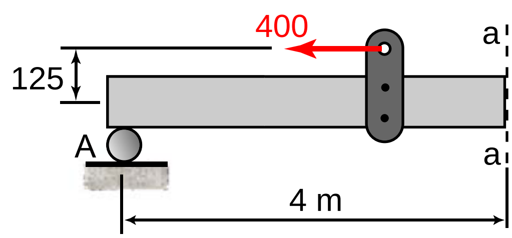
Example

Compute the maximum tension and compression stresses located at section a-a.   Units: N, mm (UNO).

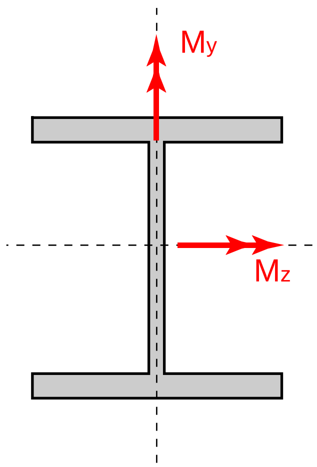




Section a-a

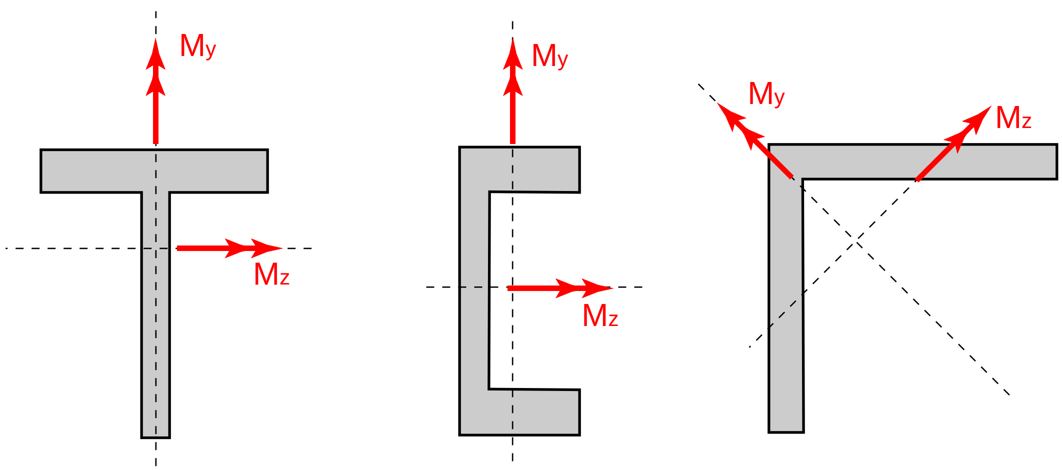


UNSYMMETRIC BENDING



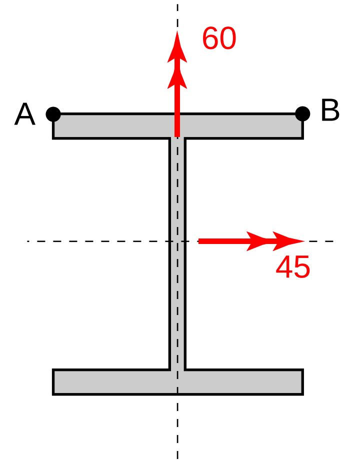
$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



Example

For the W6x20 section, determine the normal stresses at A and B. Units: kip•in.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

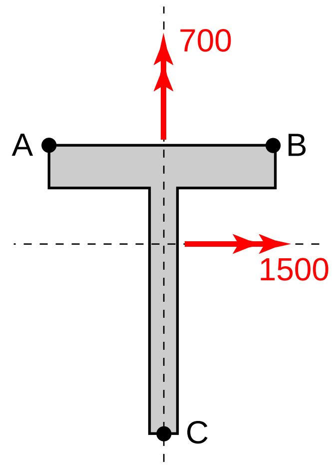
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

For the WT18x150 section, determine the normal stresses at A, B and C. Units: k•in.



WT18x150

Area, $A = 44.10^{in^2}$

Depth, $d = 18.4^{in}$

Flange Width, $b_f = 16.7^{in}$

Flange Thickness, $t_f = 1.68^{in}$

Web Thickness, $t_w = 0.945^{in}$

$I_x = 1230^{in^4}$

$I_y = 648^{in^4}$

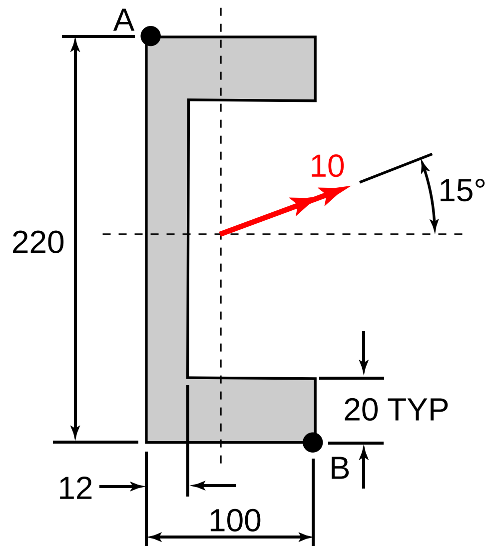
$S_x = 86.1^{in^3}$

$S_y = 77.8^{in^3}$

$\overline{y} = 4.13^{in}$

Example

For the channel section, determine the normal stresses at A and B.
Units: kN•m, mm.



Example

For the L76x76x12.7 angle section, determine the normal stresses at A and B. Units: N•m.

$\text{Area, A} = 1770^{mm^2}$

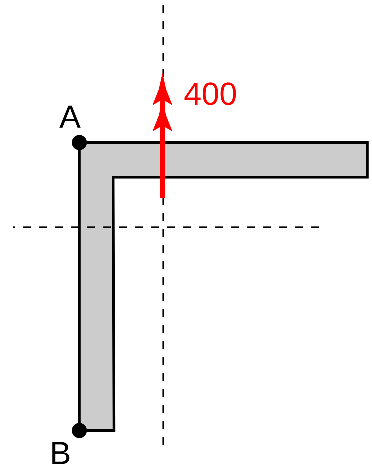
$d = b = 76^{mm}$

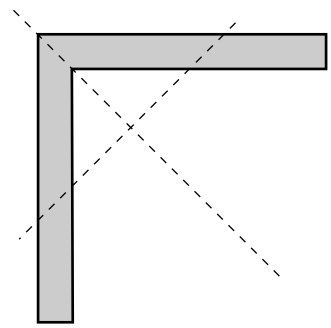
$\bar{x} = \bar{y} = 23.6^{mm}$

$\text{Thickness, t} = 12.7^{mm}$

$I_x = I_y = 0.915x10^{6 mm^4}$

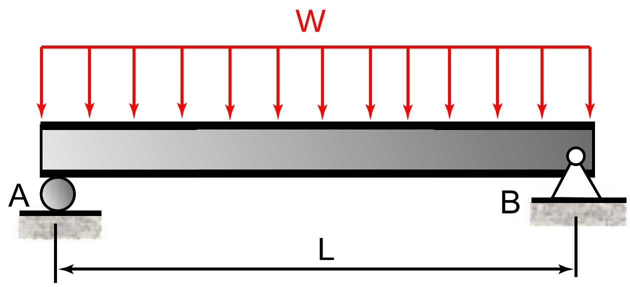
$r_z = 14.8^{mm}$





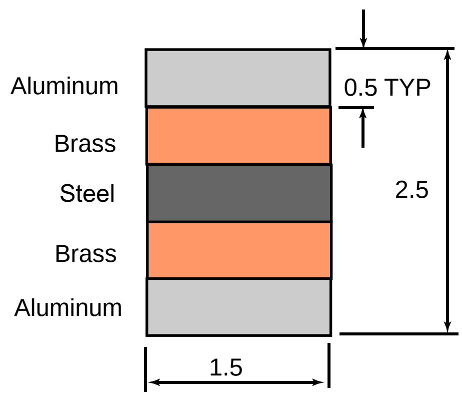
SUMMARY

Bending Stress



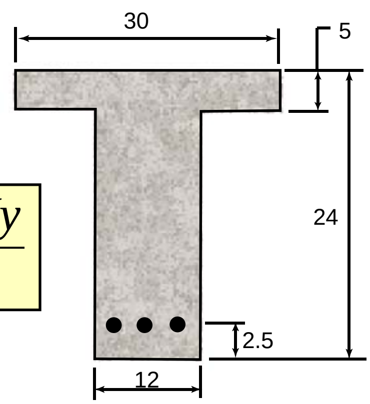
$$\sigma_x = -\frac{My}{I} = -\frac{M}{S}$$

Bending of Members made of Several Materials



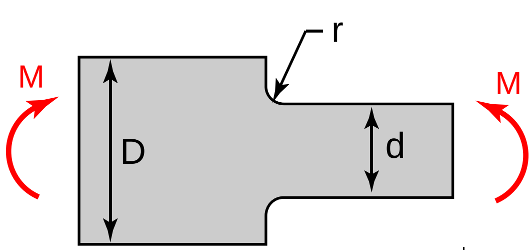
Cross-section, in

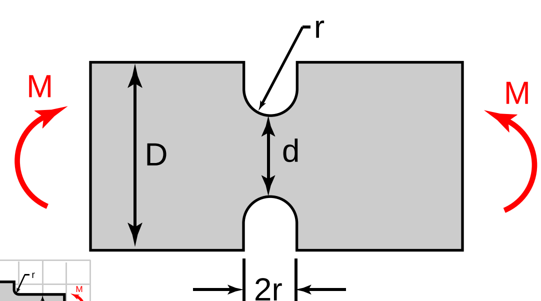
$$\sigma_x = -n\frac{My}{I}$$

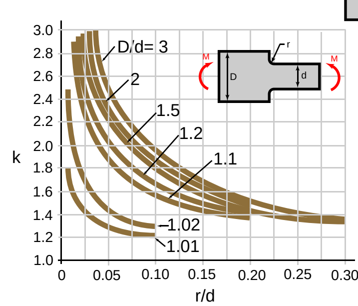


Cross-section, in

Stress Concentrations



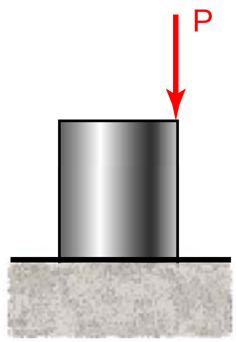




Stress-concentration factors for flat bars with fillets under pure bending

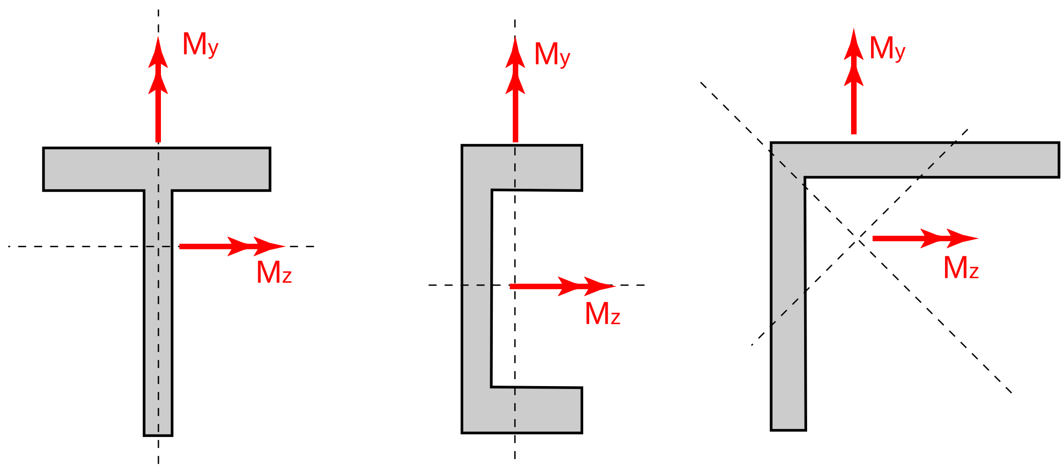
SUMMARY

Eccentric Axial Loading in a Plane of Symmetry



$$\sigma_x = \frac{P}{A} - \frac{My}{I}$$

Unsymmetric Bending



$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$