THE DISTRIBUTIVE PROPERTY

Introduction

One of the most useful properties of algebra is the Distributive Property, which is used for evaluating the product of a number and a sum or difference. This algebraic property shows how multiplication is related to addition.

THE DISTRIBUTIVE PROPERTY

$$3(4+7) = ?$$

Calculate inside the parentheses first, then multiply:

$$3(4+7) = 3 \cdot 11$$
$$= 33$$

or

Distribute the product across the sum, then multiply before you add:

$$3 \cdot 4 + 3 \cdot 7 = 12 + 21$$
$$= 33$$

The answers are the same.

$$5(14-3) = ?$$

Calculate:

$$5(14-3) = 5 \cdot 11$$
$$= 55$$

or

Calculate:

$$5 \cdot 14 - 5 \cdot 3 = 70 - 15$$
$$= 55$$

Again, the answers are the same.

The **Distributive Property** is a formal way of stating and generalizing what the calculations above illustrate.

Distributive Property:

For all numbers a, b, and c, $a(b+c) = ab + ac$

and

$a(b-c) = ab - ac$.

THE DISTRIBUTIVE PROPERTY

The Distributive Property can be used when there are more than two terms inside the parentheses:

$$a(b+c+d)=ab+ac+ad$$

$$a(b+c+d+e)=ab+ac+ad+ae$$

EXAMPLE A

Use the Distributive Property to multiply: $5(x+7)$.

$$\begin{aligned} 5\cdot(x+7) &= 5\cdot x+5\cdot 7 \\ &= 5x+35 \end{aligned}$$

EXAMPLE B

Use the Distributive Property to multiply: $2(3x^2-5x)$.

$$\begin{aligned} 2\cdot(3x^2-5x) &= 2\cdot 3x^2-2\cdot 5x \\ &= 6x^2-10x \end{aligned}$$

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THE DISTRIBUTIVE PROPERTY

EXAMPLE C

Use the Distributive Property to multiply: $5(2xy+3x-4y)$.

Distribute 5.

$$5\cdot(2xy+3x-4y)=5\cdot 2xy+5\cdot 3x-5\cdot 4y$$

Then, simplify.

$$=10xy+15x-20y$$

EXAMPLE D

Multiply: $3x\left(4x^2-2xy+5y^2\right)$.

Distribute $3x$.

$$3x\cdot\left(4x^2-2xy+5y^2\right)=3x\cdot 4x^2-3x\cdot 2xy+3x\cdot 5y^2$$

Then, simplify.

$$=12x^3-6x^2y+15xy^2$$

EXAMPLE E

Multiply: $2a^2b(5b-3a+2ab)$.

Distribute $2a^2b$.

$$2a^2b\cdot(5b-3a+2ab)=2a^2b\cdot 5b-2a^2b\cdot 3a+2a^2b\cdot 2ab$$

Then, simplify.

$$=10a^2b^2-6a^3b+4a^3b^2$$

THE DISTRIBUTIVE PROPERTY

Question: Multiply: $5a\left(3a^2+2ab-11b^2\right)$.

Question: Multiply: $-2v\left(-2u^2+3uv-4v^2\right)$.

THE DISTRIBUTIVE PROPERTY

EXAMPLE F

Simplify: $8x(x+3y)-3x(3x+2y)$.

Distribute $8x$ and $-3x$.

$$= 8x \cdot (x+3y) - 3x \cdot (3x+2y)$$
$$= 8x \cdot x + 8x \cdot 3y - 3x \cdot 3x - 3x \cdot 2y$$
$$= 8x^2 + 24xy - 9x^2 - 6xy$$

Then, combine like terms.

$$= 8x^2 + 24xy - 9x^2 - 6xy$$
$$= -x^2 + 18xy$$

EXAMPLE G

Simplify: $6a(2a+3b)-5b(7a-2b)$.

Distribute $6a$ and $-5b$.

$$6a \cdot (2a+3b) - 5b \cdot (7a-2b) = 6a \cdot 2a + 6a \cdot 3b - 5b \cdot 7a - 5b \cdot (-2b)$$
$$= 12a^2 + 18ab - 35ab + 10b^2$$

Then, combine like terms:

$$= 12a^2 + 18ab - 35ab + 10b^2$$
$$= 12a^2 - 17ab + 10b^2$$
$$= 12a^2 - 17ab + 10b^2$$

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THE DISTRIBUTIVE PROPERTY

Extended Example 1a

Simplify: $7u(3u-4v)-9u(2u-v)$.

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THE DISTRIBUTIVE PROPERTY

Notice that using the Distributive Property in reverse gives you

$$ab + ac = a(b + c)$$

or

$$ba + ca = (b + c)a \text{ .}$$

The process above is called **factoring**. It shows why we can combine like terms:

$$2c + 3c = (2 + 3)c = 5c$$

EXAMPLE H

Use the Distributive Property to simplify: $3x^2 + 5x + 11 - 9x^2 - 8x$.

Group like terms.

$$3x^2 + 5x + 11 - 9x^2 - 8x = 3x^2 - 9x^2 + 5x - 8x + 11$$

Use the Distributive Property. $= 3x^2 - 9x^2 + 5x - 8x + 11$

$$= (3 - 9)x^2 + (5 - 8)x + 11$$

Simplify. $= -6x^2 - 3x + 11$

Note:

- Although Example H shows how the Distributive Property is used to combine like terms, you don't need to show this step in your work. It's faster to go directly to the solution, as shown below. But now you know that the Distributive Property is at work behind the scenes when you combine like terms.

$$3x^2 + 5x + 11 - 9x^2 - 8x = -6x^2 - 3x + 11$$

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THE DISTRIBUTIVE PROPERTY

The Distributive Property can be used to factor expressions. To **factor an expression** is to write the expression as the product of other expressions.

For example, to factor $6xy - 8x^2$, first find the **greatest common factor** of the two terms. In this example, the greatest common factor is $2x$.

Next, rewrite the expression using this common factor:

$$6xy - 8x^2 = 2x \cdot 3y - 2x \cdot 4x$$

Then, use the Distributive Property:

$$\begin{aligned} &= 2x \cdot 3y - 2x \cdot 4x \\ &= 2x \cdot (3y - 4x) \\ &= 2x(3y - 4x) \end{aligned}$$

EXAMPLE I

Factor out the greatest common factor: $8y - 12x$.

The greatest common factor is 4.

$$\begin{aligned} 8y - 12x &= 4 \cdot 2y - 4 \cdot 3x \\ &= 4 \cdot (2y - 3x) \\ &= 4(2y - 3x) \end{aligned}$$

THE DISTRIBUTIVE PROPERTY

EXAMPLE J

Factor completely: $9t^2 - 6t$.

The greatest common factor is $3t$.

$$\begin{aligned} 9t^2 - 6t &= 3t \cdot 3t - 3t \cdot 2 \\ &= 3t \cdot (3t - 2) \\ &= 3t(3t - 2) \end{aligned}$$

EXAMPLE K

Factor completely: $2m^2n^3 - 6mn^2 + 4m^2n^2$.

The greatest common factor is $2mn^2$.

$$\begin{aligned} 2m^2n^3 - 6mn^2 + 4m^2n^2 &= 2mn^2 \cdot mn - 2mn^2 \cdot 3 + 2mn^2 \cdot 2m \\ &= 2mn^2 \cdot (mn - 3 + 2m) \\ &= 2mn^2(mn - 3 + 2m) \end{aligned}$$

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Question: Factor completely: $12a^2b - 15a$.

Question: Factor completely: $20a^4b^2 - 30a^6b^2 + 50a^4b^6$.

END OF LESSON

Evaluate both $x(y+z)$ and $xy+xz$ for: $x=\frac{5}{8}$, $y=-\frac{4}{15}$, $z=\frac{17}{10}$

Use the distributive law to simplify: $16ab+13ab-4ab$

Use the distributive law to simplify: $11r(4c-2d)-7s(c-4d)$

Use the distributive law to factor out what is common: $33t - 22t^2 + 44t^4$

Use the distributive law to factor out what is common: $12S^3 - 8S^4 + 6S^2$

Use the distributive law to factor out what is common:
$18x^3 - 12x^2 + 8x^4 - 4x$

MULTIPLYING BINOMIALS

Introduction

This lesson explains how to multiply two binomials using the FOIL method. Remember, a binomial is a polynomial with two unlike, nonzero terms.

Here are some examples of binomials:

$$3+2x \qquad 3x^2y-5xy \qquad \pi r^2+2\pi rh$$

MULTIPLYING BINOMIALS

To find the product of two binomials, $(a+b)(x+y)$, you can use the Distributive Property three times.

First, we use the Distributive Property in the form $A\cdot(B+C) = A\cdot B + A\cdot C$:

$$A\cdot(B+C) = A\cdot B + A\cdot C$$
$$(a+b)\cdot(x+y) = (a+b)\cdot x + (a+b)\cdot y$$

Now we use the Distributive Property in the form $(B+C)\cdot A = B\cdot A + C\cdot A$ twice (once with $(a+b)\cdot x$ and once with $(a+b)\cdot y$):

$$\begin{aligned}(B+C)\cdot A &= B\cdot A + C\cdot A\\ &= (a+b)\cdot x + (a+b)\cdot y\\ &= a\cdot x + b\cdot x + a\cdot y + b\cdot y\\ &= ax + bx + ay + by\end{aligned}$$

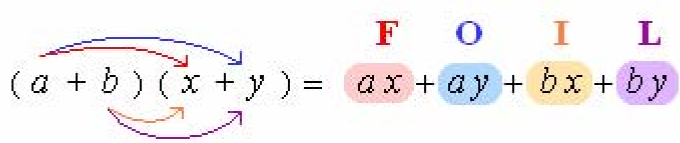
$$(a+b)(x+y) = (a+b)x + (a+b)y = ax + bx + ay + by$$

MULTIPLYING BINOMIALS

You can avoid all of the steps shown on the previous screen by using a method known as **FOIL**. **FOIL** stands for **F**irst **O**uter **I**nner **L**ast. The example below shows how this method works.

$(a+b)(x+y)=?$							
Product of the First terms $(a+b)(x+y)$		Product of the Outer terms $(a+b)(x+y)$		Product of the Inner terms $(a+b)(x+y)$		Product of the Last terms $(a+b)(x+y)$	
ax	$+$	ay	$+$	bx	$+$	by	
$(a+b)(x+y)=ax+ay+bx+by$							

Using the FOIL method, we were able to obtain the correct result in just one step; the order of the terms does not matter.



MULTIPLYING BINOMIALS

EXAMPLE A

Use the FOIL method to find the product: $(a+x)(y+2)$.

First		Outer		Inner		Last
$a \cdot y$	+	$a \cdot 2$	+	$x \cdot y$	+	$x \cdot 2$
$(a+x)(y+2) = ay + 2a + xy + 2x$						

EXAMPLE B

Use the FOIL method to find the product: $(x+3)(x-7)$.

First		Outer		Inner		Last
$x \cdot x$	+	$x \cdot (-7)$	+	$3 \cdot x$	+	$3 \cdot (-7)$
$(x+3)(x-7) = x^2 - 7x + 3x - 21$						

Then, simplify by combining like terms:

$$= x^2 - 7x + 3x - 21$$

$$= x^2 - 4x - 21$$

$$= x^2 - 4x - 21$$

Note:

- After you multiply binomials, be sure to combine any like terms.

MULTIPLYING BINOMIALS

EXAMPLE C

Use the FOIL method to multiply: $(b+2)(b+5)$.

First		**O**uter		**I**nner		**L**ast
$b \cdot b$	$+$	$b \cdot 5$	$+$	$2 \cdot b$	$+$	$2 \cdot 5$
$(b+2)(b+5) = b^2 + 5b + 2b + 10$						

Simplify by combining like terms:

$$= b^2 + 5b + 2b + 10$$
$$= b^2 + 7b + 10$$
$$= b^2 + 7b + 10$$

Extended Example 1a

Find the product: $(3a-2b)(4a+b)$.

MULTIPLYING BINOMIALS

EXAMPLE D

Multiply: $(2r-5)(x+4)$.

Use the FOIL method.

$$(2r-5)(x+4) = \overbrace{2r \cdot x}^{F} + \overbrace{2r \cdot 4}^{O} + \overbrace{(-5) \cdot x}^{I} + \overbrace{(-5) \cdot 4}^{L}$$
$$= 2rx + 8r - 5x - 20$$

There are no like terms to combine.

Question: Multiply: $(x+2)(y-11z)$.

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MULTIPLYING BINOMIALS

Question: Multiply: $(3r+5)(2s-3)$

Question: Multiply: $(u-5v)(3w-u)$.

MULTIPLYING BINOMIALS

EXAMPLE E

Multiply: $\left(3x^2+y\right)\left(2x-3y^2\right)$.

$$\left(3x^2+y\right)\left(2x-3y^2\right) = \overbrace{3x^2\cdot 2x}^{\text{F}} + \overbrace{3x^2\cdot\left(-3y^2\right)}^{\text{O}} + \overbrace{y\cdot 2x}^{\text{I}} + \overbrace{y\cdot\left(-3y^2\right)}^{\text{L}}$$

$$= 6x^3-9x^2y^2+2xy-3y^3$$

EXAMPLE F

Multiply: $\left(4ax-3y^2\right)\left(2az-9yz\right)$.

$$\left(4ax-3y^2\right)\left(2az-9yz\right) = \overbrace{4ax\left(2az\right)}^{\text{F}} + \overbrace{4ax\left(-9yz\right)}^{\text{O}} + \overbrace{\left(-3y^2\right)2az}^{\text{I}} + \overbrace{\left(-3y^2\right)\left(-9yz\right)}^{\text{L}}$$

$$= 8a^2xz-36axyz-6ay^2z+27y^3z$$

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MULTIPLYING BINOMIALS

Extended Example 2a

Multiply: $\left(u^2 - 2w\right)\left(5w^2 + u\right)$.

MULTIPLYING BINOMIALS

EXAMPLE G

Find the product: $(a+a)(c+d)$.

Although the FOIL method can be used to expand this product, this time it's simpler to start by adding like terms in the first set of parentheses:

$$(a+a)(c+d) = (2a)(c+d)$$

Then use the Distributive Property:

$$\begin{aligned} &= 2a \cdot (c+d) \\ &= 2a \cdot c + 2a \cdot d \\ &= 2ac + 2ad \end{aligned}$$

END OF LESSON

Use FOIL to multiply: $(2a-3)(a+b)$

Use FOIL to multiply: $(5-2t)(s-t)$ Answer: $5s-5t-2st+2t^2$

Use FOIL to multiply: $(2a-3b)(a+2b)$

Use FOIL to multiply: $\left(4x^2 - 2y\right)\left(x + y^2\right)$

Use FOIL to multiply: $\left(ab - 5\right)\left(3ab + 2\right)$

Use FOIL to multiply: $\left(m - n\right)\left(mz + nz\right)$

SQUARING BINOMIALS

Introduction

In this lesson, you will learn how to multiply a binomial by itself (how to square a binomial). We will begin by using the FOIL method, which will lead us to a useful formula for squaring binomials. You will also learn a formula for multiplying two binomials of the form $(a+b)(a-b)$.

SQUARING BINOMIALS

EXAMPLE A

Use the FOIL method to find $(t+3)^2$.

$$(t+3)^2 = (t+3)(t+3)$$

F O I L

$$= t^2 + 3t + 3t + 9$$

$$= t^2 + 6t + 9$$

When you square a binomial, the two middle terms are ALWAYS the same. In this example, the two middle terms are $3t$.

EXAMPLE B

Use the FOIL method to find $(x-5)^2$.

$$(x-5)^2 = (x-5)(x-5)$$

F O I L

$$= x^2 - 5x - 5x + 25$$

$$= x^2 - 10x + 25$$

Notice that the two middle terms are the same ($-5x$).

SQUARING BINOMIALS

The following generalizes what Examples A and B illustrate.

The square of the first term	Twice the product of the two terms	The square of the last term

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$

$$(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$$

When you square a binomial, the two middle terms will ALWAYS be the same. To find the middle term you can double the product of a and b by multiplying by 2. This is why there's a 2 in the middle term of each of the formulas below.

Squaring a Binomial

$(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$

A Word to the Wise

A common mistake students make is to square only the first and last terms (setting $(a+b)^2$ equal to $a^2 + b^2$), but that is NOT correct.

Remember: $(a+b)^2 \neq a^2 + b^2$ and $(a-b)^2 \neq a^2 - b^2$

SQUARING BINOMIALS

EXAMPLE C

Use the rule for squaring a binomial to find $(n+7)^2$.

To apply the binomial squaring formula, $(a+b)^2 = a^2 + 2ab + b^2$, we must first identify a and b. The first term of the binomial is a and the second term is b. So,

$$\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2 \\ (n+7)^2 &= n^2 + 2 \cdot n \cdot 7 + 7^2 \\ &= n^2 + 14n + 49\end{aligned}$$

EXAMPLE D

Use the rule for squaring a binomial to find $(3p+10q)^2$.

$$\begin{aligned}(a+b)^2 &= a^2 + 2ab + b^2 \\ (3p+10q)^2 &= (3p)^2 + 2(3p)(10q) + (10q)^2 \\ &= 9p^2 + 60pq + 100q^2\end{aligned}$$

SQUARING BINOMIALS

Extended Example 1a

Use the rule for squaring a binomial to find $(2x+7y)^2$.

SQUARING BINOMIALS

EXAMPLE E

Find $(x-11)^2$.

Identify a and b and use the rule for squaring a binomial of the form $(a-b)^2$.

$$(a-b)^2 = a^2 - 2\,a\,b + b^2$$

$$(x-11)^2 = x^2 - 2 \cdot x \cdot 11 + 11^2$$

$$= x^2 - 22x + 121$$

EXAMPLE F

Find $(2r-3s)^2$.

$$(a-b)^2 = a^2 - 2\,a\,b + b^2$$

$$(2r-3s)^2 = (2r)^2 - 2 \cdot 2r \cdot 3s + (3s)^2$$

$$= 4r^2 - 12rs + 9s^2$$

SQUARING BINOMIALS

Extended Example 2a

Find $(7x-8)^2$.

SQUARING BINOMIALS

EXAMPLE G

Find $\left(x^2+3\right)^2$.

$$\left(x^2+3\right)^2 = \left(x^2\right)^2 + 2\cdot x^2\cdot 3 + 3^2$$
$$= x^4 + 6x^2 + 9$$

EXAMPLE H

Find $\left(9st-8u^3\right)^2$.

$$\left(9st-8u^3\right)^2 = \left(9st\right)^2 - 2\left(9st\right)\left(8u^3\right) + \left(8u^3\right)^2$$
$$= 81s^2t^2 - 144stu^3 + 64u^6$$

Note:

- The square of a binomial, expanded and simplified, is usually a trinomial. Recall that a trinomial is a polynomial with three unlike terms.

SQUARING BINOMIALS

Extended Example 3a

Find $\left(5x^3 - 6y^2\right)^2$.

SQUARING BINOMIALS

EXAMPLE I

Use the rule for squaring a binomial to find $(2a+3a)^2$.

$$(2a+3a)^2 = (2a)^2 + 2(2a)(3a) + (3a)^2$$
$$= 4a^2 + 12a^2 + 9a^2$$
$$= 25a^2$$

Note:

- In Example I, the result is a monomial. The terms inside the parentheses in the given problem were like terms. It's much simpler to combine like terms as a first step:

$$(2a+3a)^2 = (5a)^2 = 25a^2$$

A Word to the Wise

Remember to combine like terms whenever possible!

SQUARING BINOMIALS

Another important and extremely useful formula appears when we multiply two binomials of the form $(a+b)(a-b)$, the sum of two terms multiplied by their difference. Using the FOIL method, we get:

$$\begin{aligned} & \quad\ \text{F} \quad \text{O} \quad \text{I} \quad \text{L} \\ (a+b)(a-b) &= a^2 - ab + ba - b^2 \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - \cancel{ab} + \cancel{ab} - b^2 \\ &= a^2 - b^2 \end{aligned}$$

As the process above shows, $(a+b)(a-b)$ equals the square of the first term minus the square of the second term: $(a+b)(a-b) = a^2 - b^2$. Due to the commutative property of multiplication, $(a-b)(a+b) = a^2 - b^2$ is also true. This rule is sometimes referred to as the "difference of squares" rule.

Difference of Squares

$$(a+b)(a-b) = a^2 - b^2 \quad \text{or} \quad (a-b)(a+b) = a^2 - b^2$$

SQUARING BINOMIALS

EXAMPLE J

Multiply: $(x+3)(x-3)$.

To apply the difference of squares formula, $(a+b)(a-b)=a^2-b^2$, we must first identify a and b. In this case, a is x and b is 3.

$$(a+b)(a-b)=a^2-b^2$$
$$(x+3)(x-3)=x^2-3^2$$
$$=x^2-9$$

EXAMPLE K

Multiply: $(2x+y)(2x-y)$.

$$(a+b)(a-b) = a^2 - b^2$$
$$(2x+y)(2x-y)=(2x)^2-y^2$$
$$=4x^2-y^2$$

SQUARING BINOMIALS

Extended Example 4a

Multiply: $(x+10)(x-10)$.

END OF LESSON

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$(5u+2v)^2 = ?$

Find the square.

$(r^2-3)^2$

$(mn-n^2)^2 = ?$

$(x^3+y^2)^2 = ?$

$$\left(3\frac{1}{2}\right)^2 = ?$$

Find the product.

$$(3x+2)(3x-2)$$

$$\left(4xy^2+5y\right)\left(4xy^2-5y\right) = ?$$

ADDING AND SUBTRACTING POLYNOMIALS

Introduction

Recall that terms of a polynomial are called **like terms** if they have identical variable parts and identical exponents. Adding or subtracting polynomials involves combining these like terms.

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE A

Add the two polynomials: $(8x^2 - 3x + 4) + (5x^2 + 8x - 9)$.

Begin by removing the parentheses:

$$(8x^2 - 3x + 4) + (5x^2 + 8x - 9) = 8x^2 - 3x + 4 + 5x^2 + 8x - 9$$

Then combine like terms:

$$= 8x^2 - 3x + 4 + 5x^2 + 8x - 9$$
$$= 13x^2 + 5x - 5$$
$$= 13x^2 + 5x - 5$$

Note that you can also add two polynomials by vertically aligning like terms (pay attention to the signs!).

$$\begin{array}{r} 8x^2 - 3x + 4 \\ \underline{5x^2 + 8x - 9} \\ 13x^2 + 5x - 5 \end{array}$$

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE B

Add the polynomials $3x^3 + 2x^2 - 5x - 17$ and $x^3 - 5x^2 - 12x + 6$ vertically.

Vertically align each power of x.

$$\begin{array}{r} 3x^3 + 2x^2 - 5x - 17 \\ \underline{x^3 - 5x^2 - 12x + 6} \end{array}$$

Add the corresponding coefficients. Recall that to add numbers with opposite signs is to subtract and use the sign of the larger number.

$$\begin{array}{r} 3x^3 + 2x^2 - 5x - 17 \\ \underline{x^3 - 5x^2 - 12x + 6} \\ 4x^3 - 3x^2 - 17x - 11 \end{array}$$

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ADDING AND SUBTRACTING POLYNOMIALS

Let's look at what happens when a negative sign precedes parentheses. Recall that $-A = (-1) \cdot A$. So,

$$\begin{aligned} -(15x - 8) &= (-1) \cdot (15x - 8) \\ &= (-1) \cdot 15x - (-1) \cdot 8 \\ &= -15x + 8 \end{aligned}$$

Rule for Negative Sign Preceding Parentheses
When a set of parentheses is preceded by a negative sign, distribute the negative sign to each term inside the parentheses by multiplying by -1. Positive terms will become negative and negative terms will become positive.

EXAMPLE C

Subtract the binomials: $(42y + 16) - (17y - 8)$.

First, change the signs of the terms in the second set of parentheses by multiplying each by -1. Remove the parentheses.

$$(42y + 16) - (17y - 8) = 42y + 16 - 17y + 8$$

Remember: A negative number multiplied by a negative number equals a positive number. A negative number multiplied by a positive number equals a negative number.

Finally, combine like terms:

$$\begin{aligned} &= 42y + 16 - 17y + 8 \\ &= 25y + 24 \end{aligned}$$

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE D

Subtract the polynomial $5x^2+8x-9$ from the polynomial $8x^2-3x+4$.

Start by changing the sign of each term in the second pair of parentheses:

$$\left(8x^2-3x+4\right)-\left(5x^2+8x-9\right)=8x^2-3x+4-5x^2-8x+9$$

Notice how $5x^2+8x-9$ becomes $-5x^2-8x+9$ as the -1 is distributed to each term.

Finally, combine like terms:

$$=8x^2-3x+4-5x^2-8x+9$$

$$=3x^2-11x+13$$

We can also solve Example D vertically.

First, change the sign of each term of the subtracted polynomial. Then add:

$$\begin{array}{r} 8x^2-3x+4 \\ \underline{-5x^2-8x+9} \\ 3x^2-11x+13 \end{array}$$

ADDING AND SUBTRACTING POLYNOMIALS

Extended Example 1a

Subtract: $(2ab - 5a + 3b) - (6ab + 2a - 7b)$.

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE E

Simplify: $\left(4x^2-7\right)+\left(3x-6\right)-\left(x^2-x\right)$

$$\begin{aligned}\left(4x^2-7\right)+\left(3x-6\right)-\left(x^2-x\right) &= 4x^2-7+3x-6-x^2+x\\ &= 4x^2-7+3x-6-x^2+x\\ &= 3x^2+4x-13\end{aligned}$$

EXAMPLE F

Simplify $\left(4x^2-7\right)+\left(3x-6\right)-\left(x^2-x\right)$ by adding vertically.

This time, only align like terms and then add them. You may need to leave a few spaces open:

$$\begin{array}{l}4x^2 \qquad -7\\ \qquad 3x-6\\ \underline{-x^2+x \qquad\quad}\\ 3x^2+4x-13\end{array}$$

Notice how multiplying by the -1 in front of $\left(x^2-x\right)$ causes the signs to change so we end up with $-x^2+x$ in the vertical addition above.

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE G

Simplify: $(4ab + a + 3) + (3ab - 2a)$.

$$(4ab + a + 3) + (3ab - 2a) = 4ab + a + 3 + 3ab - 2a$$
$$= 4ab + a + 3 + 3ab - 2a$$
$$= 7ab - a + 3$$

EXAMPLE H

Simplify: $3(r^2 - 2r + 2) - 2(2r^2 - 4r + 3)$.

Distribute the 3 and the -2:

$$3(r^2 - 2r + 2) - 2(2r^2 - 4r + 3) = 3r^2 - 6r + 6 - 4r^2 + 8r - 6$$

Combine like terms.

$$= 3r^2 - 6r + 6 - 4r^2 + 8r - 6$$
$$= -r^2 + 2r$$

EXAMPLE I

Simplify: $x(y^2 - 3y + 4) - y(x^2 + 4x - 2)$.

$$x(y^2 - 3y + 4) - y(x^2 + 4x - 2) = xy^2 - 3xy + 4x - x^2y - 4xy + 2y$$
$$= xy^2 - 3xy + 4x - x^2y - 4xy + 2y$$
$$= xy^2 - 7xy - x^2y + 4x + 2y$$

ADDING AND SUBTRACTING POLYNOMIALS

Extended Example 2a

Simplify: $x\left(2x^2 - 3x + 7\right) - 10\left(x^3 - 2x^2 - 3x + 7\right)$.

EXAMPLE J

Simplify: $20xy-(5x+2y)^2$.

First, square the binomial $(5x+2y)^2$ on the right side of the expression using the formula learned earlier, $(a+b)^2 = a^2+2ab+b^2$:

$$(5x+2y)^2 = (5x)^2+2(5x)(2y)+(2y)^2 = 25x^2+20xy+4y^2$$

So,

$$20xy-(5x+2y)^2 = 20xy-\left(25x^2+20xy+4y^2\right).$$

Notice that the binomial must be squared <u>before</u> distributing the -1. Negative one does not get squared, only the binomial in the parentheses.

$$\begin{aligned}
&= 20xy-\left(25x^2+20xy+4y^2\right)\\
&= 20xy-25x^2-20xy-4y^2\\
&= 20xy-25x^2-20xy-4y^2\\
&= 0-25x^2-4y^2\\
&= -25x^2-4y^2
\end{aligned}$$

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE K

Simplify: $(2x-y)^2-(x-2y)^2$.

First, multiply each binomial using the formula $(a-b)^2=a^2-2ab+b^2$:

$$(2x-y)^2=(2x)^2-2(2x)y+y^2=4x^2-4xy+y^2$$

and

$$(x-2y)^2=x^2-2x(2y)+(2y)^2=x^2-4xy+4y^2$$

So,

$$\begin{aligned}(2x-y)^2-(x-2y)^2 &= \left(4x^2-4xy+y^2\right)-\left(x^2-4xy+4y^2\right)\\ &= 4x^2-4xy+y^2-x^2+4xy-4y^2\\ &= 4x^2-4xy+y^2-x^2+4xy-4y^2\\ &= 3x^2-0-3y^2\\ &= 3x^2-3y^2\end{aligned}$$

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE L

Simplify: $\left(5m^2+n-3\right)-2\left[3\left(m^2-4\right)-5\left(m^2+n-1\right)\right]$.

First, distribute the 3 and -5 inside the brackets:

$$=\left(5m^2+n-3\right)-2\left[3m^2-12-5m^2-5n+5\right]$$

Next, combine all like terms within the brackets:

$$=\left(5m^2+n-3\right)-2\left[3m^2-12-5m^2-5n+5\right]$$
$$=\left(5m^2+n-3\right)-2\left[-2m^2-5n-7\right]$$

Then, distribute the -2:

$$=5m^2+n-3+4m^2+10n+14$$

Finally, combine like terms:

$$=5m^2+n-3+4m^2+10n+14$$
$$=9m^2+11n+11$$

END OF LESSON

$$\left(5E - 2mc^2\right) - \left(4E - mc^2\right) = ?$$

$$\left(rs^2 - 2r^2s + 3s^3\right) - \left(rs^2 - 4r^2s\right) = ?$$

$$5x - \left[4 - 3\left(2x - 1\right) - \left(3x - 2\right)\right] = ?$$

$$a(a+b)-b(a-b)-\left[(a+b)(a-b)\right]=?$$

$$u\left(2u^2-3uv+v^2\right)-v\left(3u^2-4uv+2v^2\right)=?$$

$$(2x-y)^2-(x-3y)^2=?$$

DIVIDING POLYNOMIALS

Introduction

In this lesson, we'll examine a way to divide polynomials using long division that's quite similar to the ordinary long division process of arithmetic.

DIVIDING POLYNOMIALS

It will help to recall the first form of long division you ever learned, where you stop once you get a remainder. For example, we'll calculate $\frac{9}{7}$:

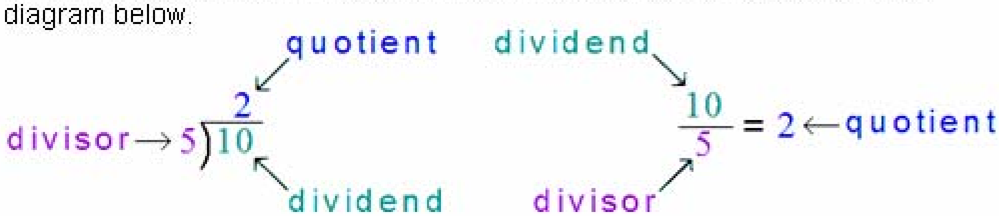
$$\begin{array}{r} 1 \\ 7\overline{)9} \\ -7 \\ \hline 2 \end{array}$$

"**Seven** goes into **nine** **one** time with a remainder of **two**". In other words:

$$\frac{9}{7} = 1 + \frac{2}{7} .$$

> **A Word to the Wise**
>
> In arithmetic, if you leave out the ' + ' sign in the expression above, the answer will be in mixed number notation: $\frac{9}{7} = 1\frac{2}{7}$. In algebra we always write the ' + ' sign.

Some terms you may encounter that are related to division are **divisor**, **dividend**, and **quotient**. The meanings of these terms are illustrated in the diagram below.



DIVIDING POLYNOMIALS

Now we'll see how to perform the long division process with polynomials. This process can be performed whenever the degree of the polynomial in the numerator (the dividend) is not less than the degree of the polynomial in the denominator (the divisor). Study the example below.

EXAMPLE A

Divide, using long division: $\dfrac{5x+3}{x-2}$.

Set up the divisor and dividend to begin the division process: $x-2\overline{)5x+3}$.

Start by asking, how many times does x go into $5x$? The answer is, 5 times:

$$\begin{array}{r} 5 \\ x-2\overline{)5x+3} \end{array}$$

As with ordinary long division, we next multiply that 5 times the divisor, $x-2$, and write the resulting product under the dividend:

$$\begin{array}{r} 5 \\ x-2\overline{)5x+3} \\ 5x-10 \end{array}$$

Also as with ordinary long division, we subtract next. Put parentheses around the expression being subtracted.

$$\begin{array}{r} 5 \\ x-2\overline{)5x+3} \\ -(5x-10) \\ \hline 13 \end{array}$$

Subtracting -10 is the same as adding 10 , and $5x$ minus $5x$ is zero. We're done at this point since x doesn't go into 13 . The remaining 13 is called the **remainder**. So, we now know that: $\dfrac{5x+3}{x-2} = 5 + \dfrac{13}{x-2}$.

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Extended Example 1a

Use long division to divide $\dfrac{7x+4}{x+2}$.

DIVIDING POLYNOMIALS

EXAMPLE B

Divide, using long division: $\frac{12x-5}{6x-11}$.

Set up the divisor and dividend to begin the long division process:

$$6x-11\overline{)12x-5}$$

Start by asking, how many times does $6x$ go into $12x$? The answer is 2 times:

$$\begin{array}{r} 2 \\ 6x-11\overline{)12x-5} \end{array}$$

As with ordinary long division, we next multiply that 2 times the divisor, $6x-11$, writing the resulting product under the dividend:

$$\begin{array}{r} 2 \\ 6x-11\overline{)12x-5} \\ 12x-22 \end{array}$$

Also as with ordinary long division, we next subtract. Put parentheses around the expression (under the dividend) being subtracted.

$$\begin{array}{r} 2 \\ 6x-11\overline{)12x-5} \\ \underline{-(12x-22)} \\ 17 \end{array}$$

Notice that subtracting a negative 22 is the same as adding 22. The $12x$ minus $12x$ leaves zero; they cancel each other out. We are done at this point, since $6x$ doesn't go into 17, which is thus the remainder. The result of the long division process is that we now know that

$$\frac{12x-5}{6x-11}=2+\frac{17}{6x-11}.$$

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DIVIDING POLYNOMIALS

Extended Example 2a

Use long division to divide $\frac{4x-1}{2x+5}$.

DIVIDING POLYNOMIALS

EXAMPLE C

Divide, using long division: $\dfrac{3x^2+2x-1}{x-3}$.

Prepare for long division: $x-3\overline{\big)3x^2+2x-1}$.

How many times does x go into $3x^2$? The answer is $3x$:

$$\begin{array}{r}3x\\ x-3\overline{\big)3x^2+2x-1}\end{array}$$

Multiply that $3x$ times $x-3$:

$$\begin{array}{r}3x\\ x-3\overline{\big)3x^2+2x-1}\\ \underline{3x^2-9x}\end{array}$$

Then subtract. Put parentheses around the expression being subtracted.

$$\begin{array}{r}3x\\ x-3\overline{\big)3x^2+2x-1}\\ \underline{-\left(3x^2-9x\right)}\\ 11x\end{array}$$

Notice that subtracting $-9x$ is the same as adding $9x$.

Bring down the next term:

$$\begin{array}{r}3x+11\\ x-3\overline{\big)3x^2+2x-1}\\ \underline{-\left(3x^2-9x\right)}\downarrow\\ 11x-1\end{array}$$

continued...

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DIVIDING POLYNOMIALS

Example C, continued...

How many times does x go into $11x$? The answer is 11 times:

$$\begin{array}{r l} & 3x + 11 \\ x-3 & \overline{)\,3x^2 + 2x - 1} \\ & -\left(3x^2 - 9x\right) \\ \hline & \qquad 11x - 1 \end{array}$$

Multiply that 11 times $x - 3$:

$$\begin{array}{r l} & 3x + 11 \\ x-3 & \overline{)\,3x^2 + 2x - 1} \\ & -\left(3x^2 - 9x\right) \\ \hline & \qquad 11x - 1 \\ & \qquad 11x - 33 \end{array}$$

Then subtract, putting parentheses around the expression being subtracted.

$$\begin{array}{r l} & 3x + 11 \\ x-3 & \overline{)\,3x^2 + 2x - 1} \\ & -\left(3x^2 - 9x\right) \\ \hline & \qquad 11x - 1 \\ & \quad -\left(11x - 33\right) \\ \hline & \qquad\qquad 32 \end{array}$$

Notice that subtracting -33 is the same as adding 33. We are done at this point, since x doesn't go into 32, the remainder.

$$\frac{3x^2 + 2x - 1}{x - 3} = 3x + 11 + \frac{32}{x - 3}$$

DIVIDING POLYNOMIALS

Extended Example 3a

Use long division to divide $\dfrac{4x^2 - 5x - 3}{x - 2}$.

DIVIDING POLYNOMIALS

EXAMPLE D

Divide, using long division: $\dfrac{25x^2 - 20x - 34}{5x - 8}$

Prepare for long division: $5x - 8 \overline{\smash{)}\,25x^2 - 20x - 34}$.

How many times does $5x$ go into $25x^2$? The answer is $5x$:

$$\begin{array}{r l} & 5x \\ 5x - 8 & \overline{)\,25x^2 - 20x - 34} \end{array}$$

Multiply that $5x$ times $5x - 8$:

$$\begin{array}{r l} & 5x \\ 5x - 8 & \overline{)\,25x^2 - 20x - 34} \\ & \underline{25x^2 - 40x} \end{array}$$

Then subtract. Put parentheses around the expression being subtracted.

$$\begin{array}{r l} & 5x \\ 5x - 8 & \overline{)\,25x^2 - 20x - 34} \\ & \underline{-\left(25x^2 - 40x\right)} \\ & 20x \end{array}$$

Notice that subtracting $-40x$ is the same as adding $40x$.

continued...

DIVIDING POLYNOMIALS

Example D, continued...

Bring down the next term:

$$\begin{array}{r} 5x \\ 5x-8\,\big)\overline{25x^2-20x-34} \\ -\left(25x^2-40x\right) \downarrow \\ \hline 20x-34 \end{array}$$

How many times does $5x$ go into $20x$? The answer is 4 times:

$$\begin{array}{r} 5x\ +\ 4 \\ 5x-8\,\big)\overline{25x^2-20x-34} \\ -\left(25x^2-40x\right) \\ \hline 20x-34 \end{array}$$

Multiply that 4 times $5x-8$:

$$\begin{array}{r} 5x\ +\ 4 \\ 5x-8\,\big)\overline{25x^2-20x-34} \\ -\left(25x^2-40x\right) \\ \hline 20x-34 \\ 20x-32 \end{array}$$

continued...

DIVIDING POLYNOMIALS

Example D, continued…

$$
\begin{array}{r|l}
 & 5x + 4 \\ \hline
5x-8 & 25x^2 - 20x - 34 \\
 & -\left(25x^2 - 40x\right) \\ \cline{2-2}
 & \qquad 20x - 34 \\
 & \qquad 20x - 32
\end{array}
$$

Then subtract, putting parentheses around the expression being subtracted.

$$
\begin{array}{r|l}
 & 5x + 4 \\ \hline
5x-8 & 25x^2 - 20x - 34 \\
 & -\left(25x^2 - 40x\right) \\ \cline{2-2}
 & \qquad 20x - 34 \\
 & \qquad -\left(20x - 32\right) \\ \cline{2-2}
 & \qquad\qquad -2
\end{array}
$$

Notice that subtracting -32 is the same as adding 32. We are done at this point, since x doesn't go into -2, the remainder.

$$\frac{25x^2 - 20x - 34}{5x-8} = 5x + 4 + \frac{-2}{5x-8}$$

or

$$\frac{25x^2 - 20x - 34}{5x-8} = 5x + 4 - \frac{2}{5x-8}.$$

DIVIDING POLYNOMIALS

Extended Example 4a

Use long division to divide $\dfrac{21x^2 - 20x - 2}{3x - 2}$.

DIVIDING POLYNOMIALS

With patience, the long division process can be carried out with dividends of any degree.

Extended Example 5

Use long division to divide $\dfrac{2x^3 + 3x^2 - 13x + 11}{2x - 3}$.

"0" Placeholders

So far, the polynomials we've divided haven't been missing any terms.

For example, the polynomial

$$x^4 - 4x^3 + 6x^2 - 4x + 1$$

has terms with degrees of $0, 1, 2, 3$ and 4; there are no missing terms.

When the polynomial in a numerator (the dividend) is missing terms, it's essential to use 0 placeholder terms for the long division process.

For example, the polynomial

$$x^4 + 4x^2 - 13$$

has two missing two terms, which would need to be included, with 0 coefficients:

$$x^4 + 0x^3 + 4x^2 + 0x - 13.$$

These zero terms act as placeholders in the long division process.

Example E on the next screen illustrates the use of placeholders.

EXAMPLE E

Divide, using long division: $\dfrac{x^3+1}{x+1}$.

END OF LESSON

Perform the long division process. What is the remainder?

$$\frac{5x^2 + 28x - 37}{5x - 7}$$

Perform the long division process.
Write your final answer in algebraic notation.

$$\frac{21x^2 - 2x + 2}{7x - 3}$$

Perform the long division process.
Write your final answer in algebraic notation.

$$\frac{6x^3 - 5x^2 + 6x + 3}{3x + 2}$$