SOLVING LINEAR EQUATIONS BY ADDITION AND SUBTRACTION

Introduction

The Golden Rule to keep in mind when solving equations is: "Do to the right side the same as you do to the left side." That is, if you perform an operation on one side of an equation, then you must perform the same operation on the other side of the equation in order for the two sides of the equation to remain equal.

In this lesson, we will practice the Golden Rule to solve linear equations using addition and subtraction.

SOLVING LINEAR EQUATIONS BY ADDITION AND SUBTRACTION

Linear Equations

A **linear equation** is an equation that involves only first-degree polynomials. In other words, in a linear equation, a variable appears only as a first power—not as a square or higher power, not as a square root, and not in a denominator.

Examples:

$3a+5=7$ is a linear equation; a has been raised to a first power only.

$3a^2+5=7$ is not a linear equation because the variable a is squared.

$y=3x+2$ is a linear equation.

$y=5x^6-2x^2+8$ is not a linear equation because it's not a polynomial of degree 1.

$\frac{5}{x}-\frac{4}{y}=2$ is not a linear equation because the variables x and y appear in the denominators.

$s=\sqrt{w+1}$ is not a linear equation because it involves the square root of a variable.

$V=\pi r^2 h$ is not a linear equation; although it is linear in variable h, variable r has been squared.

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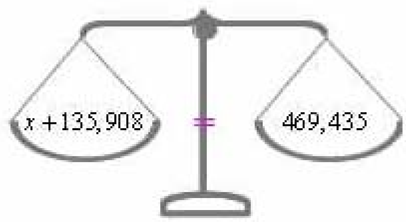
Solving Equations

To **solve** an equation means to end up with an equation of the form " $x =$ a number or expression." This is called the **solution.** Notice that when an equation is solved, the variable is alone on one side of the equal sign. How do we get the variable alone on one side of the equal sign? We follow the Golden Rule.

EXAMPLE A

Solve: $x + 135,908 = 469,435$.

Again, the image of a scale can help us visualize how to isolate x on one side of the equation to reach the solution.



Here's another way to look at what we did to solve this equation:

$$\begin{array}{rr} x + 135,908 = & 469,435 \\ -135,908 & -135,908 \\ \hline x = & 333,527 \end{array}$$

SOLVING LINEAR EQUATIONS BY ADDITION AND SUBTRACTION

EXAMPLE B

Solve: $x - 19,803 = 37,527$.

To solve for x, we have to add $19,803$ to the left side of the equation. Since we add $19,803$ to the left side we must also add $19,803$ to the right side.

$$\begin{array}{rr} x - 19,803 = & 37,527 \\ +19,803 & +19,803 \\ \hline x = & 57,330 \end{array}$$

To get the variable onto one side of the equation by itself, we had to perform an opposite operation. Since $19,803$ was subtracted from variable x in the given equation, we added the same amount. Doing so cancelled the number out since $-19,803 + 19,803 = 0$. You can use addition to undo subtraction, and subtraction to undo addition.

Question: Solve: $x - 20 = 80$.

SOLVING LINEAR EQUATIONS BY ADDITION AND SUBTRACTION

Question: Solve: $b + 213 = 91$.

Question: Solve: $31 + W = 63$.

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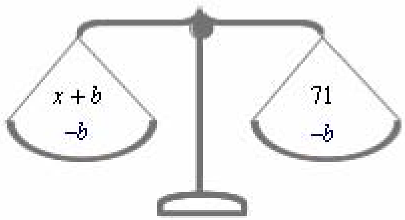
SOLVING LINEAR EQUATIONS BY ADDITION AND SUBTRACTION

Sometimes equations involve more than one variable. When they do, we have to specify which variable we want to isolate or solve for.

EXAMPLE C

Solve for x: $x+b=71$.

Since we want to get x by itself on one side of the equal sign, we will subtract b from both sides:



$$\begin{array}{rcl} x+b & = & 71 \\ -b & & -b \\ \hline x & = & 71-b \end{array}$$

EXAMPLE D

Solve for a: $a-b+x=3x-b$.

Use the Golden Rule to isolate a: add b and subtract x from both sides of the equation.

$$\begin{array}{rcl} a-b+x & = & 3x-b \\ +b-x & & -x+b \\ \hline a & = & 2x \end{array}$$

SOLVING LINEAR EQUATIONS BY ADDITION AND SUBTRACTION

Question: Solve for x: $x + 2a - 3z = 5a - z$.

Question: Solve for C: $5A - B + C = 2A - 3B$.

EXAMPLE E

Solve for x: $45 - x = 28$.

This example imposes one extra difficulty—the variable x appears with a minus sign but we need "positive $x =$ something" for our solution. One way to deal with this is to add x to both sides as a first step.

$$\begin{array}{r} 45 - x = 28 \\ +x \qquad +x \\ \hline 45 = 28 + x \end{array}$$

Now we must solve the equation $45 = 28 + x$ by subtracting 28 from both sides.

$$\begin{array}{r} 45 = \quad 28 + x \\ -28 \quad -28 \qquad \\ \hline 17 = x \qquad \end{array}$$

Notice that it doesn't matter which side of the equal sign our variable ends up on, as long as it's all alone!

Extended Example 1a

Solve: $57 - x = 43$.

EXAMPLE F

Solve for p : $ab + y = cd - p$.

We want to end up with $p =$ something.

Step 1: Add p to both sides to make it positive:

$$\begin{array}{r} ab + y = cd - p \\ +p \qquad +p \\ \hline ab + y + p = cd \end{array}$$

Step 2: Subtract ab and y from both sides:

$$\begin{array}{r} ab + y + p = cd \qquad\quad \\ -ab - y \qquad\qquad -ab - y \\ \hline p = cd - ab - y \end{array}$$

Extended Example 2a

Solve for y : $abc + 2x = 5x - y$.

END OF LESSON

Solve for x: $y-7=x+3$

Solve for x: $12+t=8-x$

Solve for x: $7w-x=8p+q$

Solve for x : $pq - x = 16z + q$

Solve for x : $t = x + ty + t$

Solve for x : $rt + gh - x = -gh - rt$

SOLVING LINEAR EQUATIONS BY MULTIPLICATION AND DIVISION

Introduction

This lesson explains how to solve linear equations using multiplication and division. The Golden Rule still applies—you must perform the same operation on both sides of an equation in order for the two sides of the equation to remain equal.

SOLVING LINEAR EQUATIONS BY MULTIPLICATION AND DIVISION

Using Division to Solve an Equation

We have seen that you can use addition to "undo" subtraction and subtraction to undo addition in order to isolate a variable. Similarly, you can use multiplication to undo division and division to undo multiplication for the same purpose.

Recall that solving an equation for a variable is the process of getting that variable alone on one side of the equal sign.

EXAMPLE A

Solve: $35z = 70$.

Variable z is being multiplied by 35. Since the opposite of multiplication is division, we can use the Golden Rule to divide both sides of the equation by 35 to get rid of the unwanted multiplication:

$$\frac{35z}{35} = \frac{70}{35}$$

$$\frac{\cancel{35}\,z}{\cancel{35}} = 2$$

$$z = 2$$

SOLVING LINEAR EQUATIONS BY MULTIPLICATION AND DIVISION

Question: Solve: $21b = 987$.

Question: Solve: $25x = 125$.

Question: Solve: $-33z = 99$.

Using Multiplication to Solve an Equation

Now we'll see how you can use multiplication to undo division to solve an equation.

EXAMPLE B

Solve for s: $\frac{s}{6} = 120$.

Variable s is being divided by 6. The opposite of division is multiplication. So, we can use the Golden Rule to isolate s by multiplying both sides of the equation by 6:

$$6 \cdot \frac{s}{6} = 6 \cdot 120$$

$$\frac{\cancel{6} \cdot s}{\cancel{6}} = 720$$

$$s = 720$$

Question: Solve: $\frac{t}{23} = 10$.

SOLVING LINEAR EQUATIONS BY MULTIPLICATION AND DIVISION

Question: Solve: $\frac{w}{17} = -3$.

Question: Solve: $\frac{h}{-100} = -5$.

EXAMPLE C

Solve for C: $\frac{5}{7}C = 11$.

In this case, we could first multiply both sides of the equation by 7. Then we'd have $5C$ on the left side, and we would need to divide both sides by 5 to isolate C.

Instead, we can isolate C in one step by multiplying both sides of the equation by $\frac{7}{5}$, which is the reciprocal of $\frac{5}{7}$.

$$\frac{7}{5}\cdot\frac{5}{7}C = \frac{7}{5}\cdot 11$$

$$\frac{\cancel{7}\cdot\cancel{5}C}{\cancel{5}\cdot\cancel{7}} = \frac{7\cdot 11}{5}$$

$$C = \frac{77}{5}$$

SOLVING LINEAR EQUATIONS BY MULTIPLICATION AND DIVISION

Question: Solve: $\frac{2}{3}x = 13$.

Question: Solve: $\frac{-3}{5}y = \frac{6}{5}$.

SOLVING LINEAR EQUATIONS BY MULTIPLICATION AND DIVISION

Question: Solve: $\frac{11}{4}Z = -\frac{13}{2}$.

Question: Solve: $\frac{2}{3}y = 28$

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SOLVING LINEAR EQUATIONS BY MULTIPLICATION AND DIVISION

EXAMPLE D

Solve for a: $23a - 18 = 51$.

Before worrying about the $23a$ term, notice that 18 is subtracted on the left side of the equation. Take care of this first to isolate the term with the variable on the left side—add 18 to both sides of the equation:

$$\begin{aligned} 23a - 18 &= 51 \\ +18 \quad &\quad +18 \\ 23a &= 69 \end{aligned}$$

Now, we can isolate a by dividing both sides of the equation by 23.

$$\frac{23a}{23} = \frac{69}{23}$$

$$\frac{\cancel{23}\,a}{\cancel{23}} = 3$$

$$a = 3$$

In general, it's best to isolate the term with the variable by itself on one side of the equal sign before multiplying or dividing to cancel its coefficient.

Extended Example 1a

Solve: $5x + 17 = 232$.

END OF LESSON

Solve for w: $\frac{12}{w} - 12 = 0$

Solve for the variable.

$14r - 7 = 21$

Solve for x: $\frac{1}{2}x + \frac{1}{2}x = 3$

Solve for the variable.

$3 - \frac{2}{9}q = 12$

Solve for q: $3-\frac{2}{9}q=12$

Solve for the variable.

$\frac{1}{2}x+\frac{1}{2}x=3$

Solve for t: $\frac{5}{t}+8=-7$

SOLVING GENERAL LINEAR EQUATIONS

Introduction

This lesson presents a series of examples that show how to solve various linear equations. There are also a few examples that show how to solve nonlinear equations by "converting" them to linear equations.

SOLVING GENERAL LINEAR EQUATIONS

EXAMPLE A

Solve: $35x - 82 = -29x + 6$.

The variable x appears on both sides of the equation. To get all the x terms together on the same side of the equation, we will combine the variable terms on the side with the larger coefficient.

Step 1: Move all variable terms to the left side of the equation by adding $29x$ to both sides:

$$\begin{array}{r} 35x - 82 = -29x + 6 \\ \underline{+29x \qquad\quad +29x} \\ 64x - 82 = \quad 6 \end{array}$$

Step 2: To isolate the term with the variable, add 82 to both sides:

$$\begin{array}{r} 64x - 82 \;=\; 6 \\ \underline{\quad +82 \quad +82} \\ 64x = 88 \end{array}$$

Step 3: To isolate x, divide both sides of the equation by 64.

$$\frac{64x}{64} = \frac{88}{64}$$

$$\frac{\cancel{64}x}{\cancel{64}} = \frac{88}{64} = \frac{8 \cdot 11}{8 \cdot 8}$$

$$x = \frac{\cancel{8} \cdot 11}{\cancel{8} \cdot 8}$$

$$x = \frac{11}{8}$$

SOLVING GENERAL LINEAR EQUATIONS

Extended Example 1a

Solve: $3x + 15 = 8x - 70$.

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SOLVING GENERAL LINEAR EQUATIONS

EXAMPLE B

Solve: $3(4w-8)+14=5w-31$.

Step 1: Distribute the 3 to remove the parentheses:

$$12w-24+14=5w-31$$

Step 2: Add $-24+14$: $\quad 12w-10=5w-31$

Step 3: Move all variable terms to the left side of the equation by subtracting $5w$ from both sides.

$$\begin{array}{r} 12w-10=5w-31 \\ -5w \qquad\quad -5w \\ \hline 7w-10=-31 \end{array}$$

Step 4: Isolate the variable term by adding 10 to both sides of the equation:

$$\begin{array}{r} 7w-10=-31 \\ +10 \quad +10 \\ \hline 7w=-21 \end{array}$$

Step 5: Isolate the w by dividing both sides of the equation by 7.

$$\frac{7w}{7}=\frac{-21}{7}$$

$$\frac{\cancel{7}w}{\cancel{7}}=-3$$

$$w=-3$$

SOLVING GENERAL LINEAR EQUATIONS

Extended Example 2a

Solve: $5(4x-2)=15(x+2)$.

SOLVING GENERAL LINEAR EQUATIONS

EXAMPLE C

Solve for x: $5(3x-2)+pq=4(pq-3)-x$.

Remember: The goal in solving an equation is to isolate the specified variable. Don't get distracted by the other variables!

Step 1: Distribute the 5 and 4 to remove parentheses:

$$15x-10+pq=4pq-12-x$$

Step 2: Move all x terms to the left side by adding x to both sides:

$$\begin{array}{l} 15x-10+pq=4pq-12-x \\ \underline{+x \qquad\qquad\qquad\qquad\quad +x} \\ 16x-10+pq=4pq-12 \end{array}$$

Step 3: Isolate the x term, by adding 10 and subtracting pq from both sides:

$$\begin{array}{rcl} 16x-10+pq & = & 4pq-12 \\ \underline{+10-pq} & & \underline{-pq+10} \\ 16x & = & 3pq-2 \end{array}$$

Step 4: Isolate the x term by dividing both sides by 16:

$$\frac{16x}{16}=\frac{3pq-2}{16}$$

$$\frac{\cancel{16}x}{\cancel{16}}=\frac{3pq-2}{16}$$

$$x=\frac{3pq-2}{16}$$

SOLVING GENERAL LINEAR EQUATIONS

Extended Example 3a

Solve for y: $3(5x-2y)+a=x-7(a-3y)$.

SOLVING GENERAL LINEAR EQUATIONS

EXAMPLE D

Solve for x: $5 = \frac{3}{4}(8 - x) + \frac{5}{3}$.

Step 1: Distribute the $\frac{3}{4}$ and reduce the fractions to lowest terms:

$$5 = \frac{3}{4} \cdot (8 - x) + \frac{5}{3}$$

$$5 = \frac{3}{4} \cdot 8 - \frac{3}{4} \cdot x + \frac{5}{3}$$

$$5 = \frac{3 \cdot 8}{4} - \frac{3 \cdot x}{4} + \frac{5}{3}$$

$$5 = \frac{3 \cdot 2 \cdot 4}{4} - \frac{3x}{4} + \frac{5}{3}$$

$$5 = \frac{3 \cdot 2 \cdot \cancel{4}}{\cancel{4}} - \frac{3x}{4} + \frac{5}{3}$$

$$5 = 6 - \frac{3x}{4} + \frac{5}{3}$$

Step 2: Multiply both sides of the equation by 12 to cancel all the denominators (since 12 is the least common multiple of all the denominators):

$$12 \cdot 5 = 12 \cdot \left(6 - \frac{3x}{4} + \frac{5}{3}\right)$$

continued...

Example D, continued…

Step 3: Distribute the 12 to eliminate the parentheses:

$$12\cdot 5 = 12\cdot\left(6 - \frac{3x}{4} + \frac{5}{3}\right)$$

$$60 = 12\cdot 6 - \frac{12\cdot 3x}{4} + \frac{12\cdot 5}{3}$$

$$60 = 72 - \frac{4\cdot 3\cdot 3x}{4} + \frac{4\cdot 3\cdot 5}{3}$$

$$60 = 72 - \frac{\cancel{4}\cdot 3\cdot 3x}{\cancel{4}} + \frac{4\cdot \cancel{3}\cdot 5}{\cancel{3}}$$

$$60 = 72 - 9x + 20$$

Step 4: Combine like terms: $60 = 92 - 9x$

Step 5: Isolate the x term by subtracting 92 from both sides of the equation:

$$\begin{array}{rcl} 60 & = & 92 - 9x \\ -92 & & -92 \\ \hline -32 & = & -9x \end{array}$$

Step 6: Isolate x by dividing both sides of the equation by -9:

$$\frac{-32}{-9} = \frac{-9x}{-9}$$

$$\frac{32}{9} = \frac{\cancel{-9}\,x}{\cancel{-9}}$$

$$\frac{32}{9} = x$$

SOLVING GENERAL LINEAR EQUATIONS

Extended Example 4a

Solve for x: $3 = \frac{5}{2}(2 - 6x) - \frac{3}{2}$.

EXAMPLE E

Solve: $5(x-7)=0$.

We could solve this equation in the same way as before. However, in this case, we can solve the equation more easily by looking at its special structure.

The equation says that 5 times $(x-7)$ equals 0.

We know that 5 times 0 equals 0, and since $5 \neq 0$, we must have

$$x-7=0.$$

Since $7-7=0$, the solution to our equation is

$$x=7.$$

The moral of the story is: anytime a product is 0, one of the factors must be 0.

If $AB=0$, then $A=0$ or $B=0$.

Question: Solve: $0=12(x+9)$.

SOLVING GENERAL LINEAR EQUATIONS

Question: Solve: $\frac{2}{5}(x-13)=0$.

Question: Solve: $\frac{4}{7}(2x+3)=0$.

SOLVING GENERAL LINEAR EQUATIONS

EXAMPLE F

Solve: $\dfrac{2x+4}{x} = 3$.

This is a **rational equation**, which is an equation with a variable in the denominator. Sometimes such equations can be "converted" to linear equations, as shown below.

Step 1: Multiply both sides by x to end up with a linear equation:

$$x \cdot \left(\frac{2x+4}{x}\right) = x \cdot 3$$

$$\frac{x \cdot (2x+4)}{x} = 3x$$

$$\frac{\cancel{x} \cdot (2x+4)}{\cancel{x}} = 3x$$

$$2x + 4 = 3x$$

Step 2: Combine the x terms by subtracting $2x$ from both sides:

$$\begin{array}{rcl} 2x+4 & = & 3x \\ -2x \quad & & -2x \\ \hline 4 & = & x \end{array}$$

continued...

Example F, continued…

Checking our answer, $x = 4$:

$$\frac{2x+4}{x} = 3$$

$$\frac{2 \cdot 4 + 4}{4} \stackrel{?}{=} 3$$

$$\frac{8+4}{4} \stackrel{?}{=} 3$$

$$\frac{12}{4} \stackrel{?}{=} 3$$

$$3 \stackrel{?}{=} 3 \quad ✓$$

So, $x = 4$ is our solution.

A Word to the Wise:
We must check solutions to rational equations to make sure that we don't end up with a zero in the denominator, which is undefined. But it's always good practice to check your solutions when solving any equation.

EXAMPLE G

Solve: $\frac{3x+1}{x-5} = \frac{2x+6}{x-5}$.

This rational equation can be transformed to a linear one by multiplying both sides by $x-5$. This cancels out both denominators:

$$(x-5)\cdot\frac{3x+1}{x-5} = (x-5)\cdot\frac{2x+6}{x-5}$$

$$\frac{(x-5)\cdot(3x+1)}{(x-5)} = \frac{(x-5)\cdot(2x+6)}{(x-5)}$$

$$\frac{\cancel{(x-5)}\cdot(3x+1)}{\cancel{(x-5)}} = \frac{\cancel{(x-5)}\cdot(2x+6)}{\cancel{(x-5)}}$$

$$3x+1 = 2x+6$$

Combine the x terms by subtracting $2x$ from both sides of the equation:

$$\begin{array}{rcl} 3x+1 & = & 2x+6 \\ -2x & & -2x \\ \hline x+1 & = & 6 \end{array}$$

Isolate the x term by subtracting 1 from both sides of the equation:

$$\begin{array}{rcl} x+1 & = & 6 \\ -1 & & -1 \\ \hline x & = & 5 \end{array}$$

continued...

SOLVING GENERAL LINEAR EQUATIONS

Example G, continued…

Next we check our possible answer, $x = 5$, by substituting it into the original equation:

$$\frac{3x+1}{x-5} = \frac{2x+6}{x-5}$$

$$\frac{3 \cdot 5+1}{5-5} \stackrel{?}{=} \frac{2 \cdot 5+6}{5-5}$$

$$\frac{16}{0} \stackrel{?}{=} \frac{16}{0}$$

These expressions appear to be equal at first, but there is a serious problem: the denominators both equal zero. Dividing by zero is undefined. This means the only possible solution doesn't solve the equation. The equation has <u>no solution</u>.

If your algebra is correct, the <u>only</u> way a possible solution can fail to be the actual solution is if the denominator equals zero. Possible solutions that make a denominator equal zero must be eliminated since they make the original equation undefined.

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SOLVING GENERAL LINEAR EQUATIONS

Extended Example 5a

Solve: $\frac{1}{3}+\frac{1}{x}=\frac{1}{2}$.

SOLVING GENERAL LINEAR EQUATIONS

Summary

- Always do the same thing to both sides of an equation (the Golden Rule).
- Isolate the variable or the term(s) containing the variable on one side of an equation. This may involve combining like terms more than once.
- Use addition to "undo" subtraction and use subtraction to undo addition. They are opposite operations.
- Use multiplication to undo division and use division to undo multiplication. They are opposite operations.

END OF LESSON

Solve: $t-\frac{1}{2}t-8=0$

Solve for x : $\frac{4}{5}x-12=-\frac{1}{5}x$

Solve the following.

$\frac{4}{5}q-12=-\frac{1}{5}q$

Solve for x : $\dfrac{2}{x-4} - \dfrac{15}{x} = 0$

Solve for x : $3p + \dfrac{1}{2}x = 5(x-1)$

Solve for x : $\dfrac{2x}{x-4} = \dfrac{8}{x-4} + 1$

APPLICATIONS INVOLVING LINEAR EQUATIONS

Introduction

This lesson shows you various techniques to help you solve some "real-world" problems using linear equations.

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APPLICATIONS INVOLVING LINEAR EQUATIONS

Solving Word Problems

Read the problem several times to understand what you are being told and what you are being asked. Then, follow these steps:

1. **Organize the information.** Identify the known values and use variables to represent unknown values. It's very helpful to visualize the problem. If possible, draw and label diagrams with the information provided in the problem.

2. **Use a formula or write an equation.** Use this to relate known and unknown values.

3. **Solve.** Solve the equation from Step 2 above.

4. **Answer the question.** Refer to the question asked in the original problem.

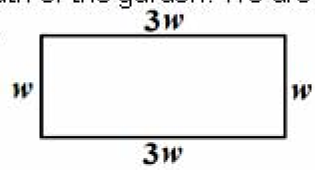
5. **Consider/check your answer.** Does it seem reasonable? If not, start over at Step 1.

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE A

The perimeter of a rectangular garden is 160 feet. The length is three times the width. What are the width and the length of the garden?

Step 1: Organize the information given and draw the garden. Use variables to represent the dimensions. Let w be the width of the garden. We are told the length is "three times the width," or $3w$.



Step 2: The perimeter of a rectangle is the sum of the lengths of all of its sides. We are told that the perimeter of this garden is 160 feet. So the equation should look like this: $160 = w + 3w + w + 3w$

$$160 = 8w$$

Step 3: Solve for w by dividing both sides of the equation by 8:

$$\frac{160}{8} = \frac{8w}{8}$$

$$20 = \frac{\cancel{8}w}{\cancel{8}}$$

$$20 = w$$

Step 4: Now you know that $w = 20\text{ ft}$, so you can find the length.

$$3w = 3(20\text{ ft}) = 60\text{ ft}.$$

Step 5: Check this answer by finding the perimeter:

$$P = 20\text{ ft} + 60\text{ ft} + 20\text{ ft} + 60\text{ ft} = 160\text{ ft}. \checkmark$$

APPLICATIONS INVOLVING LINEAR EQUATIONS

Extended Example 1a

The perimeter of a rectangular garden is 36 meters. The length is five times the width. What are the width and the length of the garden?

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE B

An investor invests $2,500 in the stock market. At the end of one year, she sells her stock for $2,800. What is her rate of return?

Step 1: Organize the information. The initial investment, or principal, is $2,500. The amount after 1 year is $2,800. We need to find the rate of return, r .

Step 2: Use the formula for simple interest learned earlier: $A = P(1+r)$.

Substitute given information: $\$2,800 = \$2,500(1+r)$.

Step 3: Solve for r :

$$2,800 = 2,500(1+r)$$

$$\begin{array}{rcl} 2,800 & = & 2,500 + 2,500r \\ -2,500 & & -2,500 \\ \hline 300 & = & 2,500r \end{array}$$

$$\frac{300}{2,500} = \frac{2,500\,r}{2,500}$$

$$\frac{3}{25} = \frac{\cancel{2,500}\,r}{\cancel{2,500}}$$

$$0.12 = r$$

Step 4: The investor's rate of return is 12%.

Step 5: Use the formula to check the answer: $A = P(1+r)$

$$2,800 = 2,500(1+0.12)$$

$$2,800 = 2,500 + 300$$

$$2,800 = 2,800 \quad \checkmark$$

APPLICATIONS INVOLVING LINEAR EQUATIONS

Extended Example 2a

An investor invests $22,000 in a business venture. At the end of one year, he sells his share of the business for $27,060. What is his rate of return?

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE C

The sum of three consecutive integers is 123. What are the three integers?

Step 1: At first, it seems as though we can use x, y, and z to represent the integers. While this may be true, we can also express them using a single variable.

Since we know that the integers are consecutive (they occur one after the other), we can let x be the first integer. The second integer can be $x+1$, and the third integer can be $x+2$.

Step 2: So, the equation is:

$$1^{st} + 2^{nd} + 3^{rd} = 123$$
$$x + x+1 + x+2 = 123$$

Step 3: Solve for x:

$$x + x + 1 + x + 2 = 123$$
$$3x + 3 = 123$$
$$\begin{array}{r} 3x + 3 = 123 \\ -3 \quad -3 \\ \hline 3x = 120 \end{array}$$
$$\frac{3x}{3} = \frac{120}{3}$$
$$x = 40$$

Step 4: Since the first integer is 40, the three integers are 40, 41, and 42.

Step 5: Check this answer: $40 + 41 + 42 = 123$. ✓

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APPLICATIONS INVOLVING LINEAR EQUATIONS

Extended Example 3a

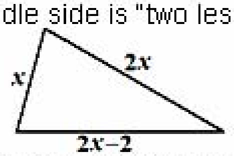
There are three consecutive even integers such that twice the sum of the first two integers is three times the third integer. What are the three numbers?

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE D

The length of the longest side of a triangle is twice the length of the shortest side. The length of the middle side is two less than the length of the longest side. If the triangle's perimeter is 23 cm, how long are the triangle's sides?

Step 1: Draw and label a picture of the triangle. Call the shortest side x. So the longest side is $2x$. The middle side is "two less than the longest side," so its length is $2x-2$.



Step 2: The perimeter, 23 cm, is the sum of these three sides, so we have:

$$x+2x+(2x-2)=23$$

Step 3: Solve:

$$5x-2=23$$

Isolate the x term by adding 2 to both sides of the equation:

$$5x-2+2=23+2$$

$$5x=25$$

Isolate x by dividing both sides by 5:

$$\frac{5x}{5}=\frac{25}{5}$$

$$\frac{\cancel{5}x}{\cancel{5}}=5$$

$$x=5$$

Step 4: Answer the question: The shortest side, x, is 5 cm long. The longest side is $2(5)=10$ cm, and the middle side is $10-2=8$ cm.

APPLICATIONS INVOLVING LINEAR EQUATIONS

Extended Example 4a

The length of the longest side of a triangle is 3 inches more than twice the length of the shortest side. The length of the middle side is 4 inches more than the length of the shortest side. The perimeter is 35 inches. How long are the sides?

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APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE E

The area of a triangle is 253 square inches, and its height is 22 inches. What is the length of the triangle's base?

Step 1: We know $A = 253 \text{ in}^2$ and $h = 22 \text{ in}$.

Step 2: The area of a triangle is one-half the base times the height or $A = \frac{bh}{2}$.

Substituting the given information, we get: $253 = \frac{b \cdot 22}{2}$.

Step 3: Solve this equation. Start by reducing the fraction to lowest terms:

$$253 = \frac{b \cdot 2 \cdot 11}{2}$$

$$253 = \frac{b \cdot \cancel{2} \cdot 11}{\cancel{2}}$$

$$253 = 11b$$

Solve for b by dividing both sides of the equation by 11:

$$\frac{253}{11} = \frac{11b}{11}$$

$$23 = \frac{\cancel{11}\, b}{\cancel{11}}$$

$$23 = b$$

Step 4: The triangle's base is 23 inches long.

Step 5: Use the formula to check: $A = \frac{23 \cdot 22}{2} = \frac{506}{2} = 253$. ✓

APPLICATIONS INVOLVING LINEAR EQUATIONS

Question: The area of a triangle is 102 square meters, and its height is 12 meters. What is the length of the triangle's base?

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE F

The area of a rectangle is 195 square microns, and its width is 13 microns. What is the rectangle's length?

Step 1: We know that $A = 195$ square microns and $W = 13$ microns.

Step 2: The area of a rectangle is the length times the width: $A = LW$.
Substituting the given information, we get: $195 = L \cdot 13$.

Step 3: Solve for L by dividing both sides of the equation by 13:

$$\frac{195}{13} = \frac{L \cdot 13}{13}$$

$$15 = \frac{L \cdot \cancel{13}}{\cancel{13}}$$

$$15 = L$$

Step 4: The rectangle's base is 15 microns long.

Step 5: Use the formula to check: $A = 13 \cdot 15 = 195$. ✓

APPLICATIONS INVOLVING LINEAR EQUATIONS

Question: The area of a rectangle is 361 square meters, and its length is 19 meters. What is the width of the rectangle's base?

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE G

If you drove for 5 hours and your speed was 65 miles per hour, how far did you travel?

Distance equals rate times time: $d = rt$. Substitute known values and solve.

$$\begin{aligned} d &= 65\frac{\text{miles}}{\text{hour}} \cdot 5 \text{ hours} \\ &= 65 \cdot 5 \frac{\text{miles}}{\text{hour}} \cdot \text{hours} \\ &= 325 \frac{\text{miles} \cdot \cancel{\text{hours}}}{\cancel{\text{hour}}} \\ d &= 325 \text{ miles} \end{aligned}$$

Question: If you drove for 3 hours and traveled 216 kilometers, what was your average speed?

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE H

Katie earns $64 per day, plus commission of 5% of total sales. If she takes home $83 on a certain day, what were her total sales for that day?

Step 1: If E represents her earnings on the day, and S represents her sales for that day, then: $E = 64 + 0.05S$. Substituting the given information, we get: $83 = 64 + 0.05S$.

Step 2: Solve. Subtract 64 from both sides, to isolate the S term:

$$83 - 64 = 64 + 0.05S - 64$$

$$19 = 0.05S$$

Divide both sides by 0.05 to solve for S:

$$\frac{19}{0.05} = \frac{0.05S}{0.05}$$

$$\frac{19}{0.05} = \frac{\cancel{0.05}\,S}{\cancel{0.05}}$$

$$380 = S$$

Katie's sales for the day were $380.

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APPLICATIONS INVOLVING LINEAR EQUATIONS

Question: Suppose you earn $88 per day, plus commission of 8% of your total sales. If you take home $159.04 on a certain day, what were your total sales for that day?

APPLICATIONS INVOLVING LINEAR EQUATIONS

EXAMPLE I

Marco earns the following scores on his algebra exams: 84, 93, 92, 82. What's the lowest score he can get on his fifth exam to have an exam average of at least 90?

Step 1: If his average is 90 and his score on the fifth exam is x, then we have:

$$\frac{84+93+92+82+x}{5}=90$$

Step 2: Solve:

$$\frac{351+x}{5}=90$$

$$5\cdot\frac{351+x}{5}=5\cdot 90$$

$$\frac{\cancel{5}\cdot(351+x)}{\cancel{5}}=450$$

$$351+x=450$$

$$351+x-351=450-351$$

$$x=99$$

Marco needs at least a 99 on his fifth exam in order to earn an exam average of 90.

APPLICATIONS INVOLVING LINEAR EQUATIONS

Question: Sondra earns the following scores on her Spanish quizzes: 73%, 90%, 72%. What's the lowest score she can get on her fourth quiz to have a quiz average Of at least 80%?

END OF LESSON

If you wish to travel 2800 miles in 5 hours, what should your average speed be? What kind of transportation should you use?

One number is 5 more than another number. Their sum is -15. What are the numbers?

The sum of two consecutive odd integers is 56. Find the integers.

The perimeter of a triangle is 24 in. The longest side is 4 more than the middle side, and the smallest side is $\frac{1}{2}$ the length of the middle side. What are the lengths of the sides of the triangle?

In the same class, Mary has a total sum of 350 points on 4 tests. If she scored 100 points on the fifth and last test, what is Marie's test average?

In a certain triangle, the longest side is five times as long as the shortest side, and the middle side is 23 more than the shortest side. If the triangle's perimeter is 72, then how long are the sides of the triangle?

An investment of $1,009 grows to $1,142.27 after one year. What is the rate of return? Round your answer to the nearest percent.

SOLVING LINEAR INEQUALITIES

Introduction

This lesson shows how to solve and graph linear inequalities. Recall the definitions of the inequality symbols:

$A < B$ means "A is less than B"

$A > B$ means "A is greater than B"

$A \leq B$ means "A is less than or equal to B"

$A \geq B$ means "A is greater than or equal to B"

When solving a linear inequality, the inequality symbol is treated just like our old friend the equal sign. So the "Golden Rule" applies as before... **EXCEPT** for one thing: When you multiply or divide both sides of an inequality by a negative number, the direction of the inequality must be reversed.

SOLVING LINEAR INEQUALITIES

EXAMPLE A

Solve: $5t + 3 < 6 - 2t$.

Step 1: Move all variable terms to the left side by adding $2t$ to both sides:

$$5t + 3 + 2t < 6 - 2t + 2t$$

$$7t + 3 < 6$$

Step 2: Isolate the variable term by subtracting 3 from both sides:

$$7t + 3 - 3 < 6 - 3$$

$$7t < 3$$

Step 3: Solve for t by dividing both sides by 7. Since 7 is positive, the direction of the inequality does not change:

$$\frac{7t}{7} < \frac{3}{7}$$

$$\frac{\cancel{7}t}{\cancel{7}} < \frac{3}{7}$$

$$t < \frac{3}{7}$$

We can graph this result on the real number line. The fact that t is less than $\frac{3}{7}$ means that all numbers to the left of $\frac{3}{7}$ are part of the solution, indicated by the red arrow in our graph (see below). Since we have "less than" (<) but not "less than or equal to" (≤), $\frac{3}{7}$ is not included in the solution. We indicate this with an open circle at $\frac{3}{7}$. Below is the graph of the solution on the real number line.

SOLVING LINEAR INEQUALITIES

Extended Example 1a

Solve $2x+10>4$ and graph the solution.

SOLVING LINEAR INEQUALITIES

EXAMPLE B

Solve the linear inequality $4x - 7 \ge 6x - 11$ and graph the solution.

Step 1: Combine the variable terms by subtracting $4x$ from both sides of the inequality:

$$4x - 7 - 4x \ge 6x - 11 - 4x$$

$$-7 \ge 2x - 11$$

Step 2: Isolate the variable term by adding 11 to both sides of the equation:

$$-7 + 11 \ge 2x - 11 + 11$$

$$4 \ge 2x$$

Step 3: Divide both sides by 2 to solve the inequality.

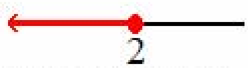
Since 2 is positive, the direction of the inequality does not change.

$$\frac{4}{2} \ge \frac{2x}{2}$$

$$2 \ge \frac{\cancel{2}x}{\cancel{2}}$$

$$2 \ge x$$

Step 4: Graph.



The filled-in circle indicates that 2 is included as part of the solution since we have x is "less than or equal to" 2.

Extended Example 2a

Solve $13 - 3a \le -17 - 5a$ and graph the solution.

SOLVING LINEAR INEQUALITIES

EXAMPLE C

Solve and graph the inequality $-5x+8\le 23$.

Step 1: Isolate the x term, by subtracting 8 from both sides of the equation:

$$-5x+8-8\le 23-8$$

$$-5x \le 15$$

Step 2: Divide both sides by -5 to solve the inequality.

NOTE: Since we are dividing by a negative number, the direction of the inequality is reversed.

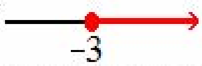
$$-5x\le 15$$

$$\frac{-5x}{-5}\ge\frac{15}{-5}$$

$$\frac{\cancel{-5}\,x}{\cancel{-5}}\ge -3$$

$$x\ge -3$$

Step 3: Graph.



Again, the arrow indicates that all numbers to the right of -3 are part of the solution and the filled-in circle indicates that -3 is part of the solution.

SOLVING LINEAR INEQUALITIES

Extended Example 3a

Solve $2 - 27m \leq -25$ and graph the solution.

SOLVING LINEAR INEQUALITIES

EXAMPLE D

Solve and graph: $-\frac{w}{3}-7<2$.

Step 1: Isolate the variable term by adding 7 to both sides:

$$-\frac{w}{3}-7+7<2+7$$

$$-\frac{w}{3}<9$$

Step 2: Solve the inequality by multiplying both sides by -3. Since we are multiplying by a negative number, we must reverse the direction of the inequality:

$$-3\cdot\left(-\frac{w}{3}\right)>-3\cdot 9$$

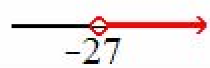
$$\frac{-3\cdot -w}{3}>-27$$

$$\frac{\cancel{3}w}{\cancel{3}}>-27$$

$$w>-27$$

We could have multiplied by 3 to eliminate the denominator, but by multiplying by -3 we got rid of the negative sign in front of the variable (since a negative times a negative is positive).

Step 3: Graph the solution:



Extended Example 4a

Solve and graph: $-\frac{2c}{5}+11<\frac{7}{10}$.

END OF LESSON

Insert >, <, or = to make the following a true statement: $\frac{8}{4} \square \frac{12}{3}$

Insert >, <, or = to make the following a true statement: $2^{10} \square 10^{3}$

Solve and graph: $4 - 6x > -8$

Solve the inequality for the indicated variable. Graph the solution.

$3(2y-1) > 2(1-3y)$

Solve and graph: $5(4-y) \geq 3(4y-2)$

Solve and graph: $\frac{3}{4}y + 2 \leq \frac{1}{3}y - 5$