FORMULAS FOR PERIMETER, AREA, AND VOLUME

Introduction

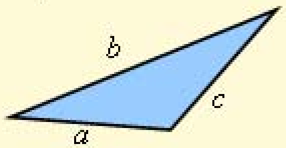
In a formula, variables are used to represent numbers. Different numerical values are substituted for the appropriate variables in a formula according to a specific situation or problem. This lesson will demonstrate the use of formulas in calculating the perimeter, area, and volume of certain shapes.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

Perimeter

The **perimeter** of a geometric figure is the sum of the length of its sides.

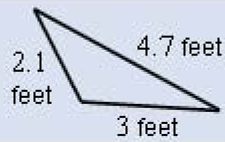
Example: The perimeter of the triangle shown below is $a + b + c$.



$$P = a + b + c$$

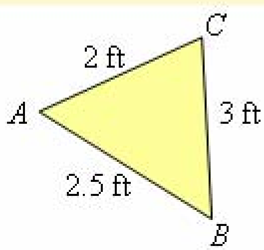
EXAMPLE A

What is the perimeter of the triangle shown?



$$P = 2.1\text{ ft} + 4.7\text{ ft} + 3\text{ ft}$$
$$= 9.8\text{ ft}$$

Question: What is the perimeter, P, of triangle ΔABC shown below



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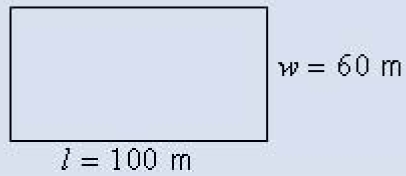
FORMULAS FOR PERIMETER, AREA, AND VOLUME

Rectangles have two pairs of sides with the same lengths. So, we can find the **perimeter of a rectangle** using the formula $P = 2l + 2w$.

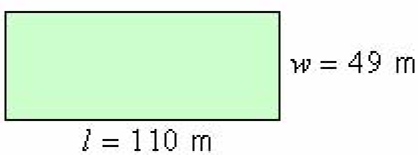
EXAMPLE B

Find the perimeter of a rectangular soccer field with a length, l , of 100 meters and a width, w , of 60 meters.



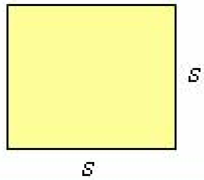
$$
\begin{aligned}
P &= 2l + 2w \\
&= 2(100\text{ m}) + 2(60\text{ m}) \\
&= 200\text{ m} + 120\text{ m} \\
&= 320\text{ m}
\end{aligned}
$$

Question: What is the perimeter of a football field with a length of 110 meters and a width of 49 meters?



FORMULAS FOR PERIMETER, AREA, AND VOLUME

Squares are rectangles with four equal sides, s. We can find the **perimeter of a square** using the formula $P = 4s$, since $s + s + s + s = 4s$.



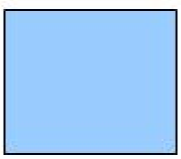
EXAMPLE C

Find the perimeter of a square with sides that are 10 inches long.

$s = 10$ in

$$P = 4s$$
$$= 4(10 \text{ in})$$
$$= 40 \text{ in}$$

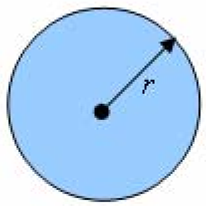
Question: What is the perimeter of a square with sides that are 7 inches long?



$s = 7$ in

FORMULAS FOR PERIMETER, AREA, AND VOLUME

The **perimeter of a circle** is called its **circumference**. The formula for finding the circumference of a circle is $C = 2\pi r$. The **radius** of the circle is represented by r. The radius is the distance from the center of the circle to any point on the circle.



$$C = 2\pi r$$

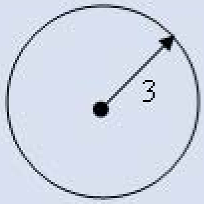
Note:

- The symbol π is called **pi**.
- Because π is a non-terminating decimal, it is sometimes rounded to 3.14. The results of calculations that involve π are usually shown with the $\approx$ symbol or the $\cong$ symbol, both of which show that the result is an approximate value, not an exact one. A more precise value for π is 3.1415926535897932384626433832795028841971693993751, but even this is an *approximate* value of π !

FORMULAS FOR PERIMETER, AREA, AND VOLUME

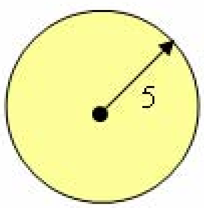
EXAMPLE D

Find the circumference of a circle with $r = 3$. (Use $\pi \cong 3.14$.)



$$
\begin{aligned}
C &= 2\pi r \\
&= 2\pi(3) \\
&= 6\pi \\
&\cong 6(3.14) \\
&\cong 18.84
\end{aligned}
$$

Question: What is the circumference of a circle with $r = 5$?

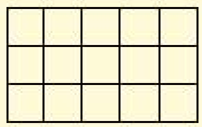


FORMULAS FOR PERIMETER, AREA, AND VOLUME

Area

You can find the **area** of a plane geometric figure by counting the number of unit squares it contains.

Example:

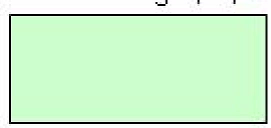


$$5 \text{ in} \times 3 \text{ in} = 15\text{in}^2$$

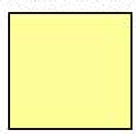
We see 15 unit squares. The area is 15 in^2, which is read "15 square inches."

The **area of a rectangle**, A, is the product of its length, l, and its width, w.

$$A = lw$$

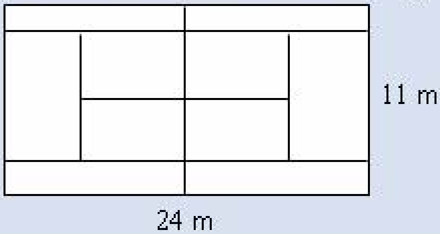


The **area of a square** with sides of length s is $A = s^2$.



EXAMPLE E

What is the area of a tennis court with a length of 24 meters and a width of 11 meters?



$A = lw$

$= (24\ \text{m})(11\ \text{m})$

$= 264\ \text{m}^2$

EXAMPLE F

What is the area of a square garden with side lengths of 12 feet?

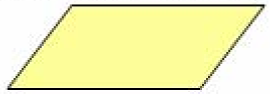
$A = s^2$

$= (12\ \text{ft})(12\ \text{ft})$

$= 144\ \text{ft}^2$

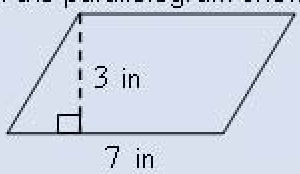
FORMULAS FOR PERIMETER, AREA, AND VOLUME

A **parallelogram** is quadrilateral (a 4-sided figure) formed with two pairs of parallel lines.



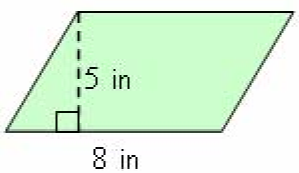
EXAMPLE G

Find the area of the parallelogram shown below.



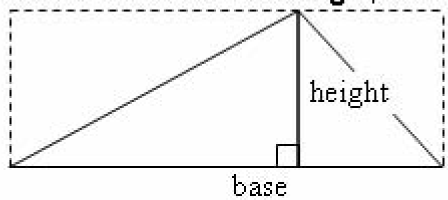
$$A = bh$$
$$= 7 \text{ in}(3 \text{ in})$$
$$= 21 \text{ in}^2$$

Question: What is the area of the parallelogram shown below?



FORMULAS FOR PERIMETER, AREA, AND VOLUME

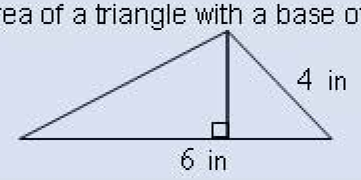
To discover a formula for the **area of a triangle**, consider the image below:



The dotted lines show that the area of the triangle equals $\frac{1}{2}$ the area of the rectangle. You know that the area of a rectangle is its length times its width, or in other words, as shown above, its base times its height $A = bh$. Because the area of a triangle is $\frac{1}{2}$ of the area of a rectangle, the formula for the **area of a triangle** is $A = \frac{1}{2}bh$. This can also be stated as $A = \frac{bh}{2}$.

EXAMPLE H

Find the area of a triangle with a base of 6 in and a height of 4 in.

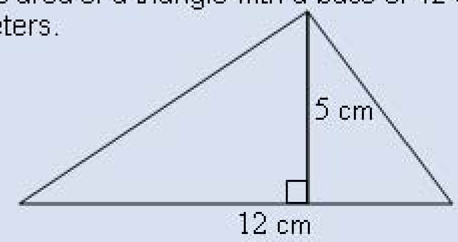


$$
\begin{aligned}
A &= \frac{1}{2}bh \\
&= \frac{1}{2}6 \text{ in}(4 \text{ in}) \\
&= 12 \text{ in}^2
\end{aligned}
$$

FORMULAS FOR PERIMETER, AREA, AND VOLUME

Extended Example 1

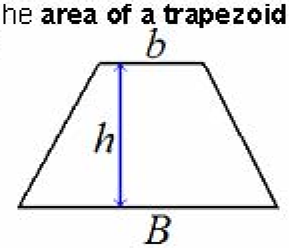
Find the area of a triangle with a base of 12 centimeters and a height of 5 centimeters.



FORMULAS FOR PERIMETER, AREA, AND VOLUME

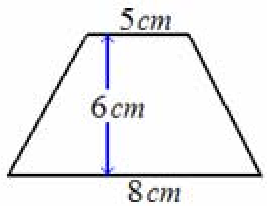
A **trapezoid** is a quadrilateral with two parallel sides called bases; the distance between the bases is the trapezoid's height. The **area of a trapezoid** is the average of the bases multiplied by the height.

$$A = \frac{b + B}{2} \cdot h$$



EXAMPLE I

Find the area of a trapezoid with bases of 5cm and 8cm, and a height of 6cm.



$$
\begin{aligned}
A &= \frac{b+B}{2}\cdot h \\
&= \frac{5\text{cm}+8\text{cm}}{2}\cdot 6\text{cm} \\
&= \frac{13\text{cm}\cdot 6\text{cm}}{2} \\
&= \frac{13\cdot 3\cdot \cancel{2}\,\text{cm}^2}{\cancel{2}} \\
&= 39\text{cm}^2
\end{aligned}
$$

The area of the trapezoid is 39 square centimeters.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

Extended Example 2

What is the area of a trapezoid with bases of 2m and 7m and a height of 5m?

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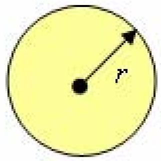
FORMULAS FOR PERIMETER, AREA, AND VOLUME

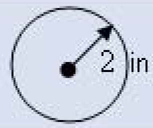
The formula for the **area of a circle** is πr^2. (Remember: $\pi \cong 3.14$.)



$$A = \pi r^2$$

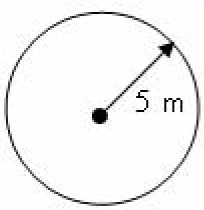
EXAMPLE J

Find the area of the circle shown ($r = 2$ in).



$$\begin{aligned} A &= \pi r^2 \\ &= \pi (2 \text{ in})^2 \\ &\cong 3.14 (4 \text{ in}^2) \\ &\cong 12.56 \text{ in}^2 \end{aligned}$$

Question: Find the area of a circle with $r = 5$ m.



FORMULAS FOR PERIMETER, AREA, AND VOLUME

Extended Example 3

A pizza has a radius of 9 inches. What is the area of the pizza to the nearest square inch?

Extended Example 4

A pizzeria offers two small pizzas with 10-inch diameters for the same price as one large pizza with a 16-inch diameter. Which deal gives you the most pizza?

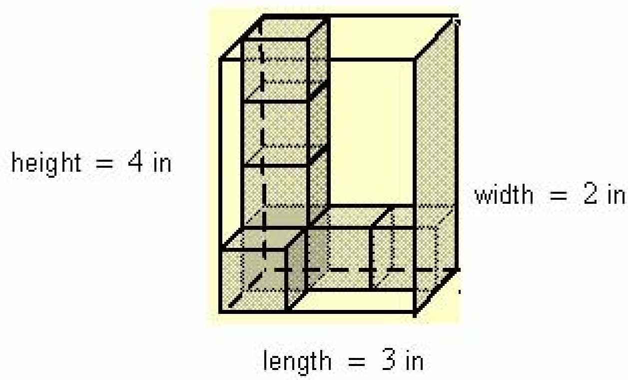
FORMULAS FOR PERIMETER, AREA, AND VOLUME

Volume

The **volume** of a 3-dimensional figure is the number of unit cubes needed to fill it up.

EXAMPLE K

What is the volume of the figure shown below?



The figure is 3 unit cubes long times 2 unit cubes wide times 4 unit cubes high. There are 24 unit cubes total:

$$\begin{aligned} V &= lwh \\ &= 3\text{ in}(2\text{ in})(4\text{ in}) \\ &= 24\text{ in}^3 \end{aligned}$$

This is read as "24 cubic inches."

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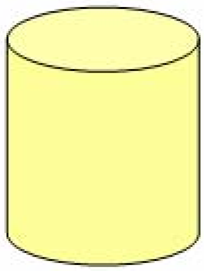
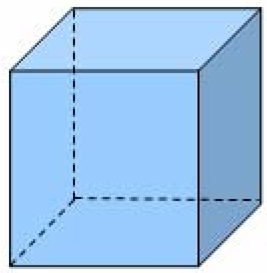
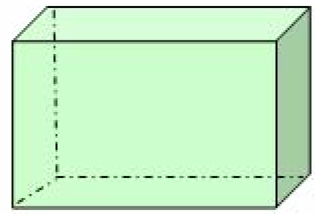
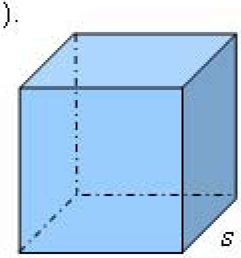
FORMULAS FOR PERIMETER, AREA, AND VOLUME

A **prism** is a 3-dimensional figure with two parallel faces. One of the parallel faces is usually called the **base** of the prism. Prisms are often named for the shapes of their bases.

Cylinder	Cube	Rectangular Prism
		

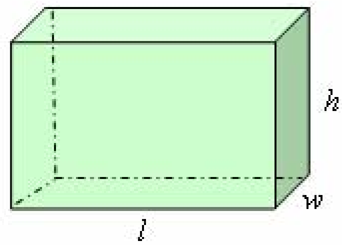
The **volume of a prism** is the area of its base times its height.

Volume of a cube: The area of the base is s^2 so the volume is $V = s^3$ (the area of the base, s^2, times the height, s).



Volume of a rectangular prism:

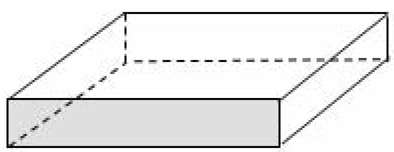
The area of the base is lw, so the volume is $V = lwh$.



EXAMPLE L

A local park is installing a sand volleyball court. The sand will cover an area 30 yards long, 20 yards wide, and 0.75 yards deep. What is the volume of sand needed to fill the court?

The problem is describing a rectangular prism:



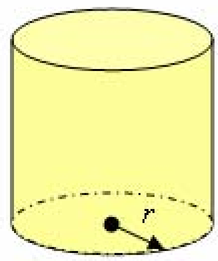
$$\begin{aligned} V &= lwh \\ &= 30 \text{ yd}(20 \text{ yd})(0.75 \text{ yd}) \\ &= 450 \text{ yd}^3 \end{aligned}$$

The court builders will need 450 cubic yards of sand to fill the court.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

Volume of a cylinder:

The area of the base (a circle) is πr^2, so the volume is $V = \pi r^2 h$.



EXAMPLE M

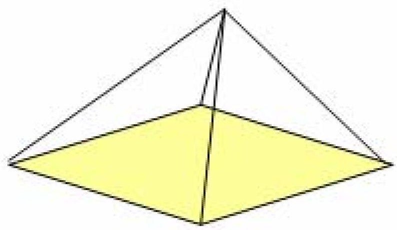
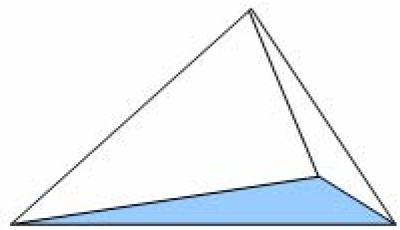
What is the volume of a cylinder with a radius of 5 inches and a height of 7 inches?

$$
\begin{aligned}
V &= \pi r^2 h \\
&\cong 3.14\left[(5\text{ in})^2(7\text{ in})\right] \\
&\cong 3.14\left[25\text{ in}^2(7\text{ in})\right] \\
&\cong 549.5\text{ in}^3
\end{aligned}
$$

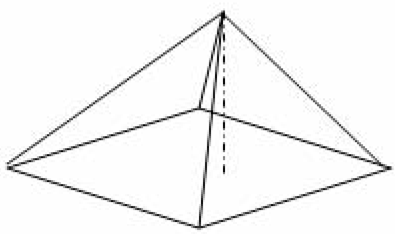
The volume is approximately 549.5 cubic inches.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

A **pyramid** is a 3-dimensional object whose base is a triangle, rectangle, etc., and whose triangular faces meet at one point.

Rectangular Pyramid	Triangular Pyramid
	

The **volume of a pyramid** is $\frac{1}{3}$ the area of its base times its height.



$$V = \frac{1}{3} \cdot (\text{base area}) \cdot \text{height}$$

FORMULAS FOR PERIMETER, AREA, AND VOLUME

EXAMPLE N

What is the volume of a rectangular pyramid whose length is 3 m, width is 5 m, and height is 15 m?

The area of the rectangular base is its length times its width, $l \cdot w$, so:

$$V = \frac{1}{3} \cdot (\text{base area}) \cdot \text{height}$$
$$= \frac{1}{3} \cdot l \cdot w \cdot h$$
$$= \frac{1}{3}(3\text{ m})(5\text{ m})(15\text{ m}) = 75\text{ m}^3$$

The volume is 75 cubic meters.

EXAMPLE O

What is the volume of a triangular pyramid whose base is 9 m, width is 4 m, and height is 10 m?

The area of the triangular base is half its length times its width, $\frac{1}{2} \cdot l \cdot w$, so:

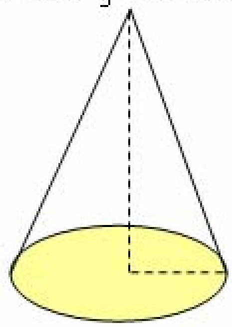
$$V = \frac{1}{3} \cdot (\text{base area}) \cdot \text{height}$$
$$= \frac{1}{3} \cdot \frac{1}{2} \cdot l \cdot w \cdot h$$
$$= \frac{1}{3} \cdot \frac{1}{2}(9\text{ m})(4\text{ m})(10\text{ m})$$
$$= \frac{1}{6}(9\text{ m})(4\text{ m})(10\text{ m}) = 60\text{ m}^3$$

The volume is 60 cubic meters.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

An object closely related to pyramids is the **cone**. Its base is a circle.

The **volume of a cone** is also $\frac{1}{3}$ the area of its base times its height.



$$V = \frac{1}{3}\pi r^2 h$$

EXAMPLE P

What is the volume of a cone whose radius is 3 inches and height is 7 inches?

The area of the cone's base is: πr^2, so:

$$\begin{aligned} V &= \frac{1}{3} \cdot \text{(base area)} \cdot \text{height} \\ &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi (3\text{in})^2 (7\text{in}) \\ &\cong \frac{1}{3}(3.14)(9\text{in}^2)(7\text{in}) \cong 65.94\text{in}^3 \end{aligned}$$

The volume is approximately 65.94 cubic inches.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

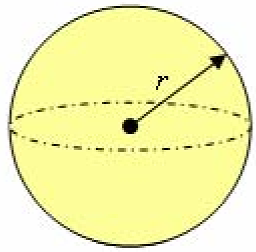
Extended Example 5

Find the volume of a traffic cone that is 12 inches tall and whose base has a radius of 6 inches, to the nearest cubic inch.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

A **sphere** is a figure shaped like a round ball.

The **volume of a sphere** with radius r is $V = \frac{4}{3}\pi r^3$.



EXAMPLE Q

What is the volume of a sphere with radius 3 inches, to the nearest inch?

$$V = \frac{4}{3}\pi r^3$$

$$\cong \frac{4}{3}(3.14)(3\text{ in})^3$$

$$\cong \frac{4}{3}(3.14)\left(27\text{in}^3\right) \cong 113.04\text{in}^3$$

The volume of the sphere is approximately 113 cubic inches.

FORMULAS FOR PERIMETER, AREA, AND VOLUME

EXAMPLE R

Some scientists believe that shortly after the big bang there was a time when the entire universe was a sphere only 10 inches in diameter. At that time, what would the volume of the universe have been? Round your answer to the nearest cubic inch.

The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. Since the radius is half the diameter, if the diameter is 10 inches, the radius is 5 inches. So:

$$V = \frac{4}{3}\pi\left(5 \text{ in}\right)^3 = \frac{4\pi \cdot 125 \text{ in}^3}{3}.$$

This time we'll use 3.14159 for π.

$$V \cong \frac{4(3.14159)\cdot 125}{3} \text{ in}^3$$

$$\cong 523.59 \text{ in}^3$$

Rounded to the nearest cubic inch: $\cong 524 \text{ in}^3$

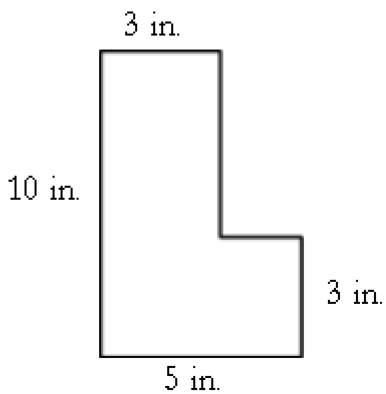
The volume of the universe was about 524 cubic inches.

Note:

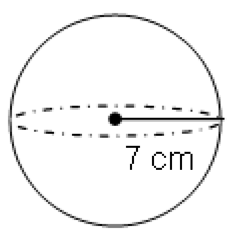
- Had we used the approximation 3.14 for π, the result would have rounded to one less than the correct answer (523 cubic inches). The small round-off error is multiplied by a large number and becomes significant. It's better to use extra precision in such calculations and to round off only in the final step.

END OF LESSON

Find the perimeter:



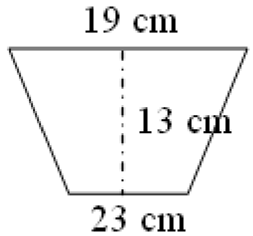
Find the volume:



Draw and label the figure for which the given formula applies. Then find each perimeter, area, or volume.

$A = lh$ $l = 33.4$ meters, $h = 16.2$ meters

Find the Area:



A tent is made from four triangular pieces of canvas. The base of each triangle is 2.6 meters; the height of each triangle is 3.1 meters. How many square meters of canvas are there in the tent?

A breakfast cereal used to be sold in a rectangular box that was 5 inches wide, 7.5 inches high, and 3.5 inches thick. To reduce the volume of food in the package, the company reduced the thickness of the box by 0.5 inch. By how much was the breakfast food in the box reduced?

The perimeter of a square is 48 centimeters. The measurement of each side is decreased by 4.5 centimeters. What is the area of the smaller square?

OTHER USEFUL FORMULAS

Introduction

In this lesson, we'll discuss several formulas that are useful in everyday life:

- Simple and Compound Interest
- Percent Discount
- Mileage
- Temperature Conversion
- Unit Price and Cost of Multiple Items

OTHER USEFUL FORMULAS

Simple Interest

When money is borrowed from a bank to purchase a home or a car, the amount borrowed is called the **principal**. In addition to the principal borrowed, a certain percentage is added to the amount owed each year. This percentage is called the **annual interest rate**.

Example:

Suppose you borrow \$100 from a friend, and agree to pay it back in one year with an extra \$5 added. In one year you would pay back your friend \$105.

This is an example of a loan with a principal of \$100 and an annual interest rate of 5%. It is also an example of **simple interest**, where interest is paid only on the original principal.

Using simple interest, the interest, I, earned on a principal amount of P dollars invested for one year at a constant annual percentage rate, r, can be calculated using the formula:

$$I = Pr.$$

OTHER USEFUL FORMULAS

EXAMPLE A

A bank pays 5% annual (yearly) interest on a money market account, using simple interest. How much interest does a $600 deposit earn in one year?

$$\begin{aligned} I &= Pr \\ &= (\$600)(5\%) \\ &= (\$600)(0.05) \\ &= \$30 \end{aligned}$$

A $600 deposit will earn $30 in one year.

Question: If $1,500 is invested in an account earning 6% annual interest, how much interest will the account earn after 1 year?

Answer ▼

OTHER USEFUL FORMULAS

We now know that simple interest earned in one year is given by $I = Pr$, where P is the original principal and r is the annual interest rate. After one year, the total amount, A, is the original principal P plus the interest I, so:

$$\begin{aligned} A &= P + I \\ &= P + Pr \end{aligned}$$

By factoring out the P on the right side of $P + Pr$, we get a formula that can be used to find the total amount in an account after one year of simple interest:

$$A = P(1+r).$$

EXAMPLE B

Using simple interest, a bank pays 5% annual interest on a money market account. How much will a $600 deposit grow to after one year?

$$\begin{aligned} A &= P(1+r) \\ &= \$600(1+0.05) \\ &= \$600(1.05) \\ &= \$630 \end{aligned}$$

The account will be worth $630 after one year.

OTHER USEFUL FORMULAS

EXAMPLE C

You agree to pay back a loan of $100 in four years at an annual interest rate of 10%, using simple interest. What will be the total owed in four years?

The following table shows each year's interest being added to the total amount owed, starting with a principal balance of $100.

Year	Principal ($)	Annual Interest ($)	Ending Balance ($)
1	100.00	10.00	110.00
2	100.00	10.00	120.00
3	100.00	10.00	130.00
4	100.00	10.00	140.00
	$100 borrowed	**Total interest $40.00**	**Total owed in 4 years: $140**

Each year, the same fixed amount of interest is added ($10 in this example). The total amount owed, A, is the principal P plus the fixed interest earned each year. This could be written as $A = P + nrP$, where n is the number of years for which an amount P is borrowed at an annual simple interest rate r. The P can be factored out on the right side of this equation, giving us the simple interest formula for the total amount after some number of years:

$$A = P(1+nr).$$

Using this formula for Example C, we get the same result as was found above:

$$\begin{aligned} A &= P(1+nr) \\ &= \$100(1+4(0.10)) \\ &= \$100(1+0.40) = \$100(1.40) = \$140 \end{aligned}$$

OTHER USEFUL FORMULAS

Extended Example 1a

$30,000 is invested in a simple interest account earning 4% interest per year. After 2 years, how much money will be in the account?

6 of 17

OTHER USEFUL FORMULAS

Compound Interest

For most loans (mortgages, student loans, etc.) interest is charged on principal and any previously accrued interest. Such loans are said to be **compounded**. Interest is usually compounded monthly but for this section we'll compound yearly. (Monthly compounding involves formulas that are more complex.)

EXAMPLE D

After four years, what will you owe on a loan of $100 at a 10% annual interest rate, using compound interest (compounded once a year)?

This table shows each year's interest calculated on the total amount owed:

Year	Principal ($)	Yearly Interest ($)	Ending Balance ($)
1	100.00	10.00	110.00
2	110.00	11.00	121.00
3	121.00	12.10	133.10
4	133.10	13.31	146.41
	$100 borrowed	**Total interest $46.41**	**Total owed in 4 years: $146.41**

Each year, the amount of interest increases, since it includes interest on interest. The formula

$$A = P(1+r)^n$$

describes A, the amount owed after n years if P dollars is borrowed at an annual interest rate r, compounded yearly. For Example D, this formula gives:

$$A = P(1+r)^n = \$100(1+0.10)^4$$

$$= \$100(1.10)^4 = \$100(1.4641) = \$146.41$$

OTHER USEFUL FORMULAS

EXAMPLE E

Using compound interest, a bank pays 5% annual interest on a money market account. How much interest will be added to a $600 deposit after 2 years?

After 2 years, the amount, A, can be calculated as follows:

$$A = P(1+r)^n = \$600(1+0.05)^2$$
$$= \$600(1.05)^2$$
$$= \$600(1.1025)$$
$$= \$661.50$$

The interest added after 2 years is $61.50.

EXAMPLE F

If $2,000 is invested in an account at an interest rate of 4% compounded annually, what will the account balance be after 3 years?

$$A = P(1+r)^n = \$2,000(1+4\%)^3$$
$$= \$2,000(1+0.04)^3$$
$$= \$2,000(1.04)^3$$
$$= \$2,000(1.124864)$$
$$= \$2249.728000 \cong \$2,249.73$$

After 3 years, the account balance will be $2,249.73. (Note that we rounded our solution to the nearest hundredth, since that makes cents!)

OTHER USEFUL FORMULAS

Extended Example 2a

If $30,000 is invested in an account at an interest rate of 4% compounded annually, what will the account balance be after 2 years?

OTHER USEFUL FORMULAS

Percent Discount

A discount is the dollar amount by which an original price is reduced. **Percent discount** is the percent taken off of the original dollar amount.

EXAMPLE G

Before being put on the 40% off rack, a pair of jeans was $42. How much are the jeans now?

First, we need to figure out the discount. We know that the discount rate is 40% of the original price.

So: $(40\%)(\$42) = (0.40)(\$42) = \$16.80$

The sale price is $\$42 - \$16.80 = \$25.20$

EXAMPLE H

Before being put on the 40% off rack, a pair of jeans was $42. How much are the jeans now?

You're not experiencing "déjà vu." We just asked this in the last example; this time we'll take a useful short-cut. The point is: if the price is 40% off, then you're paying for the remaining 60%. So the sale price is 60% of the original price, or

$$(60\%)(\$42) = (0.60)(\$42) = \$25.20.$$

OTHER USEFUL FORMULAS

EXAMPLE I

A pair of shoes was reduced from \$90 to \$63. What is the percent discount of this reduction?

The dollar amount of the reduction is \$27 because $\$90 - \$63 = \$27$. To find what percent \$27 is of \$90, we divide \$27 (the dollar amount of the discount) by \$90 (the original dollar amount): $\frac{\$27}{\$90} = \frac{3}{10} = 0.30 = 30\%$.

The shoes were reduced by 30%.

The formula for percent discount, d, on a purchase where P represents the original price and S represents the sale price is: $d = \left(\frac{P-S}{P}\right) \cdot 100\%$.

Question: What is the percent discount of an item reduced from \$120 to \$100?

OTHER USEFUL FORMULAS

Mileage

Mileage is the number of miles that a vehicle travels per gallon of gas. We divide the miles driven, d, by the number of gallons of gasoline used, g, to figure the mileage, m (which gives us "miles per gallon"):

$$m = \frac{d}{g}$$

EXAMPLE J

Starting with a full tank of gas, Eric drove 275 miles from Los Angeles to Las Vegas. In Las Vegas he filled up his tank again with 7.6 gallons. What was his mileage for the trip, rounded to the nearest tenth?

Eric drove 275 miles on 7.6 gallons of gas. His mileage was:

$$m = \frac{275}{7.6} \cong 36.2 \text{ mpg.}$$

Question: Maria drove to visit a friend who lives 150 miles away. She started with a full tank of gas. After she arrived, she filled up her tank again. It took 6 gallons. How many miles per gallon (mpg) did her car travel on the drive?

OTHER USEFUL FORMULAS

Temperature Conversion

Two temperature scales commonly used are Celsius and Fahrenheit. On the Celsius scale, the freezing point of water is $0°$ and the boiling point of water is $100°$. On the Fahrenheit scale, the freezing point of water is $32°$ and the boiling point of water is $212°$.

The relationship between these temperature scales is expressed by two formulas:

$$C = \frac{5}{9}(F - 32°) \qquad F = \frac{9}{5}C + 32°$$

Examples:

Express $68°F$ in Celsius:	Express $30°C$ in Fahrenheit:
$C = \frac{5}{9}(68° - 32°)$ $= \frac{5}{9}(36°)$ $= 20°$	$F = \frac{9}{5}30° + 32°$ $= 54° + 32°$ $= 86°$

OTHER USEFUL FORMULAS

Extended Example 3a

You drive by a bank and see that the temperature is $23°C$. What is the temperature in degrees Fahrenheit?

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OTHER USEFUL FORMULAS

Unit Price

Unit price is the cost per pound, ounce, gram, or other unit measure.

To find the unit price, u, of an item, divide the total cost, c, by the number of units, n:

$$u = c \div n \text{ or } u = \frac{c}{n}.$$

If we know the cost per unit and we want to buy multiple units, we have an easy way to find the total price.

The cost per unit u multiplied by the number of units n equals the total cost c:

$$c = nu$$

EXAMPLE K

The price of one gallon of gasoline is $1.499. How much will it cost to fill an 11.5-gallon tank? Round your answer to nearest hundredth.

$c = nu = 11.5 \times \$1.499 = \$17.2385$, rounded to $17.24.

Question: A 15.5-ounce can of soup costs $2.79. What is the cost per ounce? (Note that $n = 1 \text{ can} = 15.5 \text{ oz}$.)

OTHER USEFUL FORMULAS

Extended Example 4a

It costs $5,000 to run a factory for a month plus another $9,000 to produce 70 washing machines. How much does it cost to produce 1 washing machine?

OTHER USEFUL FORMULAS

Extended Example 5a

A 20-ounce can of olives sells for $5.99. A 12-ounce can of the same olives sells for $3.99. Which is the better buy?

END OF LESSON

Which is the better buy: A 473-milliliter bottle of salad dressing for $0.67 or a 952- milliliter bottle for $1.34?

Find the total amount of money in the savings account after the given time period, assuming simple annual interest.

$P = \$8,000$ $r = 9.0\%$ $t = 5$ years

Find the total amount of money in each savings account afte the given time period, assuming compound interest (compounded annually).

$P = \$4,755$ $r = 8.75\%$ $t = 2$ years

Find the number of miles of gasoline per gallon: distance $= 495$ miles, gallons $= 22.5$

RATIOS, RATES, AND PROPORTIONS

Introduction

In this lesson we'll define ratios, rates, proportions, and similar triangles. We'll see ways in which these concepts can be applied to real-world situations.

RATIOS, RATES, AND PROPORTIONS

Definitions

A **quotient** is a ratio of two quantities, with or without units.

A **ratio** is a quotient in which the numerator and the denominator have the same units, or both have no units at all. Actually, when the units are the same in the numerator and denominator the units can be canceled, leaving a fraction.

Example: Brooke's hummingbird feeder contains a mixture of 1 tablespoon of corn syrup and 8 tablespoons of water. The ratio of corn syrup to water is

$$\frac{1\ \text{tbsp}}{8\ \text{tbsp}} = \frac{1\ \cancel{\text{tbsp}}}{8\ \cancel{\text{tbsp}}} = \frac{1}{8}.$$

A **rate** is a quotient in which the numerator and denominator have different units.

Examples:

A car is traveling at a speed of 65 miles per hour. The car's rate of speed is

$$\frac{65\ \text{miles}}{1\ \text{hour}} = 65\ \frac{\text{miles}}{\text{hour}} = 65\ \text{mph}.$$

A robot builds 58 TVs per week. The robot's rate of TV production is

$$\frac{58\ \text{TVs}}{1\ \text{week}} = 58\ \frac{\text{TVs}}{\text{week}}.$$

A doll factory can produce 1,600 dolls per 8-hour work day. The doll factory's hourly rate of doll production is:

$$\frac{1{,}600\ \text{dolls}}{8\ \text{hours}} = \frac{200\ \text{dolls}}{1\ \text{hour}} = 200\ \frac{\text{dolls}}{\text{hours}} = 200\ \text{dolls per hour}.$$

RATIOS, RATES, AND PROPORTIONS

An equation with a quotient on each side of the equal sign is called a **proportion**.

If you can say of a set of quantities that "A is to B as C is to D," then you can write down a corresponding proportion:

$$\frac{A}{B} = \frac{C}{D}.$$

Often one of the quantities is unknown and has to be solved for.

If we multiply both sides of a proportion by the product of its denominators, all the denominators are canceled:

$$\frac{A}{B} = \frac{C}{D}$$

$$BD \cdot \frac{A}{B} = BD \cdot \frac{C}{D}$$

$$\frac{\cancel{B}DA}{\cancel{B}} = \frac{B\cancel{D}C}{\cancel{D}}$$

$$AD = BC$$

This results in each side being the product of its numerator and the opposite denominator. Jumping straight to this conclusion is called **cross-multiplying**:

$$\frac{A}{B} = \frac{C}{D} \quad \Rightarrow \quad \frac{A}{B} \times \frac{C}{D} \quad \Rightarrow \quad AD = BC$$

RATIOS, RATES, AND PROPORTIONS

EXAMPLE A

The quotient of x and 7 equals the quotient of 5 and 2. Set up a proportion and solve for x.

Step 1: "The quotient of x and 7" is $\frac{x}{7}$. We're told this equals "the quotient of 5 and 2," $\frac{5}{2}$. Set this up as a proportion by equating the two quotients:

$$\frac{x}{7} = \frac{5}{2}.$$

Step 2: Cross-multiply:

$$2x = 5 \cdot 7$$

$$2x = 35$$

Step 3: To solve for x, we need to isolate it on one side of the equation. We can do this by dividing both sides of the equation by 2:

$$\frac{2x}{2} = \frac{35}{2} \Rightarrow \frac{\cancel{2}x}{\cancel{2}} = \frac{35}{2} \Rightarrow x = \frac{35}{2} = 17.5$$

Extended Example 1a

Solve the proportion $\frac{4}{x} = \frac{12}{17}$.

Extended Example 2a

Solve the proportion $\frac{7.9}{4.1} = \frac{2.7y}{8.3}$. Round your answer to the nearest tenth.

RATIOS, RATES, AND PROPORTIONS

EXAMPLE B

If one day you purchase 5.3 gallons of gas for $12.67, and the price doesn't change, how much will 23.9 gallons cost?

This can be rephrased as "$12.67 is to 5.3 gallons as x is to 23.9 gallons," and can be written as the proportion:

$$\frac{\$12.67}{5.3 \text{ gal}} = \frac{x}{23.9 \text{ gal}}.$$

Cross-multiplying, we get

$$\$12.67(23.9 \text{ gal}) = x(5.3 \text{ gal}).$$

To solve for x we need to isolate it on one side of the equation. We can do this by dividing both sides by 5.3 gal:

$$\frac{\$12.67(23.9 \text{ gal})}{(5.3 \text{ gal})} = \frac{x(5.3 \text{ gal})}{(5.3 \text{ gal})}$$

$$\frac{\$12.67(23.9 \cancel{\text{gal}})}{(5.3 \cancel{\text{gal}})} = \frac{x\cancel{(5.3 \text{ gal})}}{\cancel{(5.3 \text{ gal})}}$$

$$\frac{\$302.813}{5.3} = x$$

$$\$57.13452830 = x$$

The price of 23.9 gallons of gasoline will be $57.13.

EXAMPLE C

If the cost to obtain a license for a boat is \$13 for every \$1,000 of the cost of the boat, how much would it cost to get a license for a sailboat that cost \$42,000?

Step 1: Write a proportion involving two ratios.

"\$13 is to \$1,000 as x is to \$42,000" → $\frac{\$13}{\$1,000} = \frac{x}{\$42,000}$

Step 2: Cross-multiply: $\$13(\$42,000) = x(\$1,000)$

Step 3: Solve for x.

$$\frac{\$13(\$42,000)}{\$1,000} = \frac{x(\$1,000)}{(\$1,000)}$$

or

$$x = \frac{\$13(\$42,000)}{\$1,000} = \$546$$

It will cost \$546 to license the sailboat.

RATIOS, RATES, AND PROPORTIONS

EXAMPLE D

The Deuce Painting Company sends two painters, Ray and Charles, to paint a house with 8 rooms. Each painter's income is based on the proportion of work he does. Ray paints 5 of the rooms and Charles paints the other 3. They are paid a total of $500. How should the money be divided between them?

Step 1: The work done by Ray over the total work done must equal the money paid to Ray over the total money paid:

$$\frac{\text{Work done by Ray}}{\text{Total work done}} = \frac{\text{\$ Paid to Ray}}{\text{Total \$ paid}}$$

$$\frac{5 \text{ rooms}}{8 \text{ rooms}} = \frac{x}{\$500}$$

Step 2: Cross-multiply: $(5 \text{ rooms})(\$500) = x(8 \text{ rooms})$

Step 3: Solve for x by dividing both sides by 8 rooms:

$$\frac{(5 \text{ rooms})(\$500)}{(8 \text{ rooms})} = \frac{x(8 \text{ rooms})}{(8 \text{ rooms})}$$

$$\frac{(5 \cancel{\text{ rooms}})(\$500)}{(8 \cancel{\text{ rooms}})} = \frac{x \cancel{(8 \text{ rooms})}}{\cancel{(8 \text{ rooms})}}$$

$$\frac{\$2500}{8} = x$$

$$x = \$312.50$$

Ray is owed $312.50. We could solve a similar proportion to find Charles' salary, but since Ray earned $312.50 and they were paid a total of $500, then Charles earned $500 - $312.50 = $187.50.

RATIOS, RATES, AND PROPORTIONS

EXAMPLE E

John and Isabella want to drive from Spokane to Seattle. They plan to drive 65 mph, and the distance is about 280 miles. How long will it take them to make the trip?

Step 1: Write a proportion (since time is the quantity represented by the variable, we'll choose the variable t, since it stands for time).

$$\frac{65 \text{ miles}}{1 \text{ hour}} = \frac{280 \text{ miles}}{t}$$

Step 2: Cross-multiply. $t(65 \text{ miles}) = 1 \text{ hour}(280 \text{ miles})$.

Step 3: Solve for t. $t = \dfrac{1 \text{ hour}(280 \text{ miles})}{65 \text{ miles}} = 4.3 \text{ hours}$

It will take them about 4.3 hours to make the drive.

It's common to express this in terms of hours and minutes:

$$\begin{aligned} 4.3 \text{ hours} &= 4 \text{ hours} + 0.3 \text{ hours} \\ &= 4 \text{ hours} + \left(0.3 \cancel{\text{hours}}\right)\left(\frac{60 \text{ min}}{1 \cancel{\text{hour}}}\right) \\ &= 4 \text{ hours} + (0.3)(60 \text{ min}) \\ &= 4 \text{ hours} + 18 \text{ min} \\ &= 4 \text{ hours } 18 \text{ min} \end{aligned}$$

RATIOS, RATES, AND PROPORTIONS

Extended Example 3a

Janine's power bill said she used 160.4 kilowatt hours of energy, and she was charged $33.68. What was the cost of 1 kilowatt hour of energy?

RATIOS, RATES, AND PROPORTIONS

Comparing Rates

In order to compare rates, it's often useful to make the denominators equal one.

For example, if you travel 300 miles on 10 gallons of gas, you're getting

$$\frac{300 \text{ miles}}{10 \text{ gallons}} = \frac{30 \text{ miles}}{1 \text{ gallon}} = 30 \text{ miles per gallon} = 30 \text{ mpg}.$$

We just divided the 300 in the numerator by the 10 in the denominator. Afterward, the 10 becomes 1.

EXAMPLE F

Suppose Yvonne travels 300 miles on 10 gallons of gas, while Sri Ram travels 468 miles on 13 gallons of gas. Who's car will use less gas on a 600-mile trip?

Yvonne's car gets 30 mpg (see the example above).

But Sri Ram's car gets $\frac{468 \text{ miles}}{13 \text{ gallons}}$.

Divide the 468 by 13, and you get 36, so Sri Ram's car gets

$$\frac{36 \text{ miles}}{1 \text{ gallon}} = 36 \text{ mpg}.$$

Sri Ram's car gets better mileage and will use less gas on a 600-mile trip.

EXAMPLE G

Jose paid $15.40 for 9.22 gallons of gas, while Marie paid $23.44 or 13.92 gallons of gas. Who got the better deal?

Divide each numerator by each denominator to make the denominators equal to 1 and you will have the price per gallon for each purchase:

Jose: $\frac{\$15.40}{9.22 \text{ gal}} = \frac{\$1.670281996}{1 \text{ gal}} \cong 1.67 \frac{\$}{\text{gallon}}$

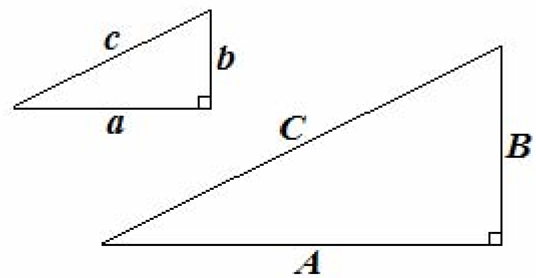
Marie: $\frac{\$23.44}{13.92 \text{ gal}} = \frac{\$1.683908046}{1 \text{ gal}} \cong 1.68 \frac{\$}{\text{gallon}}$

Jose got a slightly better deal.

RATIOS, RATES, AND PROPORTIONS

Similar Triangles

Triangles that have the same shape, but different sizes, are called **similar triangles**. Two triangles are similar whenever one is an enlargement of the other.



For such triangles, ratios of similar sides are equal. For example, for the two similar triangles above it's true that:

"A is to B as a is to b" $\rightarrow$ $\dfrac{A}{B} = \dfrac{a}{b}$

"A is to C as a is to c" $\rightarrow$ $\dfrac{A}{C} = \dfrac{a}{c}$

And also,

"a is to A as b is to B" $\rightarrow$ $\dfrac{a}{A} = \dfrac{b}{B}$

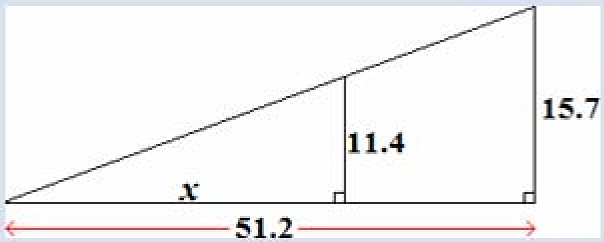
"C is to c as A is to a" $\rightarrow$ $\dfrac{C}{c} = \dfrac{A}{a}$ etc.

This principal is true for any geometric figures that vary not in their shape but only in their size. There are similar rectangles, similar pentagons, etc. For all such figures, ratios of corresponding sides are equal.

EXAMPLE H

Given the similar triangles shown in the figure, find x.

Round your answer to the nearest tenth.



"The base of the little triangle is to the base of the big triangle as the height of the little triangle is to the height of the big triangle."

"x is to 51.2 as 11.4 is to 15.7" → $\frac{x}{51.2} = \frac{11.4}{15.7}$

Cross-multiply:

$$x(15.7) = (51.2)(11.4)$$

$$15.7x = 583.68$$

Now divide both sides of the equal sign by 15.7 and round to the nearest tenth:

$$\frac{15.7x}{15.7} = \frac{583.68}{15.7}$$

$$\frac{\cancel{15.7}x}{\cancel{15.7}} = \frac{583.68}{15.7}$$

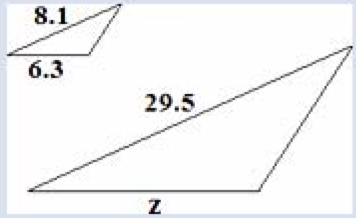
$$x \cong 37.17707006$$

$$x \cong 37.2$$

Extended Example 4a

Given the similar triangles shown in the figure, find z.

Round your answer to the nearest tenth.



END OF LESSON

Find the missing term: $\frac{x+5}{50} = \frac{3x}{75}$

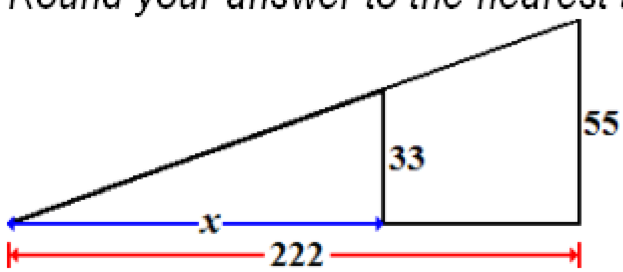
A customer sees an ad for 4 tires for $150; but she wants 5 tires so she can use the extra as a spare. How much will the 5 tires cost?

At a manufacturing plant, there are 14 defective spark plugs in each box of 500 plugs that is produced. How many defective plugs will there be in a day's production of 30,000 plugs? (Set up and solve a proportion to find the answer.)

If a six foot tall man casts a shadow that is $4\frac{1}{2}$ ft long, and a flag pole casts a shadow that is 30 ft long, how tall is the flag pole?

Stan and Emma earned $330 mowing lawns. Emma mowed 8 lawns, and Stan mowed 7. How should they divide the profits?

For the similar triangles, solve for the variable.
Round your answer to the nearest thousandth.



THE PYTHAGOREAN THEOREM

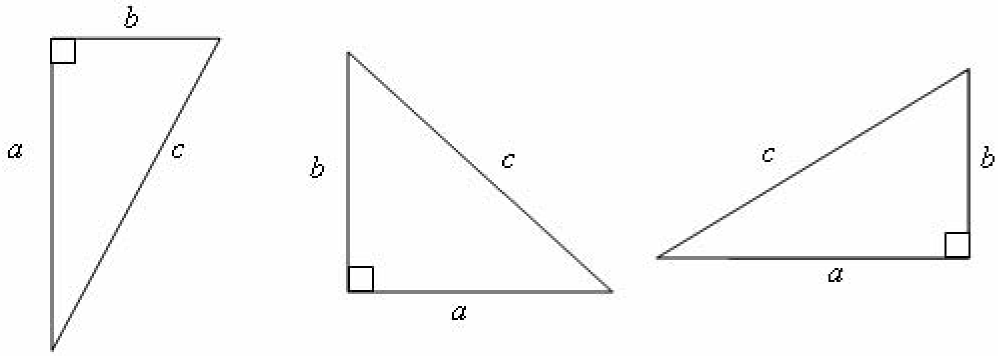
Introduction

This lesson introduces the **Pythagorean Theorem**, $c^2 = a^2 + b^2$, and shows how to use it to determine whether a given triangle is a right triangle. This theorem is one of the most famous theorems of mathematics, and one of the oldest. It was independently proven in ancient India and in China, as well as in ancient Greece.

THE PYTHAGOREAN THEOREM

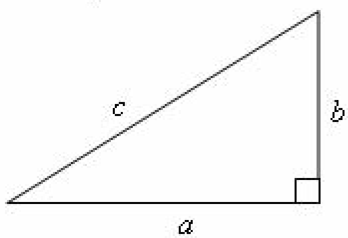
A **right triangle** is a triangle with one interior **right angle** (a $90°$ angle).

The following are examples of right triangles (the □ indicates a right angle):



The longest side, the side opposite the right angle (side c in the examples), is called the **hypotenuse**. The other two sides may be referred to as the **legs**.

The Pythagorean Theorem: In any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.



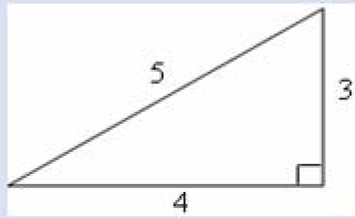
$$c^2 = a^2 + b^2$$

The converse is also true: If the square of the length of one side equals the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

THE PYTHAGOREAN THEOREM

EXAMPLE A

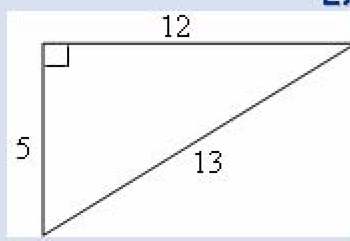
Apply the Pythagorean Theorem to determine whether the triangle shown is a right triangle.



$c^2 = a^2 + b^2$
$5^2 = 3^2 + 4^2$
$25 = 9 + 16$
$25 = 25$ ✓ Yes, it is a right triangle.

EXAMPLE B

Apply the Pythagorean Theorem to determine if the triangle shown is a right triangle.

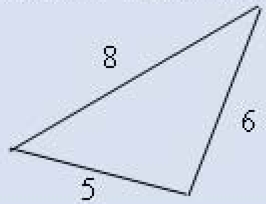


$c^2 = a^2 + b^2$
$13^2 = 5^2 + 12^2$
$169 = 25 + 144$
$169 = 169$ ✓ Yes, it is a right triangle.

THE PYTHAGOREAN THEOREM

EXAMPLE C

Apply the Pythagorean Theorem to determine if the triangle below is a right triangle.



$$c^2 = a^2 + b^2$$
$$8^2 = 5^2 + 6^2$$
$$64 = 25 + 36$$
$$64 \neq 61$$

✗ No, this is not a right triangle.

EXAMPLE D

Is a triangle with side lengths 7, 24, and 25 a right triangle?

Square all the side lengths:

$$7^2 = 49, \quad 24^2 = 576, \quad 25^2 = 625.$$

Does the sum of the two smaller squares equal the larger square?

$$7^2 + 24^2 = 49 + 576 = 625 = 25^2$$ ✓

Since the sum of the squares of the two smaller sides equals the square of the hypotenuse (the longest side), this is a right triangle.

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THE PYTHAGOREAN THEOREM

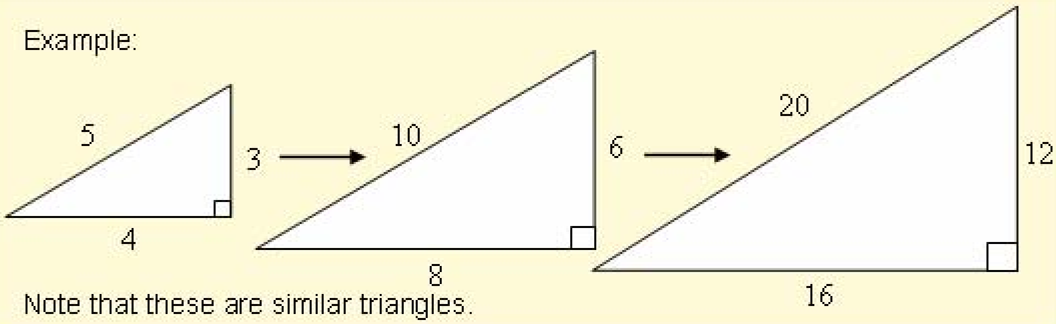
Question: Is a triangle with side lengths 5, 12, and 14 a right triangle?

Question: Is a triangle with side lengths 11, 60, and 61 a right triangle?

THE PYTHAGOREAN THEOREM

From any right triangle, you can make more right triangles by doubling, tripling, etc., the lengths of all sides.

Example:



Note that these are similar triangles.

Recall that if x is nonnegative, and $x^2 = A$, then $x = \sqrt{A}$.

For example, $x^2 = 9 \Rightarrow x = \sqrt{9} = 3$.

Sometimes square roots can't be calculated exactly. Luckily, inexpensive calculators can approximate them to great accuracy.

For example, $x^2 = 2 \Rightarrow x = \sqrt{2} \cong 1.414213562$.

Next we'll use the Pythagorean Theorem to work with right triangles that have side lengths that are not integers.

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THE PYTHAGOREAN THEOREM

EXAMPLE E

Suppose that $a = 3.4$ and $b = 5.7$ are lengths of the sides of a right triangle with hypotenuse c. Find the length of c. Round your answer to the nearest hundredth.

Using the Pythagorean theorem, we get:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= (3.4)^2 + (5.7)^2 \\ &= 11.56 + 32.49 \\ &= 44.05 \end{aligned}$$

Now we take the square root of both sides of the equation.

$$\begin{aligned} c^2 &= 44.05 \\ \sqrt{c^2} &= \sqrt{44.05} \\ c &= \sqrt{44.05} \end{aligned}$$

Use your calculator to compute $\sqrt{44.05}$:

$$\begin{aligned} c &= \sqrt{44.05} \\ &\cong 6.637017402 \\ &\cong 6.64 \end{aligned}$$

THE PYTHAGOREAN THEOREM

Extended Example 1a

Suppose that $a = 5.1$, and $b = 3.8$ are lengths of the sides of a right triangle with hypotenuse c . Find the length of c . Round your answer to the nearest hundredth.

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THE PYTHAGOREAN THEOREM

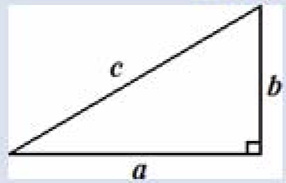
When you know the length of the hypotenuse, c, but don't know the length of one of a right triangle's legs, you can use one of the following equivalent forms of the Pythagorean theorem:

$$a^2 = c^2 - b^2 \quad \text{or} \quad b^2 = c^2 - a^2$$

EXAMPLE F

Consider the right triangle in the figure.
If $c = 8.7$ and $a = 4.2$, then $b = ?$

Round your answer to the nearest tenth.



This time we know the length of the hypotenuse, but not one of the legs, so we need to use an alternate form of the Pythagorean theorem:

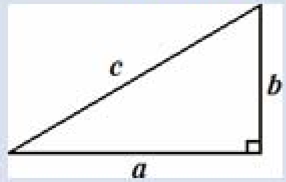
$$b^2 = c^2 - a^2$$
$$b^2 = (8.7)^2 - (4.2)^2$$
$$b^2 = 75.69 - 17.64$$
$$b^2 = 58.05$$
$$b = \sqrt{58.05}$$
$$b = \sqrt{58.05}$$
$$b \cong 7.619055060 \cong 7.6$$

THE PYTHAGOREAN THEOREM

Extended Example 2a

Consider the right triangle in the figure.
If $c = 5.3$ and $a = 3.8$, then $b = ?$

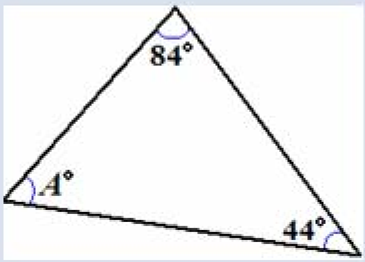
Round your answer to the nearest tenth.



An important geometric fact about triangles is that the sum of all the internal angles in any triangle always equals 180 degrees. This fact can be used to find the degree measure of a missing angle if the other two angles are known. To do so, you simply subtract the measures of the known angles from 180 degrees.

EXAMPLE G

Find the degree measure of the missing angle in the triangle shown below.



$$\begin{aligned} A &= 180^\circ - B - C \\ &= 180^\circ - 44^\circ - 84^\circ \\ &= 52^\circ \end{aligned}$$

The missing angle is 52 degrees.

THE PYTHAGOREAN THEOREM

Question: A triangle has two internal angles that are $74°$ and $21°$. Find the measure of the third angle, angle B.

Question: A triangle has two internal angles that are $28°$ and $82°$. Find the measure of the third angle, angle C.

Question: A triangle has two internal angles that are $39°$ and $93°$. Find the measure of the third angle, angle A.

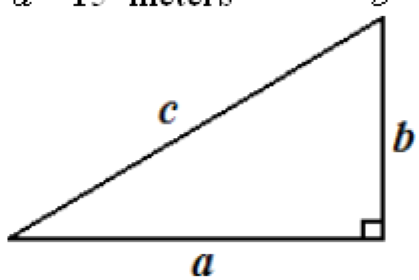
END OF LESSON

Does the following describe a right triangle? Why or why not?

The three sides are $\frac{1}{3}$ yard, 9 inches, and 15 inches.

Find the missing side c. If the answer isn't a whole number, then round your answer to the nearest hundredth.

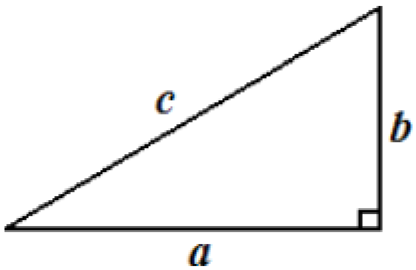
$a = 15$ meters $\qquad b = 20$ meters



Find the length of the right triangle that is not given (assume "c" is the hypotenuse): $a = 10$ cm $\qquad c = 26$ cm

Find the missing side b. If the answer isn't a whole number, then round your answer to the nearest hundredth.

$c = 13.8$ $a = 1.9$



How much distance is saved by cutting diagonally across a lot 60 feet by 80 feet instead of walking along its length and then its width? (Hint: draw a sketch.)

Introduction

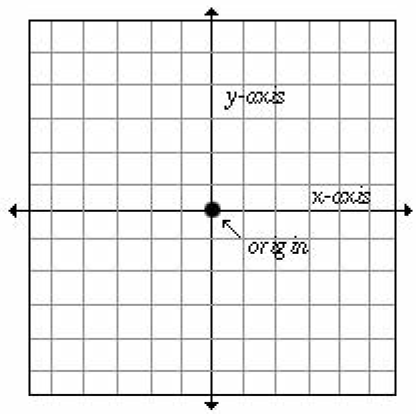
In this lesson, we will discuss rectangular coordinates known as **Cartesian coordinates.** We will also learn how to apply the distance formula to find the distance between two points.

Legend has it that the French mathematician Descartes developed the Cartesian Coordinate System while watching flies crawl over tiles on his bedroom ceiling.

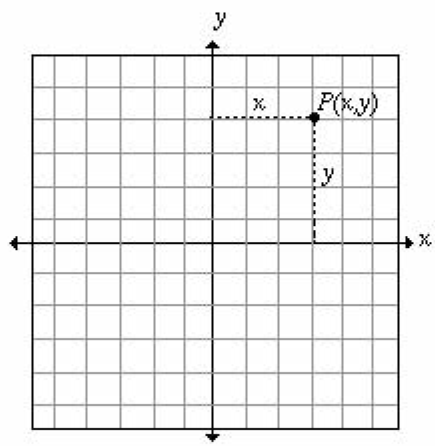
THE CARTESIAN COORDINATE PLANE AND DISTANCE FORMULA

Rectangular coordinates are used to determine the exact location of points in a **coordinate plane**. To specify points using rectangular coordinates, we use two perpendicular number lines.

The horizontal number line is called the x-**axis** of the plane, while the vertical number line is called the y-**axis**. The lines intersect at their zero points; this intersection is called the **origin**.



The rectangular coordinates of a point, P, in the plane is specified by using what is called an **ordered pair** of numbers: (x, y). The first number, x (the x-**coordinate** of point P), is the distance of P from the y-axis (the vertical one). The second number, y (the y-**coordinate** of P), measures the distance of P from the x-axis (the horizontal one).



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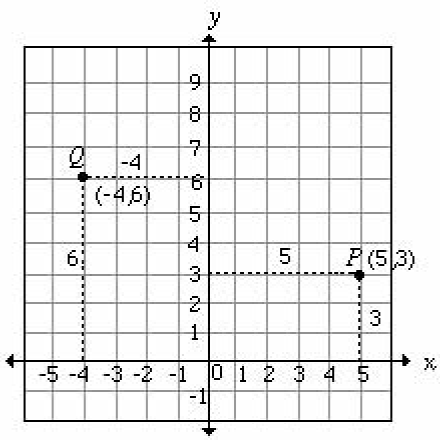
EXAMPLE A

Plot the following two points: $P = (5,3)$ and $Q = (-4,6)$.

This means to put dots at the two points specified by the given coordinates. We start at the origin. The first number tells us how far to go to the right (or left if it's negative), while the second number tells us how far to go up (or down if it's negative).

To plot point $P = (5,3)$, start at the origin and go 5 to the right (because the x-coordinate is positive), and then 3 up (because the y-coordinate is positive), and put a dot there. Similarly, to plot $Q = (-4,6)$, start at the origin and go 4 to the left (because the x-coordinate is negative), and then go 6 up (because the y-coordinate is positive), and put a dot there.



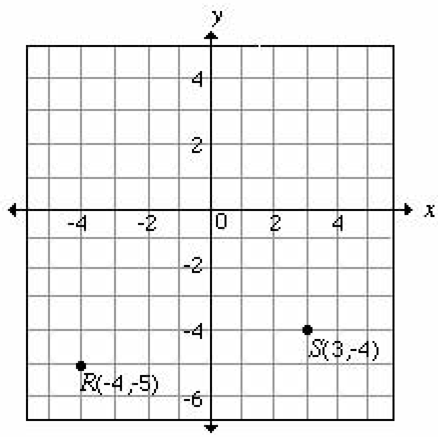
THE CARTESIAN COORDINATE PLANE AND DISTANCE FORMULA

EXAMPLE B

Plot the following two points: $R = (-4, -5)$ and $S = (3, -4)$.

This means to put dots at the two points specified by the given coordinates. The first number tells us how far to go to the right (or left if it's negative), while the second number tells us how far to go up (or down if it's negative), starting at the origin.

To plot point $R = (-4, -5)$, start at the origin and go 4 to the left (because the x-coordinate is negative), and then down 5 (because the y-coordinate is negative), and put a dot there. Similarly, to plot $S = (3, -4)$, start at the origin and go 3 to the right (because the x-coordinate is positive), and then go down 4 (because the y-coordinate is negative), and put a dot there.



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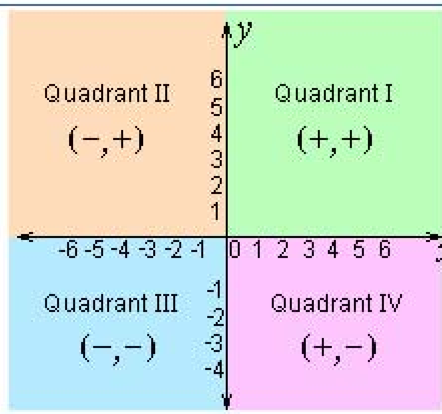
THE CARTESIAN COORDINATE PLANE AND DISTANCE FORMULA

As you saw in the examples above:

- A positive x-coordinate means that we go right, the positive direction on the x-axis.
- A negative x-coordinate means that we go left, the negative direction on the x-axis.
- A positive y-coordinate means that we go up, the positive direction on the y-axis.
- A negative y-coordinate means that we go down, the negative direction on the y-axis.

The axes divide the coordinate plane into four quadrants, which are labeled counterclockwise using Roman numerals. In the diagram below, the ordered pairs show the value of the x-coordinate and y-coordinate in each quadrant. For example, in Quadrant II, the x-coordinate is negative and the y-coordinate is positive.



THE CARTESIAN COORDINATE PLANE AND DISTANCE FORMULA

Question: Plot the following points and note the quadrant each point is in.

a) $(-4,3)$

b) $(5,4)$

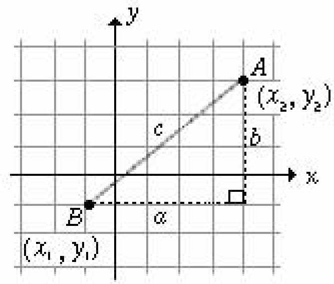
c) $(-5,-2)$

d) $(3,-5)$

THE CARTESIAN COORDINATE PLANE AND DISTANCE FORMULA

The Distance Formula

In the graph shown below, we can compute the distance between point A and point B in this way: connect the two points with a straight line segment. Make that line the hypotenuse of a right triangle, and then use the Pythagorean Theorem. This process is streamlined by using the **distance formula**, which we will now discover.



The distance between A and B corresponds to c in the Pythagorean Theorem, $c^2 = a^2 + b^2$.

The length of side a can be found by subtracting $x_2 - x_1$.

Similarly, the length of side b is found by $y_2 - y_1$.

Substituting this information into the Pythagorean Theorem gives:

$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

Taking the square root of both sides yields the **distance formula**.

$$\sqrt{c^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We replace c with d to indicate distance:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

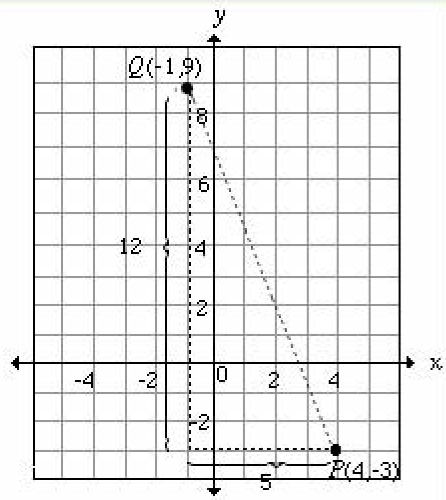
EXAMPLE C

Find the distance, d, between $P = (4, -3)$ and $Q = (-1, 9)$.

Step 1: Plot the points.

Step 2: Draw a right triangle.

Step 3: Find the length of the two legs.



Subtract the x-coordinates:
$x_2 - x_1 = -1 - 4 = -5$.
Subtract the y-coordinates:
$y_2 - y_1 = 9 - (-3) = 9 + 3 = 12$.

Step 4: Use the Pythagorean Theorem:
$d = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = \boxed{13}$

--Or--

Use the Distance Formula:

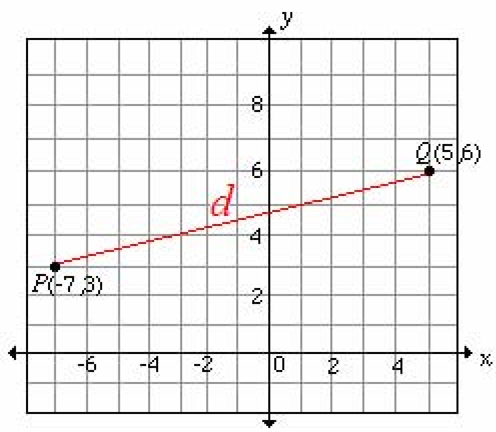
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1 - 4)^2 + (9 - (-3))^2} = \sqrt{5^2 + 12^2}$$
$$= \sqrt{25 + 144} = \sqrt{169} = \boxed{13}$$

EXAMPLE D

Find the distance, d , between $P=(-7,3)$ and $Q=(5,6)$. Round your answer to the nearest hundredth.

Using the distance formula, we get:

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \\ &= \sqrt{(5-(-7))^2+(6-3)^2} \\ &= \sqrt{(12)^2+(3)^2} \\ &= \sqrt{144+9} \\ &= \sqrt{153} \cong 12.3693169 \cong 12.37 \end{aligned}$$



THE CARTESIAN COORDINATE PLANE AND DISTANCE FORMULA

Question: What is the distance, d, between $A=(6,4)$ and $B=(-3,-2)$?
Use the distance formula.

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Extended Example 1a

Find the distance between the points $(9,7)$ and $(6,-4)$.
Approximate the distance to the nearest thousandth.

END OF LESSON

In which quadrant $(-25, 412)$ is point located?

Find the distance between these points: H $(-3, -12)$ **and** S $(3, 12)$

Find the distance between these points: L $(-7, 5)$ **and** O $(2, 2)$