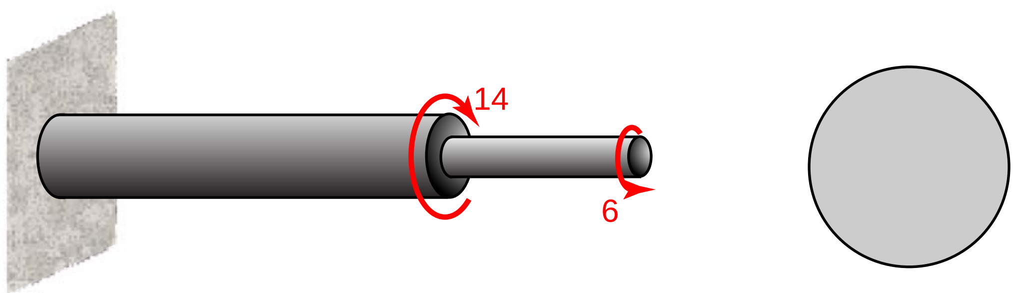
Chapter 3
Torsion

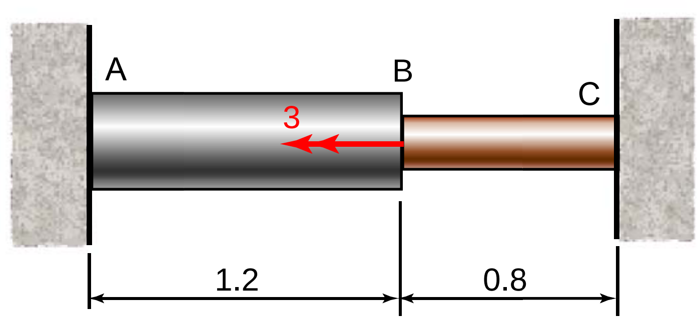
INTRODUCTION

Stresses in the Elastic Range

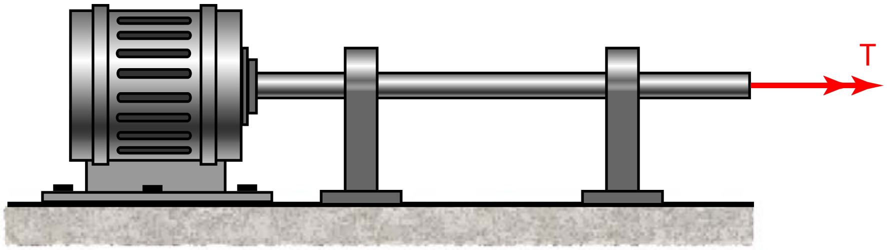
Angle of Twist in the Elastic Range



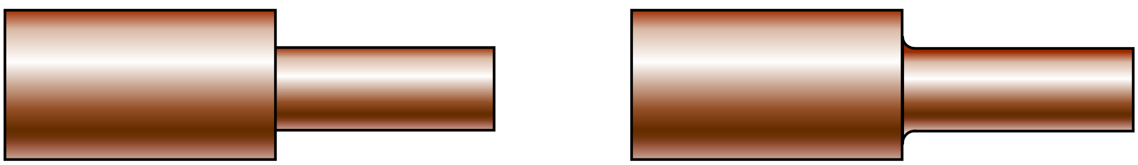
Statically Indeterminate Shafts



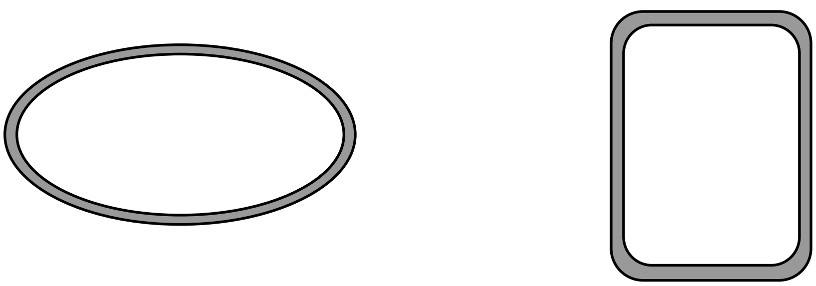
Design of Transmission Shafts



Stress Concentrations in Circular Shafts



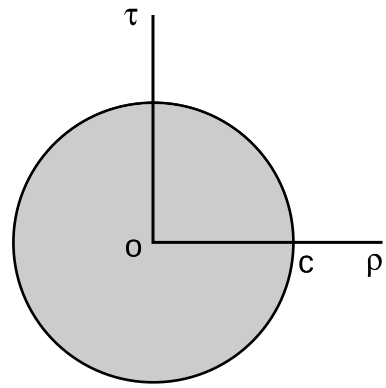
Thin-Walled Hollow Shafts

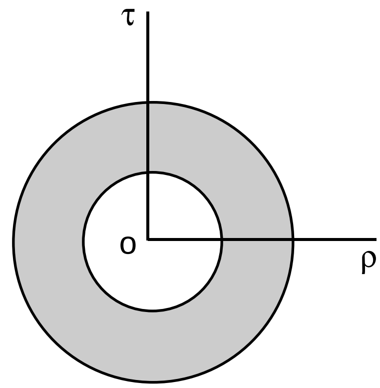


STRESSES IN THE ELASTIC RANGE

$$\tau_{\max} = \frac{Tr}{I_p} = \frac{Tc}{J}$$

$$\tau = \frac{T\rho}{I_p} = \frac{T\rho}{J}$$

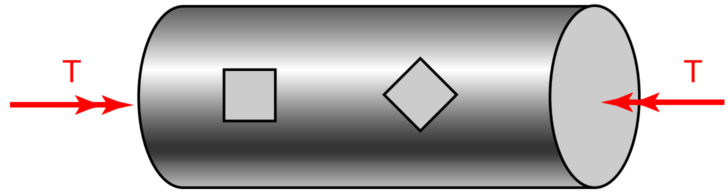




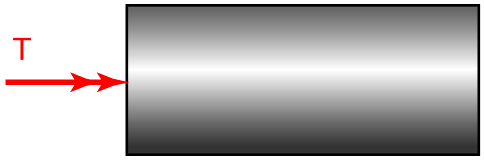
$$J = I_p = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

$$J = I_p = \frac{\pi}{32}(d_o^4 - d_i^4)$$

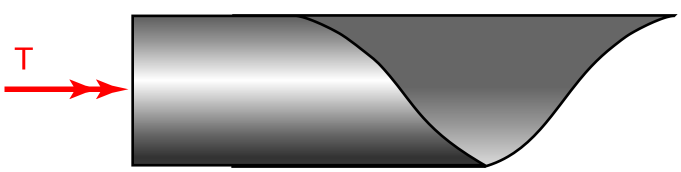
STRESSES ON INCLINED SECTIONS



$$\sigma = \tau_{max} = \frac{Tc}{J}$$



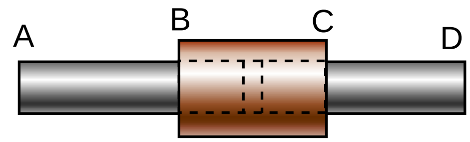
Failure due to shear stress.



Failure due to normal stress.

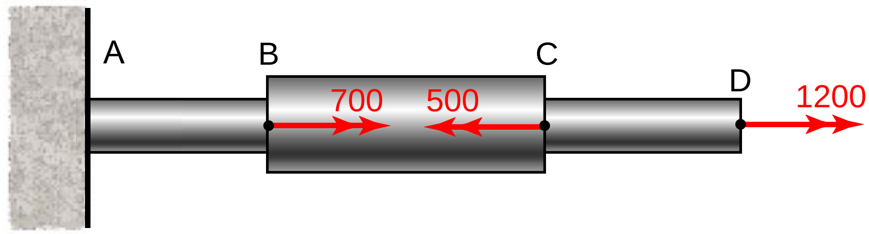
Example

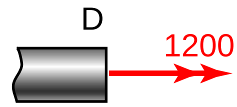
A copper coupling (BC) is used to connect two steel shafts (AB and CD). The diameter of the steel shaft is 25 mm, determine the outside diameter of the coupling so that the shear stress in it is half that of the steel shaft.
Units: kN, m.

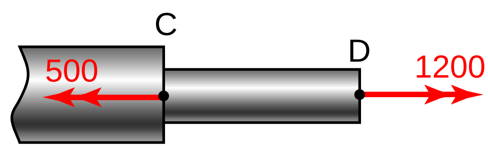


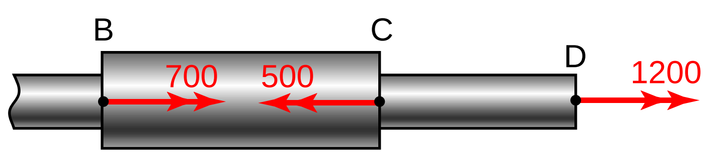
Example

Shafts AB and CD have an outside diameter of 0.75". Shaft BC has an outside diameter of 1.25". Determine the largest stress in ABCD.
Units: in-lb.



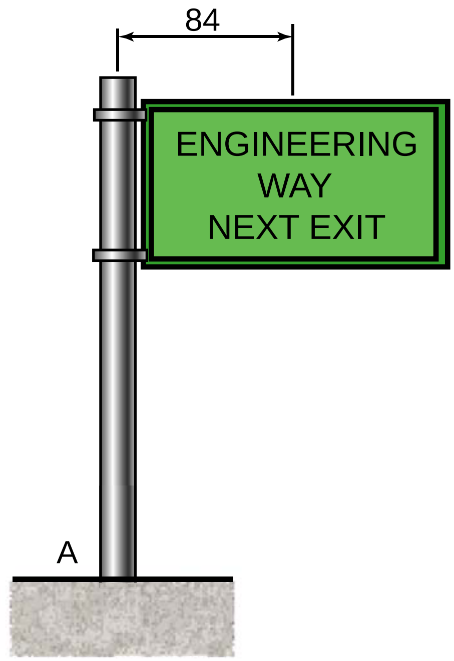






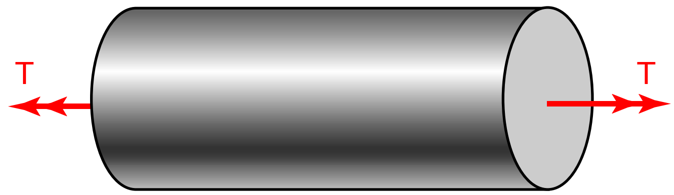
Example

The sign is subjected to a wind force of 5000 lb at it’s centroid 84" from the center of the column. The column has an outside diameter of 10" and a wall thickness of 0.125". Considering only this force, determine the torsional shear stress at A. Units: in.



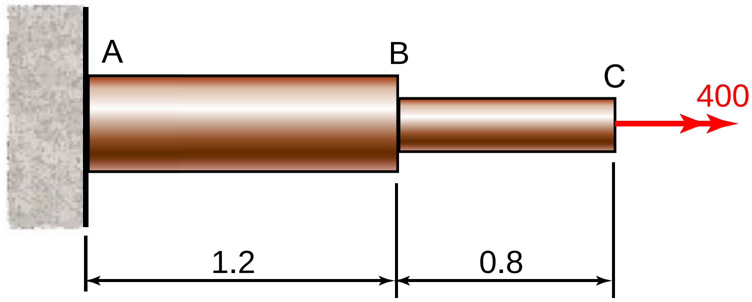
ANGLE OF TWIST IN THE ELASTIC RANGE

$$\phi = \frac{TL}{GJ}$$



Example

Shaft AB has a diameter of 60 mm. Determine the diameter of BC for which the displacement of point C will be 1.5°. G= 38 GPa.
Units: N•m, m.

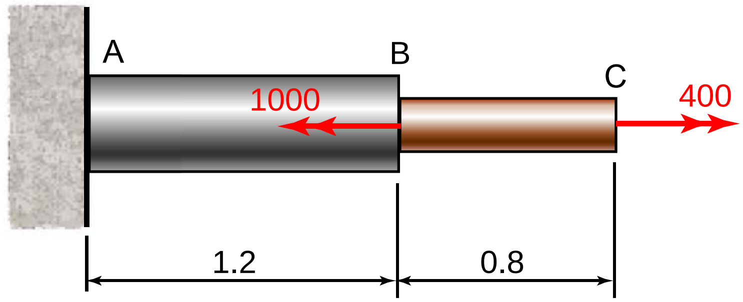


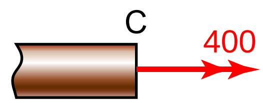
Example

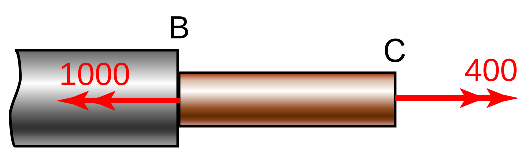
The steel shaft AB has a diameter of 60 mm and the copper shaft BC has a diameter of 45 mm. Determine the rotation of points B and C. Units: N•m, m.

G (steel)= 77.2 GPa

G (copper)= 44 GPa.





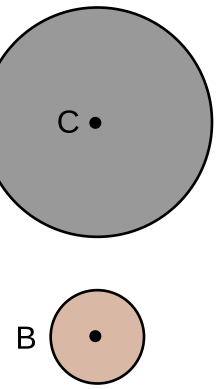


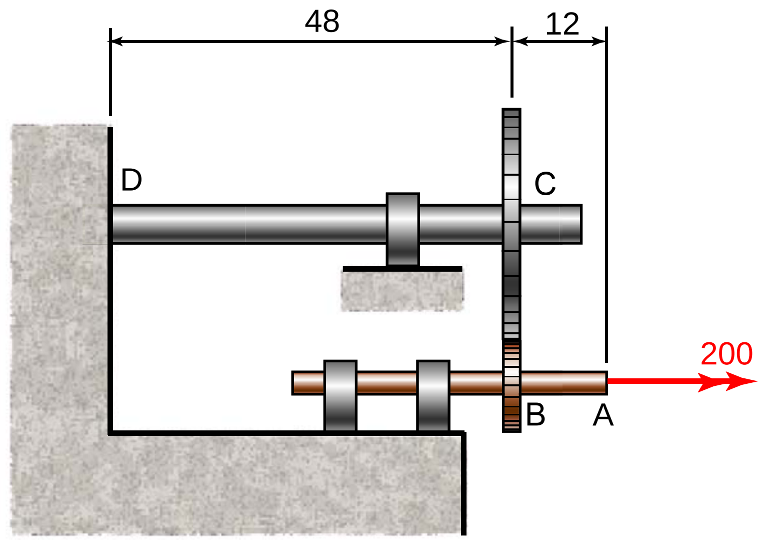
Example

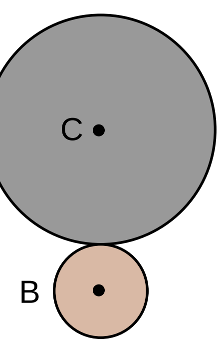
The copper shaft AB has a diameter of 1" and the steel shaft CD has a diameter of 1.75". The two shafts are connected by a 4" diameter gear at B and a 10" diameter gear at C. Point D is welded to the wall. Determine the rotation of point A. Units: lb•ft, in.

G (steel)= 11.2E6 psi

G (copper)= 6.4E6 psi

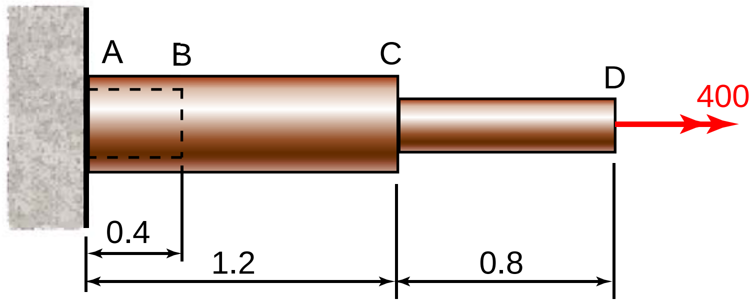






Example

Shaft ABC has a diameter of 60 mm and shaft CD has a diameter of 45 mm. The first 0.4 m of AB is hollow with a wall thickness of 4 mm. Determine the rotation of point D. G= 38 GPa.
Units: N•m, m.



STATICALLY INDETERMINATE SHAFTS

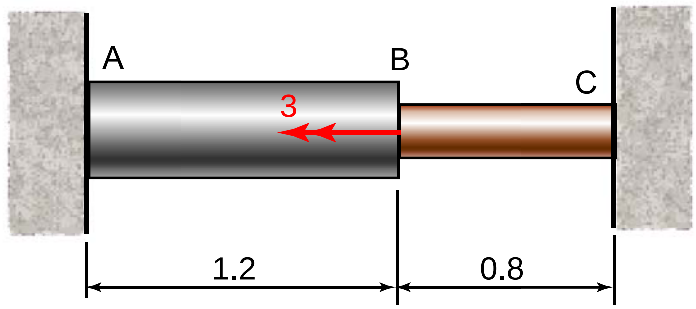
Example

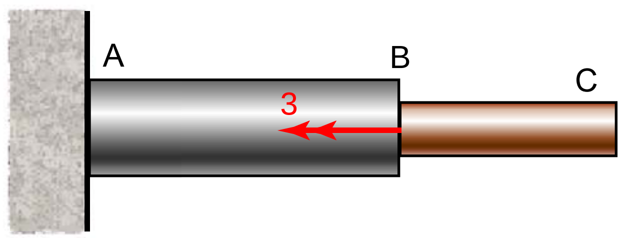
The steel shaft AB has a diameter of 60 mm and the copper shaft BC has a diameter of 45 mm. Determine the reactions at A and C.

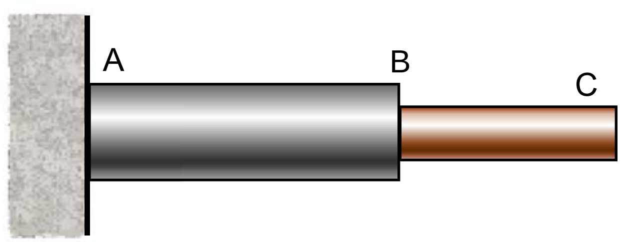
Units: kN•m, m.

G (steel)= 77.2 GPa

G (copper)= 44 GPa.





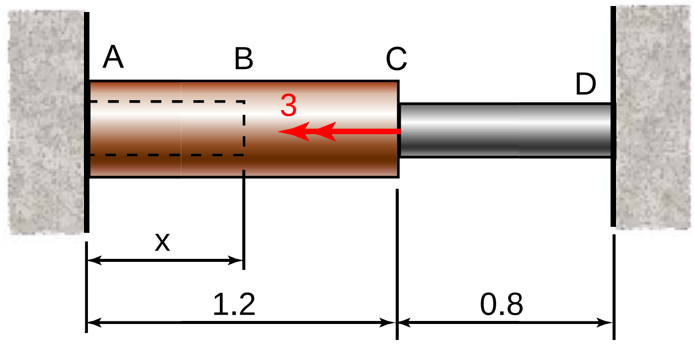


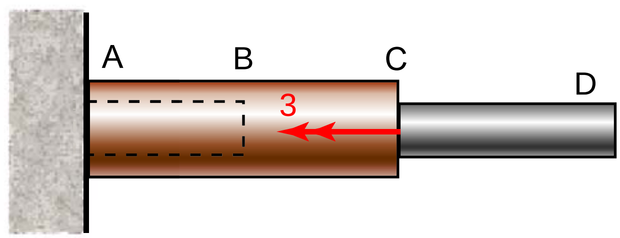
Example

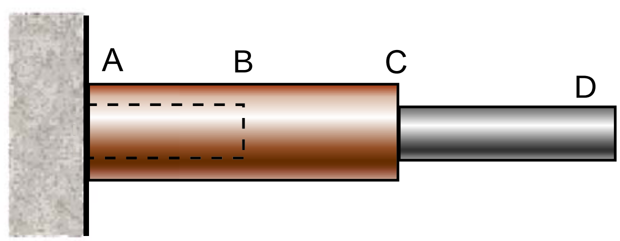
The copper shaft ABC is hollow between A and B, and solid between B and C. Shaft ABC has an outer diameter of 60 and an inside diameter between A and B of 52 mm. The solid steel shaft CD has a diameter of 45. Determine the length x so that the reactions at A and D are equal. Units: kN•m, m.

G (steel)= 77.2 GPa

G (copper)= 44 GPa.







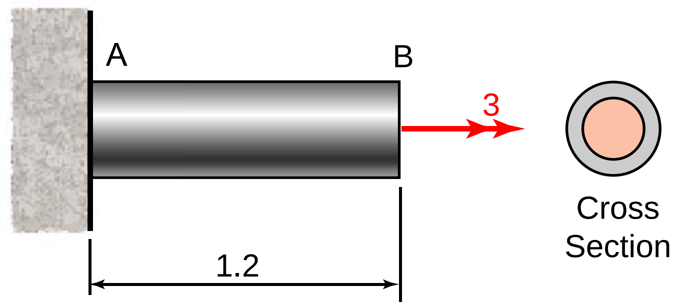
Example

The shaft AB has an outer shell of steel and an inner core of copper. The outside diameter of the steel shaft is 30 mm and the copper core has a diameter of 20 mm. The two materials are firmly connected along their lengths. Determine the torque in each material.

Units: kN•m, m.

G (steel)= 77.2 GPa

G (copper)= 44 GPa.



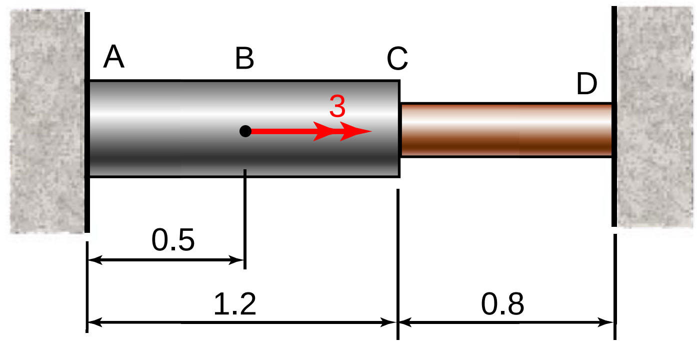
Example

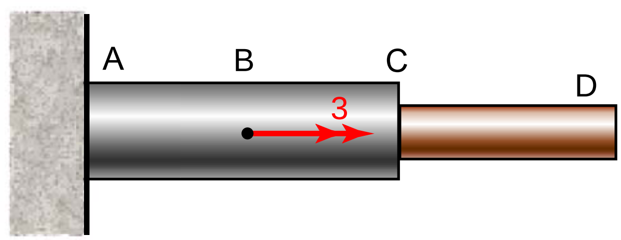
The steel shaft ABC has a diameter of 60 and the copper shaft CD has a diameter of 45 mm. Determine the reactions at A and D.

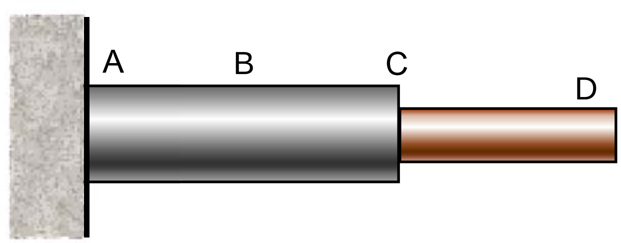
Units: kN•m, m.

G (steel)= 77.2 GPa

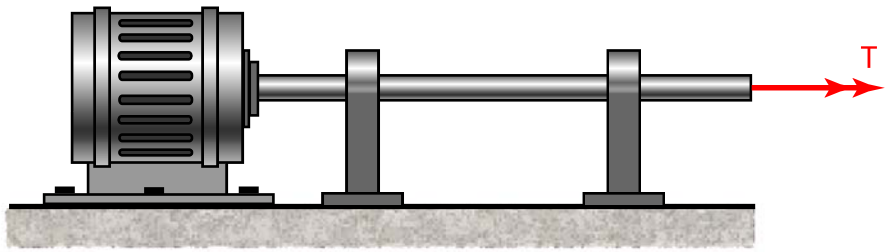
G (copper)= 44 GPa.







DESIGN OF TRANSMISSION SHAFTS



$$P = T\omega$$

$$P = 2\pi fT$$

$$T = \frac{P}{2\pi f}$$

$$T = \frac{60P}{2\pi n} = \frac{30P}{\pi n}$$

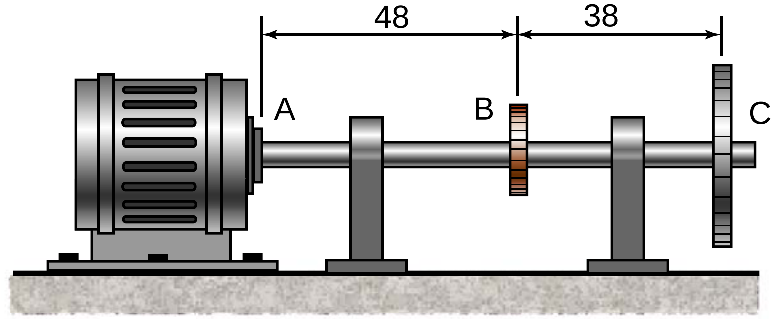
$$H = Horsepower = \frac{2\pi nT}{33,000}$$

Example

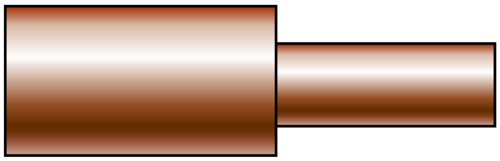
A pipe is designed to transmit 90 kW at 15 Hz. The inside diameter of the shaft is to be three-fourths of the outer diameter. a) Calculate the minimum required diameter d if the maximum shear stress is 50 MPa. b) Find the diameter if the allowable normal stress is 65 MPa.

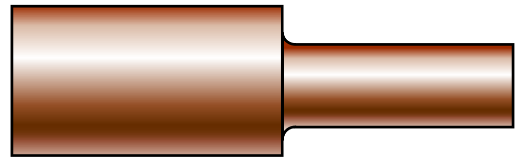
Example

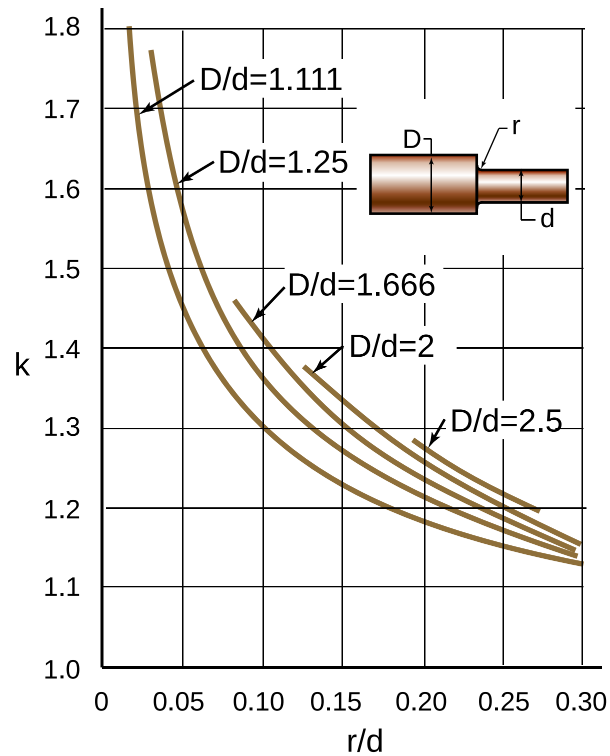
The motor generates 150 hp at 100 rpm and gears B and C consume 100 and 50 hp respectively. Determine the diameter of the solid uniform shaft if the maximum shear stress is limited to 14,000 psi and the maximum rotation at C is 1.75°. G= 11.2E6 psi. Units: lbs, in.



STRESS CONCENTRATIONS IN CIRCULAR SHAFTS





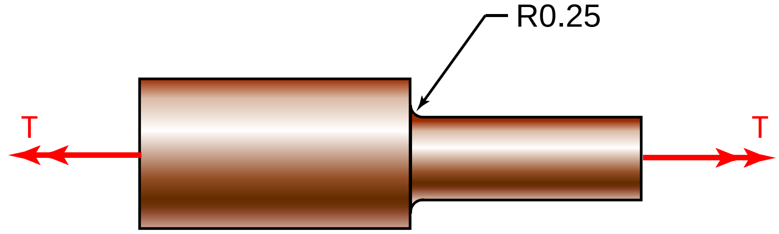


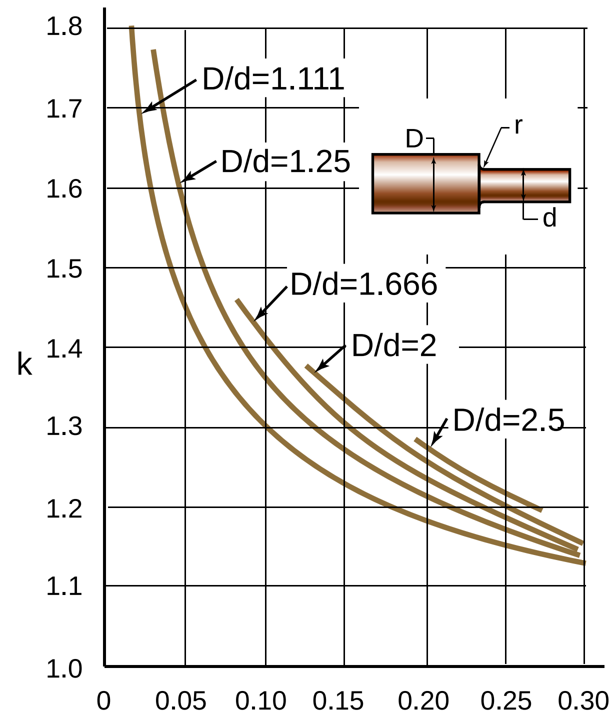
Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

Determine the maximum torque that can be applied if the allowable shear stress is 9500 psi. The diameters of the shafts are 2.06 and 1.25 in. Units: lbs, in.



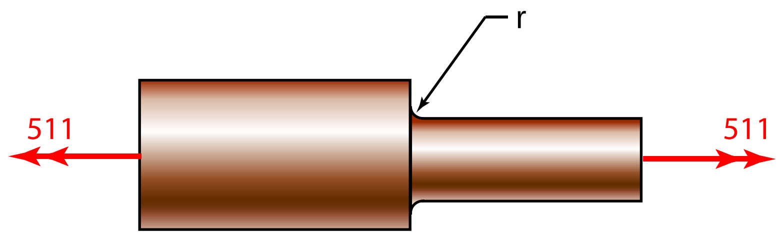


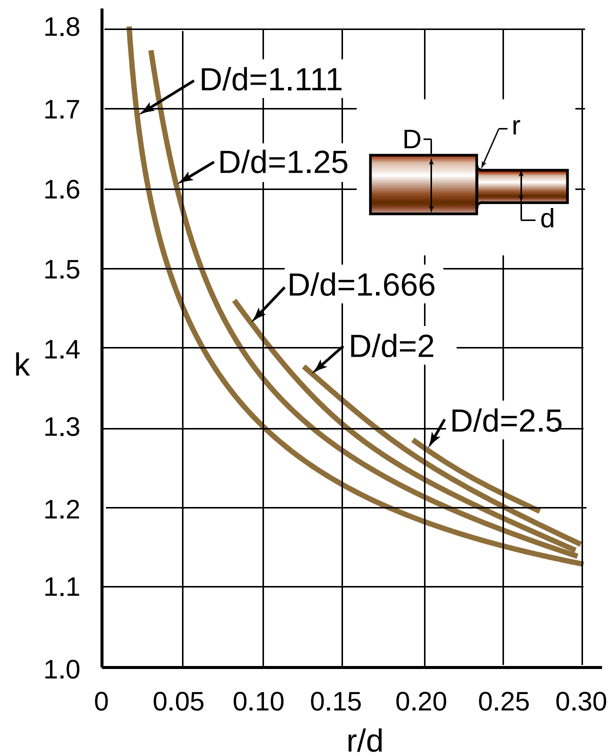
Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

Determine the smallest fillet size if the allowable shear stress is 135 MPa. The diameters of the shafts are 50 and 30 mm. Units: N•m.





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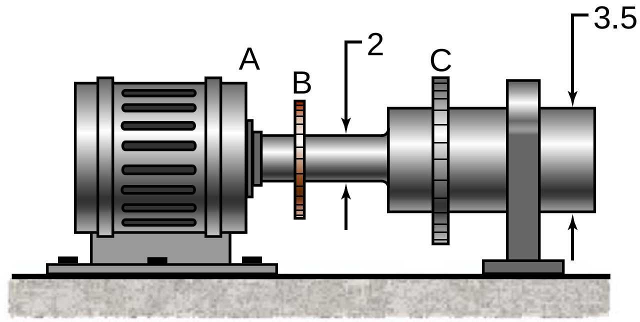
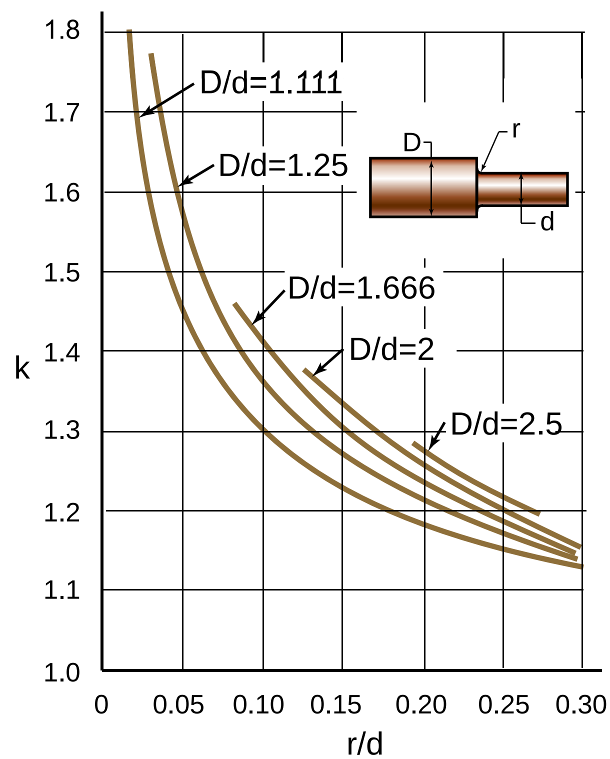
Example

The motor generates 2,000 ft-lb of torque to the shaft at A. The gears at B, and C consume 500, and 1,500 ft-lb respectively. Determine the maximum shear stress at the .25" fillet between B and C.
Units: in.





Stress-concentration factors for fillets in circular shafts

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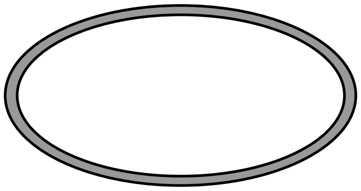
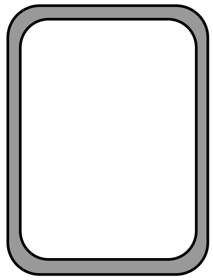
THIN-WALLED HOLLOW SHAFTS





$$\tau = \frac{T}{2tA_m}$$

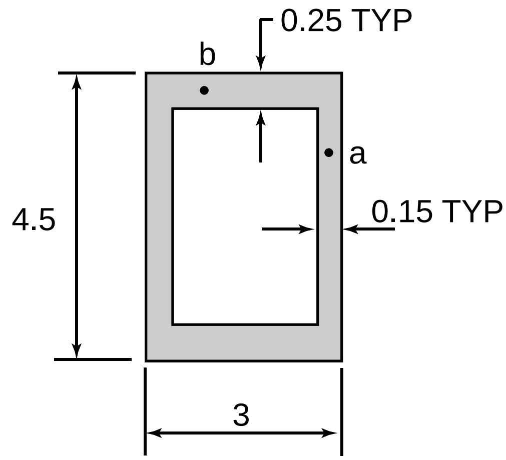
$$\phi = \frac{TL}{4A^2G}\oint\frac{ds}{t} = \frac{TL}{GJ}$$

Where

$$J = \frac{4A_m^2}{\sum \frac{L_m}{t}}$$

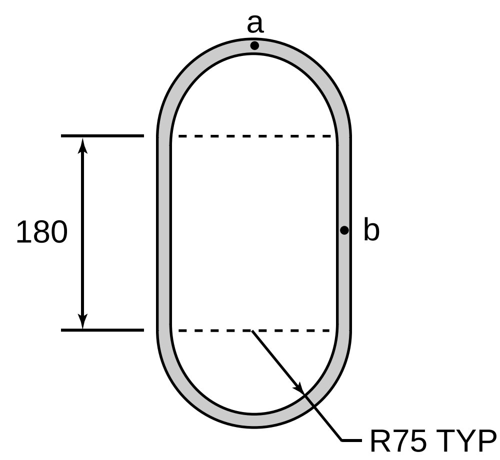
Example

A 45 kip-in torque is applied to the hollow shaft. Neglecting stress concentrations, determine the shear stress at points a and b.
Units: in.



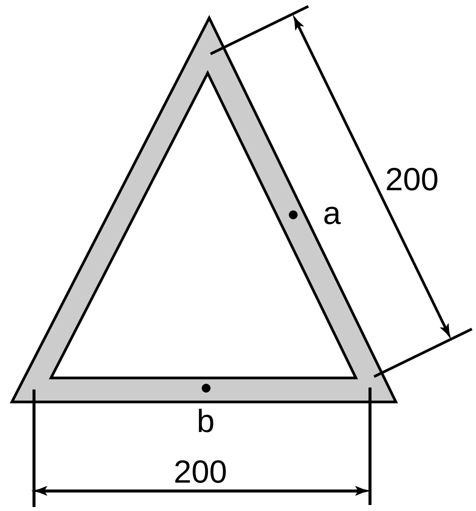
Example

A 70 kN•m torque is applied to the hollow 10 mm uniform shaft. Neglecting stress concentrations, determine the shear stress at points a and b. Units: mm.



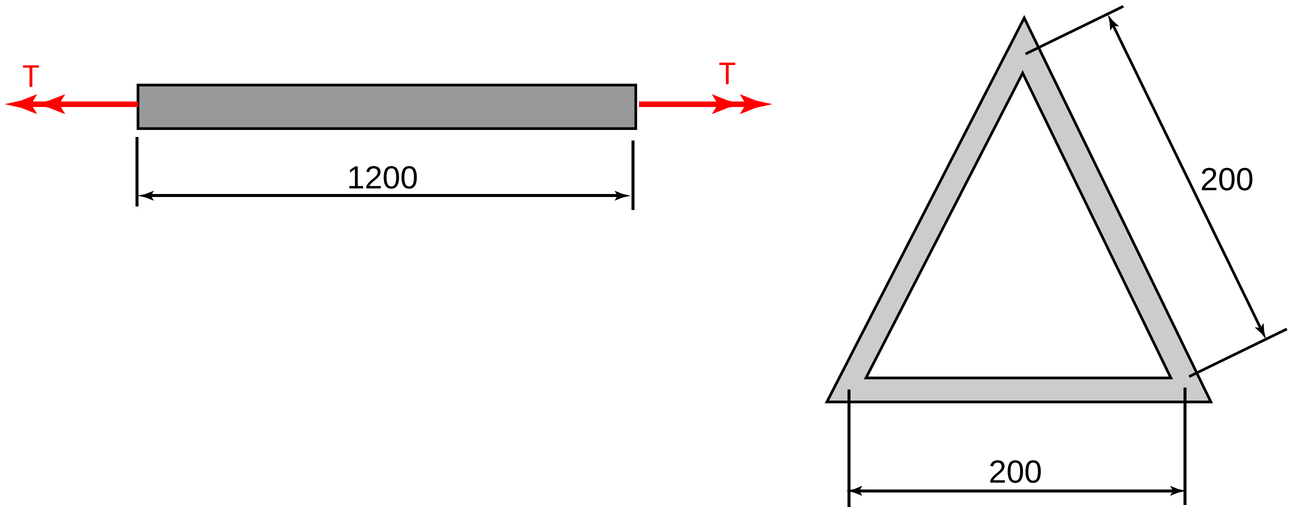
Example

A 17 kN•m torque is applied to the hollow 8 mm uniform shaft. Neglecting stress concentrations, determine the shear stress at points a and b. Units: mm.



Example

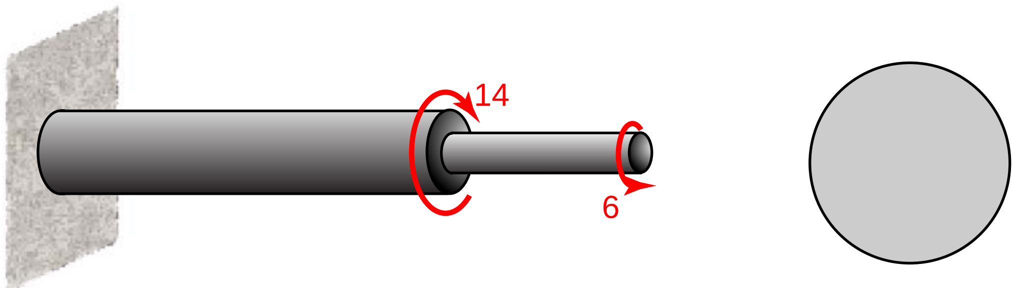
A 17 kN•m torque is applied to the hollow 8 mm uniform shaft. Determine the rotation of the 1200 mm shaft.
G= 77.2 GPa. Units: mm.



SUMMARY

Stresses in the Elastic Range

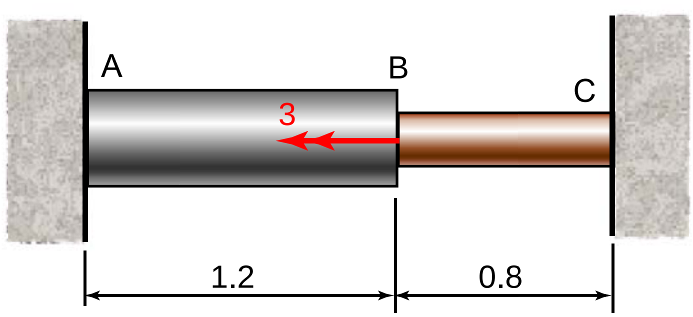
Angle of Twist in the Elastic Range



$$\tau_{max} = \frac{Tr}{I_p} = \frac{Tc}{J}$$

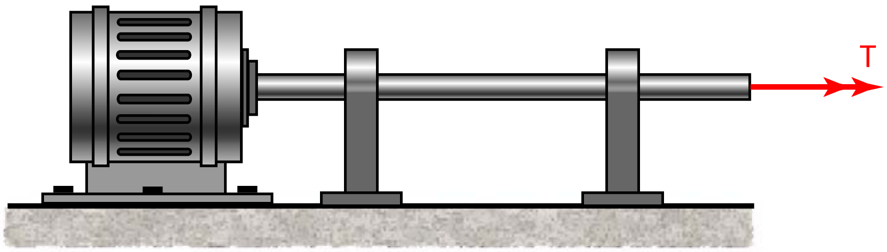
$$\phi = \frac{TL}{GJ}$$

Statically Indeterminate Shafts



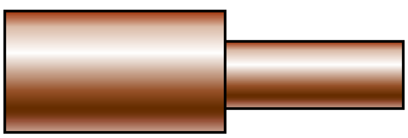
SUMMARY

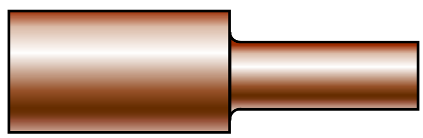
Design of Transmission Shafts

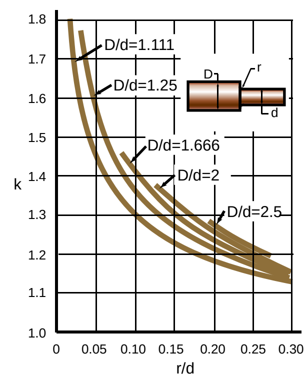


$$T = \frac{P}{2\pi f}$$

Stress Concentrations in Circular Shafts



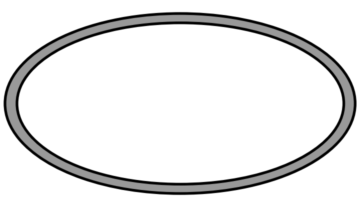


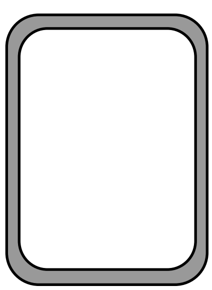


Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Thin-Walled Hollow Shafts





$$\tau = \frac{T}{2tA_m}$$

$$\phi = \frac{TL}{4A^2G}\oint\frac{ds}{t} = \frac{TL}{GJ}$$

Where

$$J = \frac{4A_m^2}{\sum \frac{L_m}{t}}$$