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Chapter 1
Kinematics of Particles

INTRODUCTION

Particle
A particle has mass but does not have dimensions.

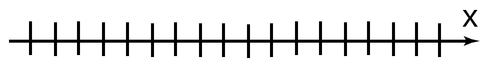
Rigid Body
A rigid body has mass and dimension, and the distance between any two points on the body remain constant.

Kinematics
Kinematics studies the motion of a body or particle without regard to the forces causing the motion.

Kinetics
Kinetics studies the motion of a body or particle and how it is influenced by forces.

POSITION, VELOCITY, AND ACCELERATION

Position



Position = x(t)

Velocity

$$v_{ave} = \frac{\Delta x}{\Delta t}$$

$$v = \frac{dx}{dt}$$

Acceleration

$$a = \frac{dv}{dt}$$

$$a = \frac{dv}{dx}\frac{dx}{dt}$$

$$a = v\frac{dv}{dx}$$

Solving Rectilinear Motion Problems by Differentiation

When *x(t)* is given, *v(t)* can be found by direct differentiation of *x* with respect to time.

When *v(t)* is known, *a(t)* can be found by direct differentiation of *v* with respect to time.

Example

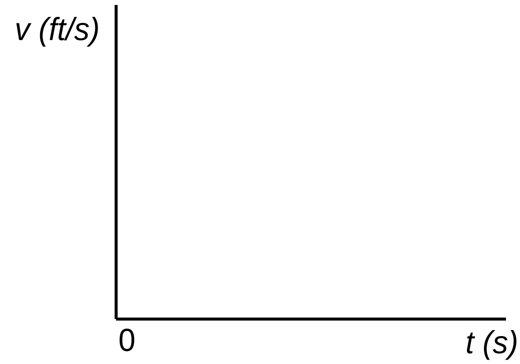
If particle A has a position which varies with time according to the relation $x = 2t^2 + 3t$, determine the following at a time of 3 seconds.
Units: m, sec:

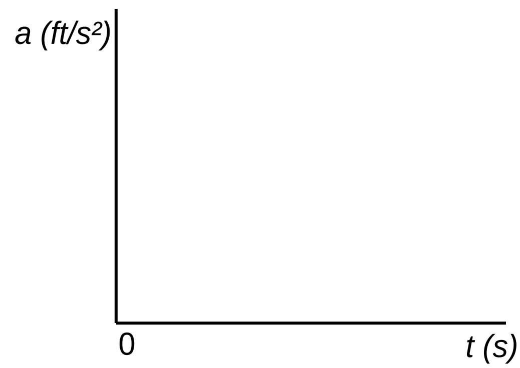
a) What is the position?
b) What is the velocity?

Example

The velocity of an object is given by the function below. Find the maximum velocity, maximum acceleration, and the times at which each occurs between 0 and 16 seconds. Units: Feet, seconds.

$$v = 5t^2 - \frac{t^3}{3}$$





Solving Rectilinear Motion Problems by Integration

If *a* is given as a function of *v*, *x*, or *t*, you may need to integrate to find other quantities.

Typically 3 variables will be involved (from *a*, *x*, *v*, or *t*) . From the equations below, choose the equation which involves the 3 variables of interest.

$$v = \frac{dx}{dt}$$

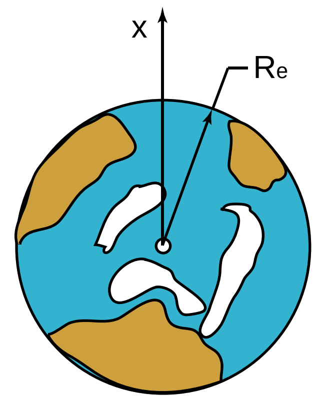
$$a = \frac{dv}{dt}$$

$$a = v\frac{dv}{dx}$$

Example

The earth's escape velocity is given below, where R_e is the earth's radius, and x is the distance to the earth's center. For a satellite which is launched from sea-level at the escape velocity, find the time for it to reach an altitude of 50,000 ft.

$$v_e = R_e \sqrt{\frac{2g}{x}} \quad \text{where: } R_e = 2.09x10^7 \text{ ft}$$



Example

A boat is moving at a speed of 4 m/s, when it shuts down its engine. The boat decelerates according to the equation below, where the acceleration is in m/s^2 and velocity is in m/s.

Determine the following:

a) How long will it take for the boat to reach a speed of 1 m/s?

b) How far will the boat go before its speed reaches 1 m/s?

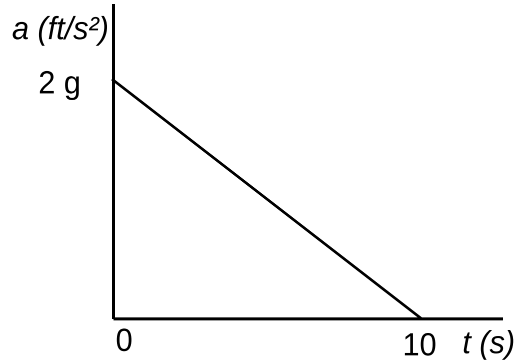
$$a = \frac{-0.1}{v^2}$$

Example

An object moving through heavy oil decelerates due to viscous drag. The deceleration, which is proportional to its velocity can be written as $a = -cv$. Find the velocity as a function of time and c.

Example

A vehicle starts from rest, then accelerates with time (see diagram below). Determine the maximum speed reached by the vehicle, and the distance traveled at the position of maximum speed.



UNIFORM RECTILINEAR MOTION

Velocity and Acceleration Equations

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} \qquad a = v\frac{dv}{dx}$$

Position Equation for *Constant Velocity*

$$x = x_0 + vt$$

Position, Velocity and Acceleration Equations for *Constant Acceleration*

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Example

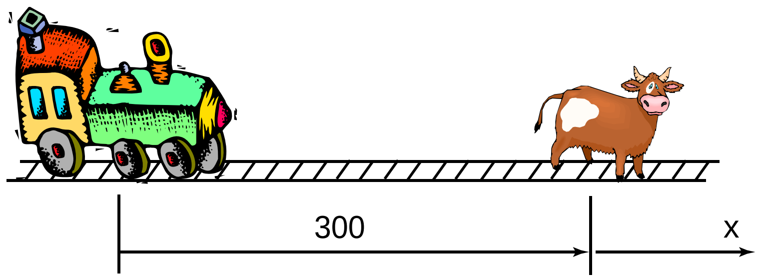
Show that $v_f - v_0 = a(t_f - t_0)$ and $v_f^2 = v_0^2 + 2a(x_f - x_0)$ if acceleration is constant.

Example

You throw a tennis ball to a friend on the second floor 15 feet above you. What should the initial velocity of the tennis ball have to just reach your friend?

Example

A train engineer traveling at 100 km/hr sees a cow standing on the tracks 300 m away. If the engineer decelerates at 0.75 m/s^2 after a reaction time of 0.2 sec and the cow leaves the road after 5 sec, does the train hit the cow? Units: meters, seconds.

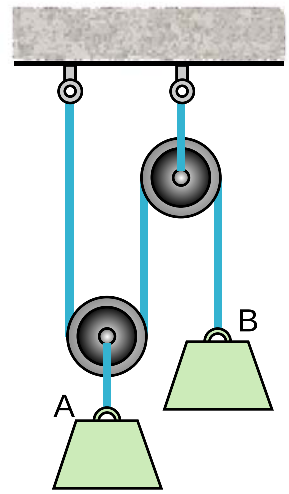


MOTION OF SEVERAL PARTICLES

Motion is constrained when the motion of one particle depends on the motion of one or more additional particles.

Example

Find the relationships between the velocities and accelerations of the particles in the system:



1. Determine the number of constraints.

2. Define fixed DATUM line for measuring positions of particles.

3. Draw position variables from DATUM to each particle.

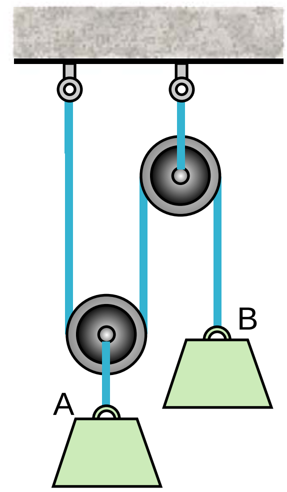
4. Create constraint equation(s) by defining the length of the constraints in terms of the variables you defined.

5. Differentiate constraint equations with respect to time to get velocity and acceleration relationships.

 # Degrees of Freedom= # Position variable - # Constraints

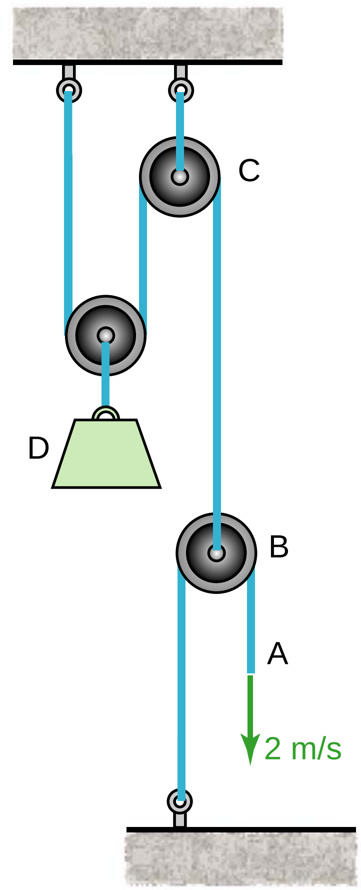
Example

If weight B is moving downward with a velocity of 4 m/s, which is decreasing at a rate of 1 m/s^2, find the speed and acceleration of weight A.



Example

The cable at A is pulled downwards at 2 m/s and is slowing at 1 m/s². Determine the velocity and acceleration of block D at this instant.



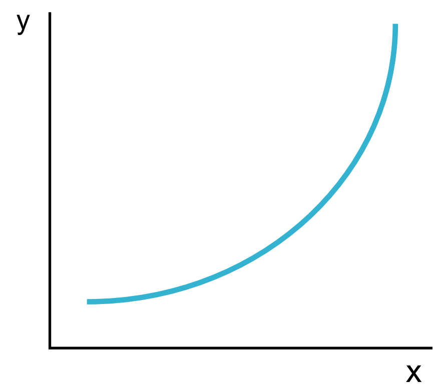
CURVILINEAR MOTION OF PARTICLES

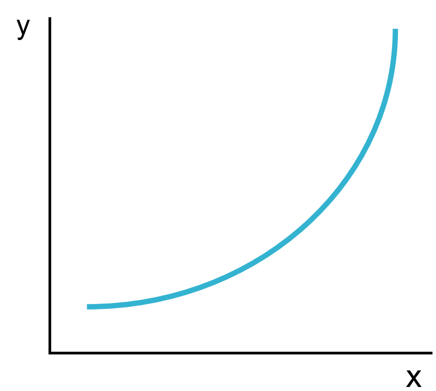
-For curvilinear motion on a plane, 2 coordinates are required to describe motion.

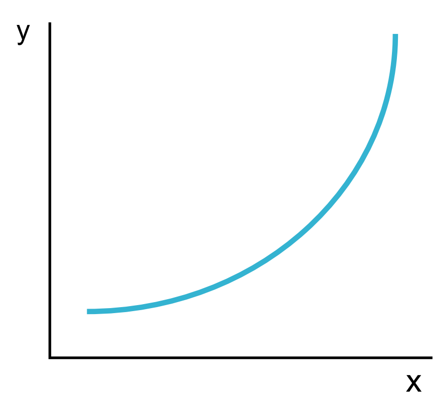
-For curvilinear motion in 3 dimensions, 3 coordinates are required to describe motion.

-For planar motion, rectangular coordinates work well if x-motion is independent of y-motion

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j}$$
$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\hat{i} + \dot{y}\hat{j} = v_x\hat{i} + v_y\hat{j}$$
$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}\hat{i} + \ddot{y}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

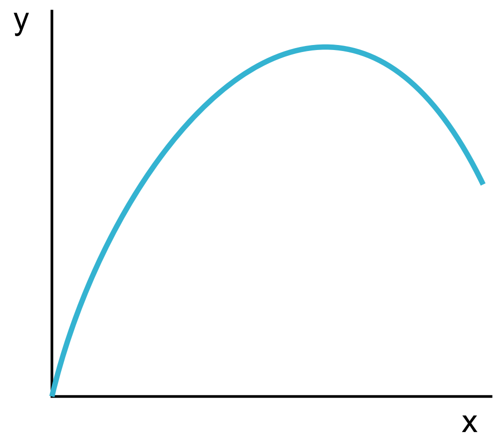






-For projectiles where gravity is the only acceleration:

Equations in y-direction



$$v_y = (v_y)_0 - gt$$
$$y = y_0 + (v_y)_0 t - (1/2)gt^2$$
$$v_y^2 = (v_y)_0^2 - 2g(y - y_0)$$

Equations in x-direction

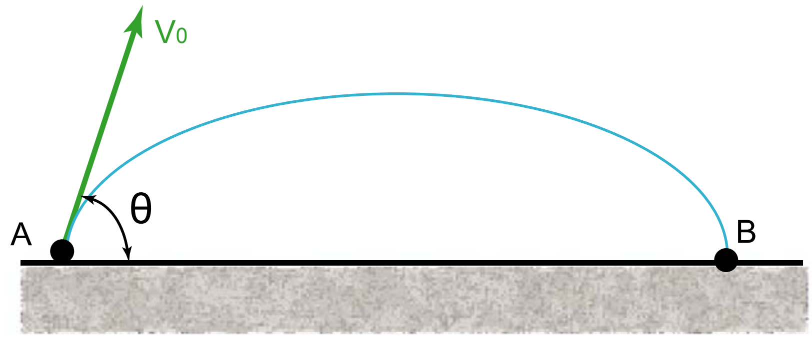
$$x = x_0 + (v_x)_0 t$$

Vector components of velocity

$$v_x = v\cos\theta$$
$$v_y = v\sin\theta$$

Example

A projectile is launched with initial velocity v_0 at a launch angle of θ. Determine an equation $y = f(x)$ which describes the parabolic path which the projectile takes.

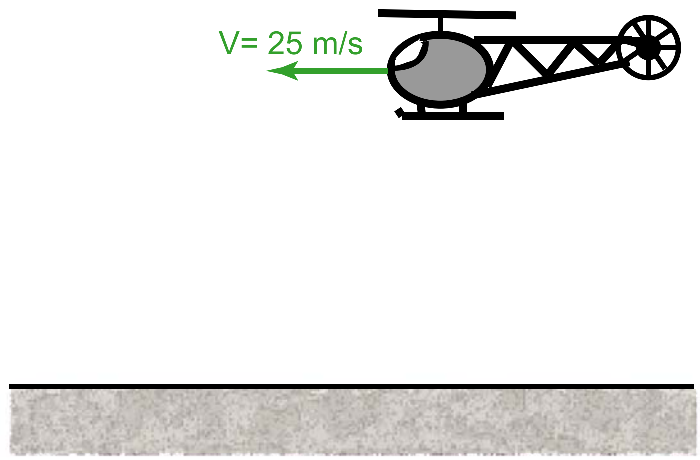


Example

A helicopter drops a box when it is flying at an altitude of 250 m with a velocity of 25 m/s. Determine the following:

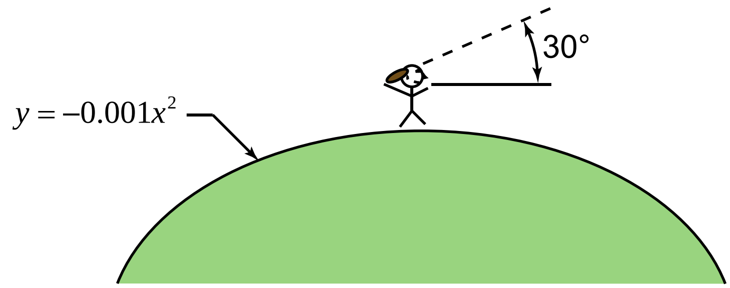
a) What is the horizontal distance from the release point at which the box lands?

b) What is the landing speed of the box?



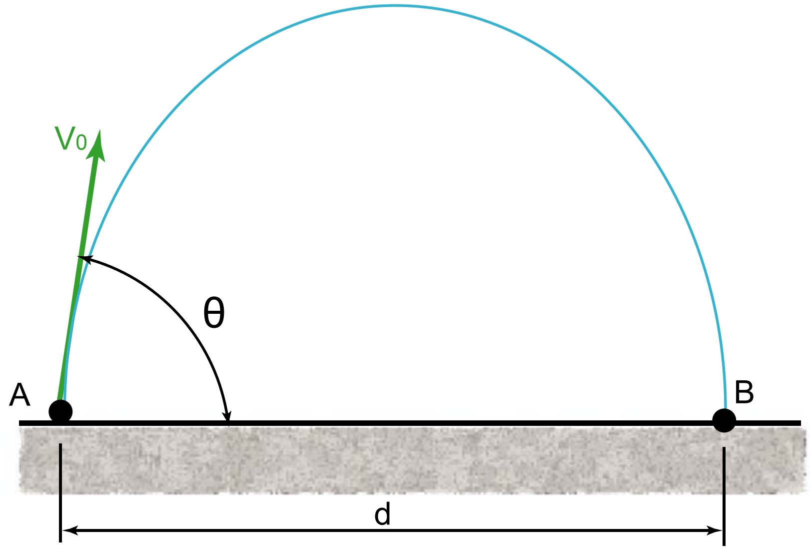
Example

A football is thrown from the top of a hill at a velocity of 75 ft/sec at an angle of 30°. The vertical profile of the hill can be approximated by the equation $y=-0.001x^2$. What are the coordinates of the point where the football lands?

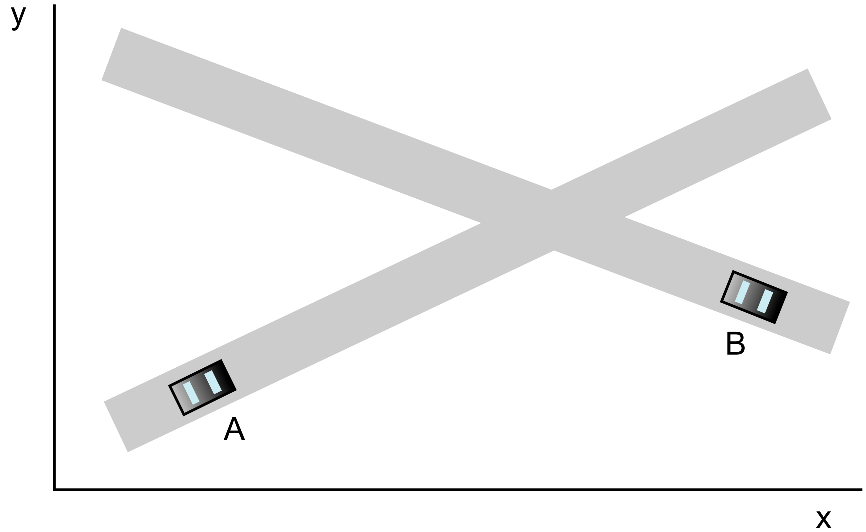


Example

A missile launched at A with the speed V_0 is aimed to hit target B. (a) Find the expression for the required angle θ of elevation. (b) Find the solution θ of the expression derived in part (a) if v= 400 ft/s and d= 2000 miles. Neglect air resistance.



MOTION RELATIVE TO A FRAME IN TRANSLATION



Relative Motion Analysis

$\vec{r}_A, \vec{r}_B$ are absolute position vectors

$\vec{r}_{B/A} = x'(t)\hat{i} + y'(t)\hat{j} =$ relative position vector

Relative Position Equation

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

Relative Velocity Equation

$$\frac{d}{dt}(\vec{r}_B = \vec{r}_A + \vec{r}_{B/A})$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

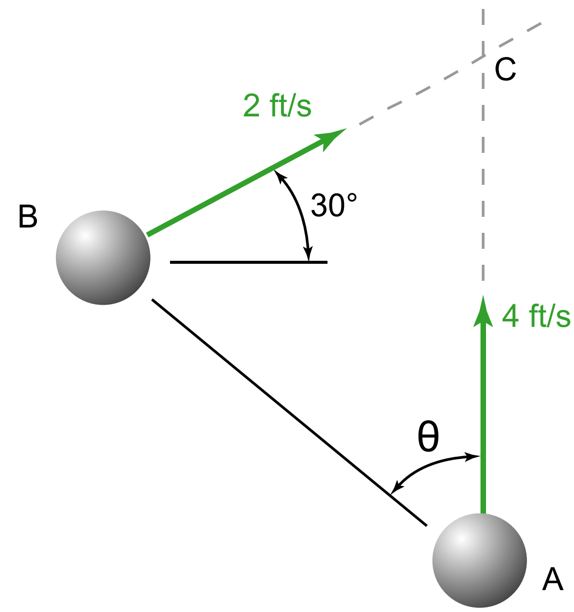
Relative Acceleration Equation

$$\frac{d}{dt}(\vec{v}_B = \vec{v}_A + \vec{v}_{B/A})$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

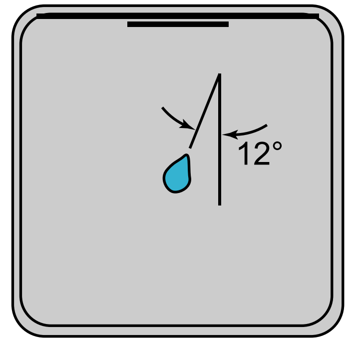
Example

Billiard balls which have the initial speeds shown move simultaneously in the paths between AC and BC. Determine θ for the balls to hit at C.



Example

When a stationary bus faces the wind, raindrops follow a downward path at an angle of 12° from vertical on the side windows. If the bus increases its velocity to 35 mph into the wind the angle increases to 65°. How fast are the raindrops moving?



Bus side window

NORMAL AND TANGENTIAL COMPONENTS

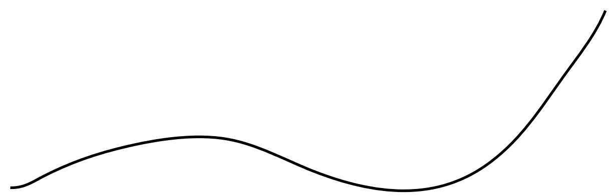
Normal and Tangential Coordinates

Position and Origin:

Unit Vectors: $\hat{e}_n, \hat{e}_t$

Velocity:

$$\vec{v} = \frac{ds}{dt}\hat{e}_t = v\hat{e}_t$$



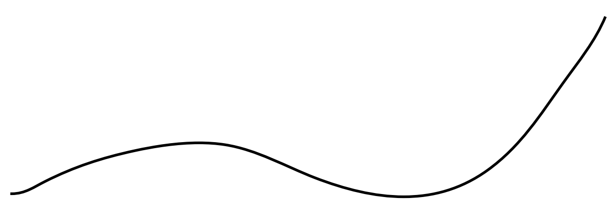
Acceleration:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v\hat{e}_t)$$

$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

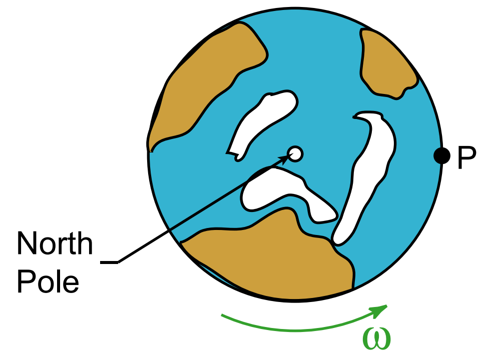
or

$$\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n$$



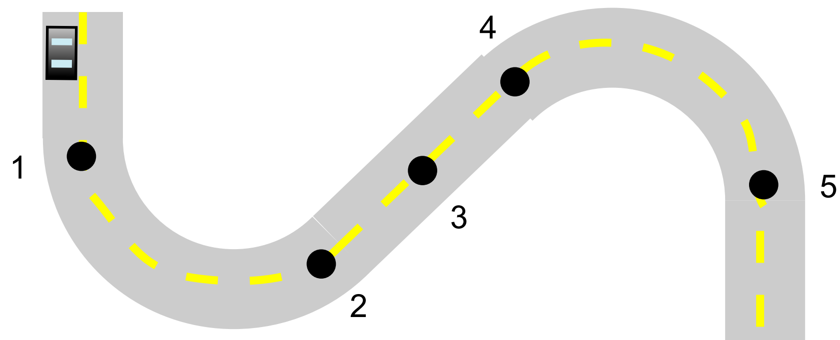
Example

The earth has an approximate radius of 6370 km. Use normal and tangential components to determine the velocity and acceleration of a point P on the earth's equator with respect to a non-rotating reference frame at the earth's core.



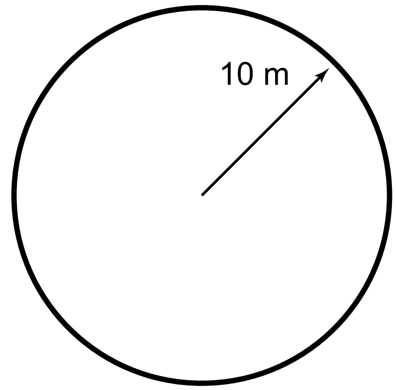
Example

A car travels on the S-shaped road shown. From point 1 to point 2, the car travels at a constant speed. It decelerates from point 2 to point 3. Then its speed increases from point 3 to point 5. At each point indicate the direction of the acceleration vector.



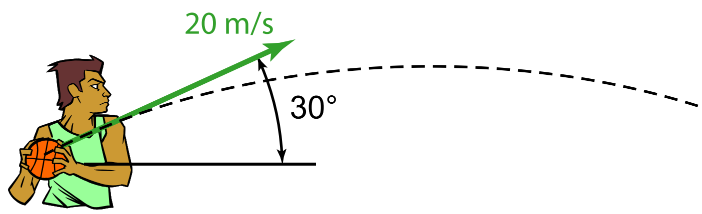
Example

An object which is initially at rest starts to move around a circle as shown. Its speed increases at a rate of 0.3t m/s². When 5 seconds have passed, what is the velocity and acceleration in terms of normal and tangential coordinates? How far does the object travel in 5 seconds?



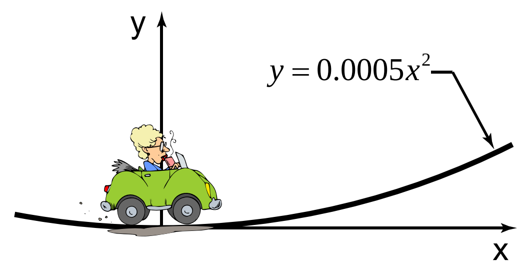
Example

A man throws a basketball with an initial velocity of 20 m/s at an angle of 30° from horizontal. Find the acceleration and velocity in terms of normal and tangential coordinates when the ball is at its highest point. What is the instantaneous radius of curvature at this point?



Example

A car which has a constant speed of 80 km/hr travels on a road whose vertical profile corresponds to the equation $y=0.0005x^2$. What are the normal and tangential components of the acceleration when $x = 350\ m$?



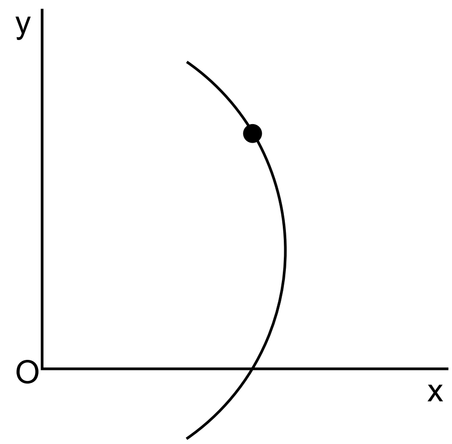
RADIAL AND TRANSVERSE COMPONENTS

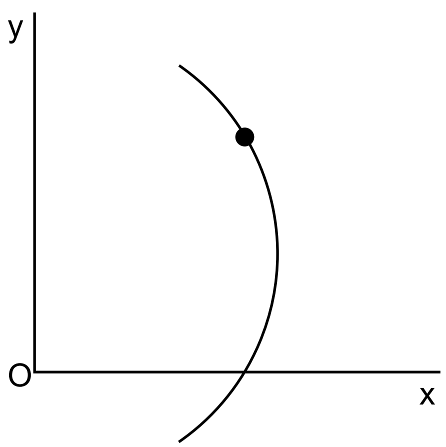
Position Vector: $\vec{r} = r\hat{e}_r$

Velocity Vector:

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

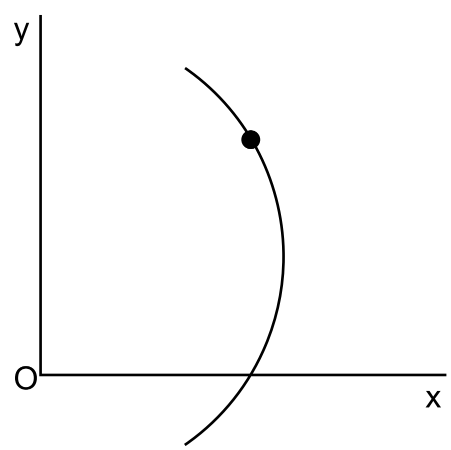




Acceleration Vector:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

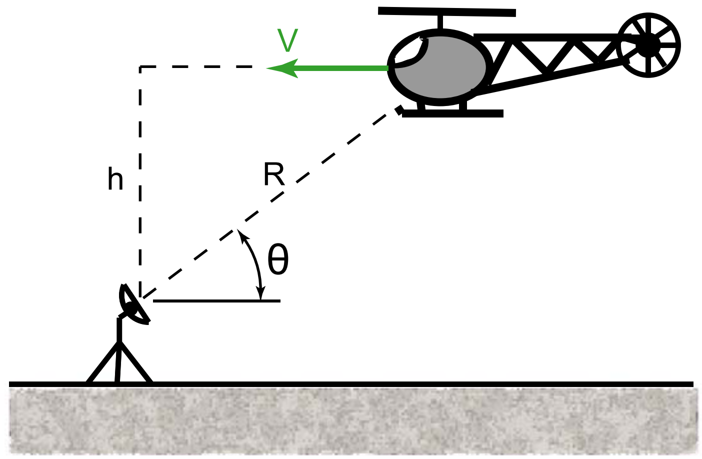
$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$



Example

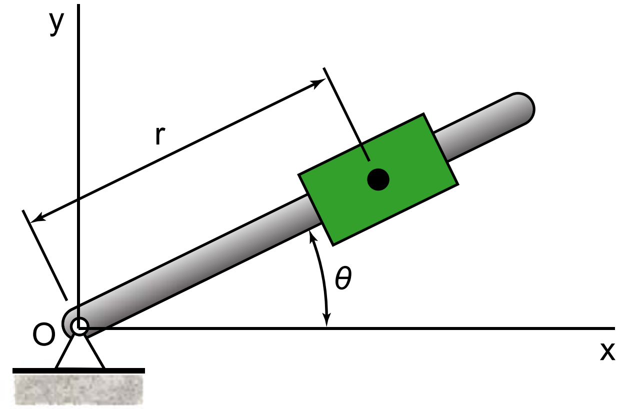
A radar tracking device follows the position of the helicopter according to its distance, angle, and angular velocity. Determine the altitude h, speed, and acceleration at which the helicopter is traveling given the following:

$$R = 1000 \text{ ft}$$
$$\theta = 45^\circ$$
$$\dot{\theta} = 0.05 \text{ rad/s}$$
$$\ddot{\theta} = 0.015 \text{ rad/s}^2$$



Example

The rod and slider are rotating in the plane, where $\theta = 2t^2$ and $r = 50t$ mm. Determine the velocity and acceleration of the slider when $t = 1$ s.

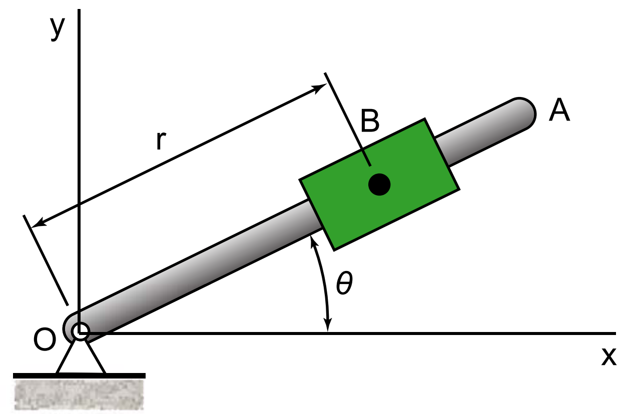


Example

The rod and slider assembly are rotating with constant angular velocity of 2 rad/s. At the same time, slider B moves along the rod at a constant speed of 3 mm/s toward A. At the point where *r= 5 mm* and *θ=0 rad*, determine the following:

a) Velocity of the slider in vector form.

b) Acceleration of the slider in vector form.



Chapter 2
Kinetics of Particles: Newton's Second Law

INTRODUCTION

Kinetics
Branch of Dynamics where forces are related to accelerations and velocities.

In Statics, bodies are in equilibrium.

$$\Sigma\vec{F} = \vec{0}$$

Newton's Second Law applies when the body is *not* in equilibrium.

$$\Sigma\vec{F} = m\vec{a}$$

Other quantities in Kinetics:

Linear Momentum: $\vec{L} = m\vec{v}$

Angular Momentum: $\vec{H} = \vec{r} \times m\vec{v}$

NEWTON'S SECOND LAW OF MOTION AND LINEAR MOMENTUM

Newton's Second Law

$$\Sigma\vec{F} = m\vec{a}$$

Restate Newton's Second Law:

Linear Momentum $\vec{L} = m\vec{v}$

$$\Sigma\vec{F} = m\frac{dv}{dt} = \dot{\vec{L}}$$

Mass Units

$$F = ma$$

$$W = mg$$

Creating Equations of Motion:

1. Create FBD and Kinetic Diagram.

$$\Sigma\vec{F} = m\vec{a}$$

2. Write Equations of Motion.

Rectangular Coordinates

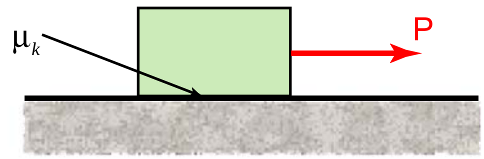
$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$

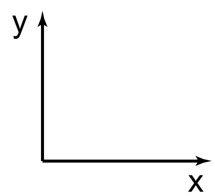
Normal and Tangential Coordinates

$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}$$
$$\Sigma F_t = ma_t = m\dot{v}$$

Example

Write the equations of motion for the moving block being pulled by force P.



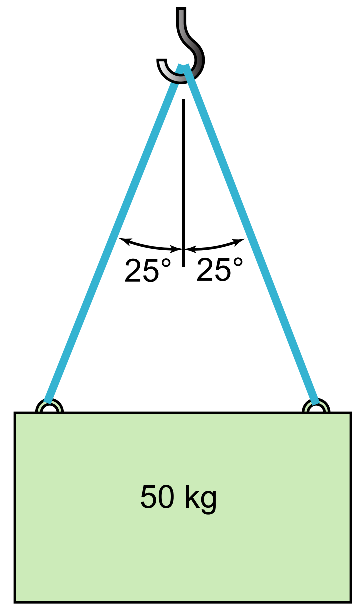


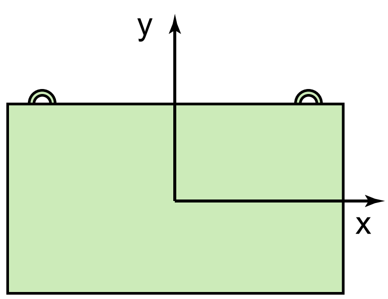
FBD

Kinetic Diagram

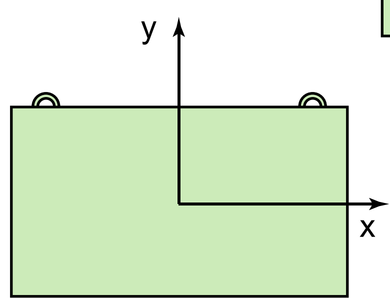
Example

A 50 kg load is initially at rest, then a hoisting mechanism raises the load 1 m over 2 s at constant acceleration. Determine the load in the two cables.





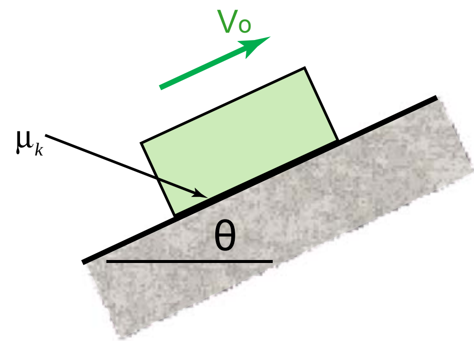
FBD

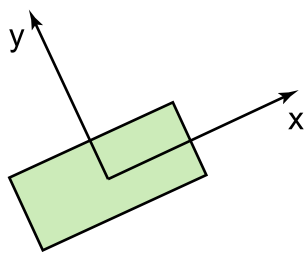


Kinetic Diagram

Example

A block of mass m is moving up a hill with the incline angle θ. The block has an initial velocity v_o. Determine the distance which the block will travel up the incline.





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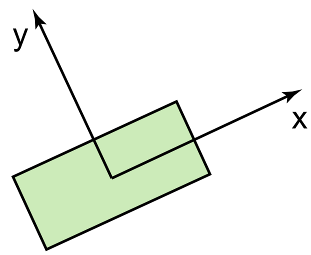
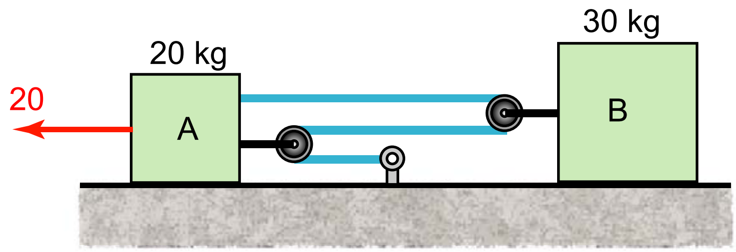
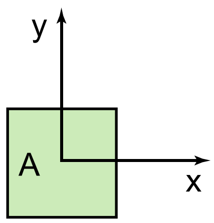


Kinetic Diagram

Example

Blocks A and B slide on a frictionless surface. If the 20 N force is applied to A as shown, what is the acceleration of each block? Also, determine the tension in the cable.





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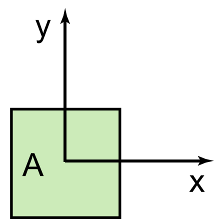
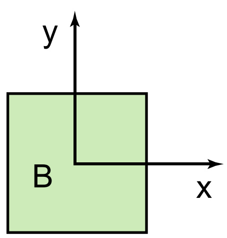


Kinetic Diagram



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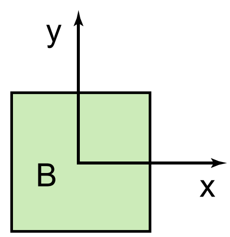
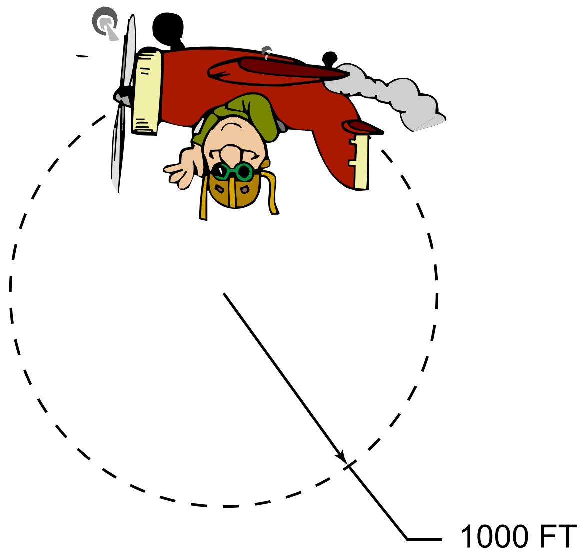
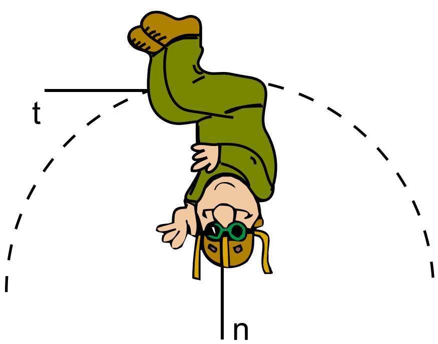


Kinetic Diagram

Example

An airplane flies in a vertical loop of radius 1000 ft at a constant speed of 200 ft/s. If the pilot weighs 175 lb, what force does the pilot's seat exert on him when the plane is at the top of the loop?





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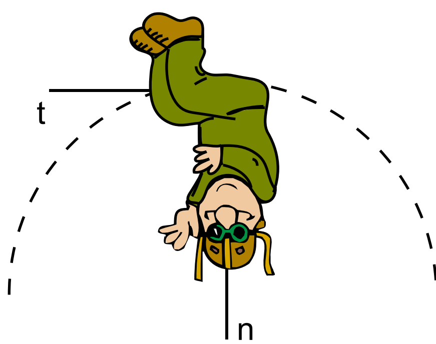
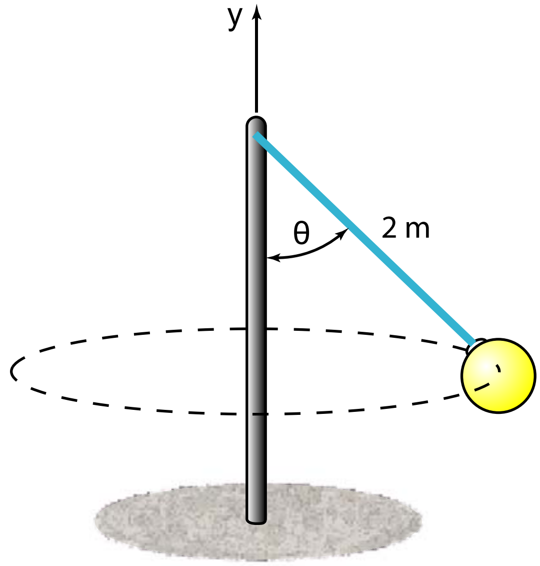
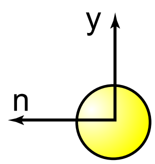


Kinetic Diagram

Example

What velocity must the 1 kg tetherball have in order for the cable assembly to make an angle θ=45° with the pole. What is the tension in the cable when θ=45°.





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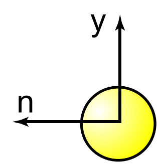
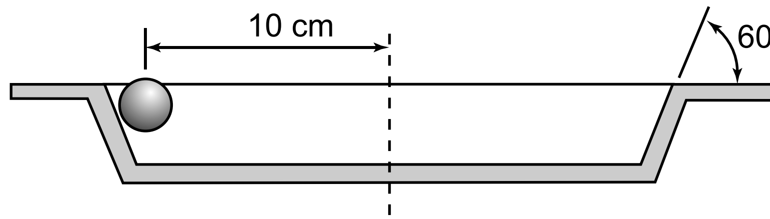
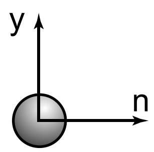


Kinetic Diagram

Example

A 10 g marble rolls in a circular path on a banked track. If the coefficient of static friction μ_s=0.3, what is the range of velocities the marble may have so that it doesn't slip?





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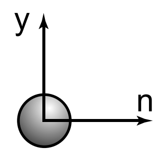
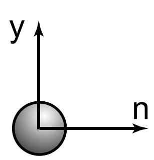


Kinetic Diagram



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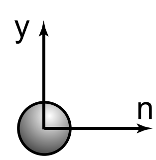
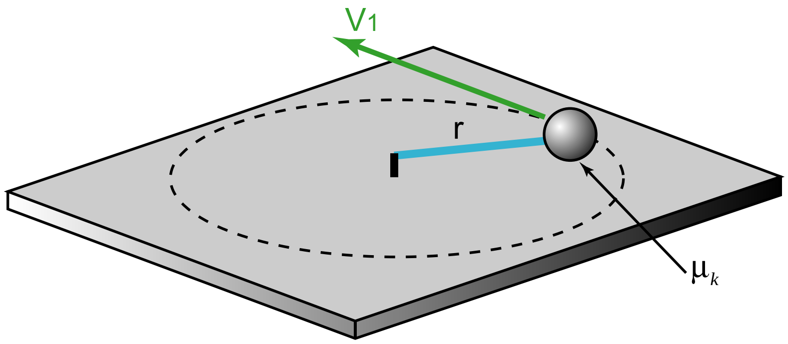
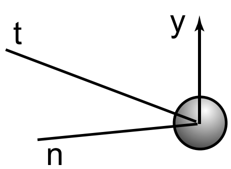


Kinetic Diagram

Example

A string of length *r* is attached to a marble of mass *m* which moves in a circular path on the horizontal table. The coefficient of kinetic friction is 0.20. If the marble is initially moving at a speed of V_1= 10 m/s, how long does it take for the marble to stop?





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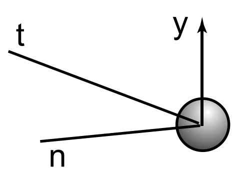


Kinetic Diagram

ANGULAR MOMENTUM OF PARTICLE AND CENTRAL FORCE MOTION

Linear Momentum

$$\vec{L} = m\vec{v}$$

Angular Momentum is the *Moment of Momentum* about a point, O.

$$\boxed{\vec{H}_o = \vec{r} \times m\vec{v}}$$

Restate Newton's Second Law:

$$\Sigma\vec{F} = m\frac{d\vec{v}}{dt} = \dot{\vec{L}}$$

-Take moments about both sides of the equations:

$$\vec{r} \times \Sigma\vec{F} = \vec{r} \times m\frac{d\vec{v}}{dt}$$

-Result:

$$\boxed{\Sigma\vec{M}_o = \dot{\vec{H}}_o}$$

Central Force Motion

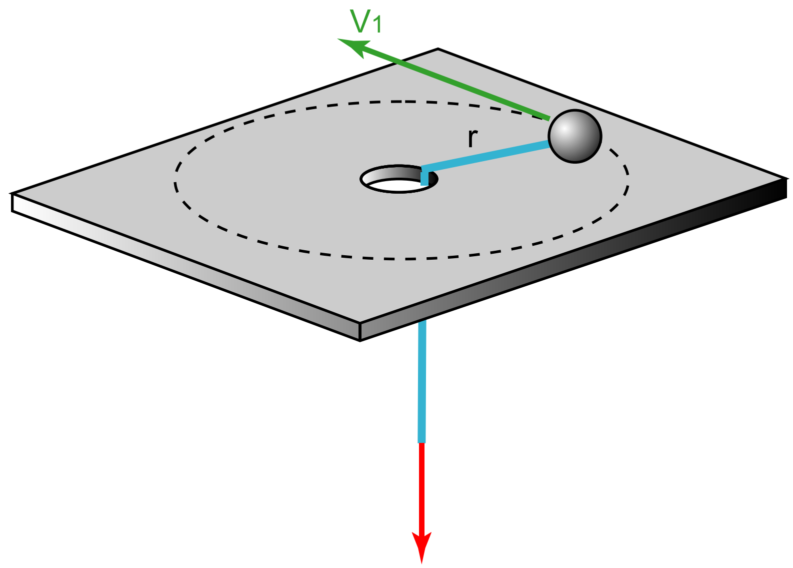
$$\Sigma\vec{M}_o = \dot{\vec{H}}_o = \vec{0}$$

$$\boxed{\vec{H}_o = \text{constant}}$$

Angular Momentum is Conserved

Example

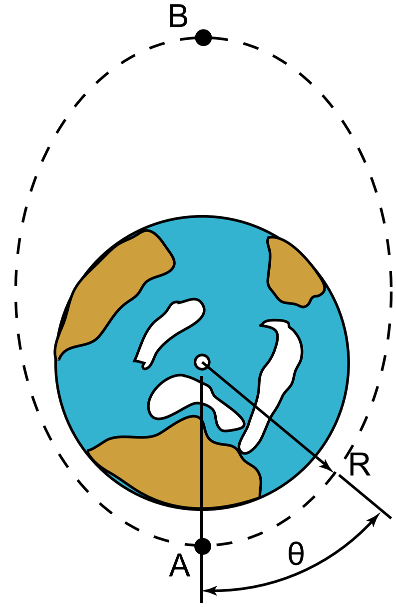
A string-and-marble assembly initially allows the marble to travel in a circular path of radius r= 1.0 meters at an initial speed of 8 m/s. If the string is pulled down such that the radius becomes 0.5 meters, what is the new velocity? (Assume friction is negligible.)



Example

A satellite travels in an elliptical path around the earth, where R is defined by the equation below. If the satellite has a speed of 1000 m/s at point A, find its speed at B. Units: meters, seconds, m/s.

$$\frac{1}{R} = (1.5x10^{-7})(1 + 0.15\cos\theta)$$



EQUATIONS OF MOTION IN TERMS OF RADIAL AND TRANSVERSE COMPONENTS

Newton's Second Law:

$$\Sigma \vec{F} = m\vec{a}$$

Equations of Motion using Polar Coordinates:

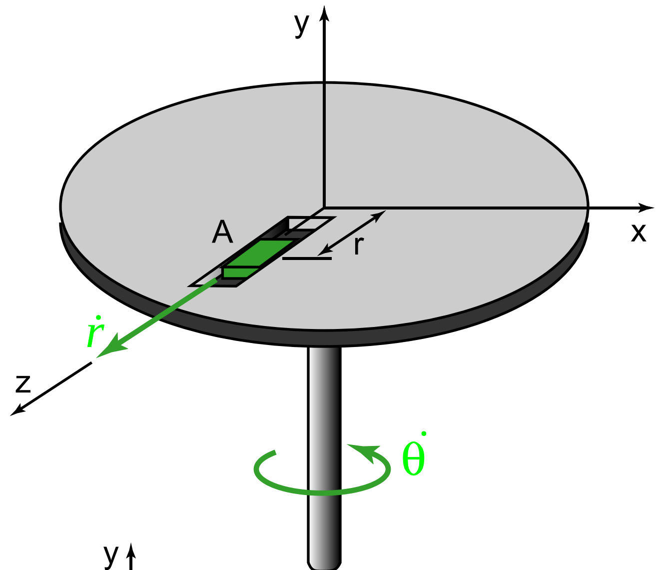
$$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2)$$
$$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

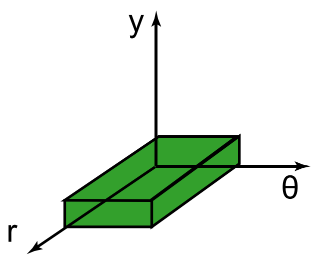
Example

A flat turntable is rotating at the constant rate of 10 rad/s. Particle A with a mass of 100 g is free to slide in the frictionless slot. If particle A is known to be at the position where *r*= 0.1 m and is moving along the slot at the rate of 0.2 m/s, determine the following:

a) What is the acceleration of A relative to the slot?

b) What is the total normal force acting on A at the instant?





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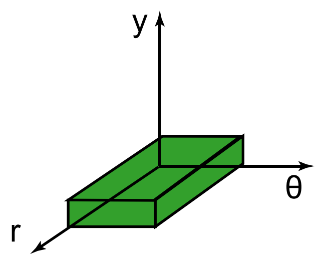
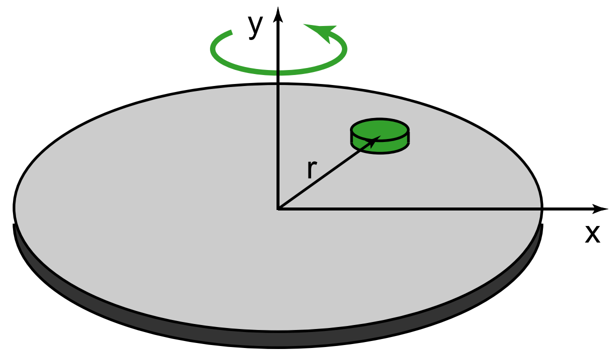
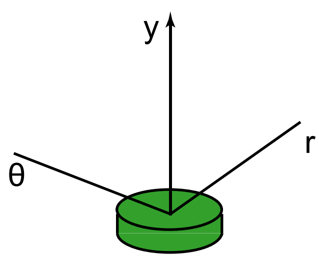


Kinetic Diagram

Example

A 0.01 kg disk is placed on a stationary turntable at r= 1 m. The static coefficient of friction between the coin and the turntable is 0.2. If the turntable is started with a constant angular acceleration of 1.5 rad/s², determine the angular velocity when the coin starts slipping.





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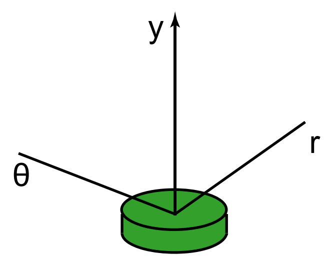
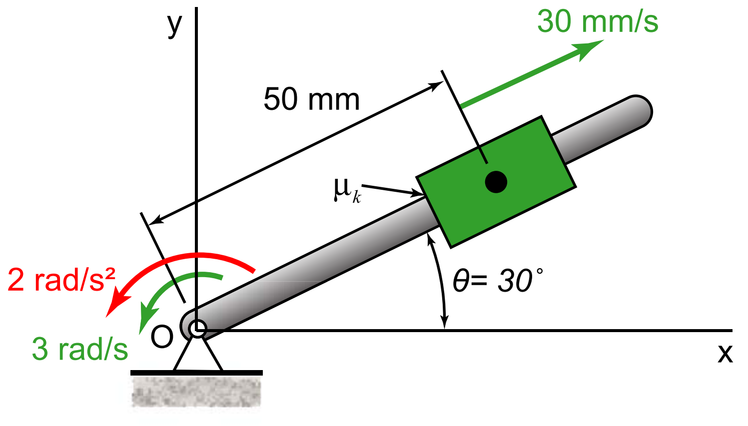
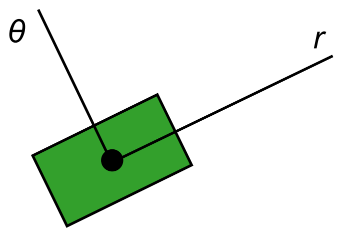


Kinetic Diagram

Example

At the instant shown a 1 kg block slides relative to the rod which is rotating in the vertical plane. Find the acceleration of the block relative to the rotating rod at this instant. What is the normal reaction?





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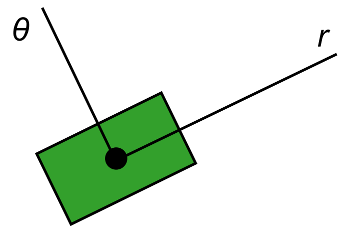


Kinetic Diagram

Chapter 3
Kinetics of Particles:
Energy and Momentum Methods

INTRODUCTION

Newton's Second Law

$$\vec{F} = m\vec{a}$$

Work and Energy Concepts

-Work of a Force

-Kinetic Energy of a Particle

-Power

-Efficiency

Impulse and Momentum Concepts

-Impulsive Motion

-Impact

WORK OF A FORCE

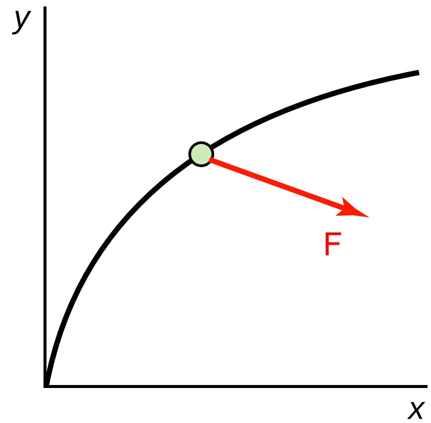
Work

$$dU = \vec{F} \bullet d\vec{r}$$

$$U_{1\to2} = \int \vec{F} \bullet d\vec{r}$$

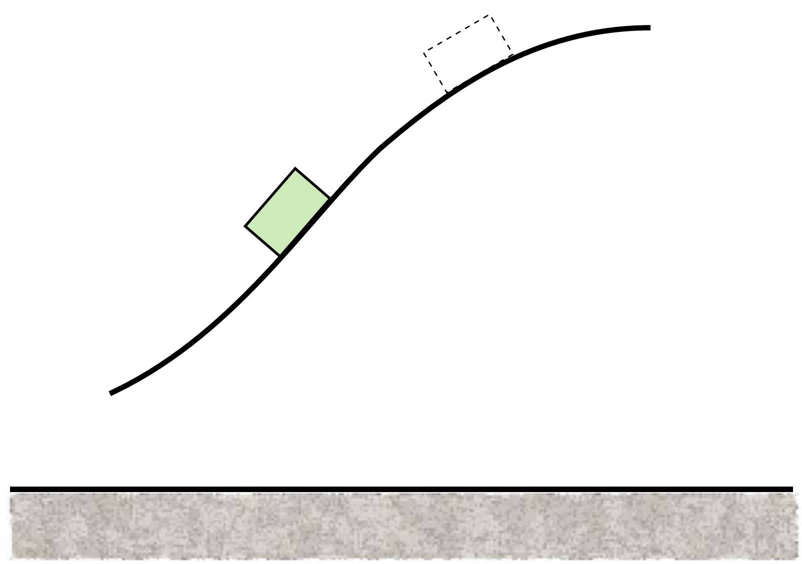
$$U_{1\to2} = \int F_t ds$$

$$U_{1\to2} = \int (F_x dx + F_y dy + F_z dz)$$

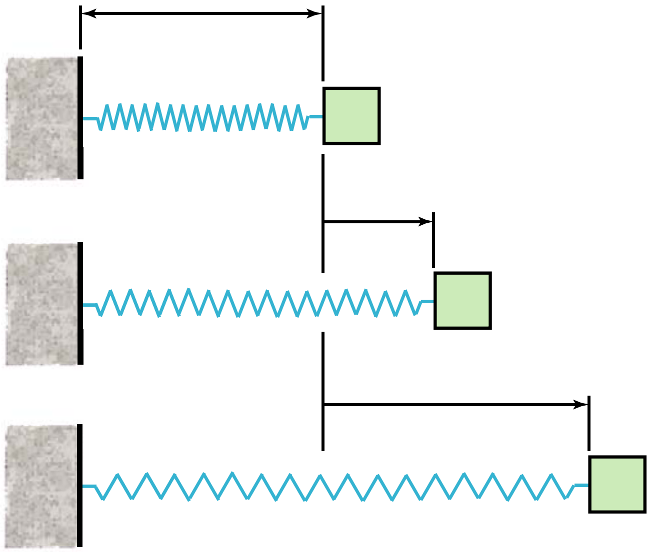


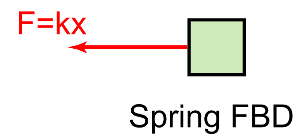
Work of Gravity

$$U_{1\to2} = -W \Delta y$$



Work of a Force Exerted by a Spring





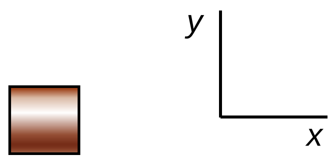
$$U_{1\to 2} = -\int kx\,dx = -\frac{k}{2}(x_2^2 - x_1^2)$$

Example

A sliding collar is attached to a spring with spring constant K. Determine the work done on the collar if it starts at position 1 and moves to 2 if:

a) the unstretched spring length is 2L.

b) the unstretched spring length is 1.5L.



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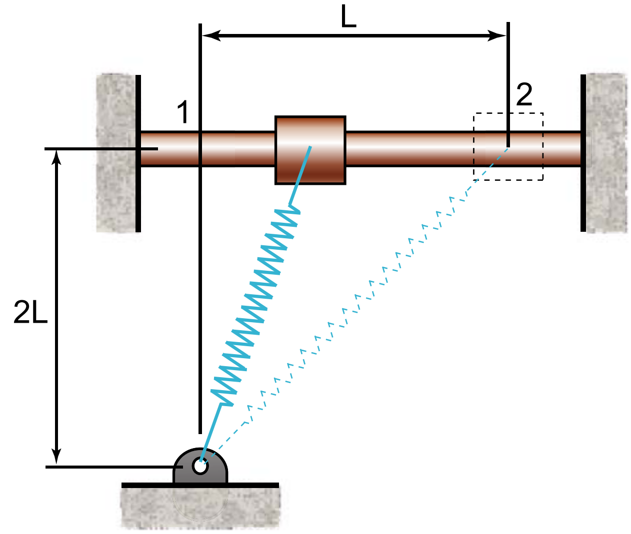


KINETIC ENERGY, PRINCIPLE OF WORK AND ENERGY

Kinetic Energy of a Particle

$$T = \frac{1}{2} m v^2$$

Principle of Work and Energy

$$U_{1\to 2} = \Delta T$$

Power

$$P = \text{Time rate of work} = \frac{dU}{dt}$$

$$P = \vec{F} \bullet \vec{v}$$

Efficiency

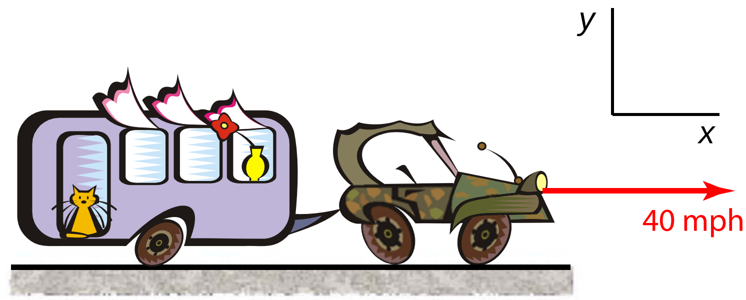
$$\eta = \frac{\text{output power}}{\text{input power}}$$

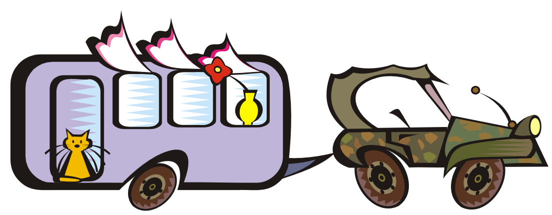
Example

A 4000 lb jeep is pulling a 3000 lb camper. The jeep and camper are traveling at a speed of 40 mph on a level section of highway.

a) How much braking force is required to stop the jeep and camper in 1000 ft?

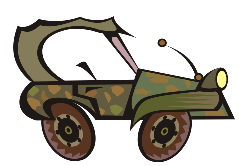
b) If all the braking force is supplied by the jeep, what is the average force in the hitch?





FBD (a)

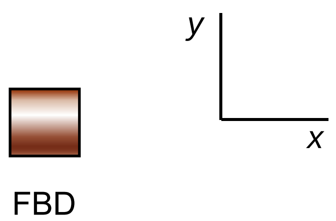
FBD (b)

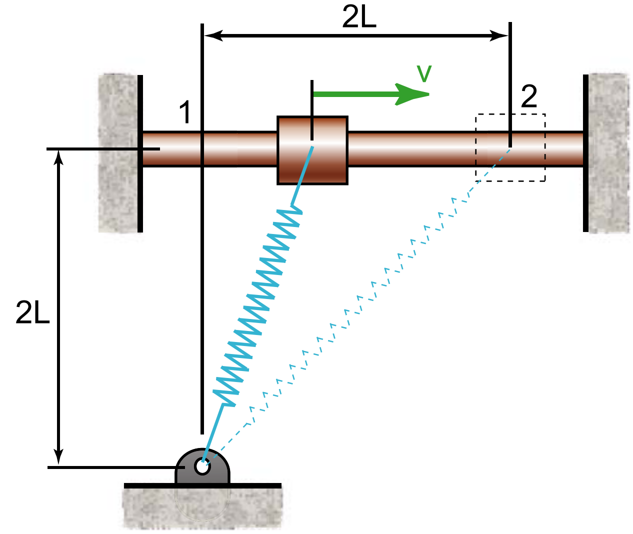


Kinetic Diagram (b)

Example

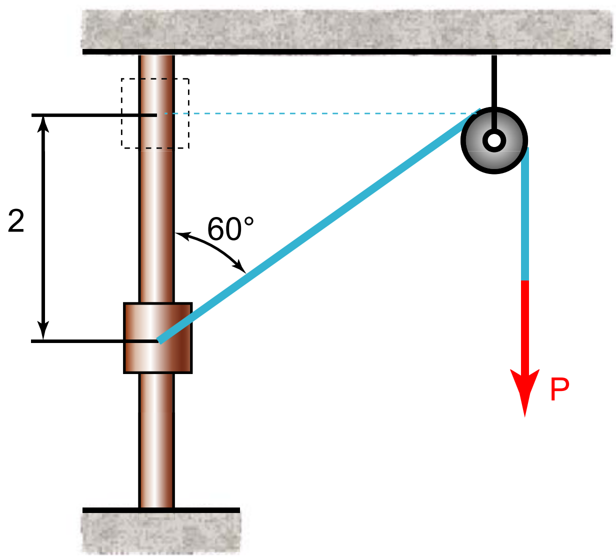
A sliding collar is attached to a spring with unstretched length of 2L and a spring constant of K. Find the expression for velocity of the collar at position 2. Assume the initial velocity at position 1 is *v* and the mass of the collar is *m*.

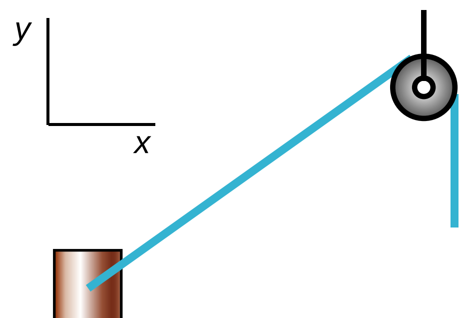




Example

A 2 lb collar moves upward as a cable pulls it with a force P= 10 lb. If the collar is initially at rest, what is its velocity after it has moved upward by 2 ft?





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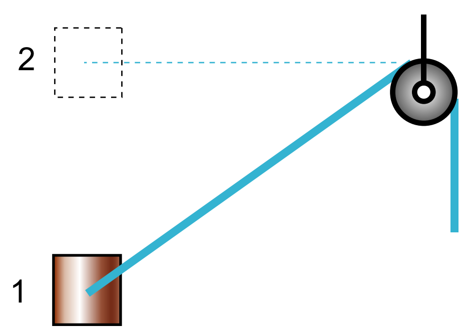


Example

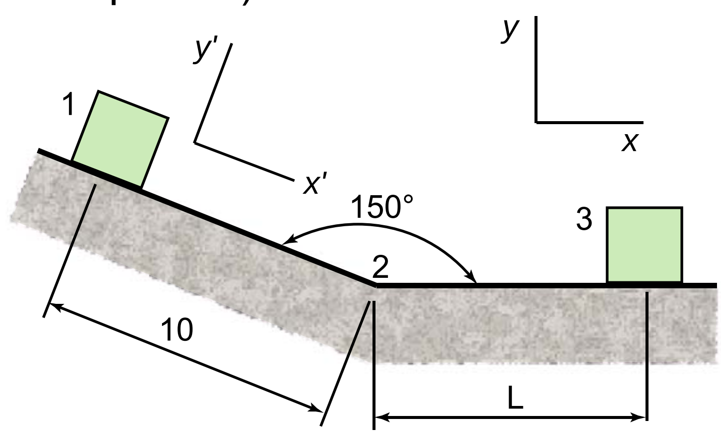
A 10 kg crate is released from rest at point 1. The coefficient of kinetic friction is 0.20.

a) What speed does the crate have at point 2?

b) How far does the crate travel before stopping at point 3?

(Assume no energy loss at impact at point 2).

Units: m, s.

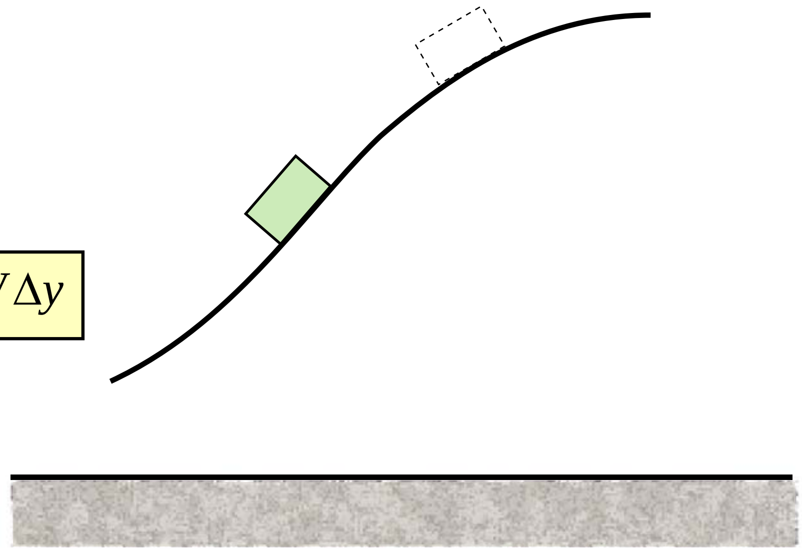


FBD@ 1 FBD@ 2 and 3

POTENTIAL ENERGY

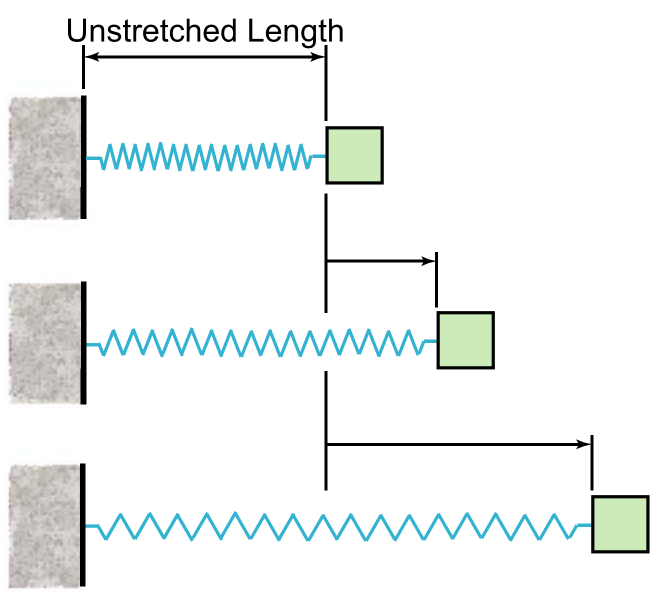
Gravitational Potential Energy

$$V_g = Wy$$



$$U_{1\to 2} = -\Delta V_g = -W\Delta y$$

Elastic Potential Energy



$$V_e = \frac{1}{2}kx^2$$

$$U_{1\to 2} = -\Delta V_e = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

WORK/ ENERGY RELATION with CONSERVATIVE FORCES

Conservative Force

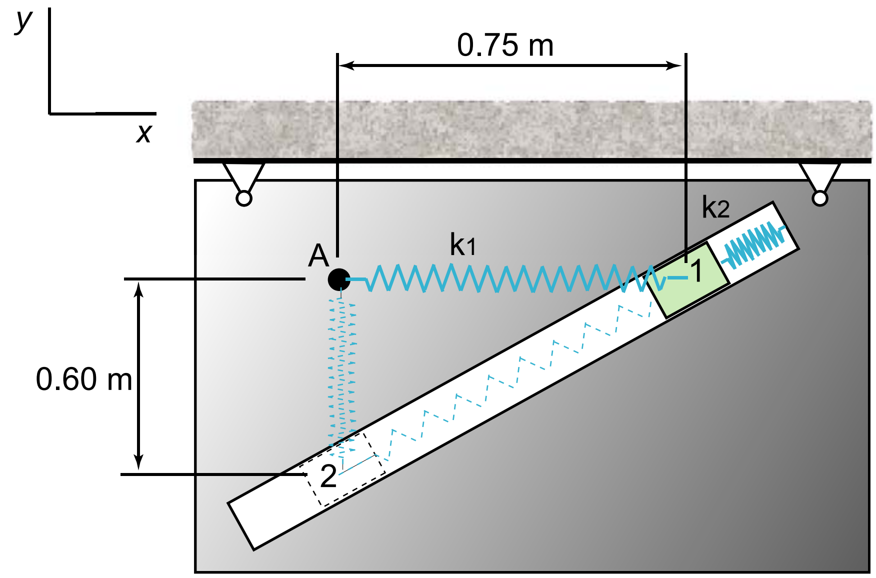
A force is conservative if work depends only on the initial and final position.

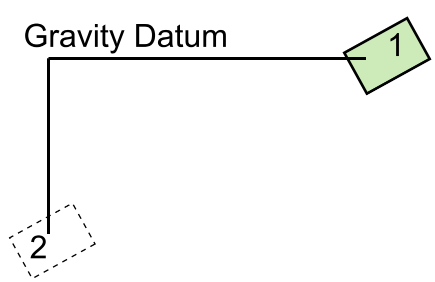
$$U_{1\rightarrow 2} = \Delta T$$

$$\boxed{T_1 + V_1 = T_2 + V_2}$$

Example

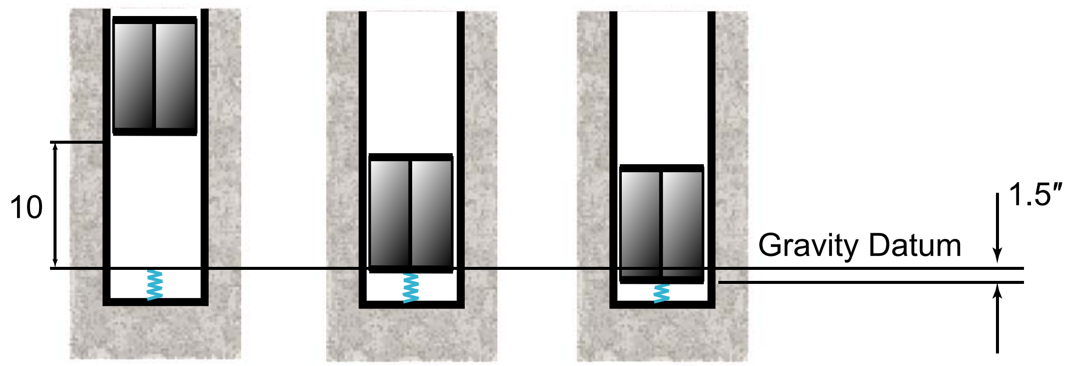
The 5 kg block is attached to the peg at A in the plate by the two springs as shown. In its initial position (1), both springs are undeformed. k_1= 750 N/m and k_2= 450 N/m. Find the required initial velocity for the block to reach the final position (2).





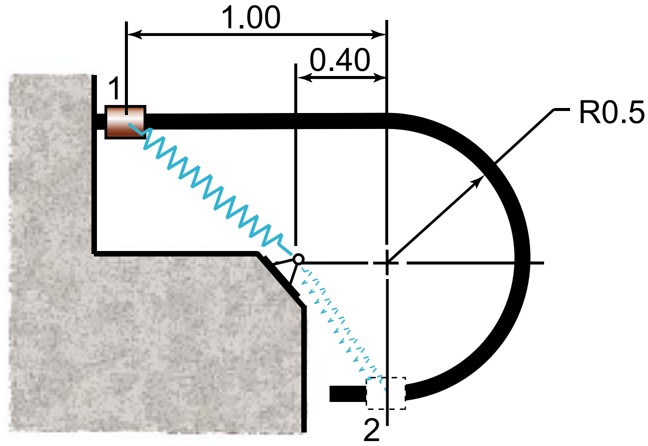
Example

A low-speed elevator uses a large “buffer” spring to stop a car moving downward in the case of a drive system failure. The 2000 lb car is initially falling at a rate of 75 ft/min at a height of 10 ft above the spring. What must the spring constant be in order to stop the car within 1.5″ after hitting the spring? Assume negligible friction between the elevator car and its guide rails. Units: Ft.



Example

A 1 kg slider moves along the bent rod and is attached to a spring as shown. The unstretched spring length is 0.50 m. The spring constant k= 1 N/mm. The collar is released from rest at position 1. Assuming friction is negligible, determine the sliders' speed at point 2. The bent rod is in the vertical plane.



PRINCIPLE OF IMPULSE AND MOMENTUM

Linear Momentum

$$= m\vec{v}$$

Linear Impulse

$$\text{Imp}_{1\rightarrow 2} = \int_1^2 \vec{F}dt$$

From Newton's Second Law:

$$\sum \vec{F} = m\vec{a}$$

$$\int_1^2 \vec{F}dt = \Delta m\vec{v}$$

or

$$m\vec{v}_1 + \int_1^2 \vec{F}dt = m\vec{v}_2$$

If no forces are acting on a system, then:

$$\int_1^2 \vec{F}dt = \Delta m\vec{v}$$

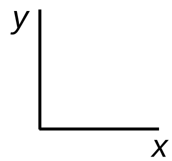
$$\vec{0} = \Delta m\vec{v}$$

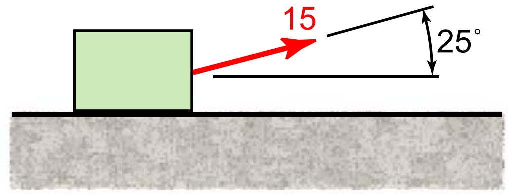
$$\boxed{m\vec{v}_1 = m\vec{v}_2}$$

This relationship is called **Conservation of Momentum.**

Example

The 20 lb crate is dragged across a surface by a constant 15 lb force. How much time is required for the crate's velocity to increase from 9 ft/s to 35 ft/s? The kinetic coefficient of friction is 0.15.





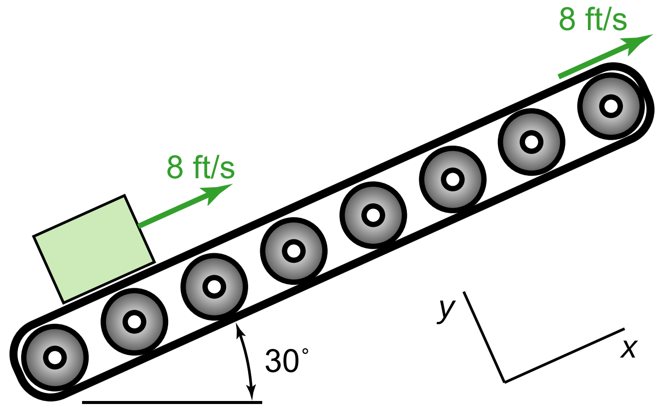
FBD

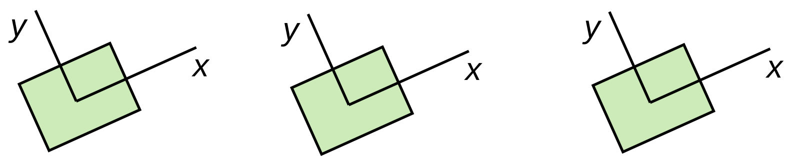
Initial Kinetic Diagram

Final Kinetic Diagram

Example

A package initially moves at 8 ft/s with the belt. Due to lack of friction, the package starts to slip. At 5 s, the package reaches its maximum displacement. What is the coefficient of friction?





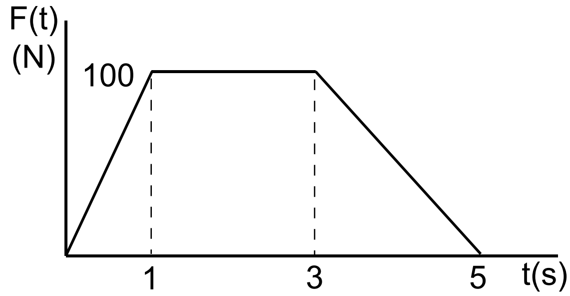
FBD

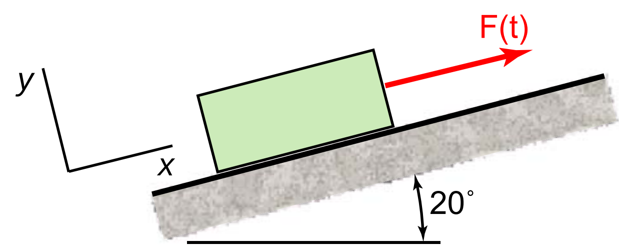
Initial Kinetic Diagram

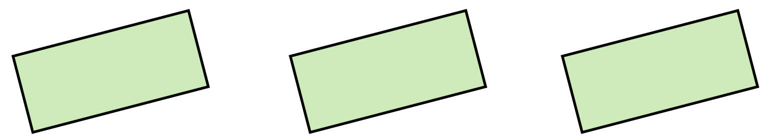
Final Kinetic Diagram

Example

A 10 kg crate is being pulled along a 20˚ incline by force F(t), which varies with time as shown in the F-t plot. Initially, the crate is moving at 1 m/s and has a kinetic coefficient of friction of 0.3. Determine the crate's velocity at time= 5 s.







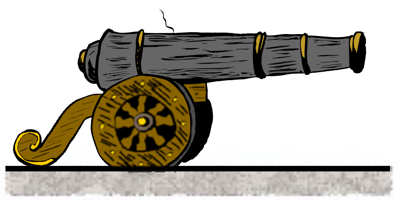
FBD

Initial Kinetic Diagram

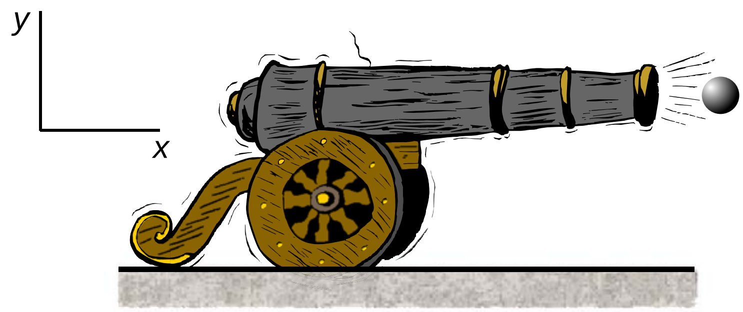
Final Kinetic Diagram

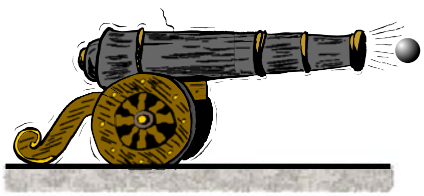
Example

The cannon has a muzzle velocity of 100 m/s for a 4kg ball. The combined weight of the system shown is 250 kg. If the system is initially at rest, what is the speed of the cannon as the ball exits the cannon?



Initial Kinetic Diagram

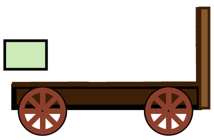




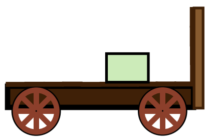
Final Kinetic Diagram

Example

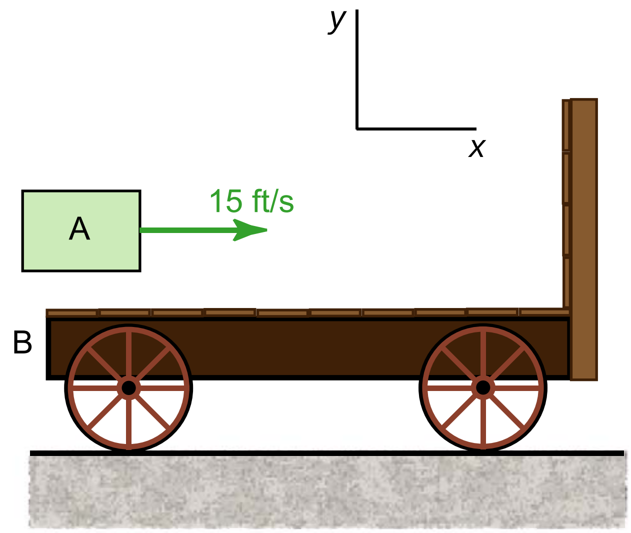
The 30 lb package A is thrown horizontally at a speed of 15 ft/s onto the 60 lb wagon B which is initially at rest. The crate slides on the wagon, then comes to rest. Find the speed of B when A no longer slides. Assume the wheels have negligible size.



Initial
Kinetic
Diagram



Final
Kinetic
Diagram



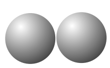
DIRECT IMPACT

$$m\vec{v}_1 + \int_1^2 \vec{F}dt = m\vec{v}_2$$

An **Impact (impulsive) Force** is a very large force which occurs over a very small increment of time.

In a **Plastic Impact,** bodies undergo permanent deformation and “stick” together.

In a **Perfectly Elastic Impact,** bodies return to original shape.



Direct Impact occurs when the impact is in the direction of motion.

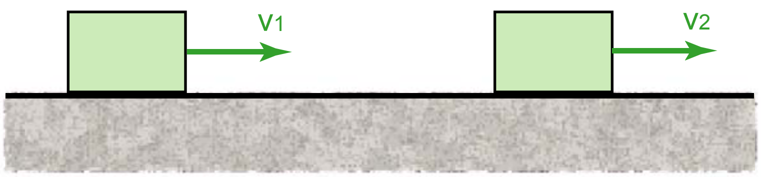
Coefficient of Restitution (Direct Impact)

$$e = \frac{v_B' - v_A'}{v_A - v_B}$$

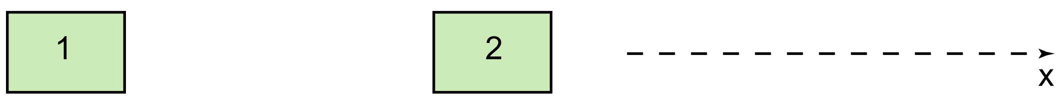
Ratio of “separation velocity” to “approach velocity”.

Example

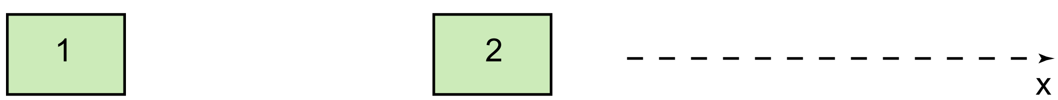
Blocks 1 and 2, with masses m_1 and m_2 respectively, are moving to the right with the velocities v_1 and v_2. The velocity of block 1, v_1, is greater than v_2, so that a collision will occur. Use the Principle of Impulse and Momentum and the coefficient of restitution to determine the equations showing the relationships between the velocities and impact force.



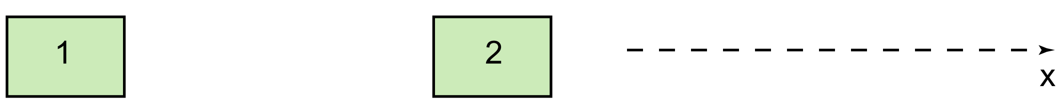
Momentum Before Impact



Forces (FBD) During Impact



Momentum After Impact



Example- Continued

Apply Impulse-Momentum in the direction of motion/ impact.

-To Block 1 only

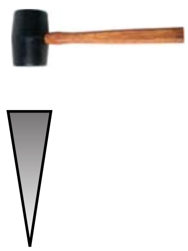
-To Block 2 only

-To system of Block 1 and Block 2

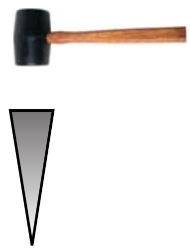
Apply Coefficient of Restitution equation

Example

An 8 lb sledge hammer is used to drive a 5 lb wedge into the log. The log resists with a force of 1500 lb. Determine: a) the necessary hammer speed to drive the wedge 0.5″ in one blow; and b) the impulse required. Assume that the collision is fully plastic. Neglect the impulse of the log against the wedge.

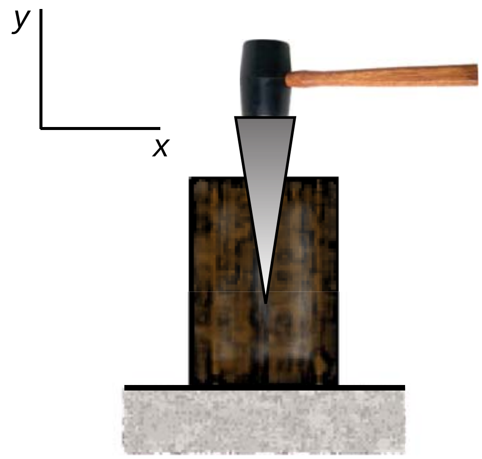


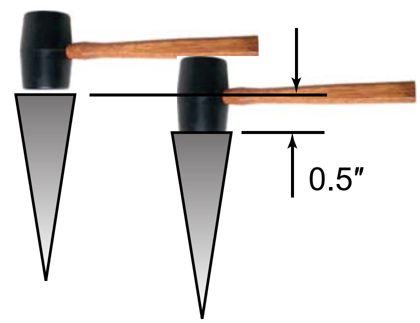
FBD During Impact



Kinetic Diagram Before Impact

Kinetic Diagram After Impact

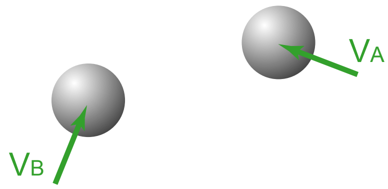




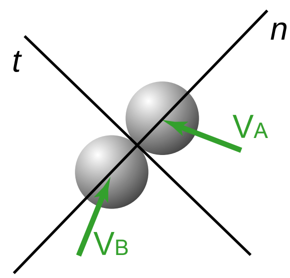
Wedge After Impact

OBLIQUE CENTRAL IMPACT

Oblique Impact occurs when the direction of impact is not along the line of action.

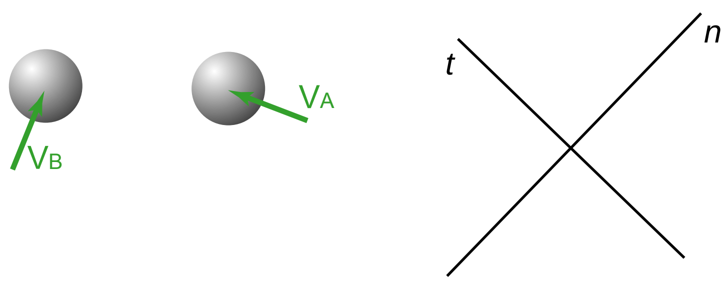


Line of Tangency (t) and Line of Impact (n)



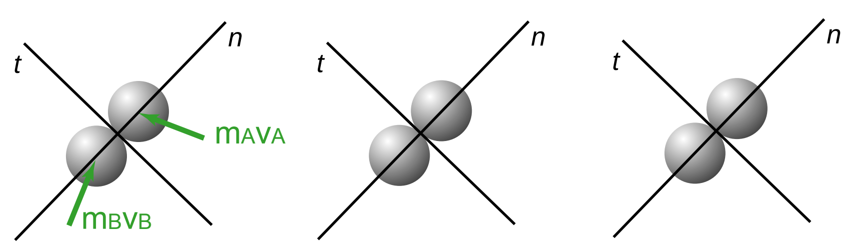
Apply the Coefficient of Restitution ONLY along the line of impact.

$$e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$



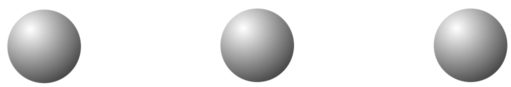
Apply the Principle of Impulse and Momentum.

$$m\vec{v}_1 + \int_1^2 \vec{F} dt = m\vec{v}_2$$

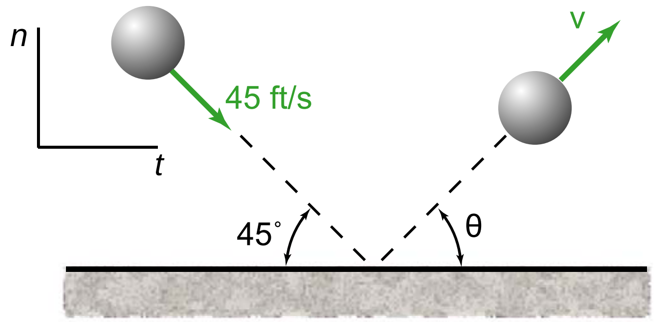


Example

A 2 lb sphere hits a smooth surface at an angle 45˚. Its velocity at impact is 45 ft/s. It is known that the impulse during the impact is 2.5 lb·s. What is the speed and angle at which the sphere leaves the surface?

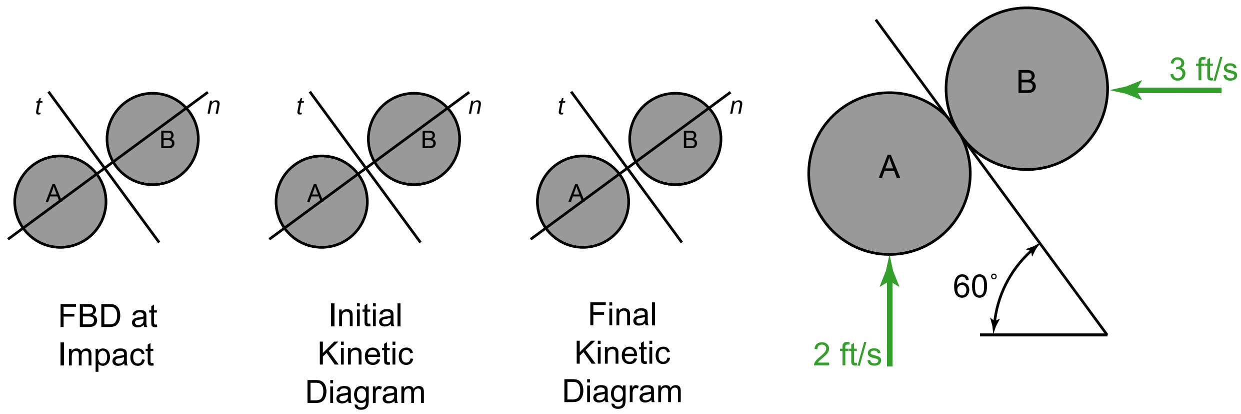


FBD | Initial Kinetic Diagram | Final Kinetic Diagram



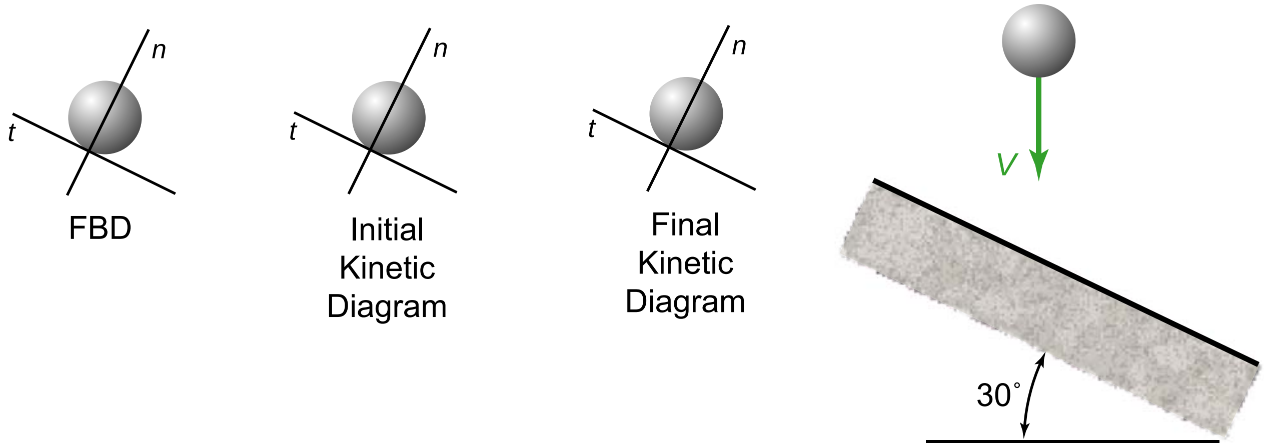
Example

Two 1 lb disks, A and B, have initial velocities of 2 ft/s and 3 ft/s, respectively. If the coefficient of restitution *e* is known to be 0.4, find the speed of each disk following impact.



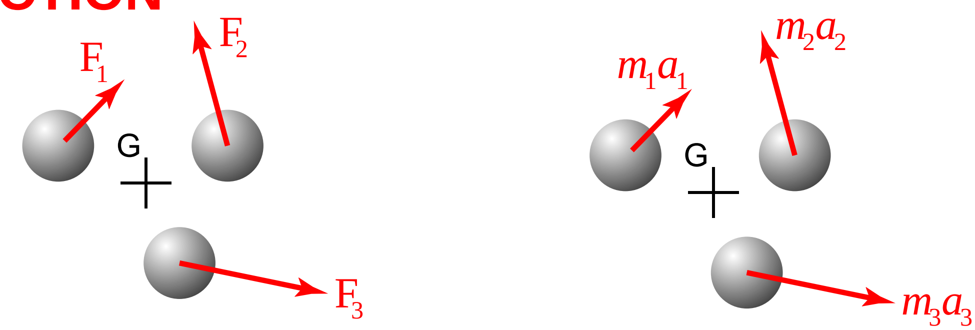
Example

A ball of mass *m* has velocity *v* before hitting the inclined surface. The coefficient of restitution is *e*. Find the velocity immediately following impact.



Chapter 4
System of Particles

INTRODUCTION



- A System of Particles has more than one particle.

- The force resultant of system of particles is equal in magnitude and direction to the resultant of the mass-acceleration vectors of the particles.

- Linear and Angular Momentum can be defined in terms of the sum of the momentums of the individual particles or in terms of the momentum of the mass center.

- Newton's 2nd Law can be defined as the time rate of change of momentum.

- The energy of a system is the sum of the energy of the individual particles.

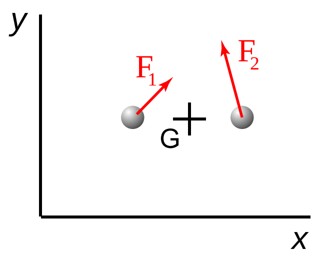
- The Principle of Work and Energy may be applied to a system of particles.

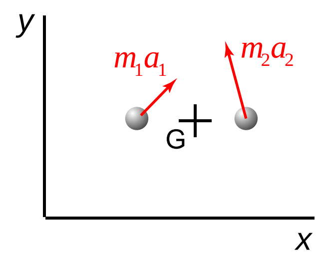
FORCE, ACCELERATION, AND MOMENTUM OF A SYSTEM OF PARTICLES

Newton's 2nd Law

$$\sum \vec{F}_i = \sum m_i \vec{a}_i$$

$$\sum (\vec{r}_i \times \vec{F}_i) = \sum (\vec{r}_i \times m_i \vec{a}_i)$$





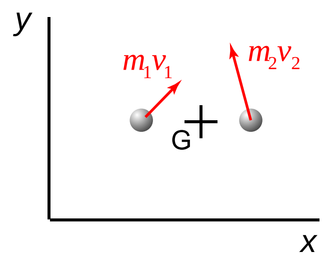
Momentum

Linear Momentum $\vec{L} = \sum m_i \vec{v}_i$

Angular Momentum $\vec{H}_o = \sum (\vec{r}_i \times m_i \vec{v}_i)$

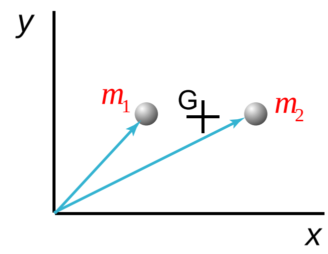
$$\sum \vec{F} = \dot{\vec{L}}$$

$$\sum \vec{M}_o = \dot{\vec{H}}_o$$



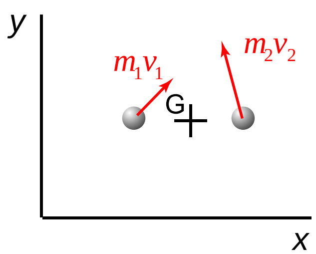
Mass Center

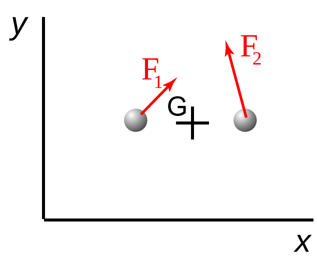
$$m\vec{\bar{r}} = \sum m_i \vec{r}_i$$

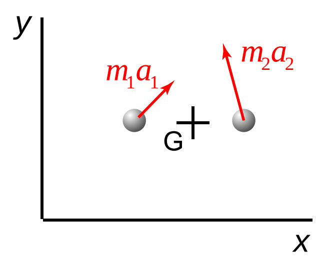


Newton's 2nd Law and Momentum at Mass Center

$$\vec{L} = m\vec{\bar{v}} \qquad \sum \vec{F} = m\vec{\bar{a}} = \dot{\vec{L}} \qquad \sum \vec{M}_G = \dot{\vec{H}}_G$$







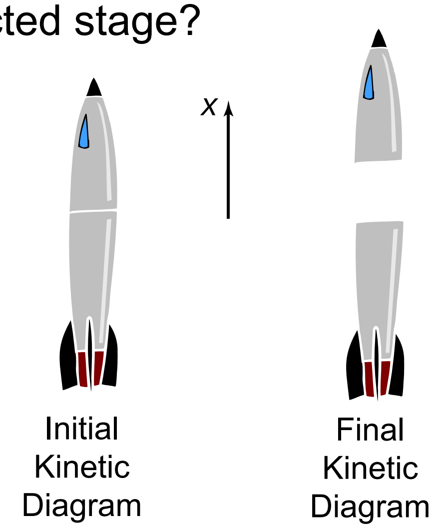
Conservation of Momentum

When $\sum \vec{F} = 0$ $\quad \vec{L}$ = constant

When $\sum \vec{M} = 0$ $\quad \vec{H}$ = constant

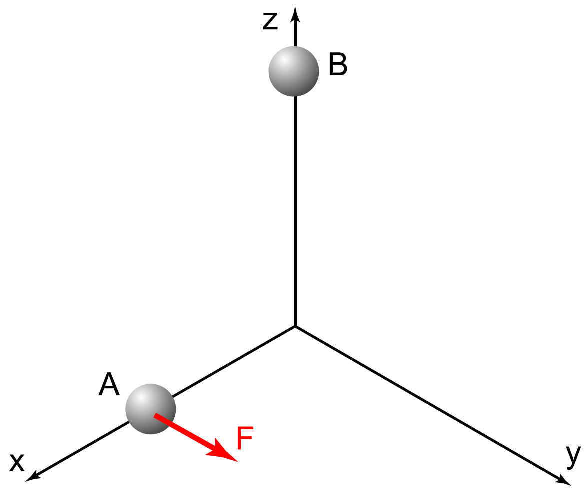
Example

A lunar vehicle with a mass of 10,000 kg travels through space at a speed of 1000 m/s. When some of the fuel is spent, the lowest stage of the vehicle, having a mass of 1500 kg, is ejected directly backwards. The rest of the vehicle continues forward at a speed of 1400 m/s. What is the speed of the ejected stage?

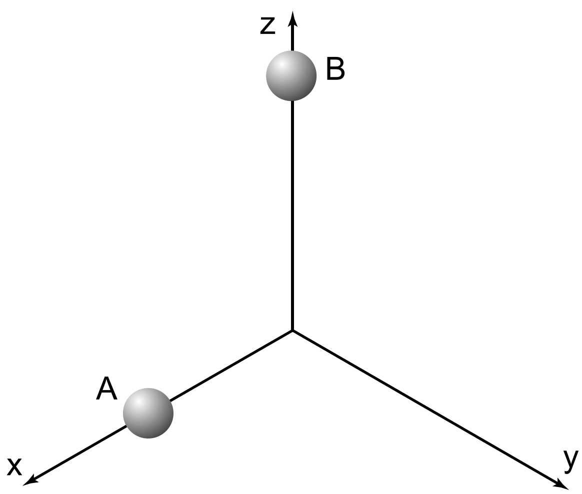


Example

Two particles, A and B, with masses of $m_A = 1$ kg and $m_B = 1.5$ kg, respectively, comprise a system. The velocity of A is $\vec{v}_A = 1\hat{i} - 2\hat{j} + 2\hat{k}$ m/s, and the velocity of B is $\vec{v}_B = -2\hat{i} - 1.5\hat{j} + 1\hat{k}$ m/s. Particle A is located by the position vector $\vec{r}_A = 1\hat{i}$ m. Particle B is located by the position vector $\vec{r}_B = 2\hat{k}$ m. Particle A is acted on by a force $\vec{F} = 3\hat{j}$ N. Determine (a) the total angular momentum about the origin, and (b) the rate of change of angular momentum.



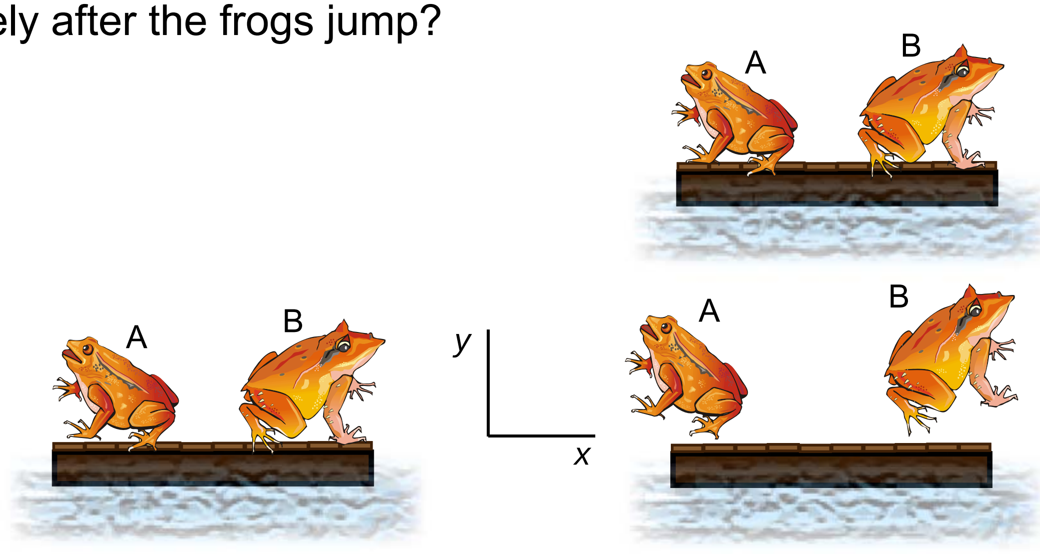
FBD



Kinetic Diagram

Example

Two frogs sit on a floating plank of mass 1 kg which is initially at rest. Frog A has a mass of 40 g and jumps to the left at a speed of 1 m/s. Simultaneously, Frog B which has a mass of 55 g jumps to the right at a speed of 1.2 m/s. What is the speed and direction of the plank immediately after the frogs jump?



Initial Kinetic Diagram

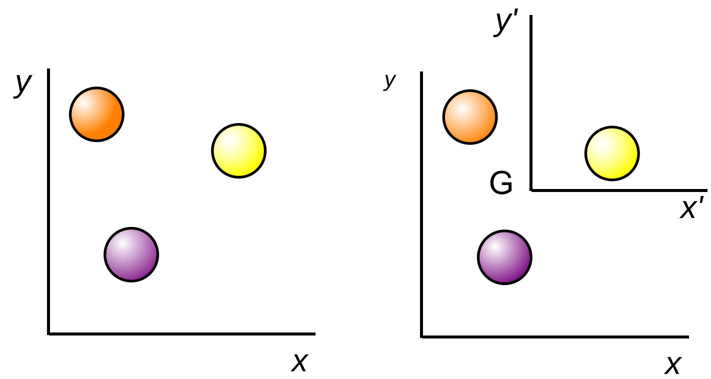
Final Kinetic Diagram

WORK, ENERGY, IMPULSE AND MOMENTUM FOR SYSTEM OF PARTICLES

Kinetic Energy

$$T = \frac{1}{2}\sum m_i v_i^{\ 2}$$

$$T = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\sum m_i v'_i{}^2$$



Work-Energy Principle

$$T_1 + U_{1\rightarrow 2} = T_2$$

Conservation of Energy

$$T_1 + V_1 = T_2 + V_2$$

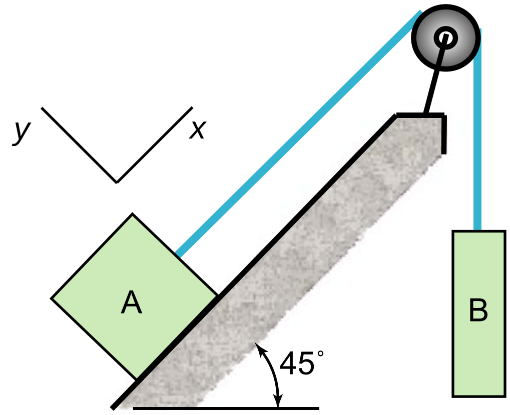
Principle of Impulse and Momentum

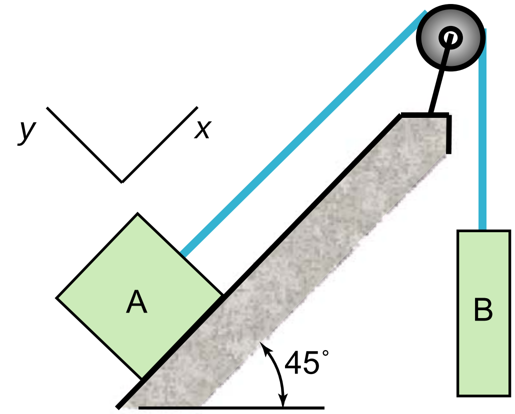
$$\vec{L}_1 + \sum \int_{t1}^{t2} \vec{F} dt = \vec{L}_2$$

$$(\vec{H}_o)_1 + \sum \int_{t1}^{t2} \vec{M}_o dt = (\vec{H}_o)_2$$

Example

Block A weighing 25 lb is released from rest and moves down the incline, which has a coefficient of kinetic friction of 0.10. Block B weighs 12 lb. Determine the velocity of A after it moves 5 ft down the incline.

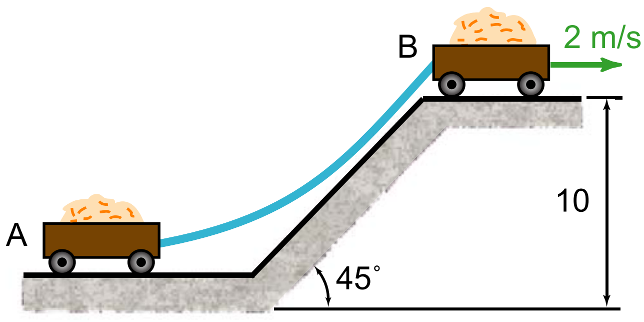


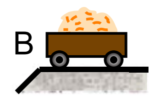


FBD

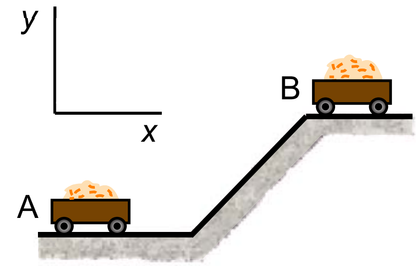
Example

Cart A, which has a mass of 4 kg, and cart B, which has a mass of 5 kg, are connected by a cable. If cart B is given a velocity to the right of 2 m/s while the cable is slightly slack, what will the velocity of the carts be when the cable becomes taut? How far will cart A travel up the hill? Assume frictionless surfaces. Units: meters

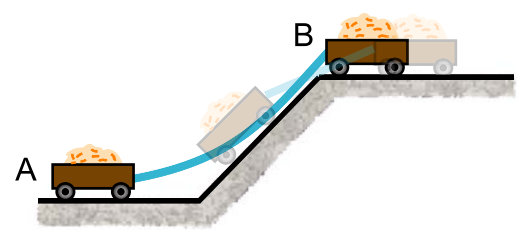




Initial Kinetic Diagram

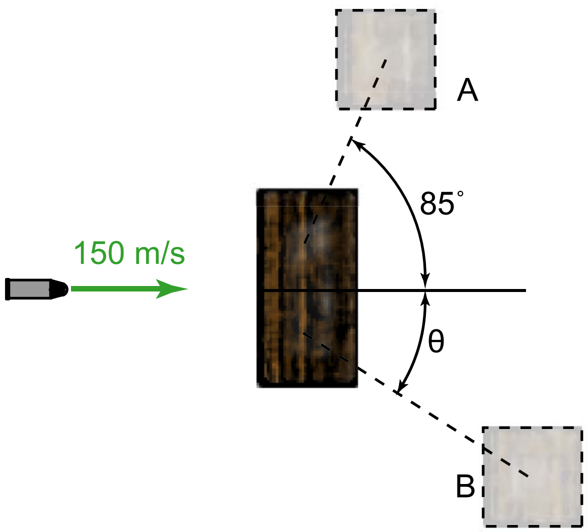


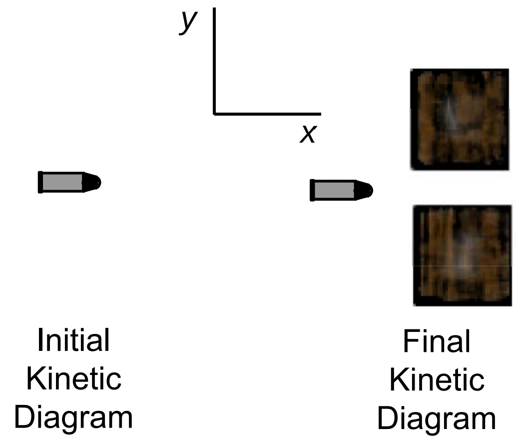
Final Kinetic Diagram



Example

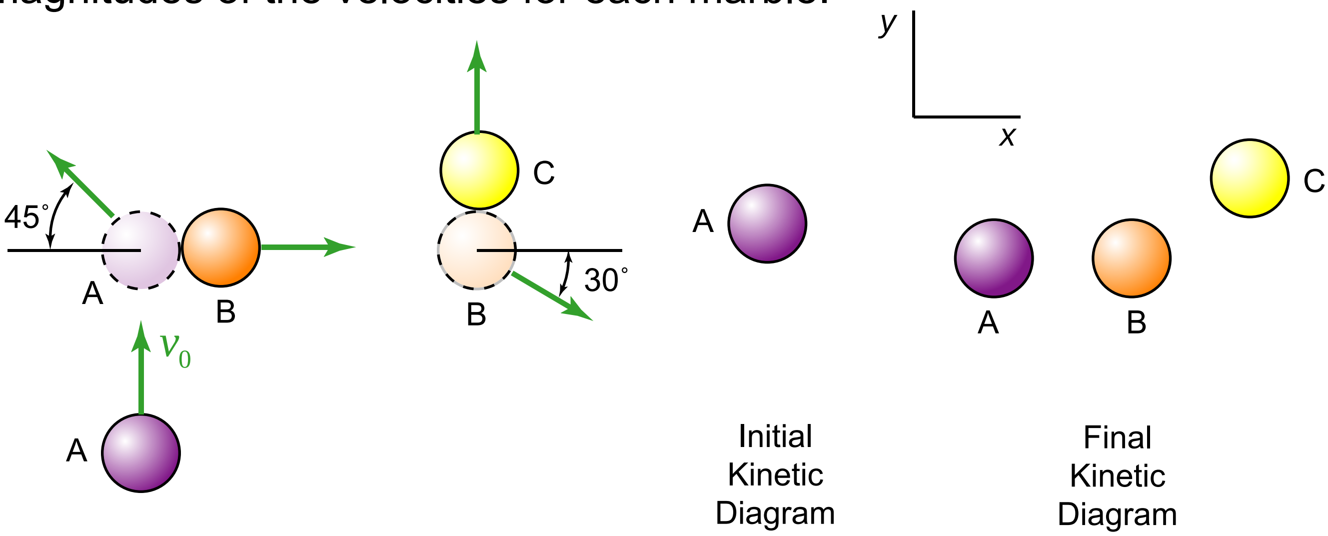
A 3 g bullet moving at 150 m/s hits a 1 kg block of wood. The wood splits into pieces A and B of mass 0.55 kg and 0.45 kg, respectively. It is known that the bullet continues in the same direction at 98 m/s. Block A moves at the 85° angle shown at a speed of 1 m/s. What is the direction and speed of block B.





Example

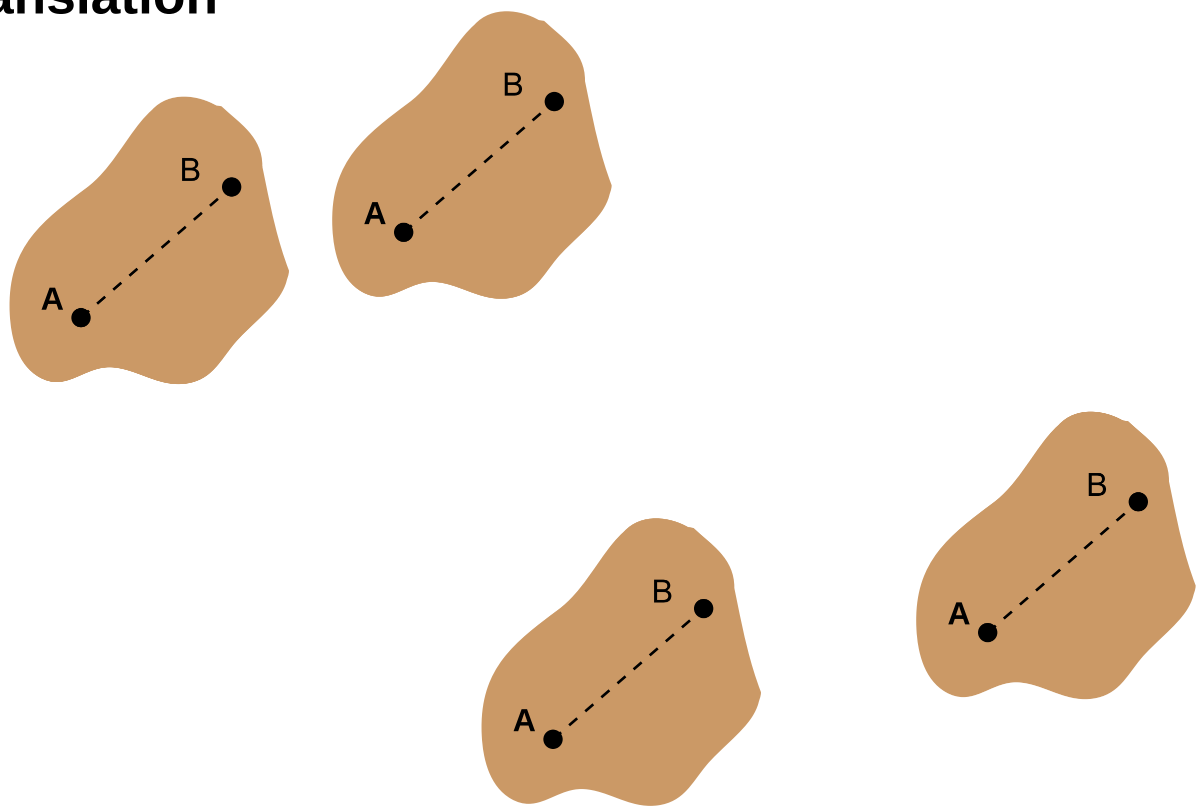
Marbles A, B, and C, each having a mass of 5 grams, are on a smooth horizontal surface. A is given a velocity of 1 m/s as shown. A hits B, causing B to hit C. Velocities after impact are in the directions shown. Assuming no friction and all perfectly elastic collisions, determine the magnitudes of the velocities for each marble.



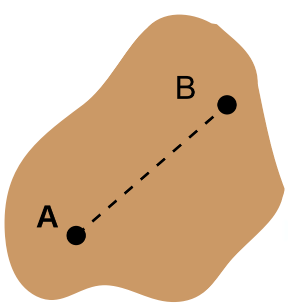
Chapter 5
Kinematics of Rigid Bodies

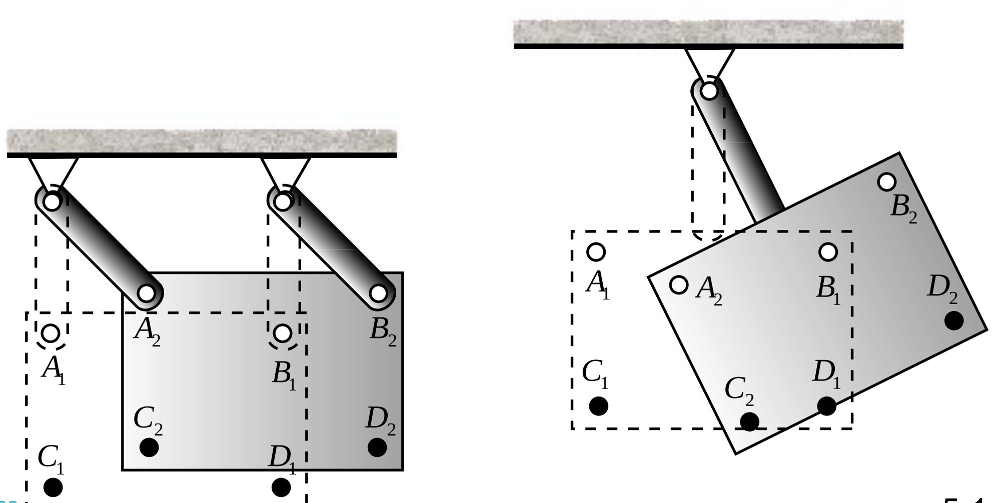
Introduction

Translation

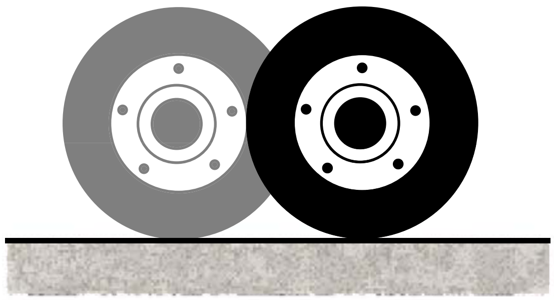


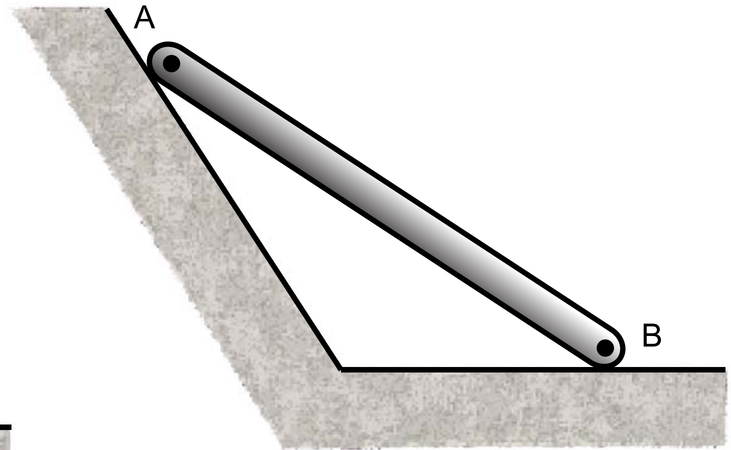
Rotation about a Fixed Axis





General Plane Motion

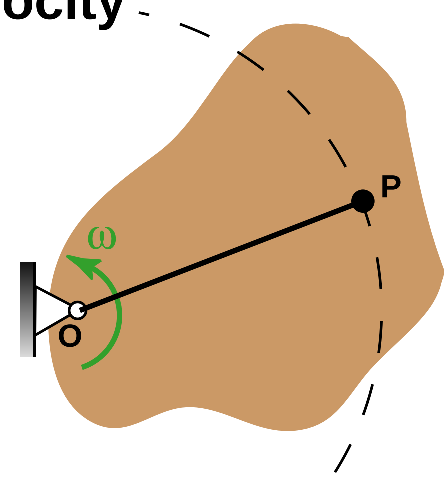




Kinematics of Rigid Bodies

ROTATION ABOUT A FIXED AXIS

Velocity



$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$\vec{v} = r\omega\hat{e}_t$$

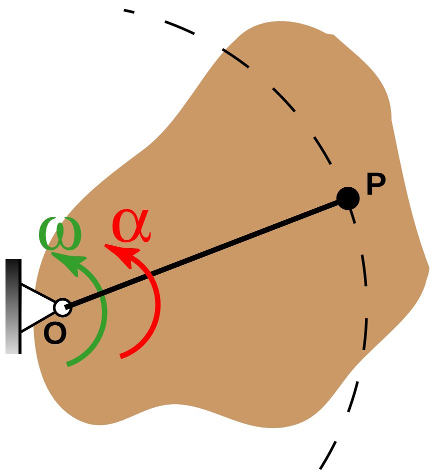
Velocity of point P which is fixed on a rigid body which rotates about a fixed pivot at "O".

Note:

$\vec{\omega}$ = angular velocity of rigid body containing points P and O.

ROTATION ABOUT A FIXED AXIS

Acceleration



$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$\vec{a} = \vec{\alpha} \times \vec{r} - (\omega^2 \vec{r})$$

$$\vec{a} = r\alpha \hat{e}_t + r\omega^2 \hat{e}_n$$

Acceleration of point P which is fixed on a rigid body which rotates about a fixed pivot at "O".

Note:

$\vec{\omega}$ = angular velocity of rigid body containing points P and O.

$\vec{\alpha}$ = angular acceleration of rigid body containing points P and O.

EQUATIONS DEFINING THE ROTATION OF A RIGID BODY ABOUT A FIXED POINT

Angular versions of the three basic kinematic equations:

Angular velocity:

$$\omega = \frac{d\theta}{dt}$$

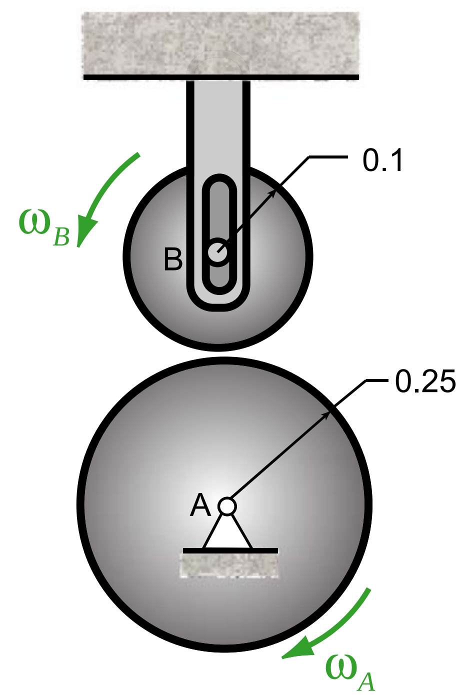
Angular acceleration:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\alpha = \omega \frac{d\omega}{d\theta}$$

Example

Before contact, A is rotating clockwise at 22 rad/s while B is at rest. During the first six seconds of contact, slipping occurs and the angular speed of each wheel changes uniformly. If the final angular speed of A is 10 rad/s, determine (a) the angular acceleration of B during the period of slipping; and (b) the number of revolutions made by B before it reaches its final speed.



Example

Starting from rest, an astronaut uses a thruster to generate an angular acceleration of 0.4 rad/sec^2 for two seconds, and then rotates with constant angular velocity. How long does it take to rotate 180°?

Example

A turbine spinning at 1200 RPM turns off and decelerates with an angular acceleration of -0.1 rad/s^2. How long does it take to for the angular speed to become 600 RPM?

Example

A flywheel starts from rest and accelerates according to the relationship: $\alpha = 0.4t + 2$ where α is in rad/s^2 and t is in seconds. (a) How long does it take to reach a speed of 200 rpm? (b) How many revolutions are required to reach that speed?

Example

A shaft has an angular acceleration of $-k\omega$. If the shaft has an initial speed of 28 rad/s and comes to rest after 5.4 revolutions, find (a) the value of k and (b) the time required for the shaft to slow to 1% of its initial speed.

Example

A rotating disk has an angular acceleration given by $\alpha=-6\theta$ where α is in rad/s² and θ is in radians. Determine the angular velocity of the disk when θ is 6 radians if the disk has a speed of 15 rad/s when $\theta=0$.

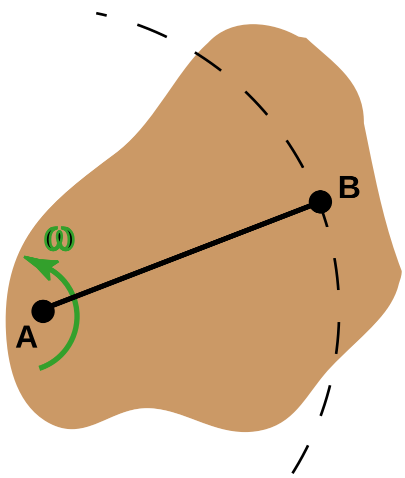
ABSOLUTE AND RELATIVE VELOCITY IN PLANE MOTION

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

When applied to two points A and B fixed to a rigid body:
the velocity of point B is the sum of the velocities due to:

-the motion of (constrained) point A and

-the rotation of point B circling point A



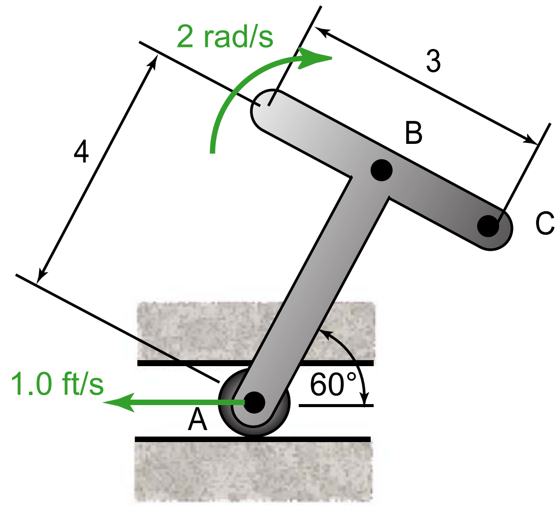
$$\vec{v}_B = \vec{v}_A + (\vec{\omega} \times \vec{r}_{B/A})$$

$\vec{v}_B$ is the velocity of point B (fixed on the rigid body) using the motion of A (also fixed on the rigid body) while the rigid body translates and rotates.

$\vec{v}_{B/A}$ is the velocity of B as it travels in a circle around A.

Example

The T-bracket has a clockwise angular speed of 2 rad/s and roller A is moving to the left in the horizontal slot with a speed of 1.0 ft/s at the instant shown. Calculate at this instant the velocity of: a) point B, b) point C. Units: in.



Example

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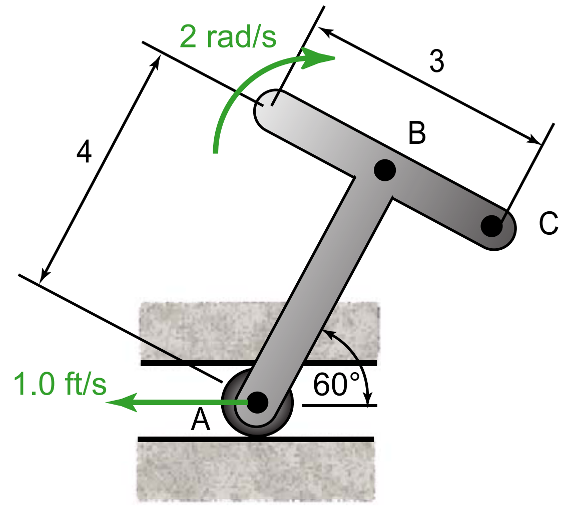
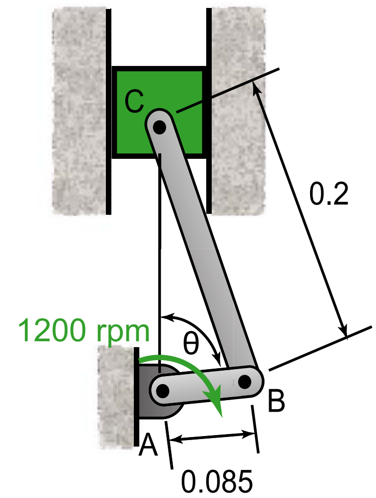


Alternative Solution:

Example

Crank AB rotates with a constant angular velocity of 1200 rpm, determine the velocity of piston C and the angular velocity of the connecting rod BC when (a) θ= 0°, (b) θ= 90°. Units: meters.



Example

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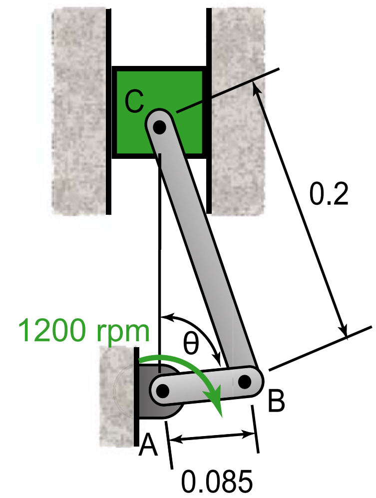
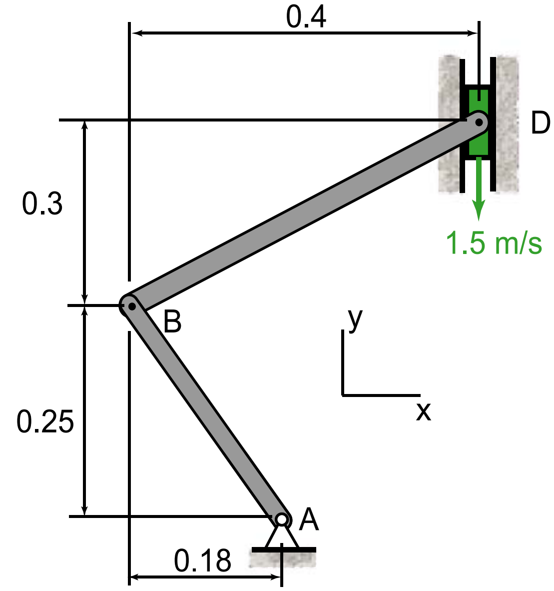


Alternative Solution:

Example

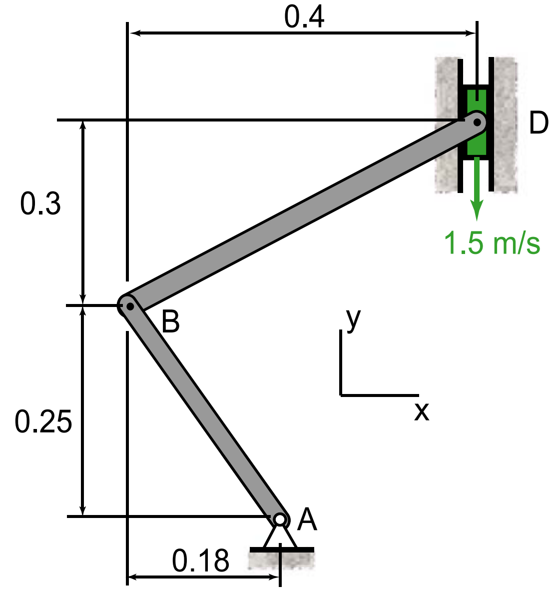
Determine the angular velocities of bars AB and BD at this instant.
Units: meters.



Example

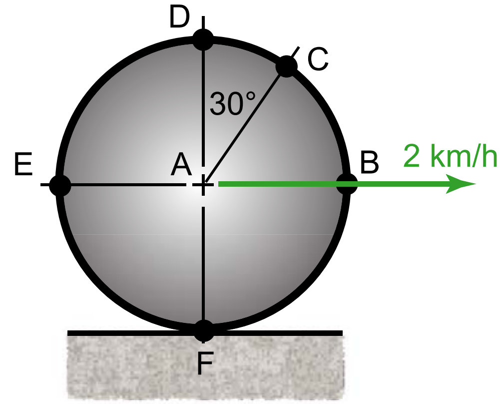
Determine the angular velocities of bars AB and BD at this instant.
Units: meters.

Alternative Solution:



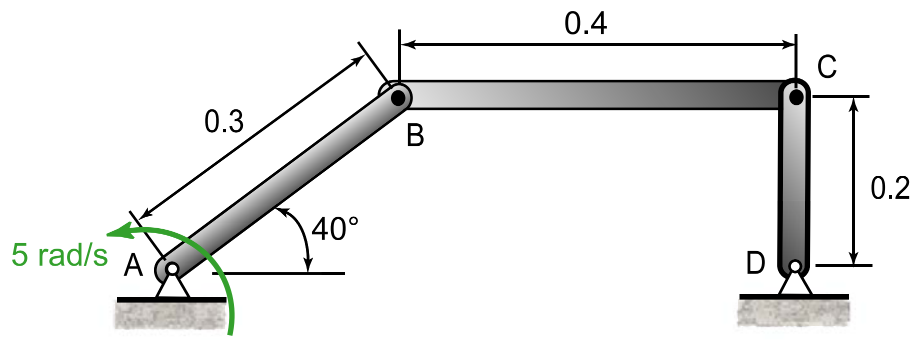
Example

A disk travels without slipping at a constant speed of 2 km/h. Determine the velocities of points B, C, D, E and F on the outside of the 100 mm diameter disk. Units: meters.



Example

Bar AB is rotating counterclockwise at the constant angular velocity of 5 rad/s. At this instant, determine the angular velocity of bar CD. Units: meters.

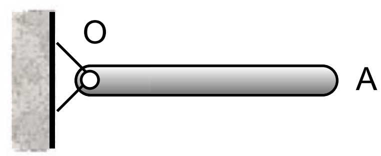


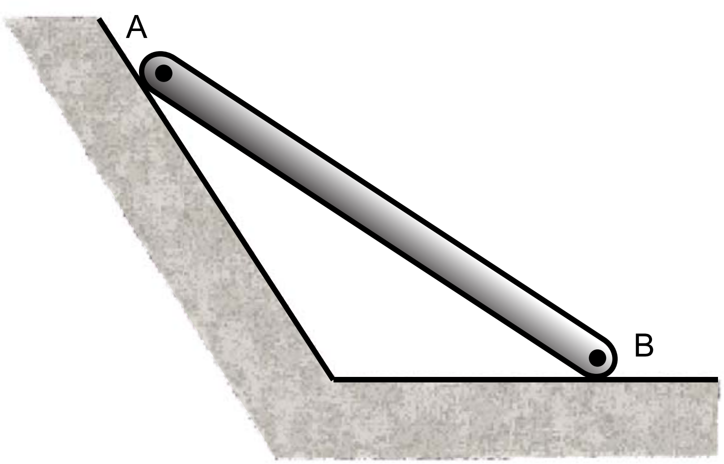
INSTANTANEOUS CENTER OF ROTATION

The point on a rigid body (or imaginary extension of the rigid body) where the velocity is zero for the instant (position) considered. NOTE: Its acceleration is usually NOT zero.

Recall, the relative velocity equation:

$$\vec{v}_B = \vec{v}_A + (\vec{\omega} \times \vec{r}_{B/A})$$

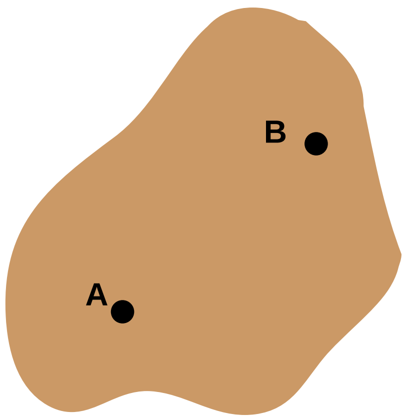




How to Locate Instant Centers

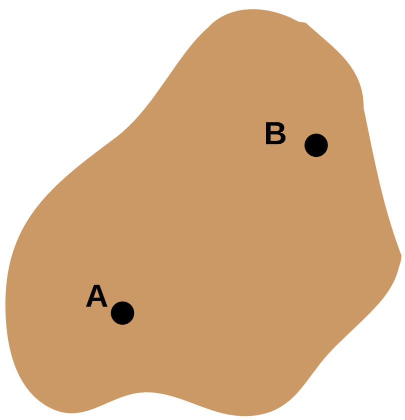
Step 1) Find the DIRECTION of the velocity of two points on the rigid body.

Case A: The two velocity vectors are NOT parallel.



Step 2)
Draw lines perpendicular to the velocity vector through the two points.

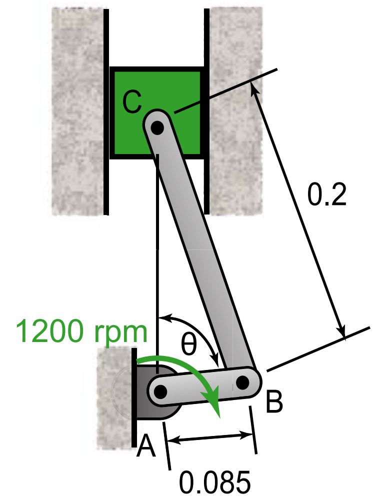
Case B: The two velocity vectors are parallel.



Step 2)
Locate the instant center by direct proportion.

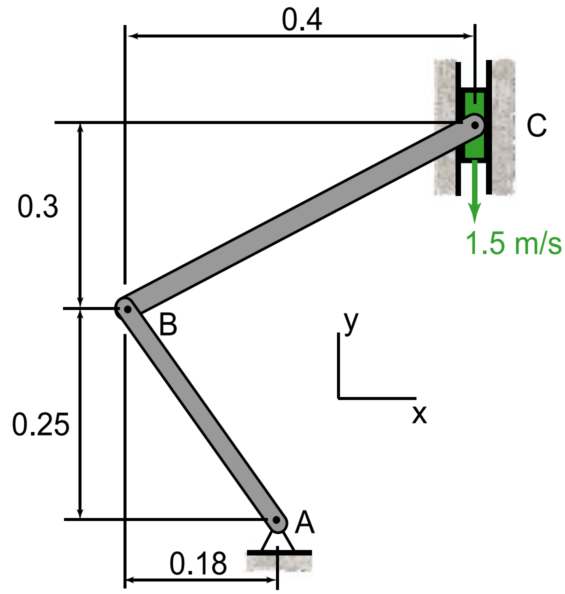
Example

Crank AB rotates with a constant angular velocity of 1200 rpm, determine the velocity of piston C and the angular velocity of the connecting rod BC when (a) θ= 0°, (b) θ= 90°. Units: meters.



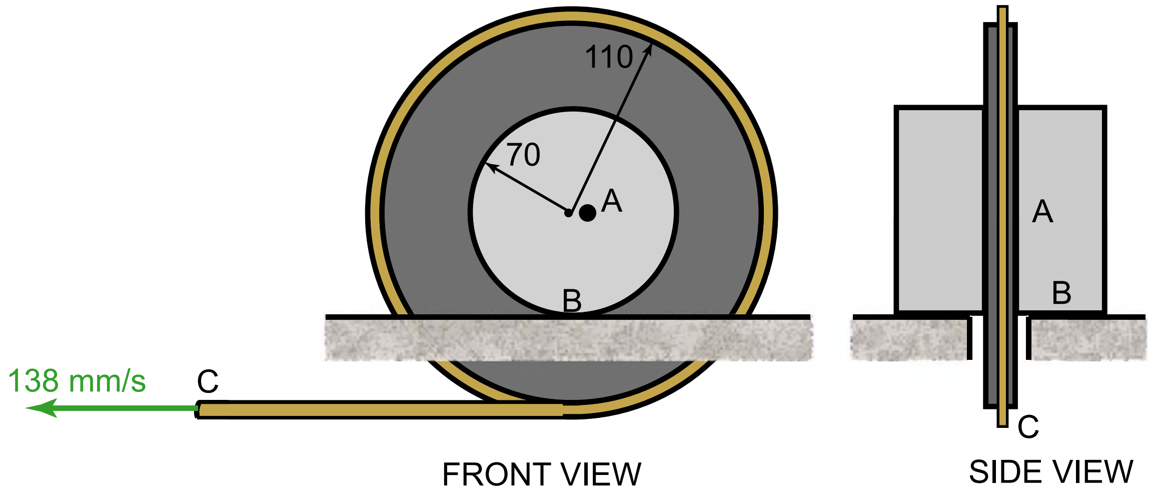
Example

The velocity of the slider C is 1.5 m/s. Determine the angular velocities of bars AB and BC at this instant. Units: meters.



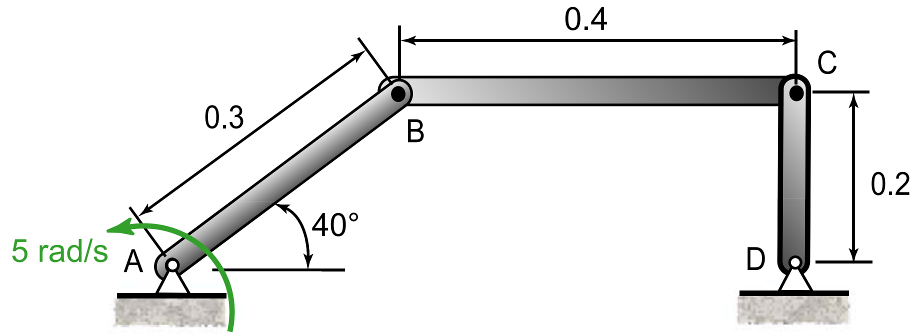
Example

Two drums are rigidly attached. The smaller drum rolls without sliding on the surface at B, and a rope is wound around the larger drum. At C, the rope is pulled to the left with a velocity of 138 mm/s, determine (a) the angular velocity of the drums, (b) the velocity of point A, (c) the length of rope wound or unwound per second. Units: mm.



Example

Bar AB is rotating counterclockwise at the constant angular velocity of 5 rad/s. At this instant, determine the angular velocity of bar CD. Units: meters.



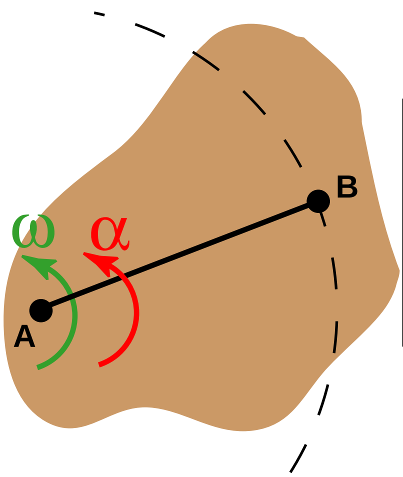
ABSOLUTE AND RELATIVE ACCELERATION IN PLANE MOTION

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

When applied to two points A and B fixed to a rigid body: the acceleration of point B is the sum of the accelerations due to:

-the motion of (constrained) point A and

-the rotation of point B circling point A.



$$\vec{a}_B = \vec{a}_A + (\vec{\alpha} \times \vec{r}_{B/A}) + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{B/A})$$

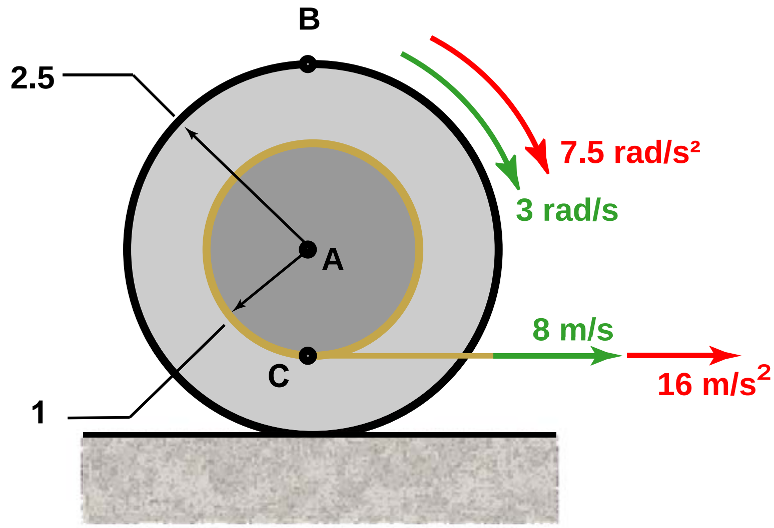
$$\vec{a}_B = \vec{a}_A + (\vec{\alpha} \times \vec{r}_{B/A}) - (\omega^2 \vec{r}_{B/A})$$

$\vec{a}_B$ is the acceleration of point B (fixed on the rigid body) using the motion of A (also fixed on the rigid body) while the rigid body translates and rotates.

$\vec{a}_{B/A}$ is the acceleration of B as it travels in a circle around A.

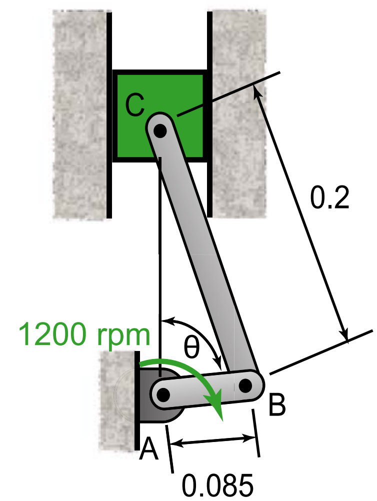
Example

A rope is wrapped around the hub of the spool. A pull at the end of the rope causes the spool to roll and slip on the horizontal plane. For the instant shown, find the acceleration of (a) point C on the spool; (b) point A; and (c) point B. Units: meters.



Example

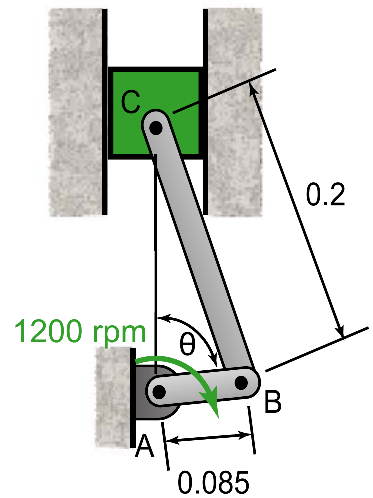
Crank AB rotates with a constant angular velocity of 1200 rpm, determine the acceleration of piston C and the angular acceleration of the connecting rod BC when (a) θ= 0°, (b) θ= 90°. Units: meters.



Example

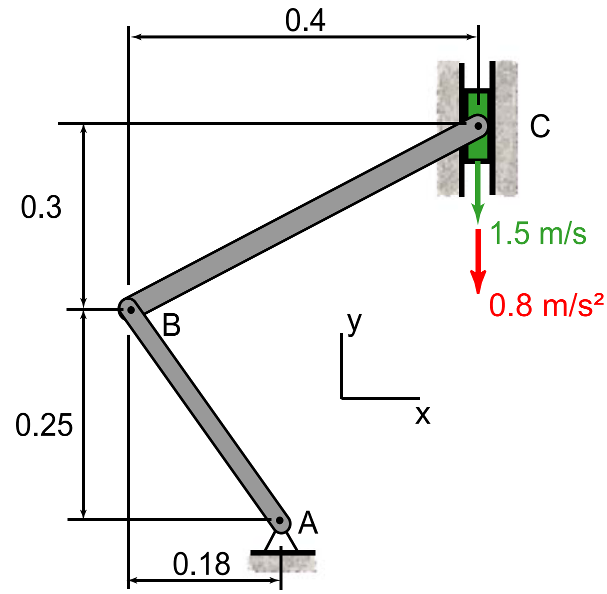
Crank AB rotates with a constant angular velocity of 1200 rpm, determine the acceleration of piston C and the angular acceleration of the connecting rod BC when (a) θ= 0°, (b) θ= 90°. Units: meters.

Alternative Solution:



Example

The velocity and acceleration of the slider C are 1.5 m/s and 0.8 m/s². Determine the angular accelerations of bars AB and BC at this instant. Units: meters.



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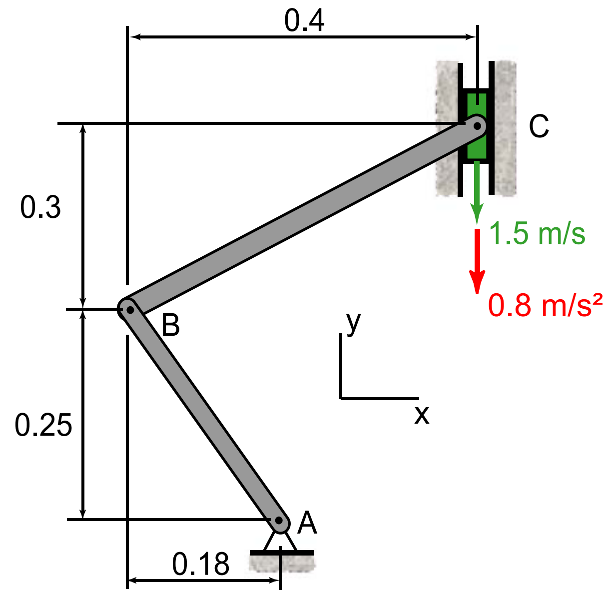
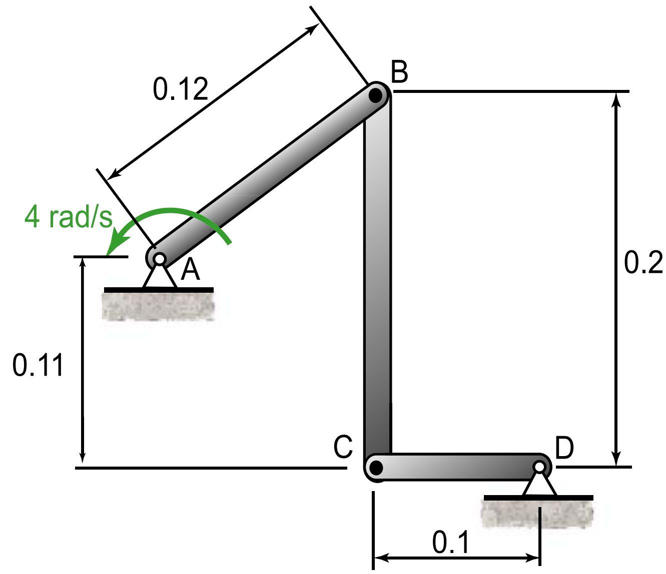


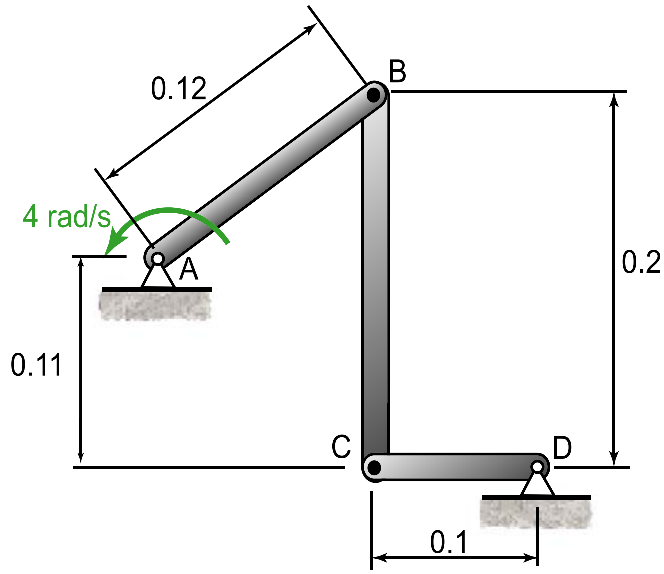
Example

Bar AB is rotating counterclockwise with a constant angular velocity of 4 rad/s. Determine the angular velocities and angular accelerations of bars BC and CD. Units: meters.



Example

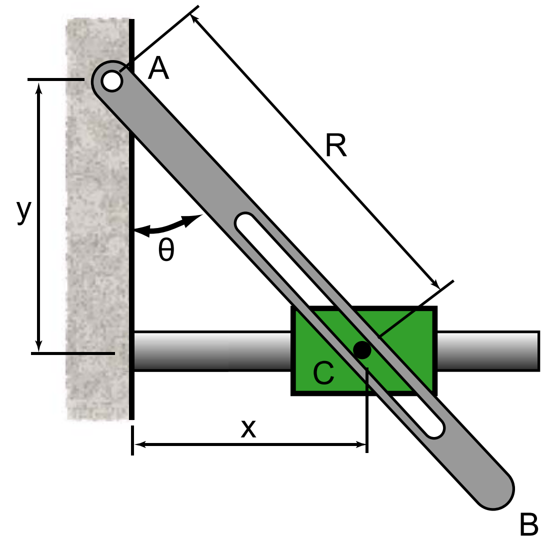
Bar AB is rotating counterclockwise with a constant angular velocity of 4 rad/s. Determine the angular velocities and angular accelerations of bars BC and CD. Units: meters.



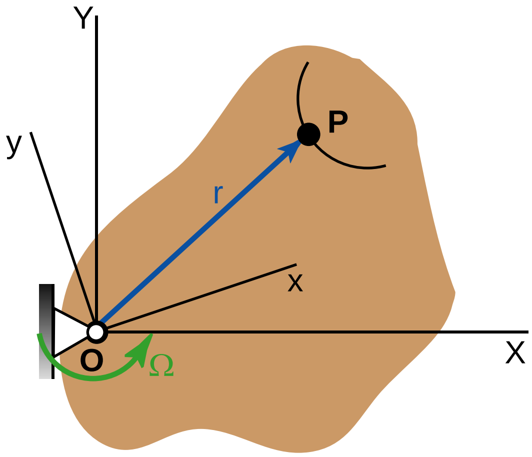
Alternative Solution:

Example

Determine the speed and acceleration of pin C in terms of θ.



PLANE MOTION OF A PARTICLE RELATIVE TO A ROTATING FRAME WITH FIXED ORIGIN



$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/F}$$

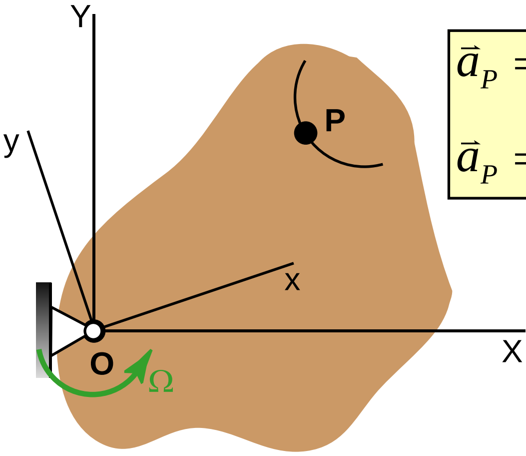
$$\vec{v}_P = (\dot{\vec{r}})_{OXY} = \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{oxy}$$

$\vec{v}_P$ = absolute velocity of P

$\vec{v}_{P'}$ =velocity of point P´ on the rotating frame F

$\vec{v}_{P/F}$ =velocity of particle P relative to the rotating frame F

PLANE MOTION OF A PARTICLE RELATIVE TO A ROTATING FRAME WITH FIXED ORIGIN



$$\vec{a}_P = \vec{a}_{P'} + \vec{a}_c + \vec{a}_{P/F}$$

$$\vec{a}_P = \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy} + (\ddot{\vec{r}})_{Oxy}$$

$\vec{a}_P$ = absolute acceleration of particle P

$\vec{a}_{P'}$ = acceleration of point P´ on the rotating frame F

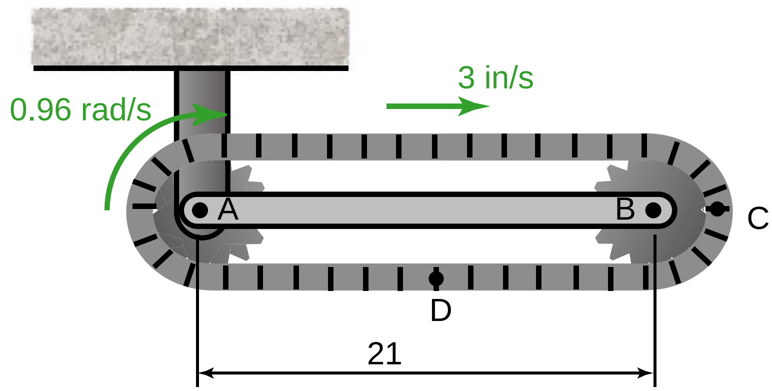
$\vec{a}_{P/F}$ = acceleration of P relative to the rotating frame F

$\vec{a}_c = 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy} = 2\vec{\Omega} \times \vec{v}_{P/F}$

= Coriolis acceleration

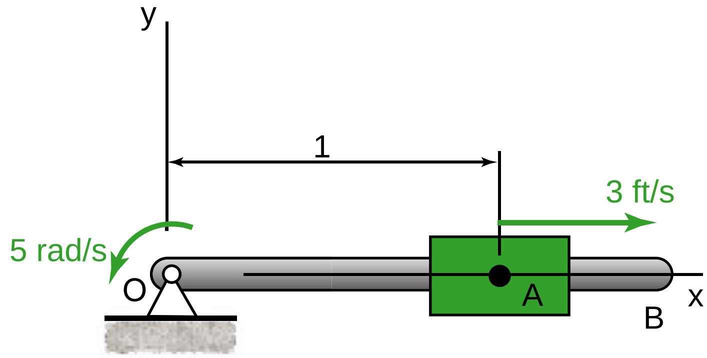
Example

A chain is looped around two gears at A and B, each with a radius of 3 in. The chain moves freely about arm AB in a clockwise direction at the constant rate of 3 in/s relative to the arm and gear A drives arm AB at a constant rate of 0.96 rad/s. Point D is located midway between A and B. At this instant, determine the acceleration of the chain at points C and D. Units: in.



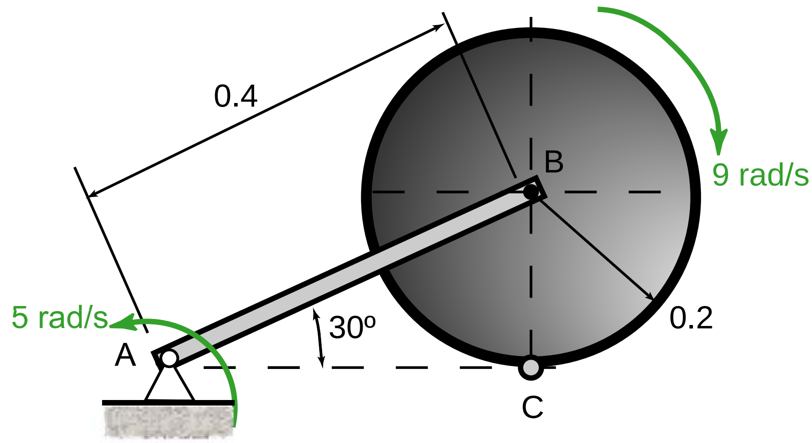
Example

Rod OB rotates counterclockwise with the constant angular speed of 5 rad/sec while collar A is sliding toward B with the constant speed 3 ft/s relative to the rod. Calculate the acceleration of the collar A. Units: Ft.

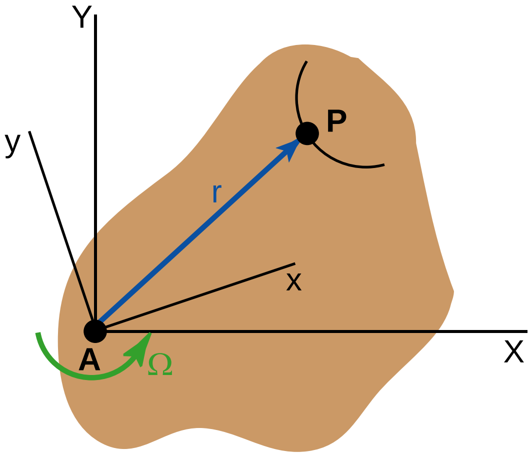


Example

Relative to AB, the disk is rotating with a constant angular speed of 9 rad/s while AB is rotating counterclockwise with the constant angular speed of 5 rad/s. Determine the acceleration of point C on the rim of the disk by (a) considering AB as a rotating reference frame; and (b) using the relative acceleration method. Units: meters.



PLANE MOTION OF A PARTICLE RELATIVE TO A ROTATING FRAME IN GENERAL MOTION



$$\vec{v}_P = \vec{v}_A + \vec{v}_{P'} + \vec{v}_{P/F}$$

$$\vec{v}_P = \vec{v}_A + \vec{\Omega} \times \vec{r} + (\dot{\vec{r}})_{Axy}$$

$\vec{v}_P$ = absolute velocity of P

$\vec{v}_A$ = velocity of point A on the rotating frame F

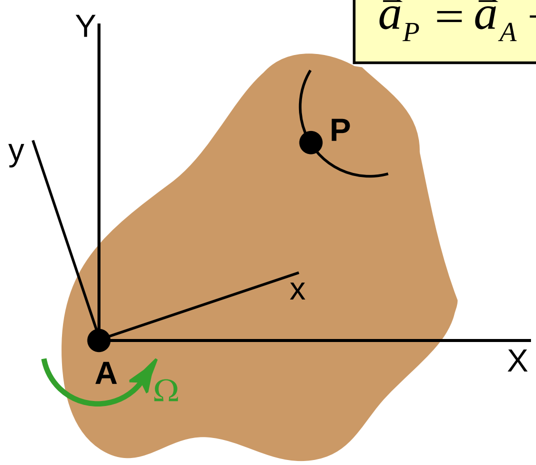
$\vec{v}_{P'}$ = velocity of point P´ on the rotating frame F

$\vec{v}_{P/F}$ = velocity of particle P relative to the rotating frame F

PLANE MOTION OF A PARTICLE RELATIVE TO A ROTATING FRAME IN GENERAL MOTION

$$\vec{a}_P = \vec{a}_A + \vec{a}_{P'} + \vec{a}_c + \vec{a}_{P/F}$$

$$\vec{a}_P = \vec{a}_A + \dot{\vec{\Omega}} \times \vec{r} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + 2\vec{\Omega} \times (\dot{\vec{r}})_{Axy} + (\ddot{\vec{r}})_{Axy}$$



$\vec{a}_P$ = absolute acceleration of particle P

$\vec{a}_A$ = acceleration of point A on the rotating frame F

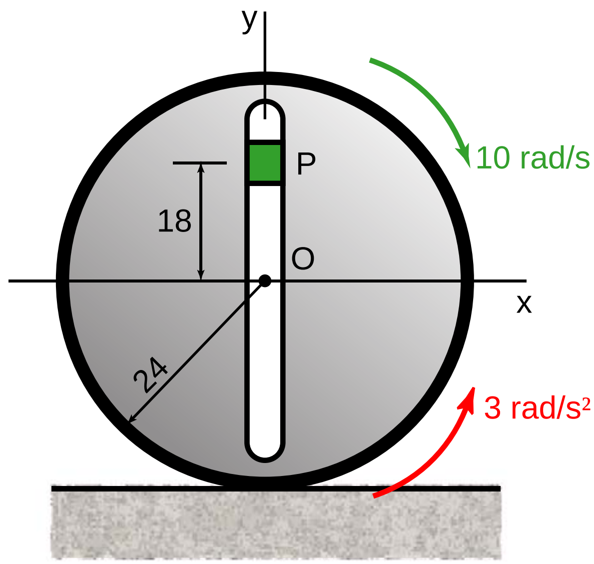
$\vec{a}_{P'}$ = acceleration of point P´ on the rotating frame F

$\vec{a}_{P/F}$ = acceleration of P relative to the rotating frame F

$$\vec{a}_c = 2\vec{\Omega} \times (\dot{\vec{r}})_{Oxy} = 2\vec{\Omega} \times \vec{v}_{P/F}$$

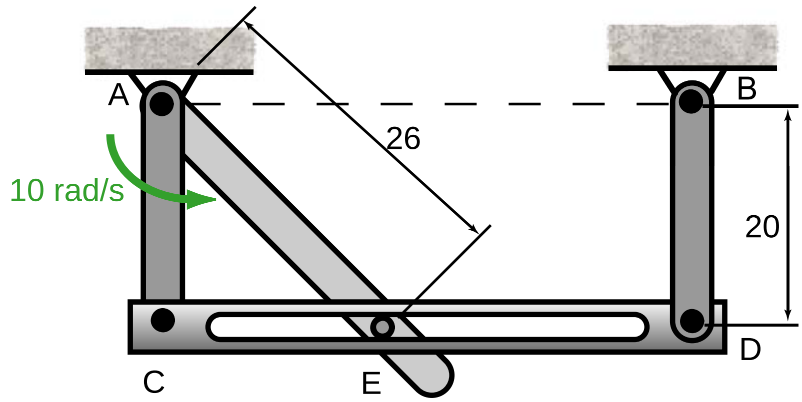
Example

The slotted disk rolls without slipping. At the instant shown, the angular velocity of the disk is 10 rad/s clockwise, and the angular acceleration is 3 rad/s^2 counterclockwise while slider P has a velocity and acceleration of 15 ft/s and 32.2 ft/s^2 (relative to the wheel), both directed downward. Find the velocity and acceleration of slider P in this position. Units: in.



Example

Bar AC has a constant angular velocity of 10 rad/s counterclockwise. Determine at this instant the angular acceleration of rod AE and the acceleration of pin E relative to bar CD. Units: in.



Chapter 6

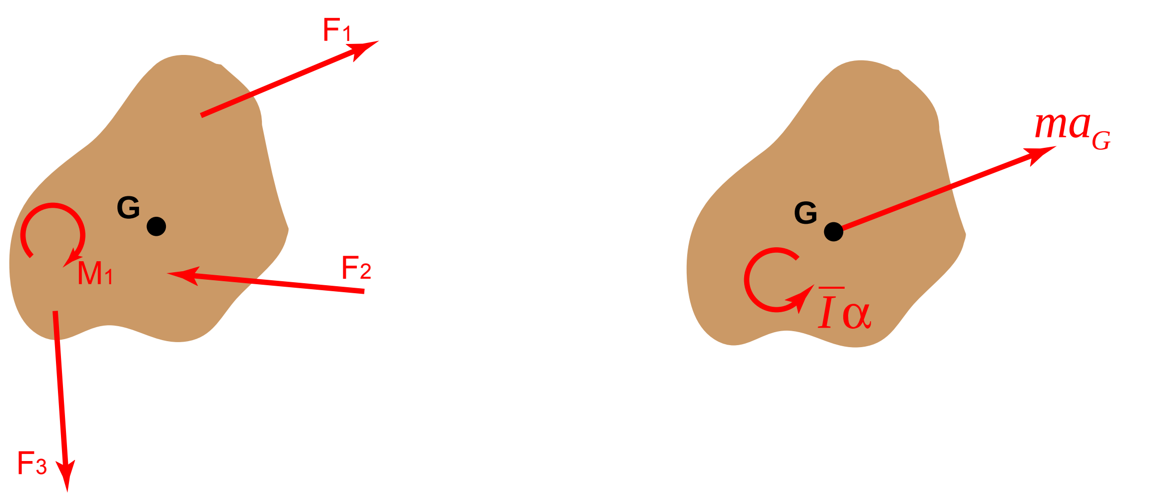
Plane Motion of Rigid Bodies: Forces and Accelerations

Equations of Motion for a Rigid Body

$$\sum \vec{F} = m\vec{a}_G$$

$$\sum \vec{M} = \bar{I}\vec{\alpha}$$

Free-Body-Diagram = Mass-Acceleration-Diagram

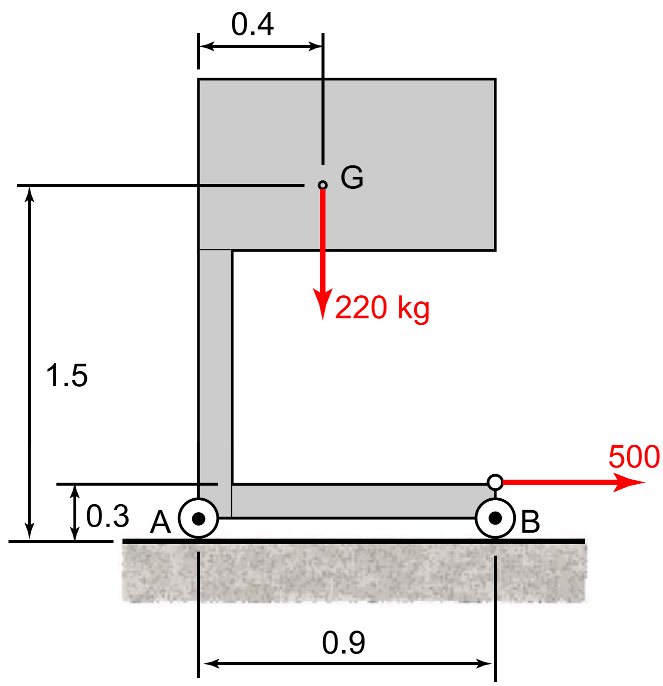


- Isolate
- Draw non-contact forces
- Draw contact forces

- Draw (straight) arrow(s) & label ma_G
- Draw curved arrow & label $\bar{I}\alpha$

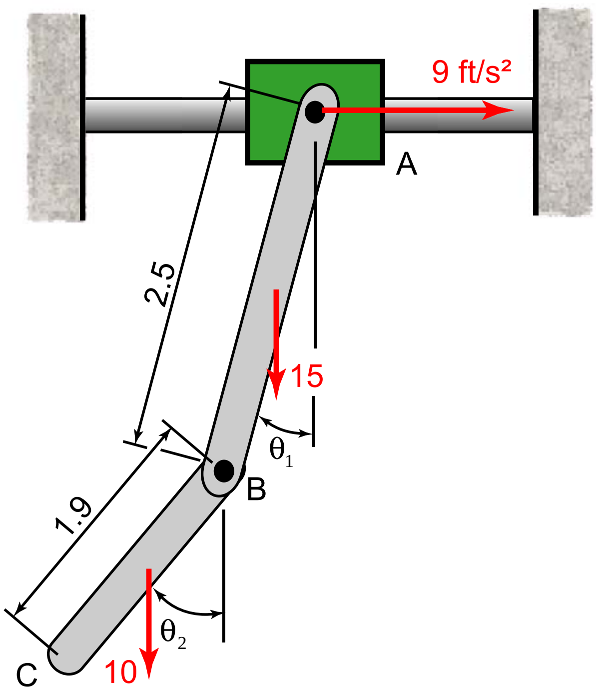
Example

The tool cabinet is supported on frictionless wheels at A and B. (a) Determine the acceleration of the cabinet assuming that it does not tip. (b) Verify that the cabinet does not tip. Units: N, m.



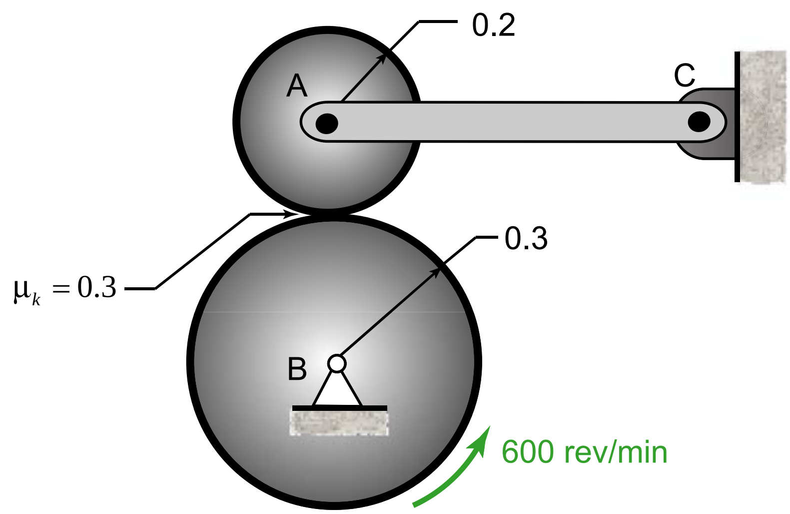
Example

The upper bar is pinned to the sliding collar at A. The collar has a constant acceleration of 9 ft/s² to the right. The two bars are homogeneous. Determine the angles θ_1 and θ_2, assuming that there is no oscillation (i.e., the angles are constant). Units: Lb, ft.



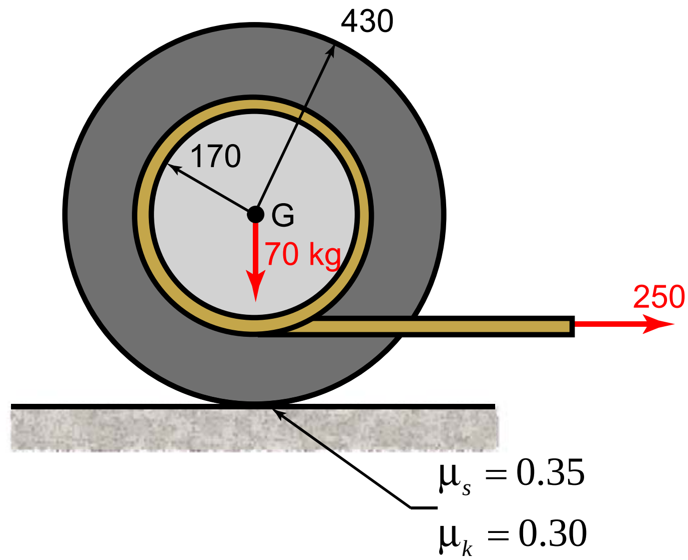
Example

Disk B is spinning freely at 600 rev/min counterclockwise when it is placed in contact with the stationary disk A. Calculate the angular acceleration of each disk during the time that slipping occurs between the disks. Neglect the mass of bar AC. Disks A and B have masses of 3 kg and 6 kg, respectively.



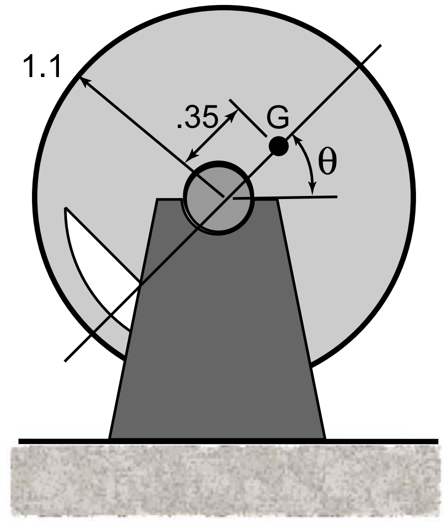
Example

The centroidal mass moment of inertia of the 70 kg spool is I =1.40 kg·m². A cable wound around the hub of the spool is pulled with the constant horizontal force of 250 N. Find the acceleration of the center of the spool. Units: N, mm.



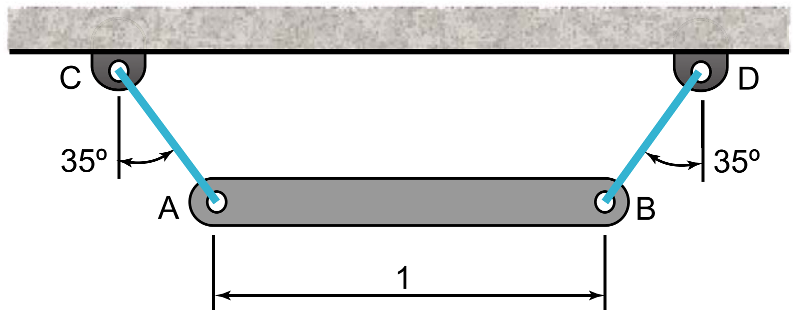
Example

The 115 kg lopsided disk has a centroidal mass moment of inertia of 14 kg·m². Its axel sits in a split bearing that supports the underside of the axel only. How fast can the disk spin without popping out of the bearing?



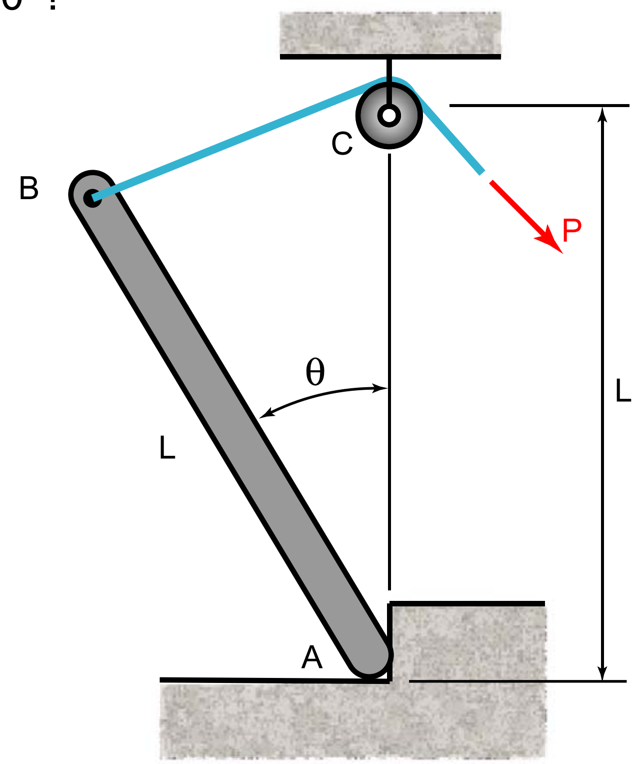
Example

The 20 kg homogeneous bar AB is at rest in the position shown when the rope BD is cut. Determine the initial values of (a) the angular acceleration of the bar; and (b) the acceleration of end B. Ropes AC and BD are 0.5 m long. Units: meters.



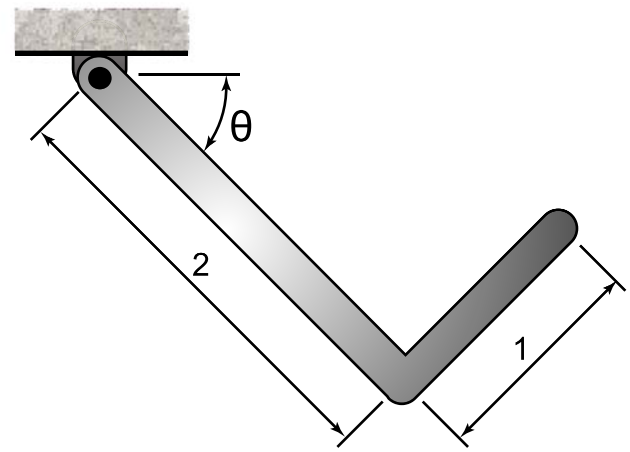
Example

The uniform pole AB of mass m is lifted from rest when θ= 90°. The force acting at the end of the cable has a constant magnitude P = mg. (a) Solve the differential equation of motion for the angular acceleration. (b) Find the angular velocity in terms of θ. (c) What is the angular velocity of the pole when θ=0°?



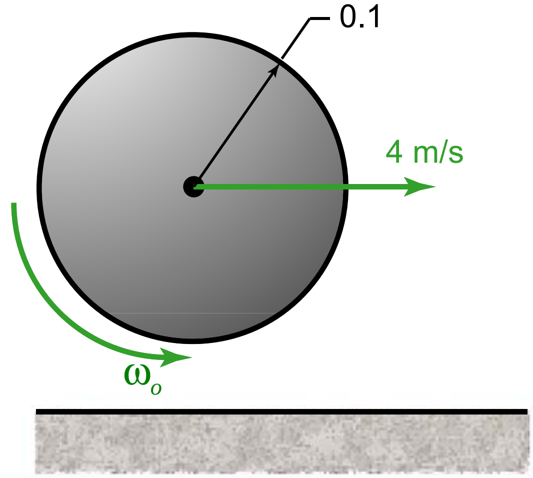
Example

The bar is released from rest when θ= 0°. (a) Derive the differential equation of motion in terms of θ. (b) Write an equation for the angular velocity in terms of θ. (c) Find the maximum value of θ. The bars have a mass of 20 kg/m. Units: meters.



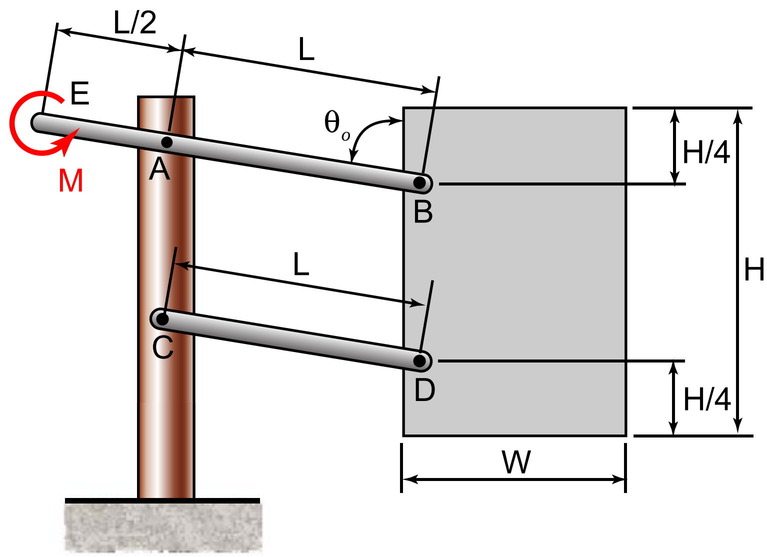
Example

The uniform disk with radius r and mass m is projected along a rough horizontal surface with the initial velocities indicated. (a) Find the initial angular velocity ω_o such that the disk stops traveling at the same instant that it stops spinning. Calculate (b) the time elaspsed and (c) the distance traveled when the disk stops. Use mass m=2 kg, radius r=0.1 meter, and kinetic coefficient of friction $\mu_k = 0.25$.



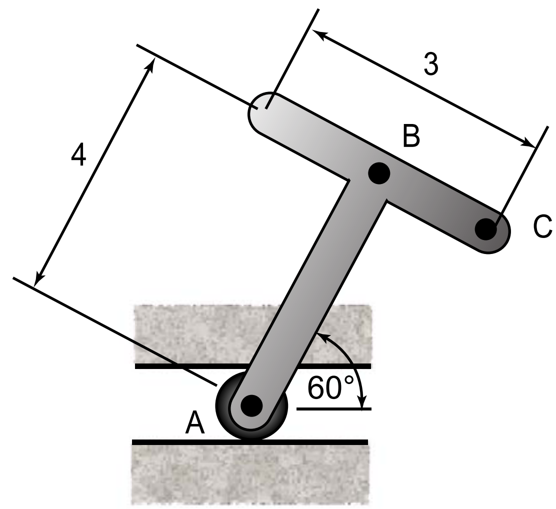
Example

The uniform rectangular plate of mass m is connected to two parallel massless linkages AB and CD. The system starts at rest in the position shown and the plate is raised under the action of the applied couple M. Calculate the forces at pin A in the initial position.



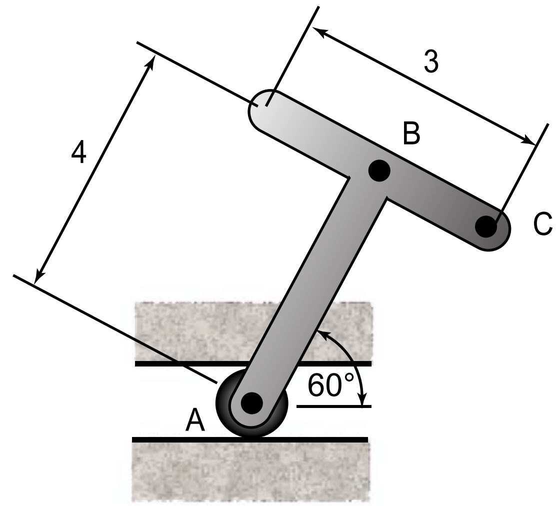
Example

The T-bracket is comprised of two slender bars weighing 0.15 pound per inch of length. The roller at A is free to move in the frictionless horizontal slot. The bracket is released from rest in the position shown and falls under its own weight. Calculate the initial angular acceleration of the bracket. Units: inches.



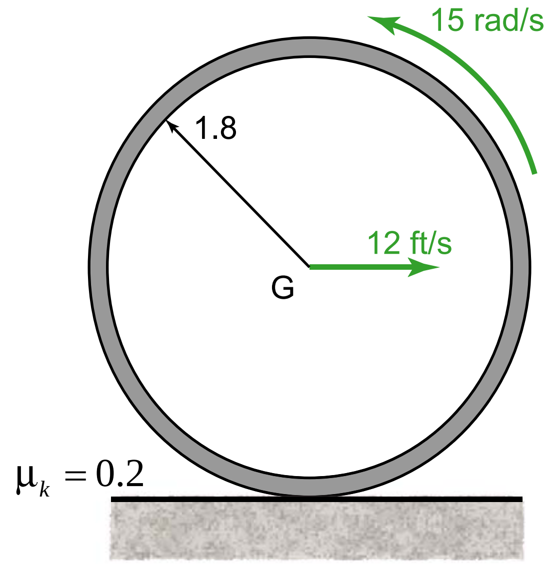
Example

The T-bracket is comprised of two slender bars weighing 0.15 pound per inch of length. The roller at A is free to move in the frictionless horizontal slot. The bracket is released from rest in the position shown and falls under its own weight. Calculate the angular acceleration of the bracket when it's rotated 60° and AB is horizontal. Units: inches.



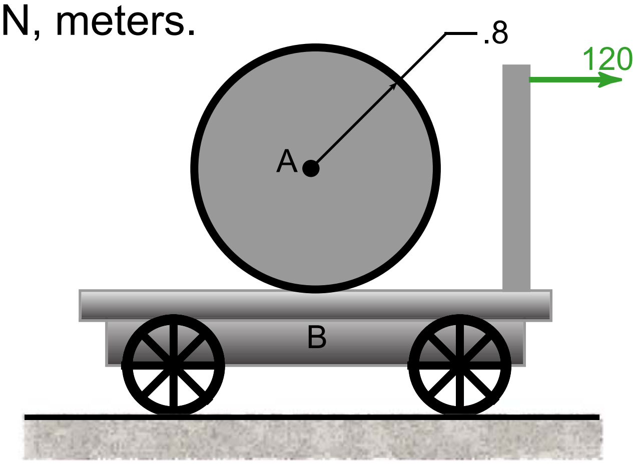
Example

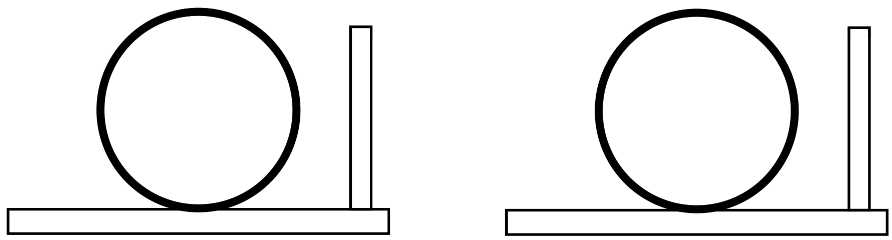
The thin 1.5 lb hoop is launched on a horizontal surface with the initial velocity of 12 ft/s and the angular velocity 15 rad/s, as shown. Calculate (a) the angular acceleration of the hoop and the acceleration of the mass center G during the period of slipping; (b) the time elapsed before slipping stops; and (c) the velocity of G when slipping stops. Units: ft.



Example

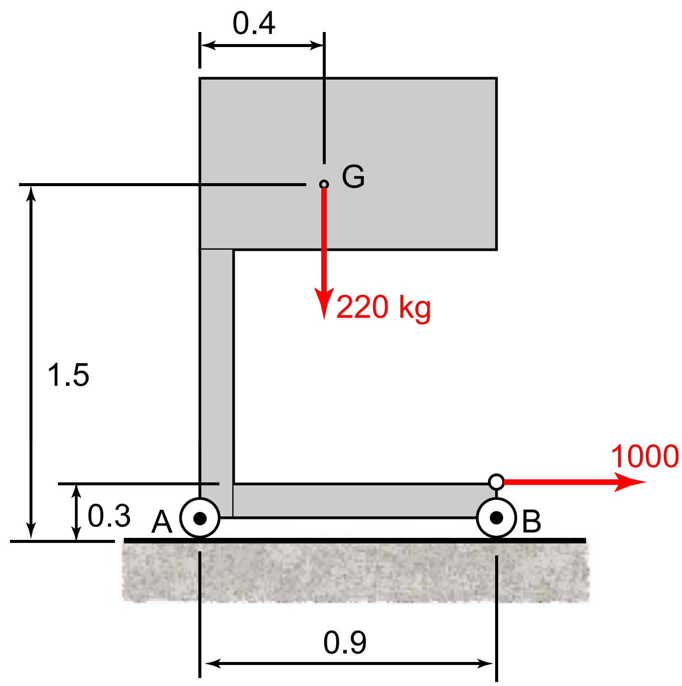
The 50 kg homogeneous disk A rests on top of the 75 kg cart B when the 120 N force is applied. Assuming that the disk does not slip, determine (a) the initial angular acceleration of the disk; and (b) the initial acceleration of the cart. Units: N, meters.





Example

The tool cabinet is supported on frictionless wheels at A and B. Its centroidal mass moment of inertia is 270 $kg \cdot m^2$. The cabinet is at rest when the 1000 N force is applied. Verify that the cabinet will tip. Find the angular acceleration and the acceleration of the mass center when the force is first applied. Units: N, m.



Chapter 7
Plane Motion of Rigid Bodies: Energy and Momentum Methods

WORK-ENERGY EQUATION FOR RIGID BODIES: Two forms:

$$U_{1\to 2} = \Sigma(T_2 - T_1)$$

$$U_{1\to 2} = \Sigma(T_2 - T_1) + \Sigma(V_{g2} - V_{g1}) + \Sigma(V_{e2} - V_{e1})$$

Work

$$U_{1\to 2} = \int \vec{F} \bullet d\vec{r}$$

$$U_{1\to 2} = \int M d\theta$$

Kinetic Energy for a Rigid Body

$$T = \frac{1}{2} m v_G^{\,2} + \frac{1}{2} \bar{I} \omega^2$$

$$T = \frac{1}{2} I_O \omega^2$$

Gravitational Potential Energy

$$V_g = m g h_G$$

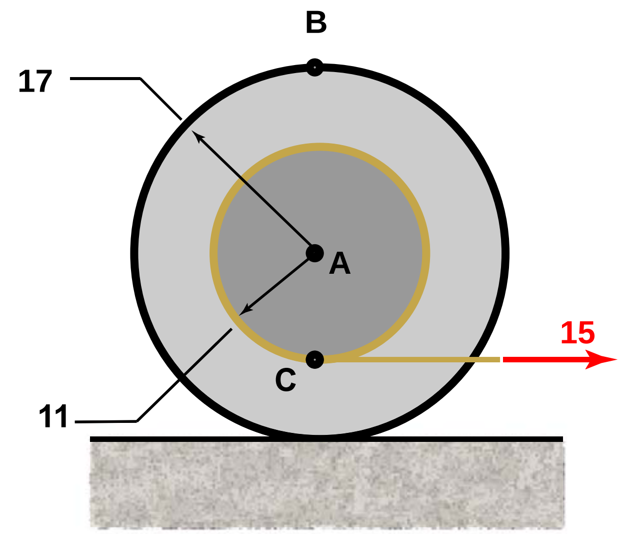
Elastic Potential Energy

$$V_e = \frac{1}{2} k x^2$$

Example

The spool is rolling to the right without slipping. A constant 15 lb force is applied to the rope that is wound around the hub. Determine the work done by this force during one revolution.

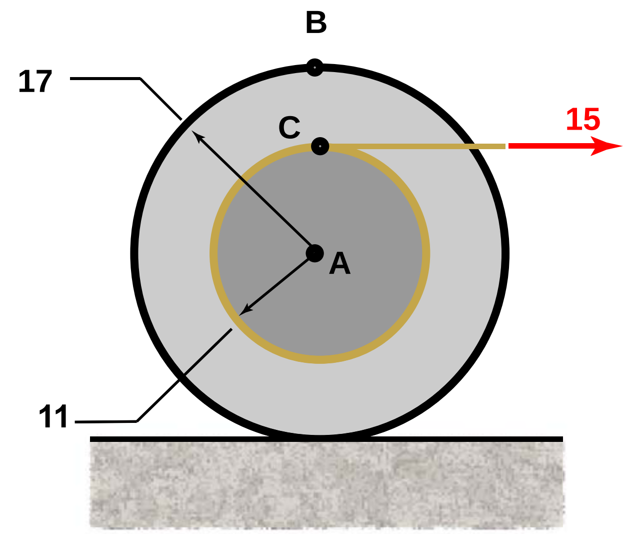
Units: Lb, in.



Example

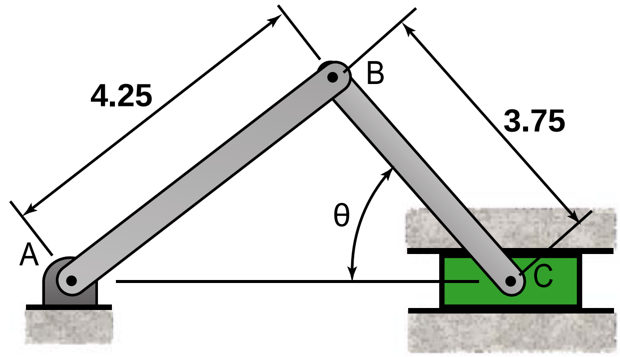
The spool is rolling to the right without slipping. A constant 15 lb force is applied to the rope that is wound around the hub. Determine the work done by this force during one revolution.

Units: Lb, in.



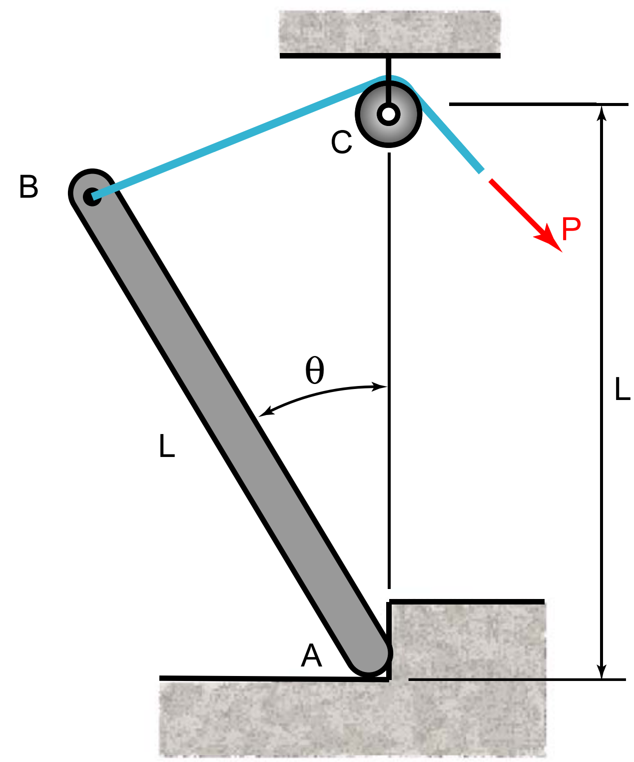
Example

Bars AB and BC of the mechanism are homogeneous and weigh 40 lb and 12 lb, respectively. Slider C weighs 3 lb. When θ= 0° bar AB is rotating at an angular velocity of 3 rad/s, calculate the total kinetic energy of the mechanism in this position. Units: Ft.



Example

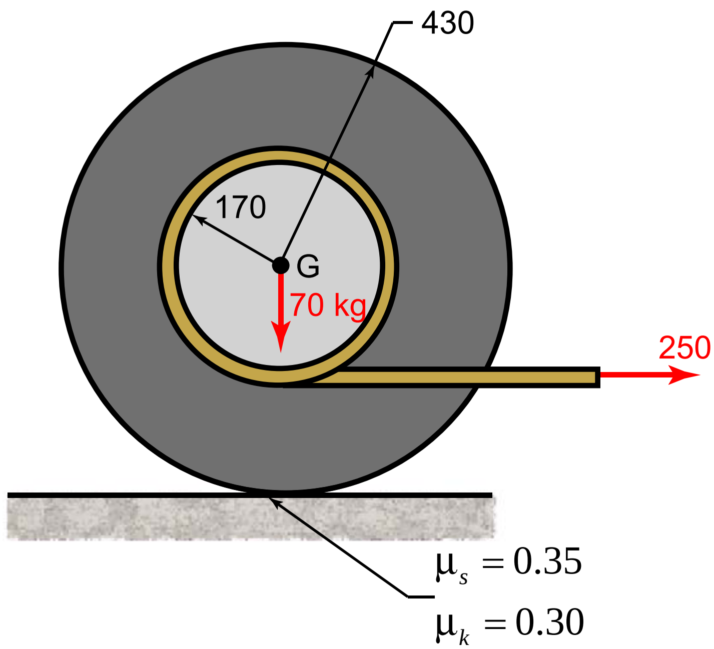
The uniform pole AB of mass m is at rest when θ= 90°. A constant force P= mg is applied to the free end of the cable to lift the pole. (a) Find the angular velocity in terms of θ. (b) What is the angular velocity of the pole when θ= 0°?



Example

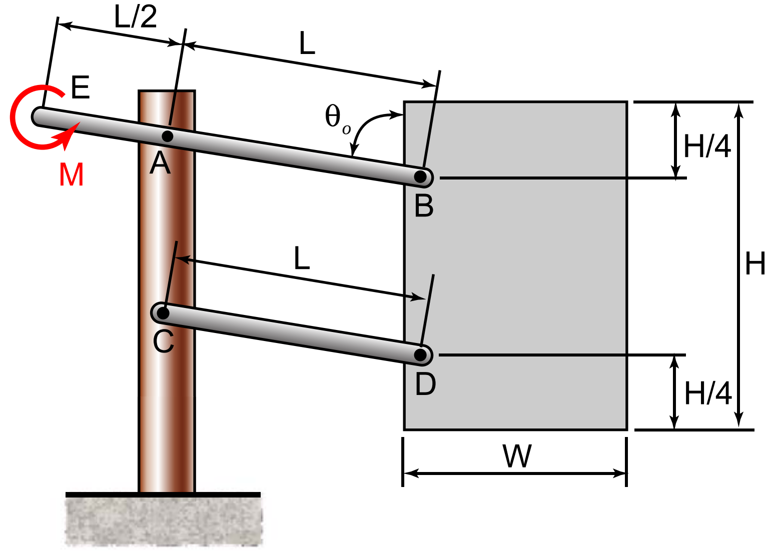
The centroidal mass moment of inertia of the 70 kg spool is I=1.40 kg·m². The system is at rest when a constant horizontal force of 250 N is applied to a cable wound around the hub of the spool. Calculate the angular velocity of the spool when it has moved through one revolution. Verify that the spool rolls without slipping.

Units: kg, N, mm.



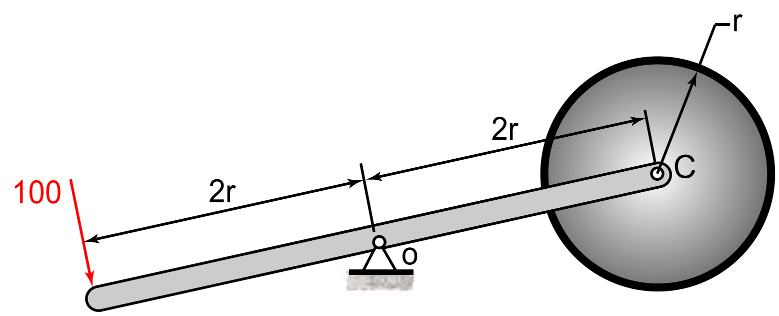
Example

The uniform rectangular plate of mass m is connected to two parallel massless linkages AB and CD. The system starts at rest with θ_o= 60° and the plate is raised under the action of the applied couple M. Calculate the velocity of the plate when the angle is 120°.



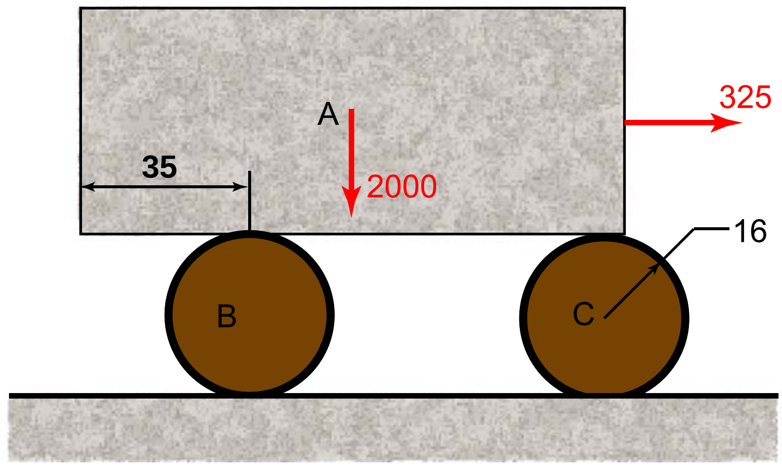
Example

The uniform slender bar has a mass of 2 kg. Welded to its end is a 6 kg uniform disk. The length r is 200 mm The system starts from rest with the bar horizontal and is acted upon by the constant 100 N force which remains perpendicular to the bar throughout the motion. Calculate the angular velocity of the system when it has moved through a 60° angle.



Example

A stone block A is supported by two uniform logs B and C weighing 400 lb each. The system is at rest in the position shown when the constant 325 lb force is applied. Determine the velocity of the block when the left corner reaches the top of log B. Units: Lb, in.



Example

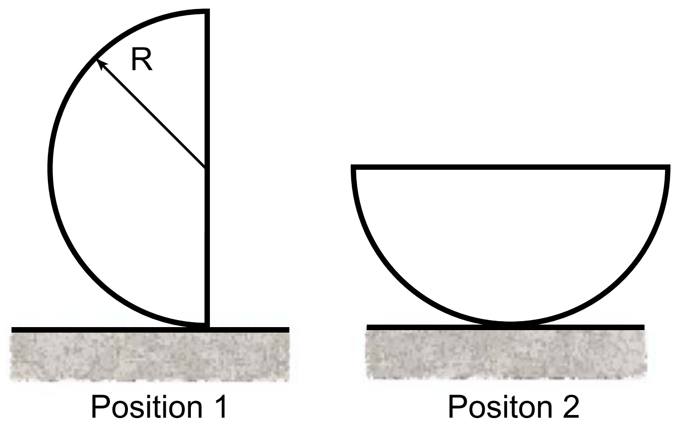
The uniform thin rod is bent into a semicircular shape as shown.

(a) Find the location of the center of gravity G.

(b) Find the centroidal mass moment of inertia.

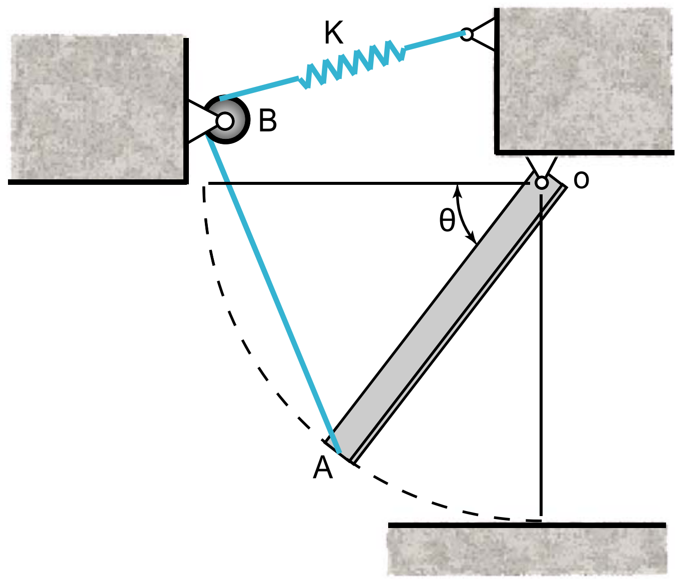
(c) Find the relationship between the velocity of the center of mass and the angular velocity when it's in position 2 and rolling without slipping.

(d) Find the angular velocity in position 2 if it's released from rest in position 1 and rolls without slipping.



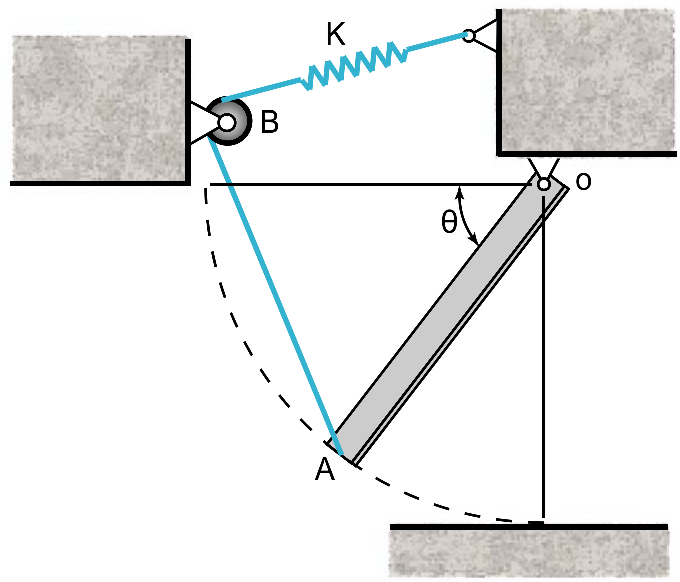
Example

The figure shows the edge-view of an industrial door which is hinged at its upper horizontal edge at O. A cable, attached to the lower edge of the door at A, passes over a small pulley at B and runs to a spring of stiffness 16 lb/ft. The spring is unstretched when θ= 0°. The door weighs 240 pounds and the edge shown is 7 feet long. The door is released from rest in the horizontal position. Calculate the angular velocity of the door just as it reaches the vertical closed position.



Example

The figure shows the edge-view of an industrial door which is hinged at its upper horizontal edge at O. A cable, attached to the lower edge of the door at A, passes over a small pulley at B and runs to a spring of stiffness 16 lb/ft. The spring is unstretched when θ= 0°. The door weighs 240 pounds and the edge shown is 7 feet long. The door is released from rest in the horizontal position. Calculate the maximum angular velocity of the door and the corresponding angle θ.



Example

The T-bracket is comprised of two slender bars weighing 0.15 pound per inch of length. The roller at A is free to move in the frictionless horizontal slot. Gravity acts downward in the figure.

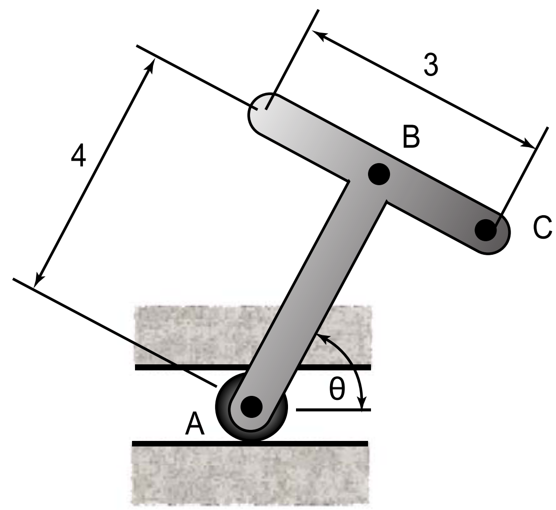
(a) Find the location of the center of gravity G of the T-bracket.

(b) Find the centroidal mass moment of inertia.

(c) Find the relationship between the velocity of the center of mass and the angular velocity when θ is zero.

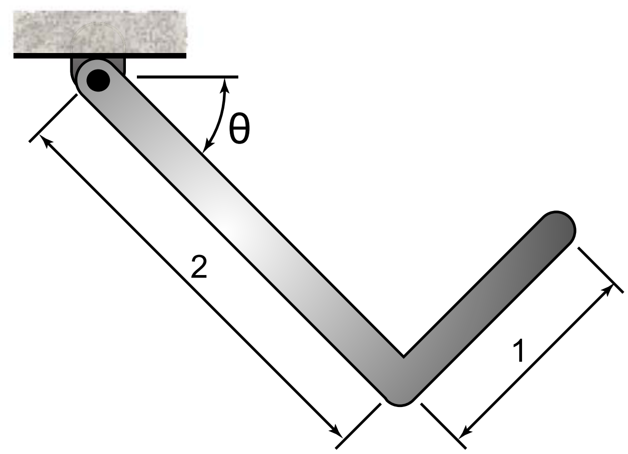
(d) Find the angular velocity when θ is zero if it's released from rest when θ is 60°.

Units: inches.



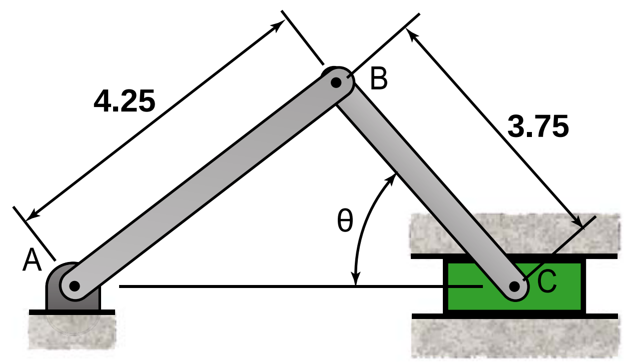
Example

The bar is released from rest when θ= 0°. (a) Write the equation for the angular velocity in terms of θ. (b) Find the maximum angular velocity and the corresponding value of θ. (c) Find the maximum value of θ. The bars have a mass of 20 kg/m. Units: meters.



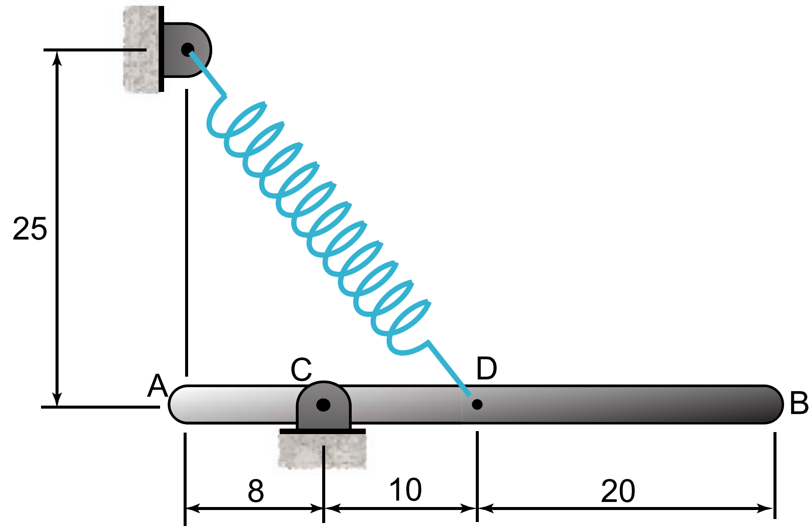
Example

Bars AB and BC of the mechanism are homogeneous and weigh 40 lb and 12 lb, respectively. Slider C weighs 3 lb. The system is released from rest at the angle θ_o. If bar AB has an angular speed of 3 rad/s when θ=0°, find the initial angle θ_o. Units: Ft.



Example

The 12 lb uniform rod AB rotates in the vertical plane about a pin at C. The spring has a stiffness of 1.25 lb/in., and a free length of 26 in. If the rod is released from rest in the position shown, determine its angular velocity when it reaches the vertical position. Units: in.



IMPULSE-MOMENTUM EQUATIONS FOR RIGID BODIES

Linear Impulse is Equal to the Change in Linear Momentum

$$\int \Sigma \vec{F} dt = m\vec{v}_{G2} - m\vec{v}_{G1}$$

Angular Impulse is Equal to the Change in Angular Momentum

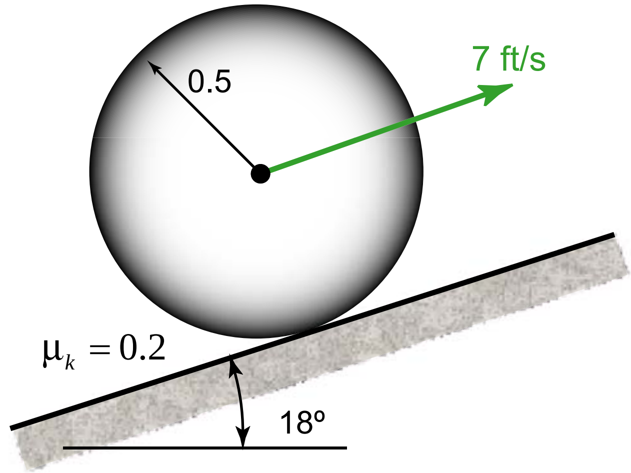
$$\int \Sigma M_G dt = \bar{I}\omega_2 - \bar{I}\omega_1$$

$$\int \Sigma M_O dt = I_O\omega_2 - I_O\omega_1$$

$$\int \Sigma M_P dt = (\bar{I}\omega_2 + mv_{G2}d_2) - (\bar{I}\omega_1 + mv_{G1}d_1)$$

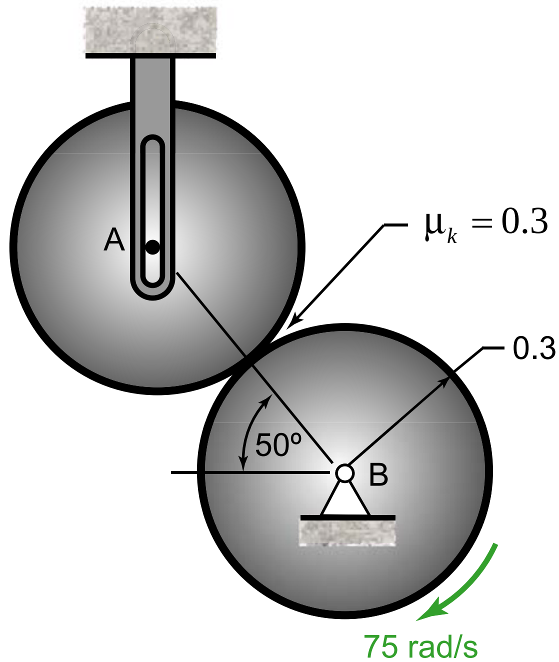
Example

The solid homogeneous ball of mass m is launched on the inclined plane at time t = 0 with the forward speed of 7 ft/s and no spin. Find the time and the angular velocity of the ball when it stops slipping. Units: in.



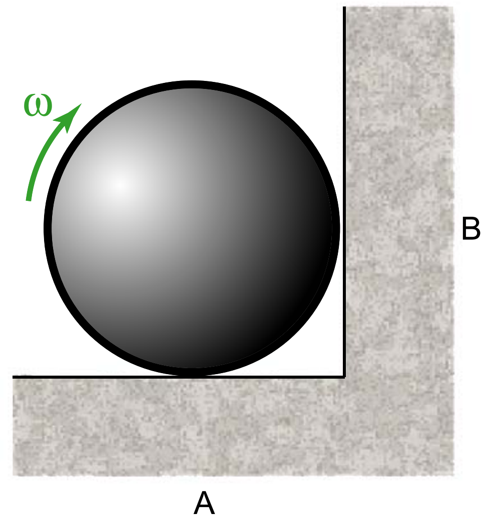
Example

Disk B is rotating at 75 rad/s when an identical stationary disk A is lowered into contact with it. The axel of disk A rides in the smooth vertical slot. Find the time and the final angular velocities when slipping between the disks stops. Units: meters.



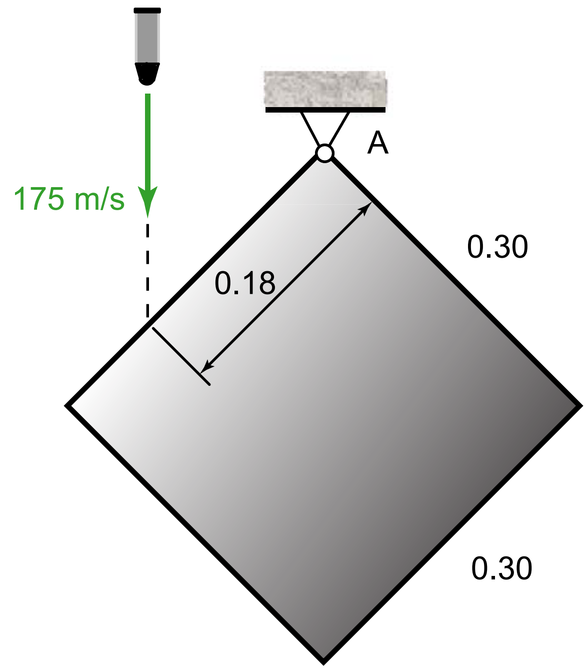
Example

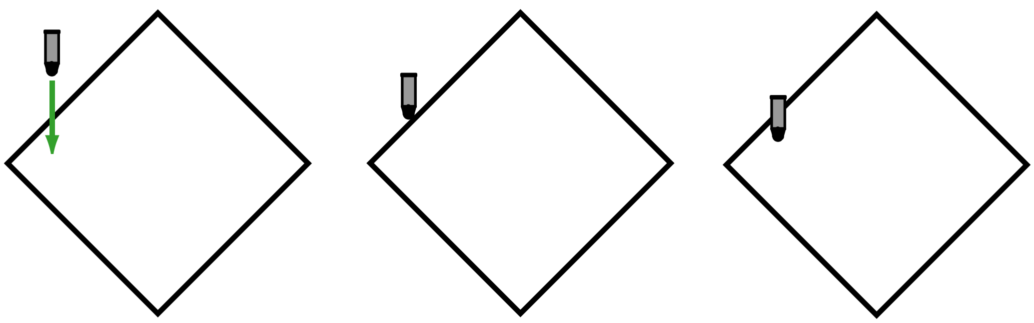
The spool is rolling with an angular speed of ω_0 when it encounters the vertical wall and begins to spin in place (no rebound). How long will it take for the spool to come to rest? The spool has a mass m, radius R, and centroidal mass moment of inertia $\bar{I}$. The coefficient of kinetic friction at both surfaces is μ.



Example

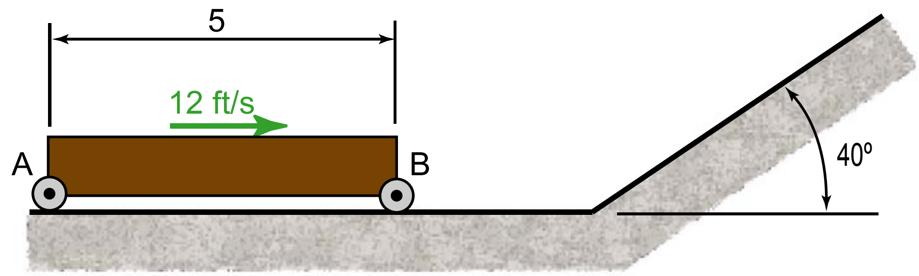
The homogeneous square plate measures 0.30 m on both sides and has a mass of 4 kg. It is suspended from a pin at A and is at rest when it is struck by the slug of mass 0.6 kg traveling vertically with a velocity of 175 m/s. Assumming the slug imbeds in the plate, determine the angular velocity of the plate immediately after impact. Units: m





Example

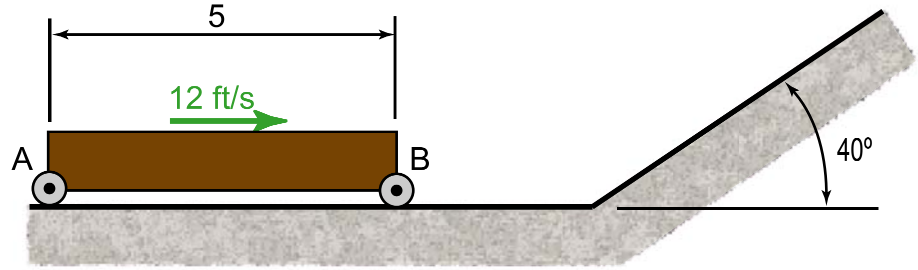
The 150 lb log AB rolls on massless wheels on the horizontal surface with the velocity of 12 ft/s when end B contacts the 40º incline. Determine the velocity of end A immediately after impact assuming both wheels stay in contact with the surfaces. Units: Ft.



Example

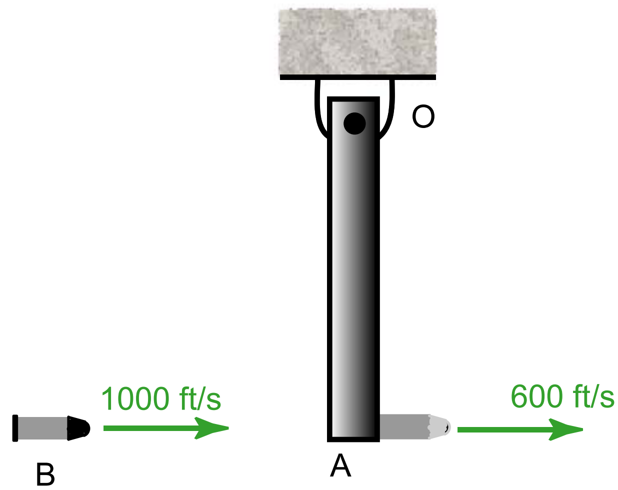
The 150 lb log AB rolls on massless wheels on the horizontal surface with the velocity of 12 ft/s when end B contacts the 40º incline. Determine the velocity of end A immediately after impact assuming both wheels stay in contact with the surfaces. Units: Ft.

Alternative Solution:



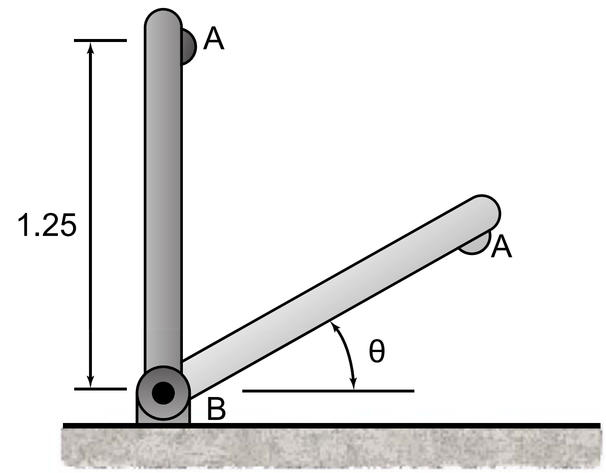
Example

The 6 ft, 200 lb rod is at rest when a 2 lb cannon ball B is fired at end A with the velocity of 1000 ft/s. The ball passes through the rod, emerging with the final velocity of 600 ft/s. Find the maximum angular displacement of the rod.



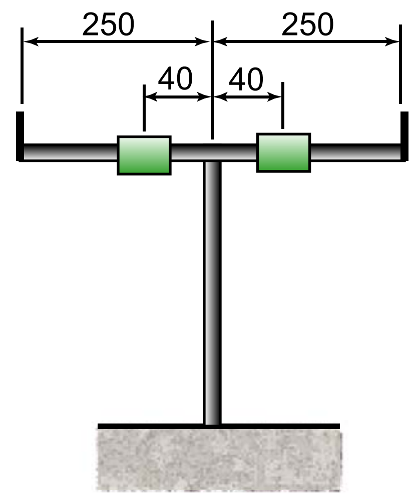
Example

The uniform slender rod AB is in equilibrium in the vertical position when end A is given a slight nudge to the right. Given the coefficient of restitution between the knob at A and the horizontal surface is 0.42, find the maximum angle of rebound θ of the rod. Units: meters.



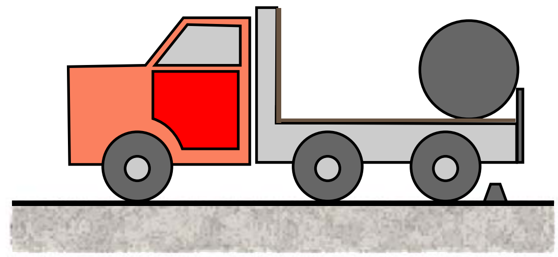
Example

The 15 mm diameter uniform rod has a mass of 0.5 kg and spins around a vertical axis. The two identical collars are initially latched at a position of 40 mm from the axis as shown and the system is spinning at a rate of 4 rad/s. An internal mechanism releases the latch and the collars move outward until they hit (and stick to) the stops at the ends of the rod. The collars are 1 kg each, 60 mm long and have an inside diameter of 15 mm and outside diameter of 40 mm. Calculate the angular velocity of the system in the new configuration. How much energy was lost? Neglect the mass of the vertical axis and hub.



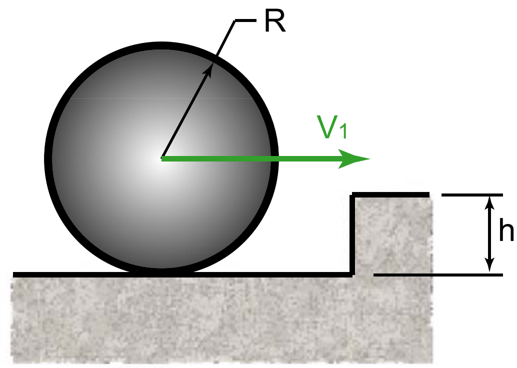
Example

The truck is backing up at a speed of v_1 when it hits a curb and comes to a sudden stop. Calculate the minimum height of the tailgate to keep the drum in the truck bed. The drum is homogeneous and has a mass m and radius R.



Example

The uniform cylinder of mass m and radius R is rolling without slipping on the horizontal surface. How fast must it be traveling to jump the curb (height h)?

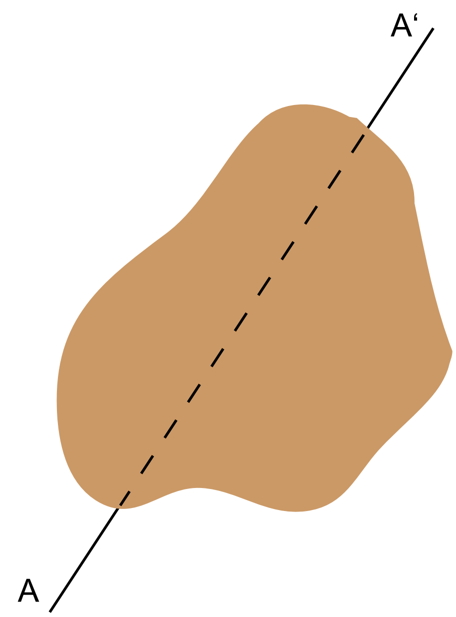


Appendix A
MOMENT OF INERTIA OF MASSES

Introduction

Just as mass (m) is a measure of resistance to linear acceleration, so Mass-moment-of-Inertia (I) is a measure of resistance to angular acceleration.

$$I_A = \int r^2 dm$$



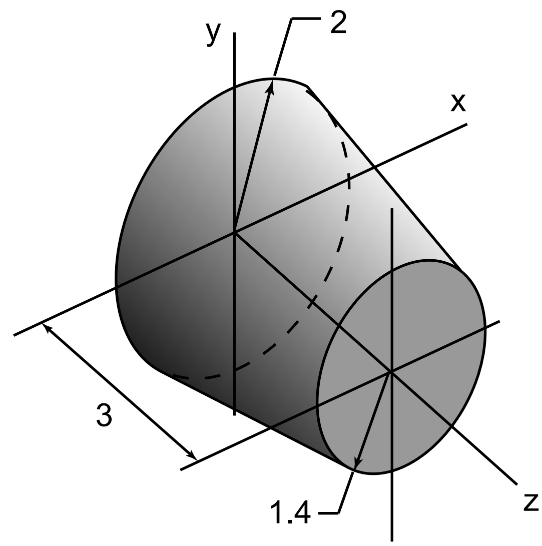
Moments of Inertia of Masses: Radius of Gyration

Radius of gyration k_{A_z} is the distance from axis Az at which all the object's mass could be concentrated to yield the same mass moment of inertia about axis Az as the original object.

$$I_{A_z} = k_{A_z}^2 m$$

Example

Determine the mass moment of inertia of the truncated cone about the z-axis. Density= 30 lb/ft³. Units: Ft.

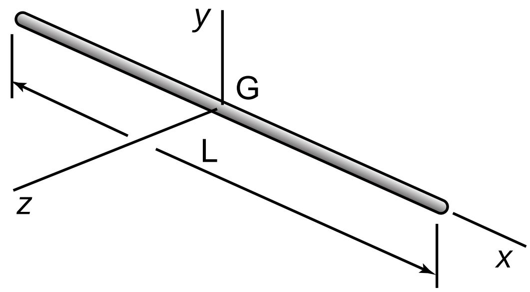
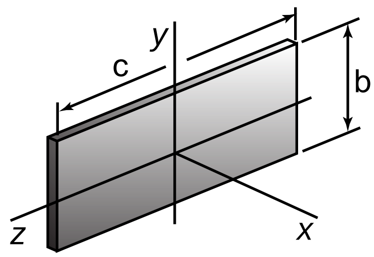
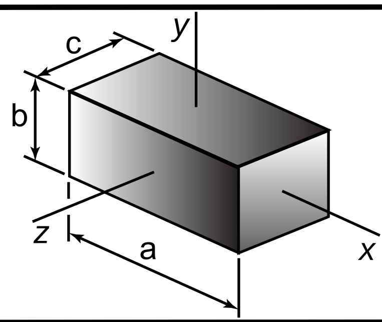
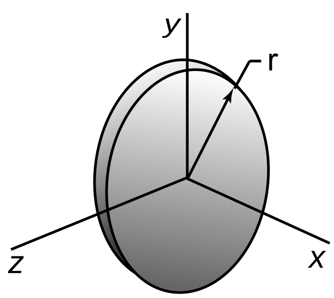
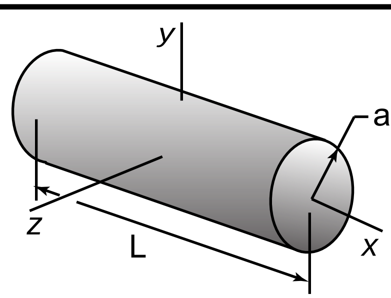


Parallel-Axis Theorem

If the centroidal mass moment of inertia is known, the mass moment of inertia about a parallel axis can be found by adding md^2 to the centroidal value. m is the mass and d is the distance between the parallel axes.

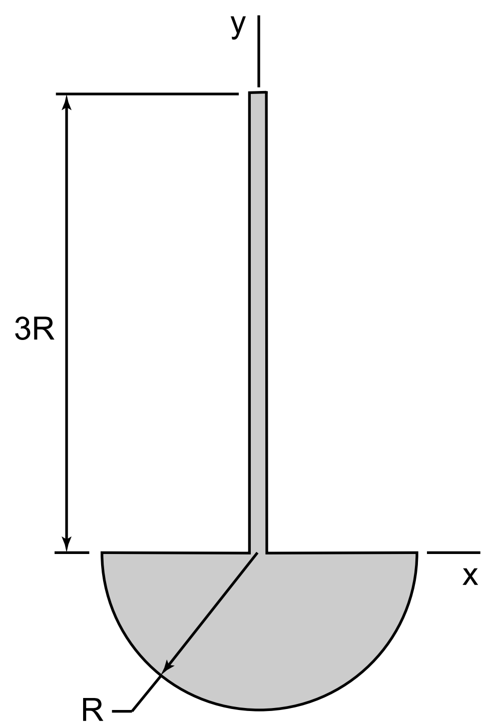
$$I_A = \bar{I} + md^2$$

Mass Moments of Inertia of Common Geometric Shapes

Shape		I
Slender rod		$I_y = I_z = \frac{1}{12}mL^2$
Thin rectangular plate		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}mc^2$ $I_z = \frac{1}{12}mb^2$
Rectangular prisim		$I_x = \frac{1}{12}m(b^2 + c^2)$ $I_y = \frac{1}{12}m(c^2 + a^2)$ $I_z = \frac{1}{12}m(a^2 + b^2)$
Thin disk		$I_x = \frac{1}{2}mr^2$ $I_y = I_z = \frac{1}{4}mr^2$
Circular cylinder		$I_x = \frac{1}{2}ma^2$ $I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$

Example

The composite body is made up of a thin uniform slender bar of length 3R welded to the uniform semicircular disk of radius R. The total mass of the composite body is m. Half of the total mass is contained in the bar and half in the disk. Find (a) the location of the center of mass of the composite body and (b) the mass moment of inertia about the z axis through that mass-center.



Example

Compute $\bar{y}$, I_z and $\bar{I}_z$ of the machine part. ρ= 8000 kg/m³. Units: m.