MONOMIALS

Introduction

This lesson introduces monomials, exponents, and associated terminology.

MONOMIALS

Definitions

Like any other subject, algebra has its own vocabulary— sets of words that are specific to the subject. To understand algebra, you must learn its vocabulary.

A **variable** is a letter or symbol used to represent a quantity that is unknown or can change. The letters x and y are the symbols most commonly used as variables but any letter can be used. Variables are also sometimes referred to as "unknowns." Common nouns can serve this purpose in the English language. For example, the word "cat" represents different cats; the variable x represents different numbers.

A **constant** is a quantity that does not change in value. For example, 3, -8, $\frac{3}{7}$, 36.5, $\sqrt{2}$, and π are all constants.

A **monomial** is a constant, a variable, or the product of constants and variables. A monomial never involves addition, subtraction, radicals of variables, or variables in a denominator.

For example, 2, $9xy$, $-4u^7v^5w^{13}$, and $\frac{1}{2}a^2b$ are all monomials. The following are <u>not</u> monomials: $9x+2y$, $2\sqrt{x}$, $\frac{5}{x}$, $\frac{1}{3\sqrt{x}}$, and $\frac{5}{3x^4}$.

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Based on the descriptions on the previous screen, try to answer the following questions yourself before revealing the answers.

Question: Is $7b$ a monomial?

Question: Is $7b + 2x$ a monomial?

Question: Is $\sqrt{2xy}$ a monomial?

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EXAMPLE A

Is $7b(2x)$ a monomial?

In this case, $7b$ and $2x$ are factors to be multiplied: $7b(2x) = 14bx$.

(Remember, when you multiply, the variables "go along for the ride.")
The result is the product of the constant 14 and the variables b and x .
So it is a monomial.

Notes:

- The monomial $14bx$ is written in **standard form**. This means that the constant comes first and the variables come second, in alphabetical order, when writing the product. In the question above, $7b(2x)$ is a monomial, but it is not in standard form.

- When a monomial is written in standard form, the constant is called the **coefficient** of the monomial. In the monomial $14bx$, 14 is the coefficient.

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Try to answer the following questions yourself before revealing the answers.

Question: Write $5m \cdot 2q$ in standard form and find the coefficient.

Question: Write $4n(5x)(2c)$ in standard form, and find the coefficient.

Question: What is the coefficient of $-xyz$?

Question: Write $-3x(-2y)(-z)$ in standard form, and find the coefficient.

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Exponential Notation

Products of monomials with repeating factors such as $x \cdot 3 \cdot x$ are not written as $3xx$. Instead, a special notation is used. The product of $x \cdot x$ is written as x^2, and read as "x to the second power." So, $x \cdot 3 \cdot x = 3 \cdot x \cdot x = 3x^2$, indicating that x is used as a factor two times.

The repeated factor, x, is called the **base**.

The number of repeated factors, 2, is called the **exponent**.

Note:

- The expression x^2 can also be read as "x squared," and the expression x^3 can be read as "x to the third power" or "x cubed."

Examples:

$x \cdot x \cdot x \cdot x \cdot x = x^5$ and is read "x to the fifth power"

$y \cdot y \cdot y = y^3$ and is read "y to the third power"

Consider the three ways of writing 3^4: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

- 3^4 is said to be in **exponential form**
- $3 \cdot 3 \cdot 3 \cdot 3$ is in **expanded form**
- 81 is in **standard form**

Question: Write 625 in exponential form, as a power of 5.

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Degree

The **degree** of a monomial with only one variable is simply the degree of that variable.

Examples:

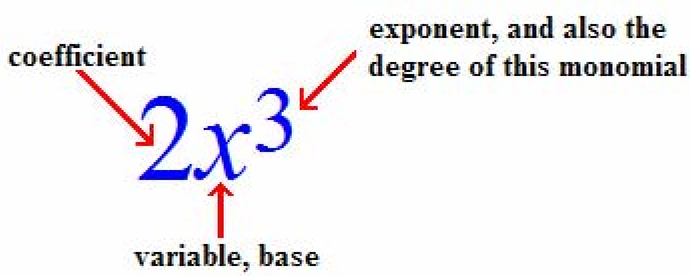
The degree of $5x^4$ is 4 .

The degree of $32y^9$ is 9 .

The degree of x is 1 since $x = x^1$.

The degree of a constant is 0 .

The following illustration may help you understand some of the terminology we've just studied:



EXAMPLE B

What is the standard form of $x^2 \cdot 4 \cdot x$, and what is this monomial's degree?

$$\begin{aligned} x^2 \cdot 4 \cdot x &= 4 \cdot x^2 \cdot x \\ &= 4 \cdot x \cdot x \cdot x \\ &= 4x^3 \end{aligned}$$

The standard form is $4x^3$. The degree of $4x^3$ is 3, since x is used as a factor three times.

EXAMPLE C

Write the monomial $x \cdot 7 \cdot x \cdot 3$ in standard form, and identify the coefficient, variable, and degree.

Standard Form	Coefficient	Variable	Degree
$x \cdot 7 \cdot x \cdot 3 = 21x^2$	21	x	2

EXAMPLE D

Write the monomial $4 \cdot y \cdot 5 \cdot y \cdot y$ in standard form, and identify the coefficient, variable, and degree.

Standard Form	Coefficient	Variable	Degree
$4 \cdot y \cdot 5 \cdot y \cdot y = 20y^3$	20	y	3

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Question: Write the monomial $5a^2 \cdot 3a^3$ in standard form; state its coefficient and degree.

EXAMPLE E

Write the monomial $6 \cdot x^3 \cdot y \cdot 4 \cdot xy^2$ in standard form.

It might help to reorganize the expression by putting the numbers and variables next to each other and in alphabetical order.

$$6 \cdot x^3 \cdot y \cdot 4 \cdot xy^2 = 6 \cdot 4 \cdot x^3 \cdot x \cdot y \cdot y^2$$
$$= 24x^4y^3$$

Since x is used as a factor 4 times and y is used as a factor 3 times, $24x^4y^3$ is the standard form of the given monomial.

EXAMPLE F

Write the product of $7x^3y^2$, $3xyz$, and $4yz^4$ as a monomial in standard form.

Since $7 \cdot 3 \cdot 4 = 84$, the coefficient of the product is 84. There are 4 factors of x, 4 factors of y, and 5 factors of z, so: $7x^3y^2 \cdot 3xyz \cdot 4yz^4 = 84x^4y^4z^5$.

The monomial $84x^4y^4z^5$ is in standard form.

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EXAMPLE G

Write the product of the monomials $2a^2c^2$, $6ab$, $2bc^3$, and abc as a monomial in standard form. (Remember that 1 is the coefficient of abc).

There are 4 factors of a, 3 factors of b, and 6 factors of c. The coefficient is $2 \cdot 6 \cdot 2 \cdot 1 = 24$.

$$2a^2c^2 \cdot 6ab \cdot 2bc^3 \cdot abc = 24a^4b^3c^6$$

EXAMPLE H

Write the product of the monomials 2^3xy, $-10x^2z$, and wxz in standard form.

$$\begin{aligned} 2^3xy\left(-10x^2z\right)wxz &= 8 \cdot (-10) \cdot w \cdot x \cdot x^2 \cdot x \cdot y \cdot z \cdot z \\ &= -80wx^4yz^2 \end{aligned}$$

Notes:

- When a product involves negative monomials, use parentheses to help keep things clear when multiplying.
- The sign of a monomial in standard form is the sign of its coefficient.

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Extended Example 1a

Write $3x^2y \cdot (-2x^2y^3z) \cdot 4xz^2$ in standard form.

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Extended Example 2a

Write $\left(-4A^2\right)\left(-5B^3\right)\left(AB\right)$ in standard form and note the coefficient.

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Extended Example 3a

Write $\left(-5u^2v^5w^3\right)\left(-20u^4v^3w^4\right)$ in standard form and note the coefficient.

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Extended Example 4a

Write $\left(-2x^2\right)\left(-3x^4\right)(7x)\left(-2x^6\right)$ in standard form. Note the coefficient and the degree.

Extended Example 5a

Simplify $\left(3a^4b^2\right)\left(-10a^2b^{11}\right)$. Write the answer in standard form.

END OF LESSON

Is the following a monomial? Why or why not?

$12m \cdot 2T \cdot (-4)$

Some of the following are monomials, and some are not.
For those that are monomials, write them in standard form.
For those that are not monomials, write N *in the space provided.*

$5m \cdot (-T) \cdot 5b$__

If the following is a monomial, write it in standard form:

$19 \cdot 3y \cdot (-8)$

Write the monomial in standard form.

$h^4 \cdot 6 \cdot g^2 \cdot 5g \cdot 2h$

If the following is a monomial, write it in standard form:

$13a \cdot (-y) \cdot 14b \cdot (-2x)$

If the following is a monomial, write it in standard form:

$(-m) \cdot 21t \cdot 2r \cdot (-3u)$

In the problem below, some monomials are listed.
Write their product as a monomial in standard form.

$-4r^3,\ 5n^2,\ -8q,\ \text{and}\ -3t^2$

BASIC RULES FOR EXPONENTS

Introduction

It is important to memorize the rules for exponents. Exponents will be used frequently in this course, as well as in the next. Try to understand each rule by carefully examining the examples in this lesson. All of the rules follow from the basic definition of exponents given on the next screen.

BASIC RULES FOR EXPONENTS

Definition of Exponents:

$$x^n = \overbrace{x \cdot x \cdot x \cdots x}^{n \text{ of these}}$$

For example, $x^2 = x \cdot x$, $x^3 = x \cdot x \cdot x$, etc.

As you study the rules of exponents in this lesson, assume all exponents are positive integers.

Rule 1: $x^1 = x$, for any real number x.

Examples: $23^1 = 23$

$A^1 = A$

Rule 2: $x^m \cdot x^n = x^{m+n}$, for any real number x and any integers m and n.

Examples: $5^4 \cdot 5^3 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^7$

or: $5^4 \cdot 5^3 = 5^{4+3} = 5^7$

$x^7 \cdot x^8 = x^{7+8} = x^{15}$

$3^5 \cdot 3^2 = 3^{5+2} = 3^7$

BASIC RULES FOR EXPONENTS

Rule 3: $(xy)^n = x^n \cdot y^n$, for any real numbers x and y and any positive integer n.

Examples:

$$(3x)^4 = 3x \cdot 3x \cdot 3x \cdot 3x = 3 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x = 3^4 x^4 = 81x^4$$

or: $(3x)^4 = 3^4 x^4 = 81x^4$

$$(2ab)^3 = 2^3 a^3 b^3 = 8a^3 b^3$$

Rule 4: $(x^m)^n = x^{m \cdot n}$, for any real number x and any integers m and n.

Examples:

$$(7^3)^4 = 7^3 \cdot 7^3 \cdot 7^3 \cdot 7^3 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^{12}$$

or: $(7^3)^4 = 7^{3 \cdot 4} = 7^{12}$

$$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x^6$$

or: $(x^2)^3 = x^{2 \cdot 3} = x^6$

BASIC RULES FOR EXPONENTS

EXAMPLE A

Multiply $m^7 \cdot m$. Write the product as a power with base m.

Use Rule 1, $x^1 = x$, to write $m = m^1$: $m^7 \cdot m^1$

Then use Rule 2, $x^m \cdot x^n = x^{m+n}$: $m^7 \cdot m^1 = m^{7+1} = m^8$.

Question: Multiply $5^2 \cdot 5 \cdot 5^4 \cdot 5^2$. Write the product as a power with base 5.

EXAMPLE B

Write $5^3 x^3$ as a power with base $5x$.

Use Rule 3, $(xy)^n = x^n \cdot y^n$: $5^3 x^3 = (5x)^3$.

EXAMPLE C

Write 16^3 as a power with base 2.

Use Rule 4, $\left(x^m\right)^n = x^{m \cdot n}$, and the fact that $16 = 2^4$:

$$16^3 = \left(2^4\right)^3 = 2^{4 \cdot 3} = 2^{12}.$$

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BASIC RULES FOR EXPONENTS

Extended Example 1a

Write the monomial $\left(4x^2y^5\right)^3$ in standard form.

BASIC RULES FOR EXPONENTS

Extended Example 2a

Write the monomial $\left(2A^4B^2C^3\right)^5$ in standard form.

BASIC RULES FOR EXPONENTS

Rule 5: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$, for any x and y, $y \neq 0$, and any integer n.

Examples:

When you multiply fractions, you multiply the numerators together and the denominators together, so this rule seems reasonable:

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{16}{81}$$

or: $$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$$

$$\left(\frac{u}{v}\right)^3 = \frac{u}{v} \cdot \frac{u}{v} \cdot \frac{u}{v} = \frac{u \cdot u \cdot u}{v \cdot v \cdot v} = \frac{u^3}{v^3}$$

or: $$\left(\frac{u}{v}\right)^3 = \frac{u^3}{v^3}$$

BASIC RULES FOR EXPONENTS

Extended Example 3a

Simplify $\left(\frac{3z^2}{t^2}\right)^2$.

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BASIC RULES FOR EXPONENTS

Extended Example 4a

Simplify $\left(\dfrac{z^2}{2t^3}\right)^3$.

Extended Example 5a

Simplify $\left(\frac{2v^4}{3w^5}\right)^3$.

BASIC RULES FOR EXPONENTS

Consider the expression $3 \cdot 2^5$.

It looks like there are two ways to calculate $3 \cdot 2^5$.

Is it $6^5 = 1{,}296$ or $3 \cdot 32 = 96$?

We could use parentheses to be absolutely clear about how we want the expression to be evaluated:

$$(3 \cdot 2)^5 \text{ or } 3 \cdot (2)^5.$$

However, an **order of operations** has been agreed upon that eliminates confusion when parentheses are not present.

The order of operations is as follows:

1. Simplify within parentheses.
2. Simplify any exponents.
3. Perform multiplication and division from left to right.
4. Perform addition and subtraction from left to right.

So, using the correct order of operations, the expression $3 \cdot 2^5$ equals

$$3 \cdot 32 = 96.$$

BASIC RULES FOR EXPONENTS

EXAMPLE D

Simplify $5+7^2-(3+9)$.

$$\begin{aligned} 5+7^2-(3+9) &= 5+7^2-12 \\ &= 5+49-12 \\ &= 54-12 \\ &= 42 \end{aligned}$$

Extended Example 6a

Simplify $-6+2^3\cdot(1+6)\div 14$.

BASIC RULES FOR EXPONENTS

Consider the following examples. Some find it helpful to keep in mind that negatives are equivalent to multiplication by -1: $-A = (-1)\cdot A$.

Simplify $(-2)^3$. $(-2)^3 = (-2)(-2)(-2) = -8$

Simplify -2^3. $-2^3 = (-1)\cdot 2^3 = (-1)\cdot 8 = -8$

Simplify $(-3)^2$. $(-3)^2 = (-3)(-3) = 9$

Simplify -3^2. $-3^2 = (-1)\cdot 3^2 = (-1)\cdot 9 = -9$

Notes:

- A negative number raised to an even exponent is positive, but a negative number raised to an odd exponent is negative. Notice the use of the order of operations in the examples above: exponents must be simplified before multiplication is performed.
- The negative sign is not part of the base unless it is included in parentheses.

EXAMPLE E

Write -16 as a power with integers as the base and exponent.

$-16 = -4^2$; also, $-16 = -2^4$. However, $(-4)^2 = 16$ and $(-2)^4 = 16$.

EXAMPLE F

Write -8 as a power with integers as the base and exponent.

$-8 = -2^3$; also, $-8 = (-2)^3$.

BASIC RULES FOR EXPONENTS

EXAMPLE G

Does $(-5)^2 = -5^2$?

No. $(-5)^2 = 25$, but $-5^2 = -25$

EXAMPLE H

Write 9 as a power of 3.

When you are asked to "write A as a power of B," ask yourself, "B to what power equals A?":

$$B^{?} = A$$

For this problem we ask, "3 to what power equals 9?"

$3^{?} = 9 \quad \Rightarrow \quad 3^2 = 9$.

Question: Write $1,000$ as a power of 10.

END OF LESSON

Write in standard form: $8x^3 \cdot 2wx \cdot 3^2 x^3 \cdot w^2$

Use the rules for exponents to simplify the monomial.

$\left(3x^2y^3\right)^2$

Write in standard form: $\left(-2t^3\right)\left(5tx^2\right)\left(-6b^2\right)\left(3bt\right)$

Use the rules for exponents to simplify the monomial.

$\left[\left(3cv\right)^2\left(2v\right)^2\right]^3$

Use the exponent rules to simplify the expression.

$$\left(\frac{2a^8}{z^9}\right)^{10}$$

Write the product of the following in standard form:

$3^4 ar^2x, -m^3rx^2, -5^2a^2m,$ and -2^2mx

Simplify: $\left(18^2 + 4 \cdot 18 \div 4 - 36\right) \div 9$

Simplify: $4^3 - 2\left[3 + 2(8-2)^2\right]$

ADDITIONAL RULES FOR EXPONENTS

Introduction

In this lesson we'll examine some additional rules that govern the behavior of exponents. The rules should be memorized; they will be used often in the remaining chapters. These additional rules also follow from the definition of exponents given below.

$$x^n = \overbrace{x \cdot x \cdot x \cdots x}^{n \text{ of these}}.$$

ADDITIONAL RULES FOR EXPONENTS

The first new rule is the familiar cancellation rule for fractions, expressed in terms of exponents.

Rule 6: $\dfrac{x^m}{x^n} = x^{m-n}$, for any real number x, $x \neq 0$, and any integers m and n.

Example:

$$\frac{2^7}{2^4} = \frac{2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}{2\cdot 2\cdot 2\cdot 2} = \frac{\cancel{2\cdot 2\cdot 2\cdot 2}\cdot 2\cdot 2\cdot 2}{\cancel{2\cdot 2\cdot 2\cdot 2}} = 2\cdot 2\cdot 2 = 2^3 = 8$$

The four 2s in the denominator canceled out four 2s in the numerator. This removed four of the seven 2s, leaving three 2s in the numerator. That is the essence of this rule – cancellation. But using Rule 6 is simpler than canceling:

$$\frac{2^7}{2^4} = 2^{7-4} = 2^3 = 8$$

Example A below shows that without Rule 6 there would be a lot of canceling!

EXAMPLE A

$$\frac{A^{555}}{A^{333}} = ?$$

$$\frac{A^{555}}{A^{333}} = A^{555-333} = A^{222}$$

ADDITIONAL RULES FOR EXPONENTS

It's clear from the examples on the previous screen that Rule 6 is true when the exponent in the numerator, m , is greater than the exponent in the denominator, n . Extending Rule 6 so that it applies for all integer values of m and n requires some new definitions.

Consider what happens when $m = n$. Clearly, $\dfrac{x^m}{x^m} = 1$.

But applying Rule 6, we get $\dfrac{x^m}{x^m} = x^{m-m} = x^0$.

This leads to a new rule. This new rule is: anything raised to the zero power equals 1 .

Rule 7: $x^0 = 1$ for any real number $x \neq 0$.

Examples:

$$5^0 = 1, \quad 917^0 = 1, \quad (-2)^0 = 1, \quad \pi^0 = 1, \quad A^0 = 1, \quad \left(\frac{A^2 - 3}{\sqrt{A^2 + 1}}\right)^0 = 1$$

ADDITIONAL RULES FOR EXPONENTS

We also need to define negative exponents to make Rule 6 true when $m < n$.

Consider: $\dfrac{x^2}{x^3} = \dfrac{x \cdot x}{x \cdot x \cdot x} = \dfrac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x} = \dfrac{1}{x}$.

On the other hand, using Rule 6, we get: $\dfrac{x^2}{x^3} = x^{2-3} = x^{-1}$.

This leads us to the conclusion: $x^{-1} = \dfrac{1}{x}$. Using this definition, all the previous rules of exponents are true even when m and n are negative. The next rule is a generalization of what we just learned.

> **Rule 8 (Definition of Negative Exponent):**
> $x^{-n} = \dfrac{1}{x^n}$, for any real number x, $x \neq 0$, and for any integer n.

Examples: $2^{-5} = \dfrac{1}{2^5} = \dfrac{1}{32}$.

$$\frac{2^4}{2^7} = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{\cancel{2 \cdot 2 \cdot 2 \cdot 2}}{\cancel{2 \cdot 2 \cdot 2 \cdot 2} \cdot 2 \cdot 2 \cdot 2} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{2^3} = \frac{1}{8}$$

As you can see, four of the 2s in the denominator canceled out the four 2s in the numerator, leaving three 2s in the denominator. Applying Rule 6 is simpler:

$$\frac{2^4}{2^7} = 2^{4-7} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

ADDITIONAL RULES FOR EXPONENTS

The table below contains all of the exponent rules (or "properties") we've seen so far. You may also want to print the table so you can refer to it for the remainder of this lesson and when working on the problems for this section.

Properties of Exponents	
Definition of Exponent: $a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ of these}}$	
1	$a^1 = a$
2	$a^m \cdot a^n = a^{m+n}$
3	$(ab)^n = a^n \cdot b^n$
4	$(a^m)^n = a^{m \cdot n}$
5	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6	$\frac{a^m}{a^n} = a^{m-n}$
7	$a^0 = 1$
8	$a^{-n} = \frac{1}{a^n}$

Note: The variable a is used to state the properties of exponents in the table above and x is used in other parts of the lesson. Remember that the rules are the same no matter what variable is used.

ADDITIONAL RULES FOR EXPONENTS

You know that multiplication is a shortcut for addition:

$$a \cdot n = \overbrace{a + a + \cdots + a}^{n \text{ of these}}.$$

In the same way, exponentiation is a shortcut for multiplication:

$$a^n = \overbrace{a \cdot a \cdot a \cdot \cdots \cdot a}^{n \text{ of these}}.$$

So, exponentiation distributes over multiplication just as multiplication distributes over addition.

- The distributive law of multiplication over addition: $(a+b) \cdot n = a \cdot n + b \cdot n$
- The distributive law of exponentiation over multiplication: $(ab)^n = a^n \cdot b^n$

You'll recognize the distributive law of exponentiation over multiplication as Rule 3 in the Properties of Exponents table.

EXAMPLE B

Rewrite $\left(8a^{-4}b^{-2}\right)^{-2}$ without using negative exponents.

$$\left(8a^{-4}b^{-2}\right)^{-2} \overset{\boxed{1}}{=} \left(8^1a^{-4}b^{-2}\right)^{-2} \overset{\boxed{3}}{=} \left(8^1\right)^{-2}\left(a^{-4}\right)^{-2}\left(b^{-2}\right)^{-2}$$

$$\overset{\boxed{4}}{=} 8^{1\cdot(-2)}a^{-4\cdot(-2)}b^{-2\cdot(-2)} = 8^{-2}a^8b^4$$

$$\overset{\boxed{8}}{=} \frac{1}{8^2} \cdot a^8b^4 = \frac{1}{64} \cdot \frac{a^8b^4}{1} = \frac{a^8b^4}{64}$$

The numbers in the boxes refer to the rules for exponents in the table.

ADDITIONAL RULES FOR EXPONENTS

There is a shorter way to perform the calculations in Example B by combining steps 3 and 4 together into one step. Use the distributive law of exponentiation over multiplication to distribute the exponent to each factor.

Distribute and multiply the exponents

$$\left(8^{1}a^{-4}\ b^{-2}\right)^{-2} = 8^{-2}a^{8}\ b^{4}$$

The exponent outside the parentheses, -2, distributes to each factor inside the parentheses:

-2 times 1 equals -2
-2 times -4 equals 8
-2 times -2 equals 4.

Notice how much less work it would have been to use this shortcut for Example B on screen 6. Next, you'll be guided step by step through simplifying problems similar to Example B, using this shortcut.

Question: Simplify $\left(5^{-2}\cdot 2^{-3}\right)^{-2}$.

ADDITIONAL RULES FOR EXPONENTS

Extended Example 1a

Simplify $\left(2^{-4} \cdot 3^{2}\right)^{-2}$, expressing it without negative exponents.

Extended Example 2a

Simplify $\left(2^{-2}x^{7}y^{-5}\right)^{-3}$, expressing it without negative exponents.

ADDITIONAL RULES FOR EXPONENTS

Extended Example 3a

Simplify $\left(4a^{-7}\right)^{-2}\left(4^{-2}a^{9}\right)^{-3}$, expressing it without negative exponents.

ADDITIONAL RULES FOR EXPONENTS

The Elevator Rule

One shortcut that you might find useful is something we can call "the **elevator rule**" (this is not a common term; it was invented by this author to help you remember the shortcut). This shortcut comes from Rule 8 combined with the rules for manipulating fractions.

Consider the expression $\frac{a^{-4}b^{-3}}{c^5d^{-6}}$. Notice that the numerator and the denominator are both completely factored as products. This is the only situation where this shortcut works; **if there is any addition or subtraction in the fraction, do NOT use this technique.**

To help you remember the elevator rule, think of the sign of the exponent as describing the "state of mind" of its corresponding variable. For example, variable a above has a negative state of mind and is unhappy. Why is a unhappy, you ask? Because she wants to be downstairs in the denominator but she's stuck up in the numerator. Similarly, b is unhappy about being upstairs, and d is unhappy about being downstairs. This is where the elevator comes in—they can each take the elevator to go wherever makes them happy. Notice that c can stay where she is because she's already happy. After they all take their elevator rides, the expression is transformed into one where everybody is happy (where there are no negative exponents):

$$\frac{a^{-4}b^{-3}}{c^5d^{-6}} = \frac{d^6}{a^4b^3c^5}$$

The negative exponents were eliminated in just one step.

ADDITIONAL RULES FOR EXPONENTS

Question: Simplify $\frac{2^{-3}x^{-1}y^{-2}}{3^{-2}z^{-8}}$, expressing it without negative exponents.

Question: Simplify $\frac{4^{-2}a^{-6}}{5^{-3}b^{-3}c^{-5}}$, expressing it without negative exponents.

Question: Simplify $\frac{7^{-2}u^{-45}v^{-28}}{3^{-4}w^{-41}z^{-62}}$, expressing it without negative exponents.

ADDITIONAL RULES FOR EXPONENTS

Extended Example 4a

Simplify $\left(\dfrac{5^{-2}u^3}{2^2z^{-6}v^{-4}}\right)^{-2}$, expressing it without negative exponents.

ADDITIONAL RULES FOR EXPONENTS

Extended Example 5a

Simplify $\left(\dfrac{2^{-3}x^{-3}}{2^{-5}x^{-2}}\right)^{-3}$, expressing it without negative exponents.

END OF LESSON

Use the rules for exponents from the Lesson to simplify
(write answer in standard form): $6^2 \cdot 6^5$ as a power with base 6

Use the rules for exponents from the Lesson to simplify
(write answer in standard form): $(4x)^5$

Use the rules for exponents from the Lesson to simplify
(write answer in standard form): $(8y)^4$

Use the rules for exponents from the Lesson to simplify
(write answer in standard form): $(2y)^2 \cdot 2y^2 \cdot 4y$

Use the rules for exponents from the Lesson to simplify
(write answer in standard form): $y \cdot 3y \cdot 3^2 y^2 \cdot 3^3 y^3$

Use the rules for exponents from the Lesson to simplify
(write answer in standard form): $\left[(3cv)^2 (2v)^2\right]^3$

Use the rules for exponents to simplify the following.
Eliminate any negative exponents.

$$\left(4^{-3}x^{-7}\right)^{-2}\left(4x^{6}\right)^{-3}$$

Use the rules for exponents to simplify the following.
Eliminate any negative exponents.

$$\left(\frac{2^3x^6y^{-8}}{2^{-2}x^{-7}y^3}\right)^{-3}$$

Use the rules for exponents to simplify the following.
Eliminate any negative exponents.

$$\left(\frac{Q^{-2}R^8}{Q^4R^{-5}}\right)^{-5}$$

FROM WORDS TO ALGEBRAIC EXPRESSIONS

Introduction

In this lesson we'll describe the process of translating certain English phrases and sentences into algebraic expressions and equations. Usually, real-world math problems are expressed first in words. Translating the words into equations allows us to apply the rules of algebra to solve the problems. This lesson introduces the process of applying mathematics to the real world. In fact, math can be found almost everywhere—some physicists have discovered that the universe itself seems to be written in the language of mathematics.

FROM WORDS TO ALGEBRAIC EXPRESSIONS

EXAMPLE A

Write an algebraic expression for the "sum of x and 9."

The word "sum" indicates addition. So, the algebraic expression is $x+9$. Note that $9+x$ is also correct.

EXAMPLE B

Write an algebraic expression for "13 decreased by y."

The phrase "decreased by" indicates subtraction. The algebraic expression for "13 decreased by y" is $13-y$. Note that $y-13$ is not correct—the order in which you subtract is important. Other word expressions for $13-y$ are "13 minus y," "y less than 13," and "y subtracted from 13."

EXAMPLE C

Write an algebraic expression for "the sum of a number and 12."

The phrase "a number" indicates an unknown quantity. We can use any variable to represent it in our algebraic expression. For example, $b+12$.

EXAMPLE D

Write an algebraic expression for "the product of s^3 and 5."

The word "product" indicates multiplication, so: $5s^3$, which is a monomial.

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FROM WORDS TO ALGEBRAIC EXPRESSIONS

EXAMPLE E

Write an expression for "9 more than the product of x and $2y$."

In this case, you add 9 to the "product of x and $2y$." So, the expression is $x \cdot 2y + 9$. The standard form for this expression is $2xy + 9$.

EXAMPLE F

Write an algebraic expression for "one-eighth of n."

A "fraction of" or "percent of" a number or variable indicates that you multiply the number by the fraction or percent. An algebraic expression for "one-eighth of n" is $\frac{1}{8}n$, which can also be written as $\frac{n}{8}$ or $n \div 8$. Notice that these are monomials whose coefficients are all $\frac{1}{8}$.

EXAMPLE G

Write an algebraic expression for "seven added to one-half of a number."

An algebraic expression is $\frac{1}{2}q + 7$ or $\frac{q}{2} + 7$.

EXAMPLE H

Write an expression for "9 times the sum of x and $2y$."

You will need parentheses to indicate that the entire "sum of x and $2y$" is multiplied by 9. The algebraic expression is $9(x + 2y)$.

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FROM WORDS TO ALGEBRAIC EXPRESSIONS

EXAMPLE I

Write an expression for "one-fifth of xy decreased by z."

Translating the phrase directly, we get $\frac{1}{5}xy - z$ which simplifies to $\frac{xy}{5} - z$.

EXAMPLE J

Write the algebraic expression for "Q minus 3, divided by 6."

The comma indicates a pause, showing that the difference is grouped together. So, the algebraic expression reflects the grouping: $\frac{Q-3}{6}$.

EXAMPLE K

Write the algebraic expression for "4 times R plus 6."

Since there is no punctuation to indicate grouping in the phrase, we translate directly and get $4R + 6$.

EXAMPLE L

Write the algebraic expression for the phrase "ax^2 plus b divided by c."

Again, since there is no grouping, this means $ax^2 + b \div c$ or $ax^2 + \frac{b}{c}$.

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FROM WORDS TO ALGEBRAIC EXPRESSIONS

We often want to say that one expression is related to another expression. Relating one expression to another results in what are called **algebraic equations** or **algebraic inequalities**.

These are the most common word expressions that indicate such relationships:

is equal to	$=$
is not equal to	$\neq$
is less than	$<$
is less than or equal to	$\leq$
is greater than	$>$
is greater than or equal to	$\geq$

These relational symbols are the verbs used to turn phrases into sentences. Phrases never contain these symbols. Complete sentences always contain one of these symbols.

EXAMPLE M

Translate "X less than Y" into an algebraic expression.

Notice that this is a phrase without a verb, not a sentence, so no inequality symbol or equal sign is used.

Often it can be helpful to substitute numbers in place of variables. In this case, think "2 less than 5." Then determine "what is two less than five?" "3 is 2 less than 5." So, how do you get 3 using the phrase "2 less than 5?" You subtract: $5-2=3$. In other words:

"2 less than 5" translates as $5-2$.

So,

"X less than Y" translates as $Y-X$.

FROM WORDS TO ALGEBRAIC EXPRESSIONS

EXAMPLE N

Translate "X is less than Y" into an algebraic expression.

This is a phrase with a verb ("is"), which makes it a sentence; in this case an inequality symbol is used. To see which direction the inequality points, remember: the alligator, $<$, always eats the bigger number. In this example Y is bigger, so the sentence "X is less than Y" translates as $X < Y$. Note that the phrase "less than" means subtraction, but the phrase "is less than" means an inequality—the verb "is" indicates that a relational symbol should be used.

EXAMPLE O

Write an algebraic equation for "8 more than x is equal to 17."

The algebraic equation is $x + 8 = 17$.

EXAMPLE P

What is an algebraic equation for "7 less than the product of 2 and n is equal to the product of 3 and n"?

It helps to insert parentheses into the sentence, to mirror the grammar:

7 less than (the product of 2 and n) = (the product of 3 and n).

Now start translating the sub-expressions in the parentheses into algebraic expressions: 7 less than $(2n) = (3n)$. So: $2n - 7 = 3n$.

EXAMPLE Q

Write an algebraic inequality for "one-fifth of x is greater than x minus 4."

The inequality is $\frac{1}{5}x > x - 4$.

FROM WORDS TO ALGEBRAIC EXPRESSIONS

Extended Example 1a

Write an algebraic inequality for "5 added to the quotient of a number and 3 is less than or equal to 7 times the sum of the number and 2." For the number, use the variable n .

FROM WORDS TO ALGEBRAIC EXPRESSIONS

Extended Example 2a

Write an algebraic equation for "4 less than the product of a number and 5 is 2 less than the sum of twice the number and 1." For the number, use variable x.

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FROM WORDS TO ALGEBRAIC EXPRESSIONS

The tables below and on the next screen show some sample word phrases and how to write them algebraically.

x plus 8 8 added to x x increased by 8 8 more than x the sum of x and 8 the total of x and 8 x and 8 together } $x+8$	8 times x the product of 8 and x x multiplied by 8 8 multiplied by x 8 xs } $8x$
x minus 8 8 subtracted from x x decreased by 8 8 less than x 8 fewer than x the difference of x and 8 } $x-8$	x divided by 8 one-eighth of x an eighth part of x the quotient of x and 8 the ratio of x and 8 } $\frac{x}{8}$ or $x \div 8$ or $\frac{1}{8}x$

FROM WORDS TO ALGEBRAIC EXPRESSIONS

is less than 8 is smaller than 8 } < 8	equals 8 is 8 is equal to 8 } $= 8$
is greater than 8 is more than 8 } > 8	equals 8 is 8 is equal to 8 } $= 8$
is greater than or equal to 8 } ≥ 8	does not equal 8 is not 8 is not equal to 8 } $\neq 8$
is less than or equal to 8 } ≤ 8	does not equal 8 is not 8 is not equal to 8 } $\neq 8$

END OF LESSON

Write the corresponding algebraic expression for:
"6 decreased by $3s$"

Write the corresponding algebraic expression for:
"7 fewer than n^2"

Write the corresponding algebraic expression for:
"Five-sixths of $-7xy$"

Write an algebraic equation or inequality for:
"12 less than the product of 4 and a number x is equal to 5 times x"

Write an algebraic equation or inequality for:
"The quotient of 17 and $2t$ is equal to the product of 4 and t"

Write an algebraic equation or inequality for: "T minus 3, divided by 5 is greater than or equal to T plus 4, divided by 7"

EVALUATING EXPRESSIONS

Introduction

We've seen how to use variables to represent numbers in algebraic expressions. In this lesson, you will learn how to **evaluate** such expressions. As you go through the lesson, notice how the value of an algebraic expression changes as the variables are assigned different values.

EVALUATING EXPRESSIONS

Evaluate means to substitute the given number for the variable and then simplify the expression.

EXAMPLE A

Evaluate the monomial $3x^2$ when $x = 5$.

If $x = 5$, then $3x^2 = 3(5)^2 = 3 \cdot 25 = 75$.

EXAMPLE B

Evaluate $3x^2$ when $x = -2$ and then when $x = 2$.

If $x = -2$, then $3x^2 = 3(-2)^2 = 3 \cdot 4 = 12$.

If $x = 2$, then $3x^2 = 3(2)^2 = 3 \cdot 4 = 12$.

EVALUATING EXPRESSIONS

EXAMPLE C

Evaluate the monomial $-5s^3$ when $s = 4$, $s = 1$, $s = -2$, and $s = -\frac{1}{2}$.

If $s = 4$, then $-5s^3 = -5(4)^3 = -5 \cdot 64 = -320$.

If $s = 1$, then $-5s^3 = -5(1)^3 = -5 \cdot 1 = -5$.

If $s = -2$, then $-5s^3 = -5(-2)^3 = (-5)(-8) = 40$.

If $s = -\frac{1}{2}$, then $-5s^3 = -5\left(-\frac{1}{2}\right)^3 = (-5)\left(-\frac{1}{8}\right) = \frac{5}{8}$.

In this case, the value of the monomial changes a lot even when the value of the variable changes very little.

EXAMPLE D

Evaluate $a - b + 7$ when $a = 14$ and $b = 3$.

Replace a with 14 and b with 3: $a - b + 7 = 14 - 3 + 7 = 18$.

EXAMPLE E

Evaluate $9xy$ when $x = 5$ and $y = 3$.

If $x = 5$ and $y = 3$, then $9xy = 9(5)(3) = 135$.

EVALUATING EXPRESSIONS

EXAMPLE F

Evaluate $\frac{m}{n}$ when $m = 57$ and $n = 6$. Write your answer in lowest terms.

$\frac{m}{n} = \frac{57}{6}$ You can reduce this fraction: $\frac{57}{6} = \frac{\cancel{3} \cdot 19}{2 \cdot \cancel{3}} = \frac{19}{2}$.

EXAMPLE G

Evaluate the monomial Ax^4 when $A = 2$ and $x = -3$, $x = -2$, $x = -1$, $x = 0$, $x = 1$, $x = 2$, and $x = 3$.

$Ax^4 = (2)(-3)^4 = 2 \cdot 81 = 162$

$Ax^4 = (2)(-2)^4 = 2 \cdot 16 = 32$

$Ax^4 = (2)(-1)^4 = 2 \cdot 1 = 2$

$Ax^4 = (2)(0)^4 = 2 \cdot 0 = 0$

$Ax^4 = (2)(1)^4 = 2 \cdot 1 = 2$

$Ax^4 = (2)(2)^4 = 2 \cdot 16 = 32$

$Ax^4 = (2)(3)^4 = 2 \cdot 81 = 162$

Note:

- When you substitute a negative number for a variable, use parentheses.
- You could have saved time by noting that $(-3)^4 = 3^4$ and $(-2)^4 = 2^4$.

EVALUATING EXPRESSIONS

Question: Evaluate $2x^2 + 3x + 5$ when $x = -2$.

Question: Evaluate $\frac{x}{3} + \frac{2y}{3}$ when $x = 4$ and $y = 7$.

EXAMPLE H

Renting a truck costs $\$19.95$ per day and $\$0.45$ for each mile driven. Write an expression for the cost, C, to rent a truck for one day and drive m miles. Then use the expression to determine the cost to rent a truck for one day and drive 200 miles.

We are assuming that m miles are driven, at $\$0.45$ per mile, so the cost for these miles is:

$$0.45m\ .$$

Add the $\$19.95$ daily fee to the cost for the miles to find the total cost.

$$C = 0.45m + 19.95$$

To rent for one day and drive 200 miles, it will cost:

$$C = 0.45m + 19.95 = 0.45 \cdot 200 + 19.95 = 90 + 19.95 = \$109.95$$

EVALUATING EXPRESSIONS

EXAMPLE I

Your boss asks you to make copies of an employee training manual. It costs $\$0.15$ per page to copy the manual and $\$2.00$ to bind each manual.

a) Write an expression for the total cost , C , to print and bind a training manual that is p pages long.

b) How much does it cost to make a copy of a 75-page manual?

c) Write an expression for the cost , C , to produce m manuals that contain p pages each.

d) How much does it cost to make 12 copies of a 50-page manual?

a) $C = \$0.15p + \$2$

b) $C = \$0.15(75) + \$2 = \$13.25$

c) $C = m(\$0.15p + \$2)$

d) $C = 12\left[\$0.15(50) + \$2\right] = \$114$

EVALUATING EXPRESSIONS

Question: José lives 150 miles east of Las Vegas. He drives west toward the city at 60 mph. Write an expression for Jose's distance, D, from the city after t hours. Then, use the expression to find his distance after 2.25 hours.

END OF LESSON

Evaluate and simplify: $\frac{5+d-e}{24f}$, for $d=15$, $e=-8$, $f=-2$

Evaluate the algebraic expression for the indicated values. Simplify thel answer.

$\frac{36z}{w-11-y}$, for $w=29, y=-3,$ and $z=10$

Evaluate and simplify: $\frac{2wx}{3yz}$, for $w=12, x=14, y=91, z=18$

Jose bought a CD storage box for D dollars and a dozen CDs for d dollars each. The store deducted these costs from his paycheck of P dollars. Write an algebraic equation for how much of Jose's paycheck was left after buying the CDs and storage box.

It costs \$0.50 per word plus \$10.50 to make a customized license plate frame for a car. Write an equation for the total cost of making p license plate frames with w words on each.

POLYNOMIALS

Introduction

This lesson introduces polynomials and like terms.

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POLYNOMIALS

As we learned earlier, a **monomial** is a constant, a variable, or the product of constants and variables. Some examples of monomials follow:

$$3, \quad -7x, \quad 2xy, \quad -\frac{2}{3}x^3, \quad 14x^6y^2z^9$$

A **polynomial** is a monomial or the sum or difference of any number of monomials. The monomials listed above are examples of polynomials, as are the following:

$$1$$

$$-9x$$

$$2x+5$$

$$3x^2-7x+1$$

$$-5x^2y+y^3z^9-2x^3z^2$$

Some examples of expressions that are not polynomials follow.

Expression	Why isn't it a polynomial?
$\frac{3x-1}{2x^4+5}$	A polynomial cannot contain a variable in a denominator.
$4x^2+3\sqrt{x}$	A polynomial cannot contain a variable under a radical sign.
$6x^2+\frac{2}{x}-3$	There is a variable in a denominator.

POLYNOMIALS

Question: Is $7t^5 - 2t + 1$ a polynomial?

Question: Is $\frac{2}{3}t + 4 - st + 9x^2$ a polynomial?

Question: Is $\frac{x^2}{4y+1} - 3xy$ a polynomial?

POLYNOMIALS

When a polynomial is written in its usual way, as a sum of monomials, then each monomial is a **term** of the polynomial. Monomials are terms, but so are many algebraic expressions that are not monomials. In general, a **term** in an algebraic expression is a monomial or any expression that is a product of factors.

If two or more terms in a polynomial have the same variables raised to the same power they are called **like terms**.

Example: In the polynomial $5x^2y - 9xy + 7x^2y - 4$, the first and third terms, $5x^2y$ and $7x^2y$, are like terms.

Notes:

- Like terms can be added. This is called **combining like terms**. When you combine like terms, you add the coefficients of each term and leave the variable parts unchanged. Thus,
$$5x^2y - 9xy + 7x^2y - 4 = 12x^2y - 9xy - 4.$$
- The distributive property, $a \cdot c + b \cdot c = (a+b) \cdot c$, justifies combining like terms. Consider the like terms that were combined above. Using the distributive property:
$$5x^2y + 7x^2y = (5+7)x^2y = 12x^2y.$$
- Polynomials with two terms are called **binomials**. Polynomials with three terms are called **trinomials**.

POLYNOMIALS

EXAMPLE A

Simplify $8x^2y - 2xy + 3x^2y - 7$ by combining like terms.

$8x^2y - 2xy + 3x^2y - 7 = 11x^2y - 2xy - 7$

EXAMPLE B

Combine the like terms in the polynomials below.

a) $3 - 9r + 2r^3 + 4r - 6$

b) $4t^2 - 2 + t - 6t^2 - t$.

a) $3 - 9r + 2r^3 + 4r - 6 = 3 - 9r + 2r^3 + 4r - 6$

$= 2r^3 - 5r - 3$

b) $4t^2 - 2 + t - 6t^2 - t = -2t^2 - 2$ In this example, the t terms add to 0.

Question: Combine like terms in the polynomial $3r + 2s + 5r - 7s$.

POLYNOMIALS

We saw earlier that the degree of a monomial with only one variable is simply the degree of that variable. When a monomial has more than one variable, the degree of the monomial is the sum of the exponents of all the variables of the monomial.

Examples: The monomial $3x^2y^3$ has a degree of 5, since $2+3=5$.

$5t^4$ has a degree of 4.

$st = s^1t^1$ has a degree of 2.

$-6xy^2 = -6x^1y^2$ has a degree of 3.

Note:

- Recall that constants such as 1, -5, and 12.9 are considered monomials. Such constant, but nonzero, monomials have degree zero. This makes sense in terms of the exponent rule, $x^0 = 1$. For example, using this peculiar way of writing the number one, we can show that the degree of the constant monomial known as the number 3 is indeed zero:

  $$3 = 3 \cdot 1 = 3 \cdot x^0 = 3x^0.$$

  The degree of 3 is zero, as is the degree of any constant.
- The only monomial whose degree is undefined is the monomial 0.

The **degree** of a polynomial in standard form is the highest degree of any of its terms (assuming its like terms have all been combined).

EXAMPLE C

What is the degree of each monomial in the polynomial $4x - 2x^2y + 3xy^2 - 4$? What is the degree of the polynomial?

a) The degree of the first term, $4x = 4x^1$, is 1.
b) The degree of the second term, $-2x^2y = -2x^2y^1$, is 3.
c) The degree of the third term, $3xy^2 = 3x^1y^2$, is also 3.
d) The degree of the fourth term, $-4 = -4 \cdot 1 = -4x^0$, is 0.
e) Since 3 is the highest degree of any term in the polynomial, the degree of this polynomial is 3.

EXAMPLE D

What is the degree of $3xy^3 - x^2y + 2x - 3xy^3 - 5 + 7y$?

The degree of this expression seems to be 4 because the highest degree of the terms shown is 4. *But wait* ... The like terms have not been combined! When $3xy^3$ and $-3xy^3$ are combined, they cancel and the result is zero. So, the expression reduces to the polynomial $-x^2y + 2x - 5 + 7y$. Now, the highest degree of these terms is 3. The polynomial actually has a degree of 3.

A Word to the Wise:
Always combine like terms before deciding the degree of a polynomial.

Extended Example 1a

Find the degree of the polynomial $2yz - xyz - 9y^4z^5 + 3yz + 9y^4z^5 + 2$.

POLYNOMIALS

EXAMPLE E

Determine the degree of each polynomial below.

a) $8x^4 + 3x^3 + 6x^2 + 6x + 9$

b) $\frac{2}{3}n + 4 - mn + 9x^2$

c) $\frac{8s}{2t} + 9x - 5 - 9x$

d) $3 - 6x^2yz + 4xyz + 6yx^2z$

e) $9n - 6x^3 \cdot 0$

a) 4
b) 2
c) This is not a polynomial, since $\frac{8s}{2t}$ is not a monomial.
d) 3, since the second and fourth terms cancel out.
e) 1, because the second term equals zero and has no degree.

We end this lesson by classifying the polynomials given as examples on Screen 2, noting the degree of each polynomial and whether it's a monomial, binomial, or trinomial:

1 . . . . degree 0 monomial

$-9x$ . . . . degree 1 monomial

$2x + 5$ . . . . degree 1 binomial

$3x^2 - 7x + 1$ . . . . . degree 2 trinomial

$-5x^2y + y^3z^9 - 2x^3z^2$ . . degree 12 trinomial

END OF LESSON

In the algebraic expression, write N if it's not a polynomial. Otherwise (after combining like terms), specify the degree of the polynomial.

$3rst + x^5 - 3rst$

Combine like terms: $3r + 5 - r^2 - 3r + 5$

Give the degree of this polynomial: $4t - 9 - 3t + 4 - t$