COLUMNS WITH PINNED-ENDS

\[ P_{cr} = \frac{\pi^2 EI}{L^2} \]
\[ \sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} \]
Example

a) Using a factor of safety of 2.5 against buckling, determine the largest load the column can support before it begins to buckle. Consider only in-plane buckling. b) Find the maximum load if the allowable axial stress is 80 MPa. The pipe has an outside diameter of 100 mm and a wall thickness of 6 mm. E=200 GPa. Units: m.
Example
Using a factor of safety of 1.85, determine the largest load the W6x20 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. E = 29E6 psi. Units: ft.

W6x20

Area, $A = 5.87 \text{in}^2$
Depth, $d = 6.20 \text{in}$
Flange Width, $b_f = 6.02 \text{in}$
Flange Thickness, $t_f = 0.365 \text{in}$
Web Thickness, $t_w = 0.260 \text{in}$
$I_x = 41.4 \text{in}^4$
$I_y = 13.3 \text{in}^4$
$S_x = 13.4 \text{in}$
$S_y = 4.41 \text{in}$
Example
Both members are identical pipe sections with an outside diameter of 100 mm and a wall thickness of 6 mm. Determine the largest load $P$ based on in-plane buckling. $E = 200 \text{ GPa}$. Units: m.
Example
Find the critical buckling load for a 28 ft pin-pin column. The two W6x20 columns are spliced together to insure they work as one. Ignore the properties of the plates used to make the splice. 
E = 30E6 psi. Units: ft.

![Diagram of W6x20 column]

W6x20
Area, A = 5.87\text{in}^2
Depth, d = 6.20\text{in}
Flange Width, b_f = 6.02\text{in}
Flange Thickness, t_f = 0.365\text{in}
Web Thickness, t_w = 0.260\text{in}
I_x = 41.4\text{in}^4
I_y = 13.3\text{in}^4
S_x = 13.4\text{in}^3
S_y = 4.41\text{in}^3
Example
Determine the radius of a round column so that it has the same buckling capacity as that of a square 30 mm column. Both columns are identical other than their cross section. Units: mm.
COLUMNS WITH OTHER END CONDITIONS

\[ P_{cr} = \frac{\pi^2 EI}{4L^2} \]
Effective Length

\[ P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 EI}{(L_e)^2} \]

Both ends pinned. 
\( K = 1 \)

One end fixed, one end free. 
\( K = 2 \)

One end fixed, one end pinned. 
\( K = 0.7 \)

Both ends fixed. 
\( K = 0.5 \)
Example
Determine the K value for each of the following conditions:

(a) (b) (c) (d)
Example
Determine the largest load the W6x20 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. E = 29E6 psi. Units: ft.

Front View
Side View

W6x20

Area, A = 5.87 in²
Depth, d = 6.20 in
Flange Width, bₕ = 6.02 in
Flange Thickness, tₕ = 0.365 in
Web Thickness, tₕ = 0.260 in
Iₓ = 41.4 in⁴
Iᵧ = 13.3 in⁴
Sₓ = 13.4 in
Sᵧ = 4.41 in
Example
Determine the largest load the W150x29.8 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. $E = 200 \text{ GPa}$. Units: m.

**W150x29.8**

- Area, $A = 3790 \text{ mm}^2$
- Depth, $d = 157 \text{ mm}$
- Flange Width, $b_f = 153 \text{ mm}$
- Flange Thickness, $t_f = 9.3 \text{ mm}$
- Web Thickness, $t_w = 6.6 \text{ mm}$
- $I_x = 17.2 \times 10^6 \text{ mm}^4$
- $I_y = 5.56 \times 10^6 \text{ mm}^4$
- $S_x = 219 \times 10^3 \text{ mm}^3$
- $S_y = 72.7 \times 10^3 \text{ mm}^3$
Example
Determine the largest load the W150x29.8 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. E = 200 GPa. Units: m.

**W150x29.8**

- Area, $A = 3790 \text{mm}^2$
- Depth, $d = 157 \text{mm}$
- Flange Width, $b_f = 153 \text{mm}$
- Flange Thickness, $t_f = 9.3 \text{mm}$
- Web Thickness, $t_w = 6.6 \text{mm}$
- $I_x = 17.2 \times 10^6 \text{mm}^4$
- $I_y = 5.56 \times 10^6 \text{mm}^4$
- $S_x = 219 \times 10^3 \text{mm}^3$
- $S_y = 72.7 \times 10^3 \text{mm}^3$
SUMMARY

Both ends pinned.  
$K = 1$

One end fixed, one end free.  
$K = 2$

One end fixed, one end pinned.  
$K = 0.7$

Both ends fixed.  
$K = 0.5$

Column $K$ values

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 EI}{(L_e)^2}$$