STRENGTH OF MATERIALS

Video Companion

Jeffrey E. Jones

Cover Photograph: Discovery Aglow
A nearly full Moon sets as the space shuttle Discovery sits atop Launch pad 39A at the Kennedy Space Center in Cape Canaveral, Florida, in the early morning hours of Wednesday, March 11, 2009.

Image Credit: NASA/Bill Ingalls

How to use this book: This video companion contains the screen shots of the problems that are solved on www.YourOtherTeacher.com. The solutions are not included here since it is the authors' belief that more can be learned from the words and gestures in a video than can ever be written. It is suggested that the students write the solutions down as presented in the videos since memory is greatly increased by the writing process.

Avoid looking for problems that are similar to your homework. This would be very short sighted. It is better to understand the concepts than to get the solution for one problem. If you understand the concepts then you can solve any problem that may appear on a test or a problem you may encounter in industry. The saying "Give a man a fish, he eats for a day, teach him how to fish, he eats for a lifetime" is the motto for YourOtherTeacher.com.

Jeffrey E. Jones: Jeffrey Jones is the recipient of the prestigious "Community College Teacher of the Year" award, Awarded by the American Society of Engineering Educators (ASEE-PSW), 2003. In 2004 he was awarded Cuesta College's highest honor "Teaching Excellence Award".

An educator since 1990, Jones has been a senior structural engineer, registered professional engineer in California, teacher, Department Chair, lead instructor, chairman and executive with 30+ years of proven experience.

Jones holds a bachelor's and master's degree in civil engineering from San Jose State University in San Jose, CA (1981, 1989). His concentration was in structural engineering and applied mechanics. He is also the author of *Statics, Video Companion,* as well as others. He has personally recorded over 300 hours of video on his website www.YourOtherTeacher.com which helps 1000's of students every year towards their goal of becoming an engineer.

Comments and suggestions are always welcomed and can be emailed to Jeff at jeff@yourotherteacher.com.

Contents

1- INTRODUCTION- CONCEPT OF STRESS

2- STRESS AND STRAIN- AXIAL LOADING

3- TORSION

4- PURE BENDING

5- ANALYSIS AND DESIGN OF BEAMS FOR BENDING

6- SHEARING STRESSES IN BEAMS AND THIN-WALLED MEMBERS

7- TRANSFORMATION OF STRESS AND STRAIN

8- PRINCIPAL STRESSES UNDER A GIVEN LOADING

9- DEFLECTION OF BEAMS

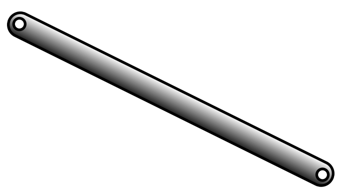
10- COLUMNS

Chapter 1
Introduction- Concept of Stress

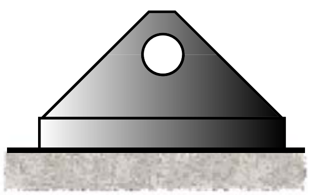
INTRODUCTION

A Review of Statics

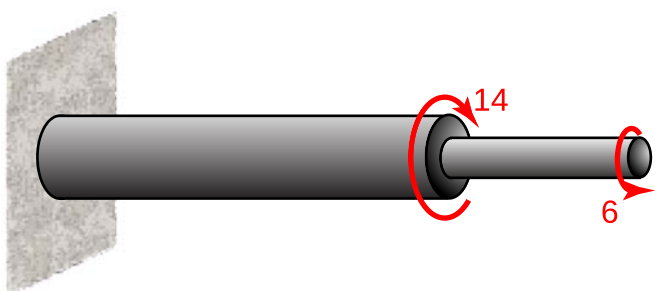
Axial Stress

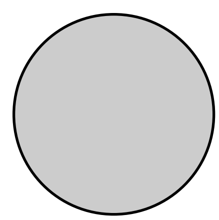


Bearing Stress

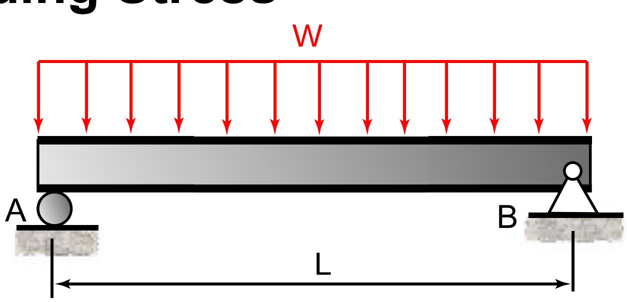


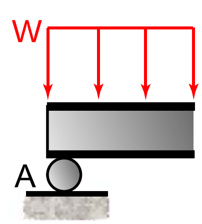
Torsional Stress



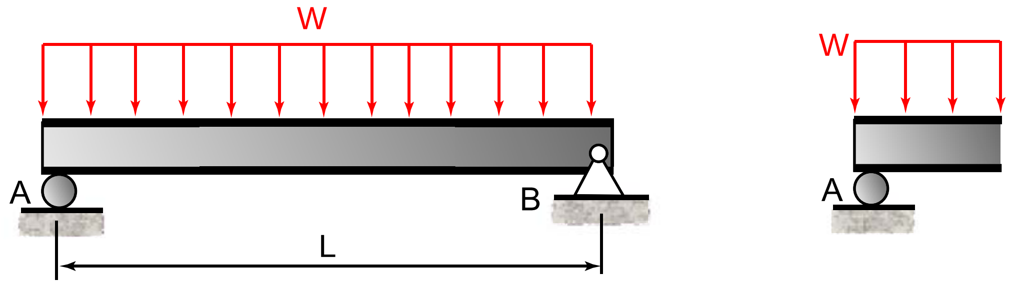


Bending Stress

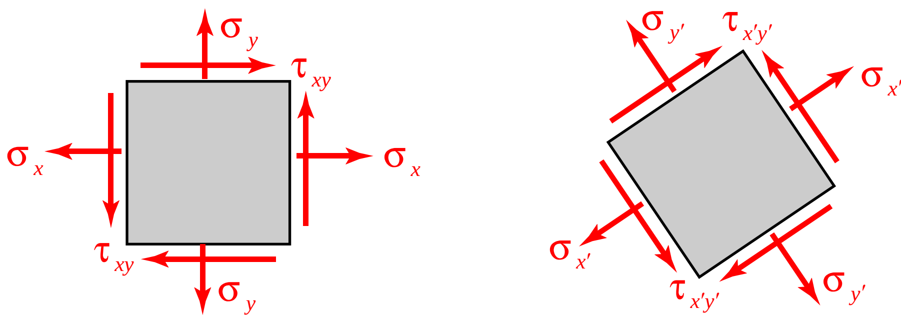




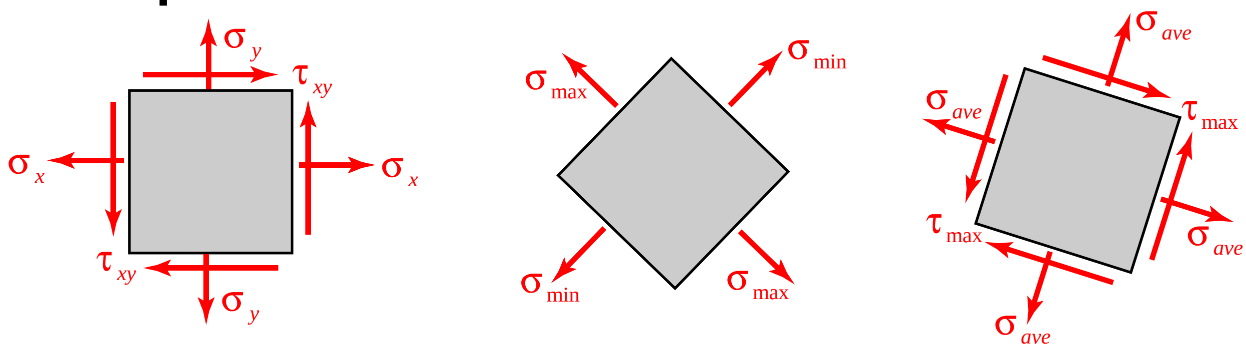
Shear Stress



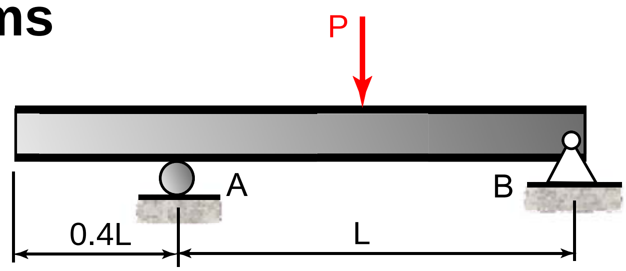
Stress and Strain



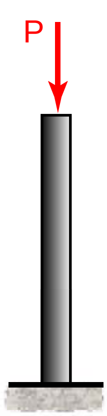
Principal Stresses



Deflection of Beams



Columns

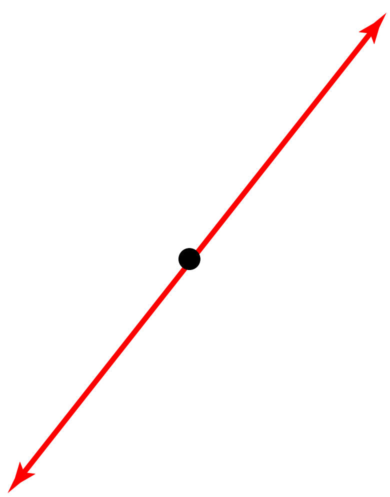


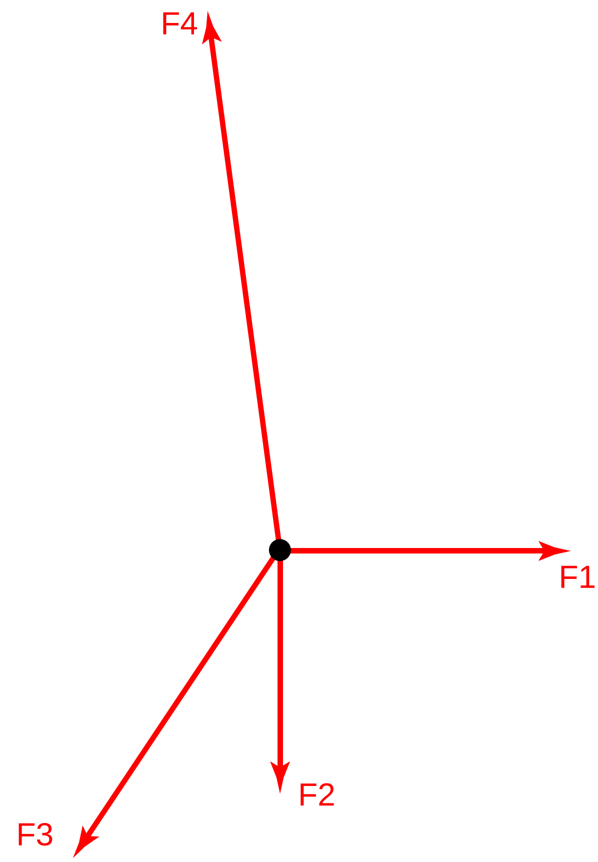
Equilibrium of a Particle

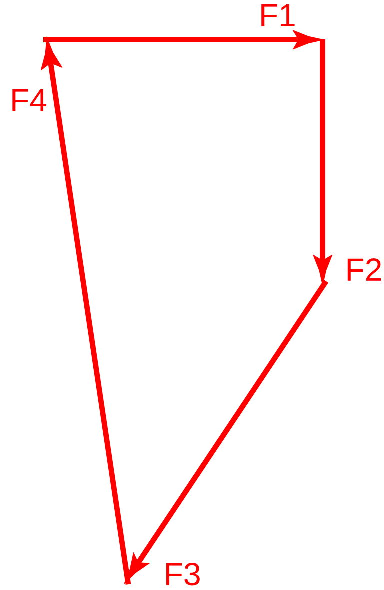
Newton's First Law

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

$$\sum F_x = 0$$
$$\sum F_y = 0$$

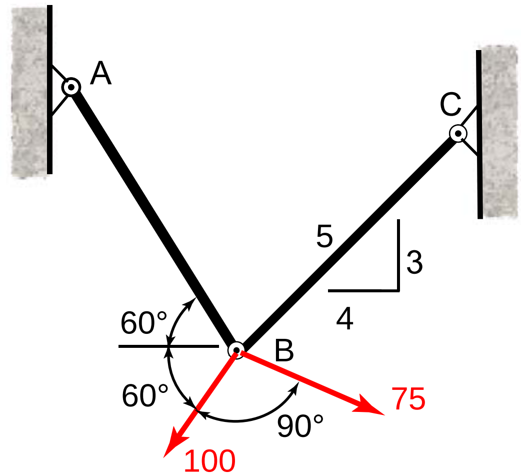






Example

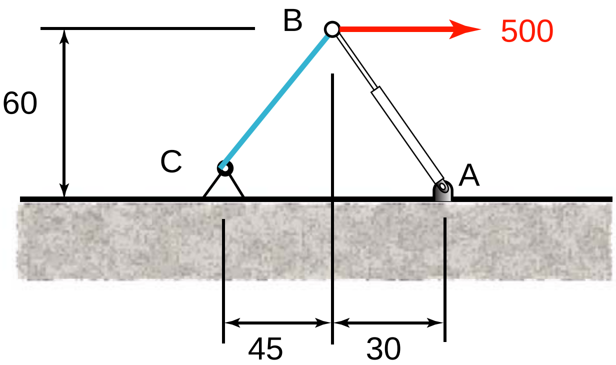
The loads are supported by two rods AB and BC as shown. Find the tension in each rod. Units: N.



Magnitude	x component	y component
100 N		
75 N		

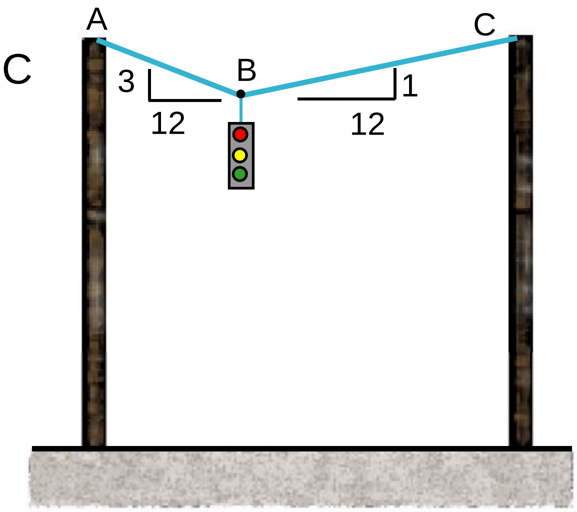
Example

Determine the forces in AB and BC.
Units: Lb, in.



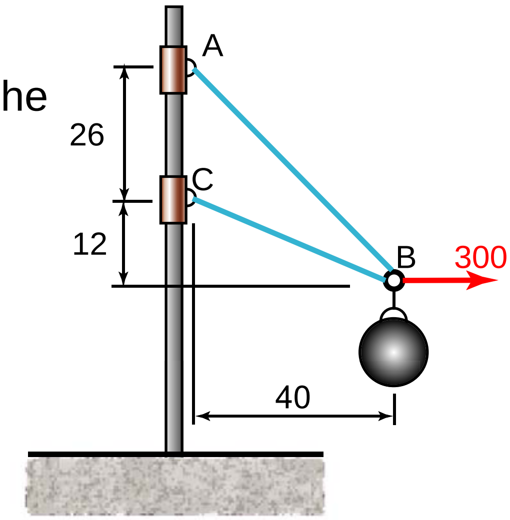
Example

Determine the forces in cables AB and BC due to the 25 lb traffic light. Units: Lb.



Example

Determine the forces in wires AB and BC. The sphere weighs 100 lbs. Units: Lb, in.

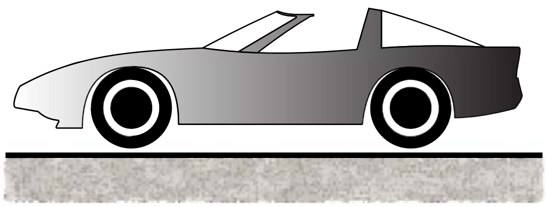


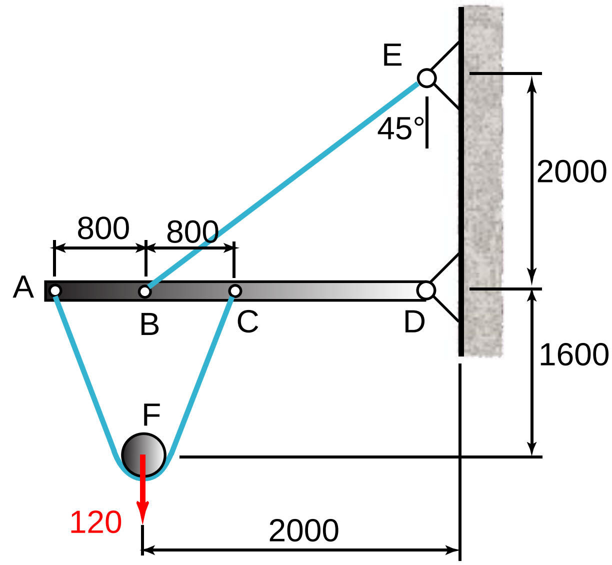
Equilibrium of Rigid Bodies

A particle remains at rest or continues to move in a straight line with uniform velocity if the resultant forces acting on it are zero, in other words:

$$\sum \vec{F} = 0$$

$$\sum \vec{M} = 0$$

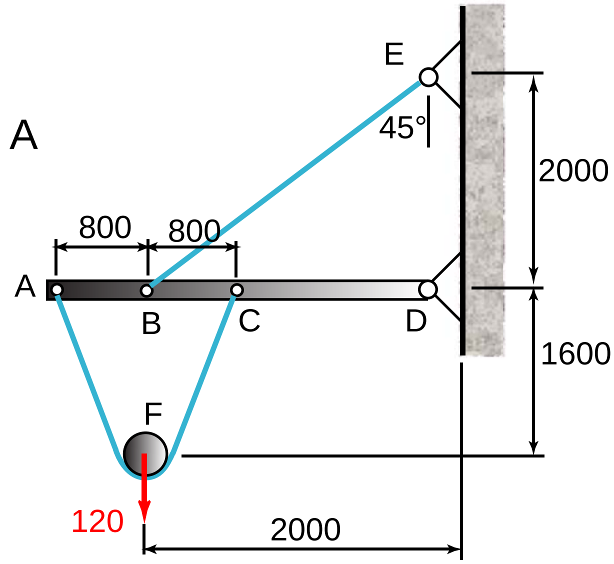




Example

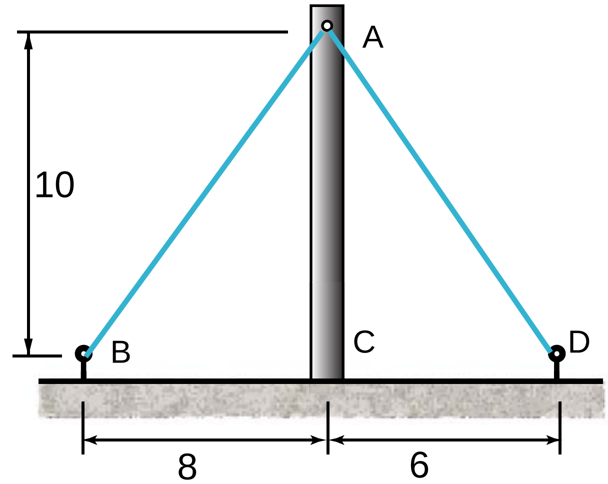
Determine the reactions at D and the tension in BE. The wire connected at A and C is continuous.

Units: N, mm.



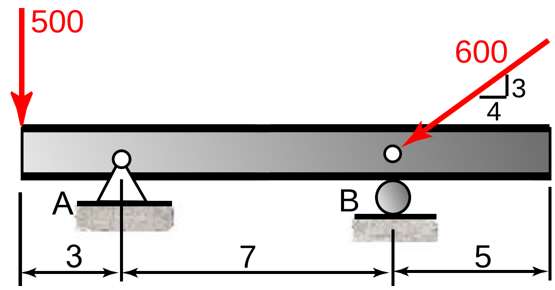
Example

The cable stays AB and AD help support pole AC. Knowing that the tension is 140 lb in AB and 40 lb in AD, determine the reactions at C. Units: Lb, ft.



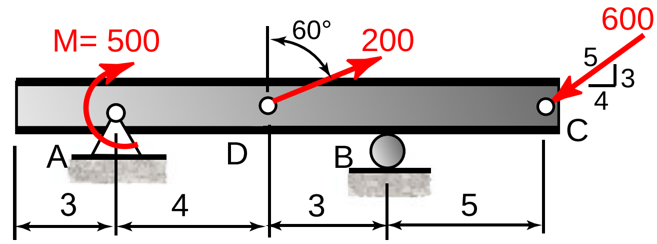
Example

Determine the reactions at supports A and B.
Units: Lb, ft.



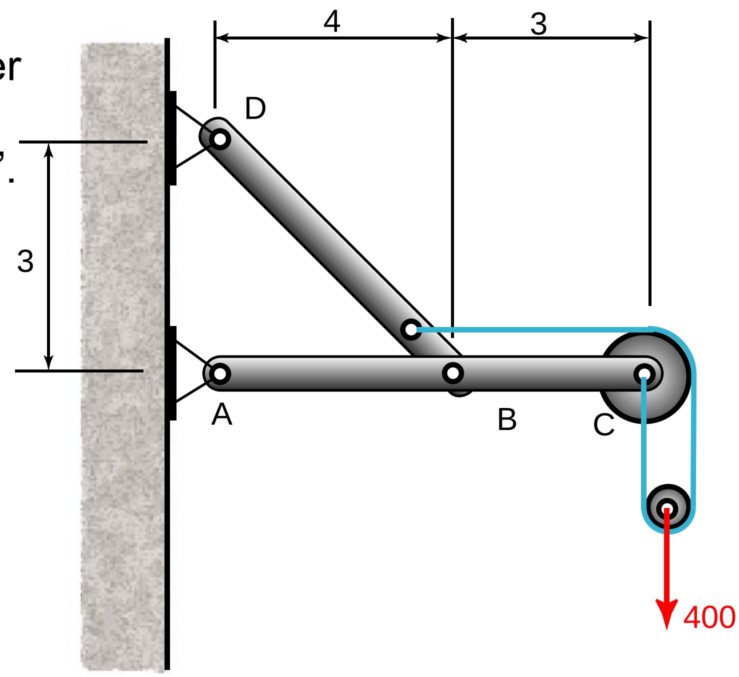
Example

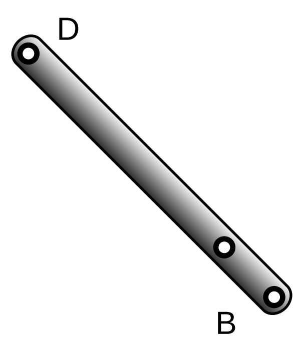
Determine the reactions at A and B. Units: Lb, ft.

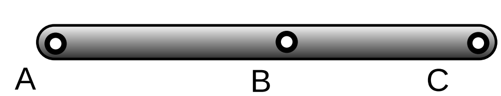


Example

Determine the forces on member ABC. The radius of the small pulley is 2.5" and the larger is 5”. Units: Lb, ft.

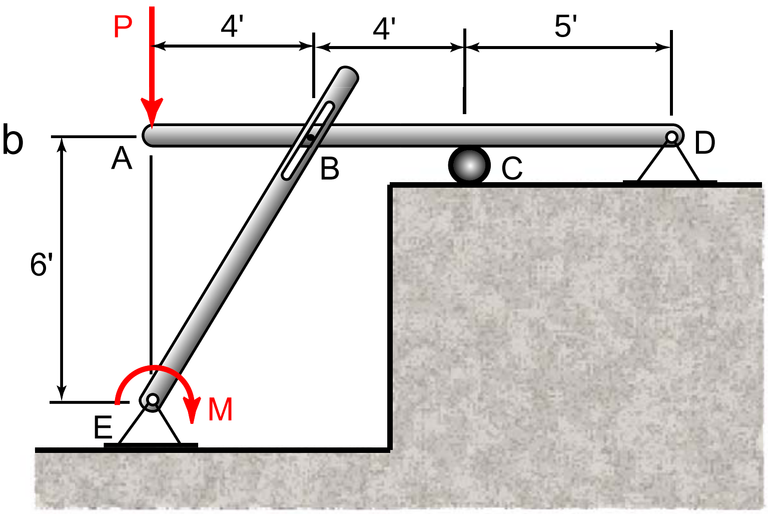


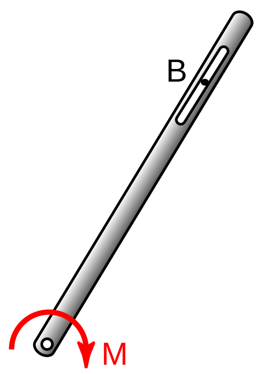


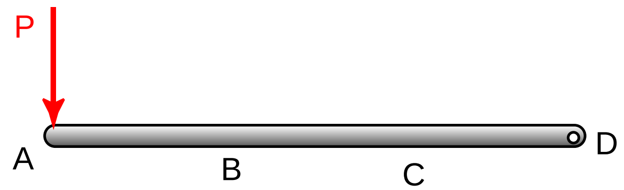


Example

Determine the forces on member ABCD due to P= 500 lb and M= 700 ft-lb. Units: Lb, ft.

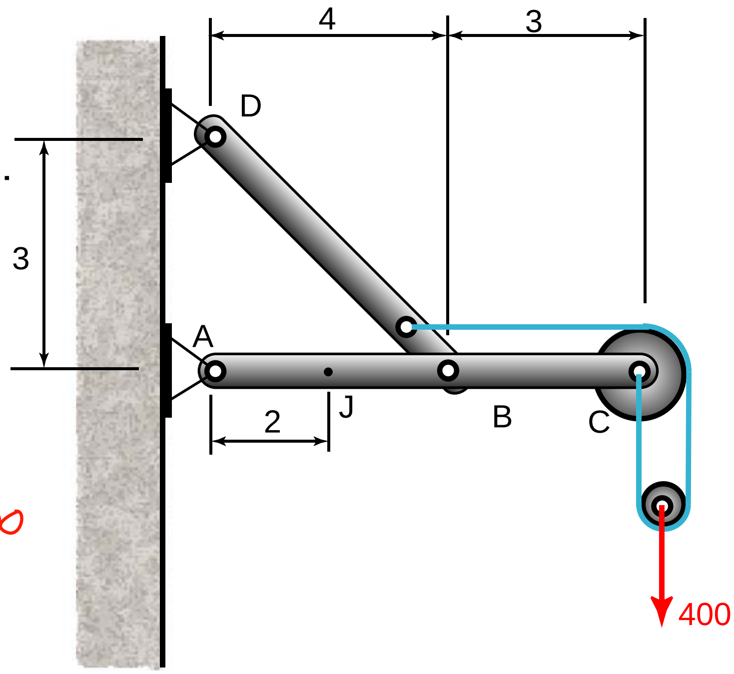




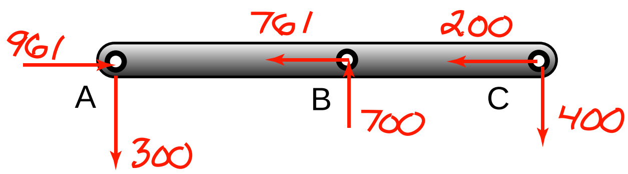


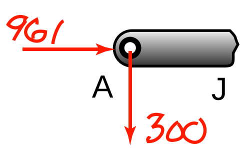
Example

Determine the internal forces at point J. The radius of the small pulley is 2.5" and the larger is 5”. Units: Lb, ft.



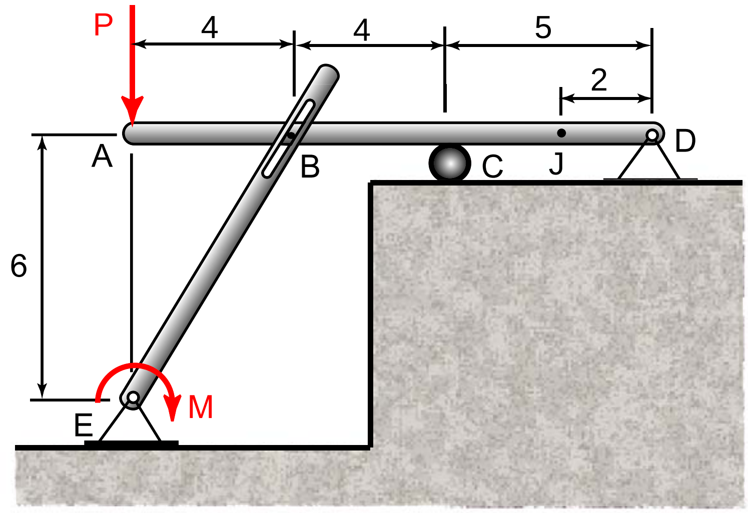
Recall from a previous solution:



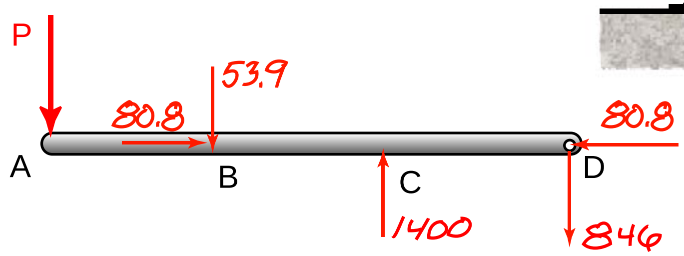


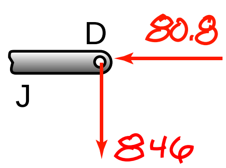
Example

Determine the internal forces at point J due to P= 500 lb and M= 700 ft-lb. Units: Lb, ft.

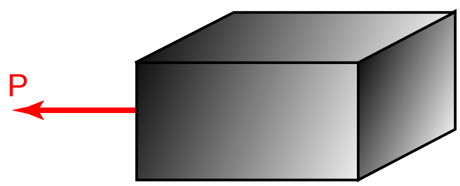


Recall from a previous solution:





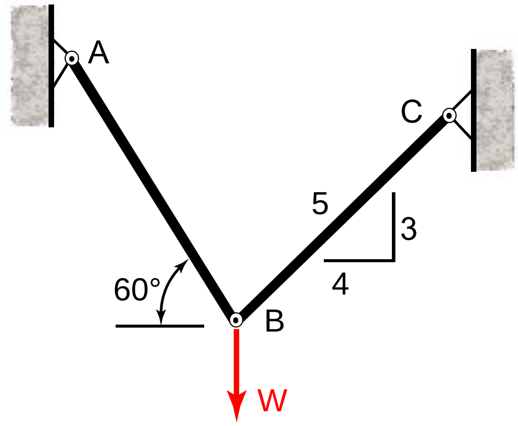
NORMAL STRESS



$$\sigma = \frac{P}{A}$$

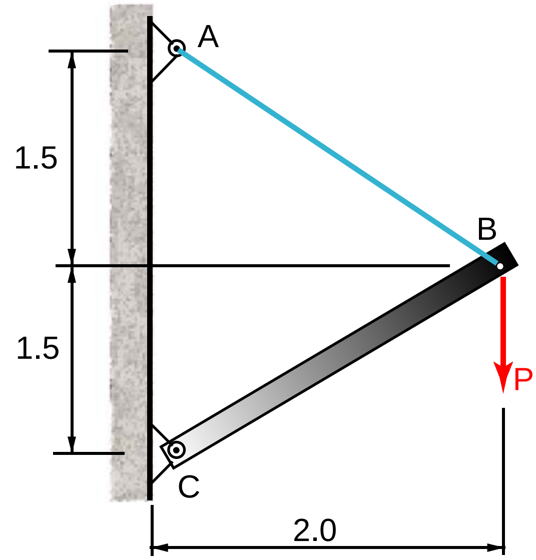
EXAMPLE

The 80-kg lamp is supported by two rods AB and BC as shown. If AB has a diameter of 10 mm and BC has a diameter of 8 mm, determine which rod is subjected to the greater normal stress?



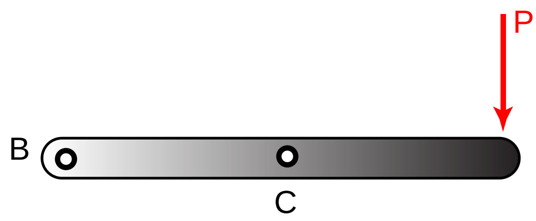
Example

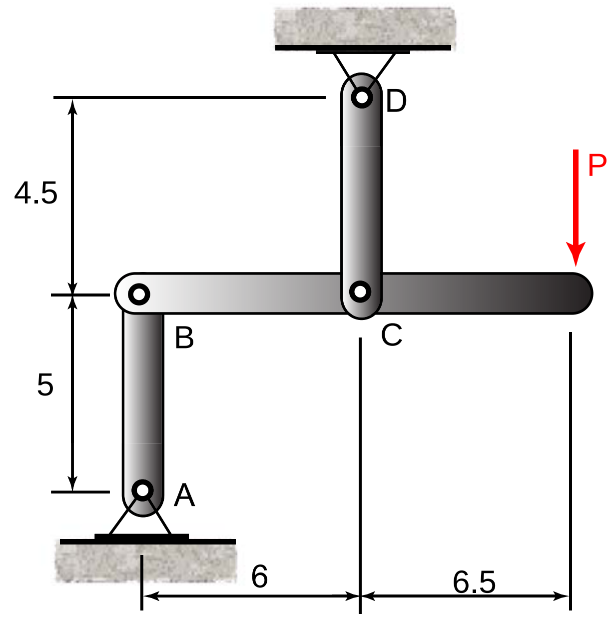
A strut and cable assembly ABC supports a vertical load P= 1.8 kN. The cable has an effective cross-sectional area of 12000 sq. mm and the strut has an area of 25000 sq. mm. Calculate the normal stress in the cable and strut, and indicate whether they are in tension or compression. Units: m



Example

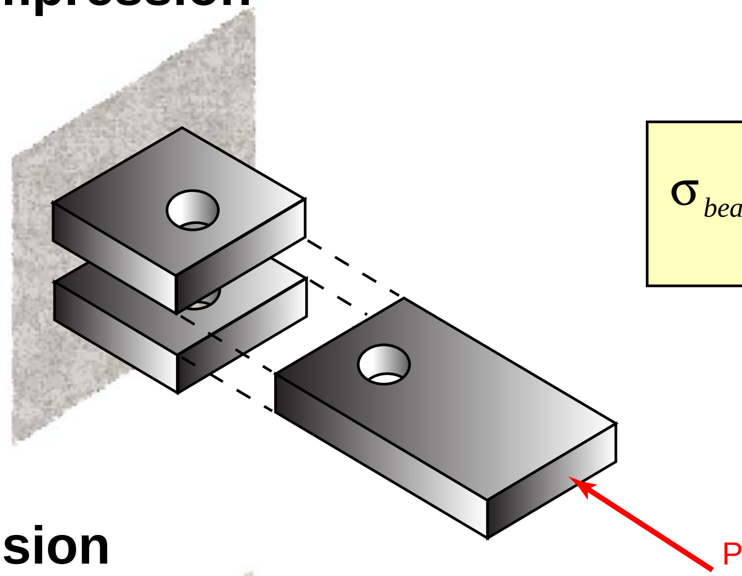
Bar AB has a cross-section of .25"x4" and CD is .60"x4". With a load of 2-k at the end, what is the axial stress in link AB and CD.
Note: The units of “k” means 1000 lbs, often referred to as “kips”.
Units: K, ft.



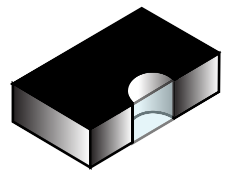


STRESSES IN CONNECTIONS

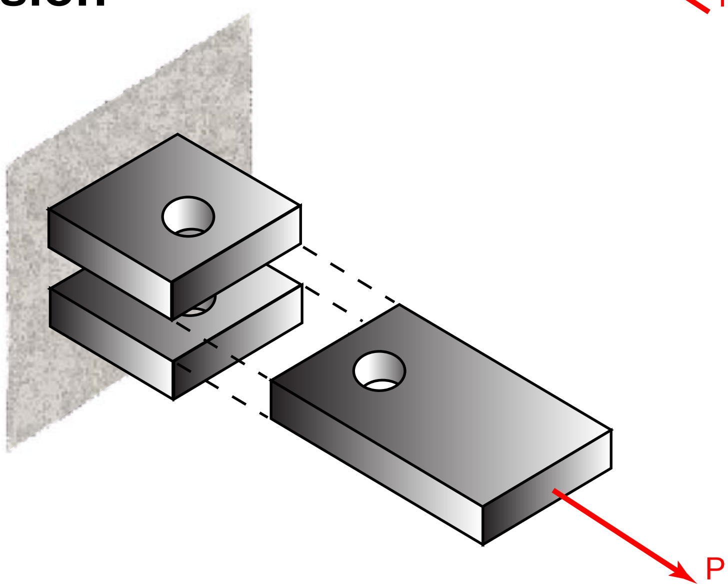
Compression

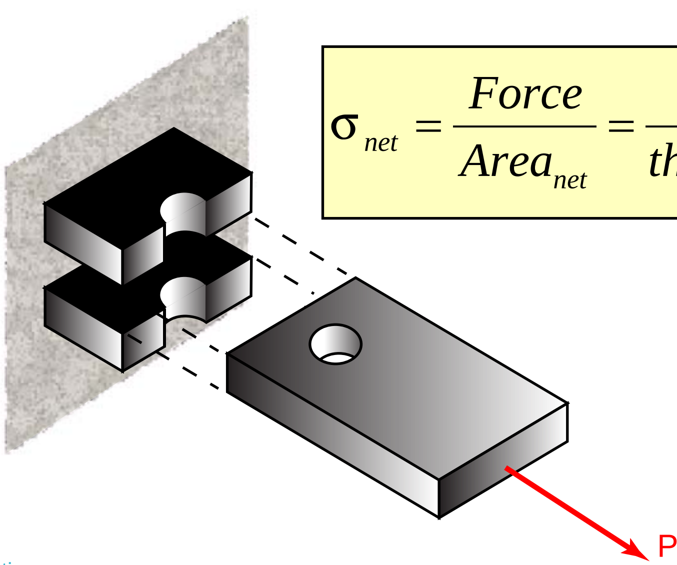


$$\sigma_{bearing} = \frac{Force}{thk_{plate} dia_{bolt}}$$



Tension

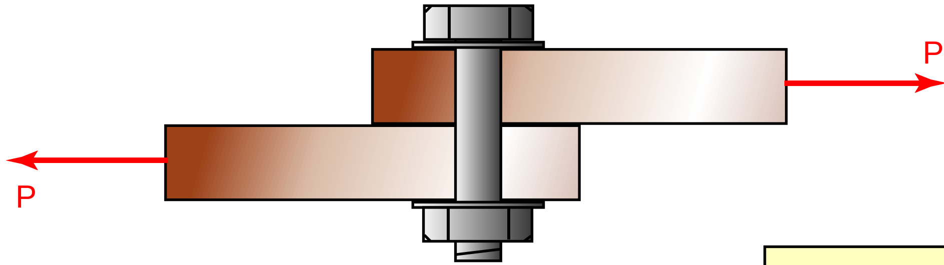




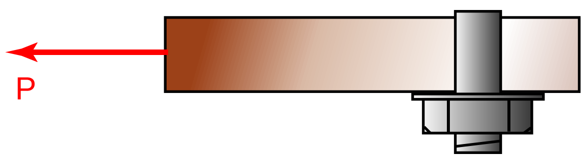
$$\sigma_{net} = \frac{Force}{Area_{net}} = \frac{Force}{thk_{plate}(width_{plate} - dia_{bolt})}$$

SHEAR STRESSES IN BOLTS

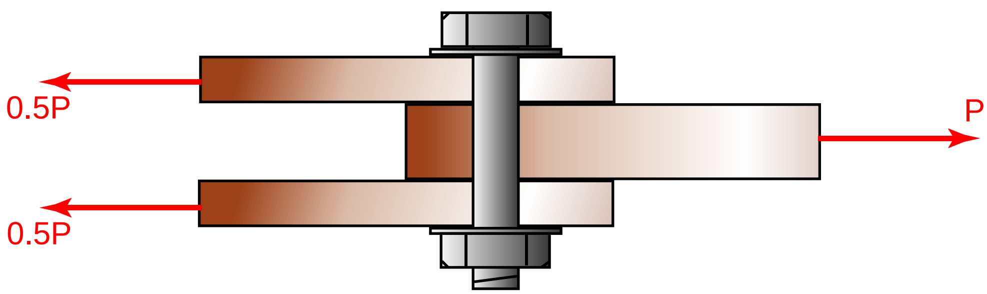
Single Shear



$$\tau_{ave} = \frac{Force_{bolt}}{A_{bolt}}$$



Double Shear



NUMERICAL ACCURACY

Numerical accuracy depends on:

-accuracy of the given data

-the accuracy of the computations

Example

I want to measure the area of my house and I'm so cheap I can't afford a tape measure. But my foot is approximately 1 foot (no pun intended) long. So I measure the length and width of the house accordingly (47.5 by 26.5 foot lengths). Find the area.

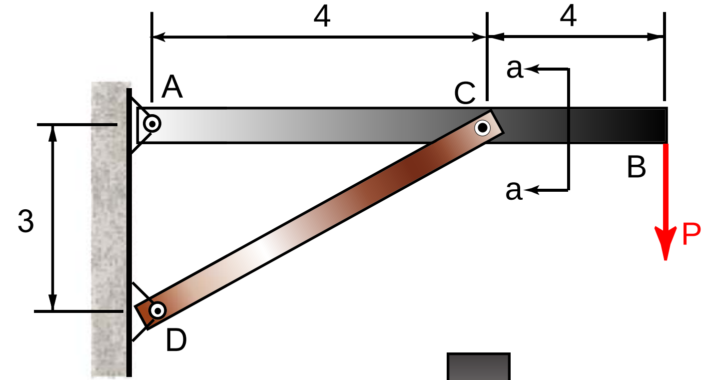
Trial and Error Solutions

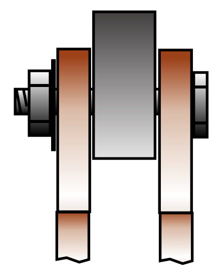
Example

Find x given: $0=73.6 - 100\sin(x) - 45\cos(x)$

Example

A beam AB is supported by a strut CD and carries a load P= 2500 lb. The strut, which consists of two bars, is connected to the beam by a bolt passing through each of the bars at joint C. (a) If the allowable average shear stress in the bolt is 14,000 psi, what is the minimum required diameter d of the bolt? (b) If the allowable bearing stress on the strut is 20 ksi and the thickness of the strut is 0.25 inches, find the minimum diameter. Units: Lbs, ft.



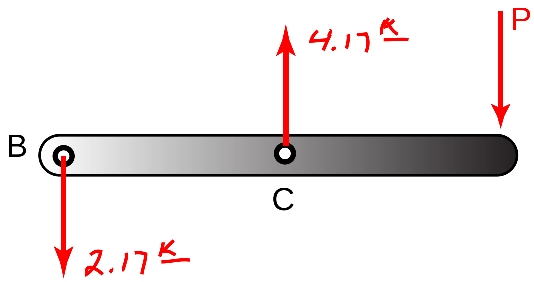


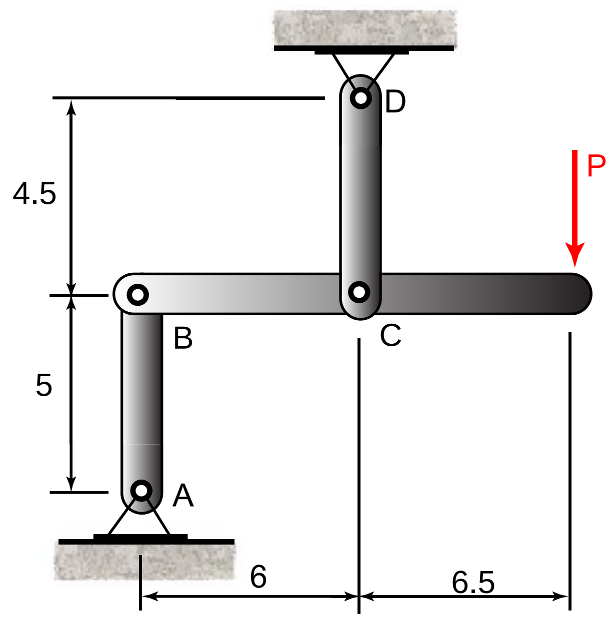
Section a-a

Example

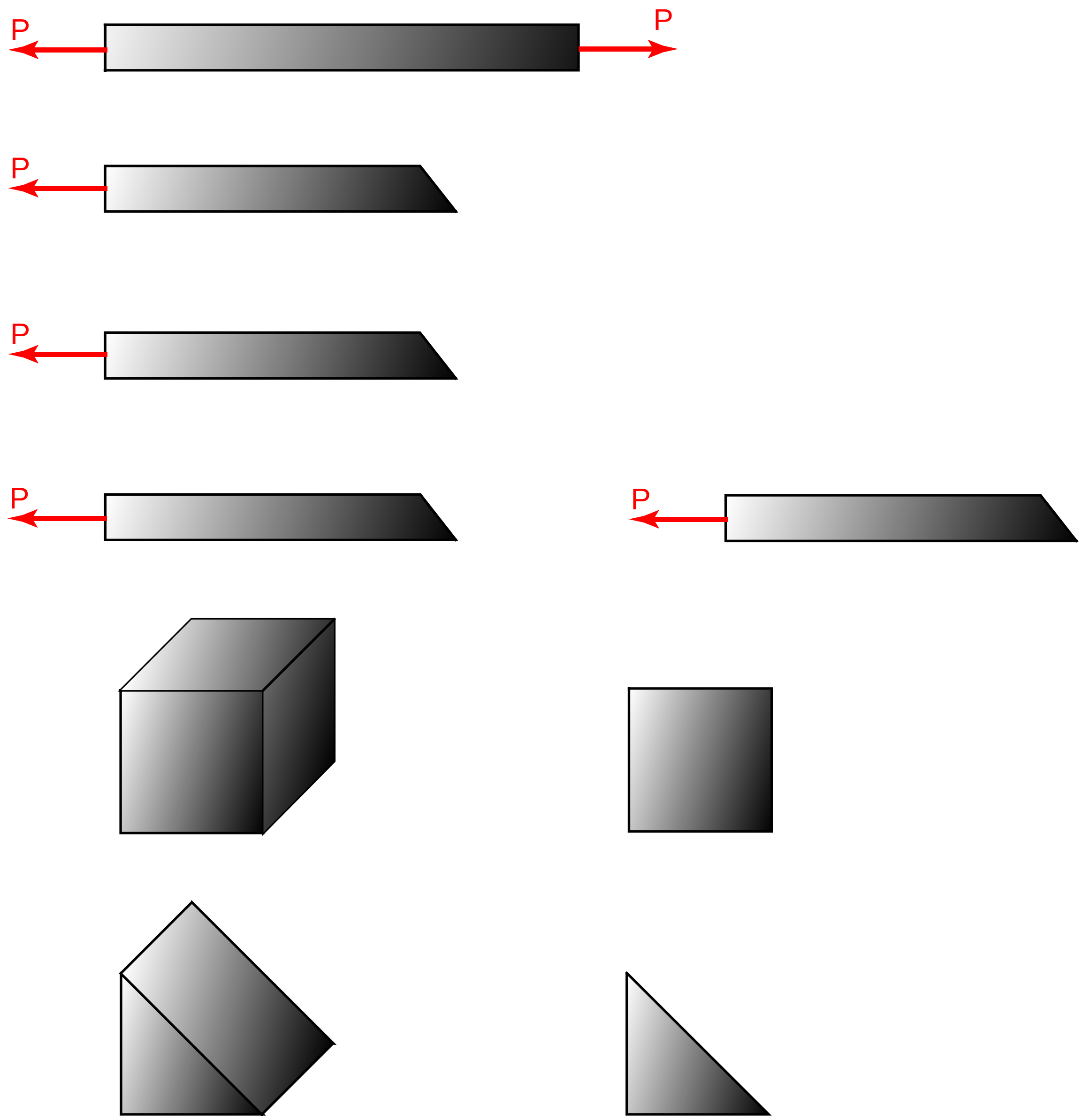
Bar AB has a cross-section of .25"x4". (a) With a load of 2-k at the end, what is the maximum bolt size at B based on a maximum net stress of 24,000 psi in member AB. (b) If the bolt has a shear stress allowable of 21,600 psi and a bearing stress allowable of 32,400 psi, find the minimum bolt size at joint B. Note: The units of "k" means 1000 lbs, often referred to as "kips". Units: Lbs, ft.

Recall from a previous solution:





STRESSES ON INCLINED SECTIONS



$$\sigma_\theta = \sigma_x \cos^2\theta$$

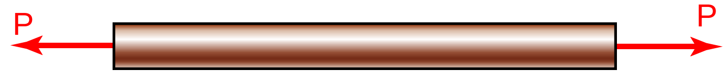
$$\sigma_{max} = \sigma_x \ @\theta = 0°$$

$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$

$$\tau_{max} = \frac{\sigma_x}{2} \ @\theta = 45°$$

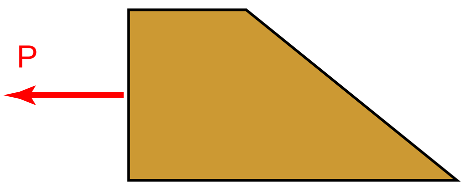
Example

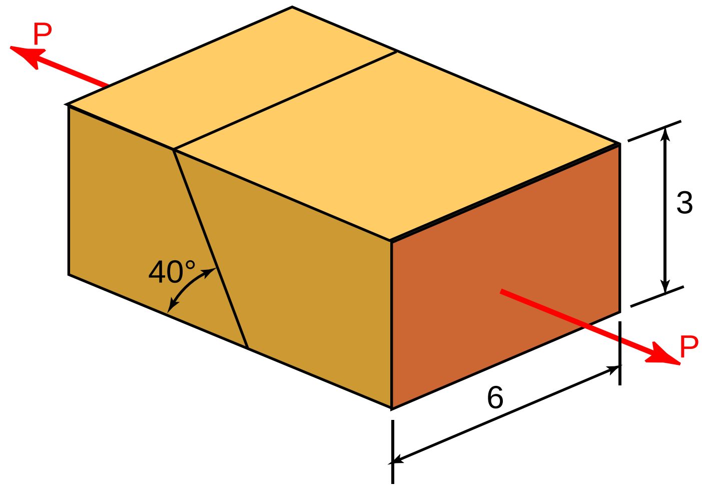
A circular steel rod is to carry a tensile load P= 140 kN. The allowable stresses in tension and shear are 120 MPa and 55 MPa, respectively. What is the minimum required diameter d of the rod?



Example

Two wooden rectangular members with a cross section of 3"x6" are joined by the simple glued 40° scarf splice shown. Knowing that the maximum allowable shearing stress in the glue splice is 90 psi and 120 psi in tension, determine the largest load P which can be safely applied. Units: Lbs, in.





DESIGN CONSIDERATIONS

Ultimate Strength

$$\sigma_U = \frac{P_U}{A}$$

Factor of Safety

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate load}}{\text{allowable load}}$$

$$\text{Factor of safety} = F.S. = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Determination:

-Variations that may occur in the properties of the member under considerations.

-The number of loading cycles that may be expected during the life of the structure or machine.

-The type of loadings that are planned for the future in the design, or that may occur in the future.

-The type of failure that may occur.

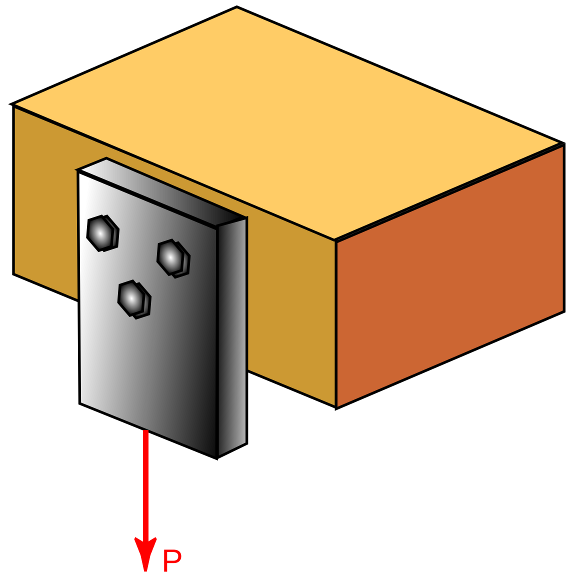
-Uncertainty due to methods of analysis.

-Deterioration that may occur in the future because of poor mainte-ance or because of unpreventable natural causes.

-The importance of a given member to the integrity of the whole structure.

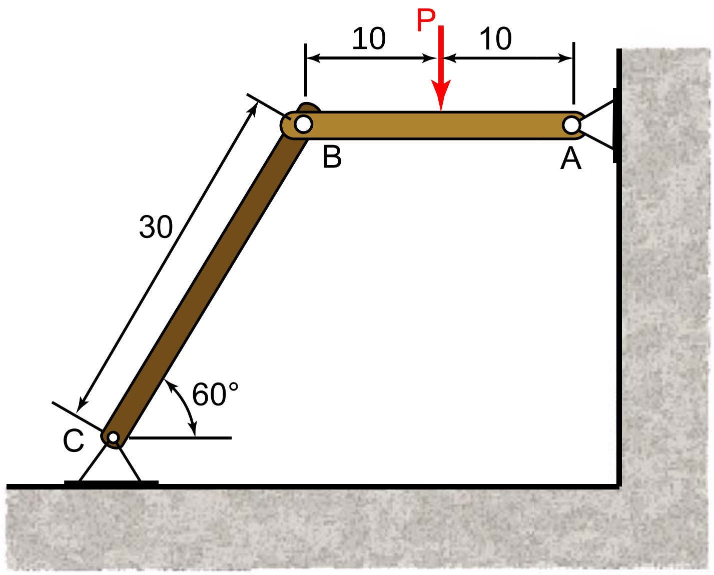
Example

Three steel bolts are to be used to attach the steel plate shown to a wooded beam. Knowing that the plate will support a 24-kip load, that the ultimate shearing stress for the bolt is 52 ksi, and a factor of safety of 3.37 is desired, determine the required diameter of the bolt.



Example

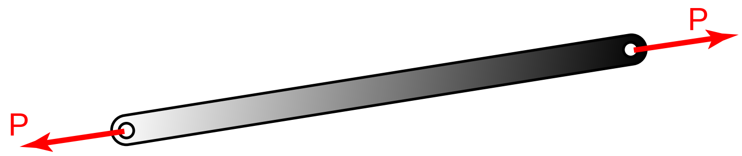
A 5/8” bolt is used at C to connect to the wooden member BC that has a cross-sectional area of 5.25 in². Knowing that the utimate shearing stress is 58 ksi for the bolt and that the ultimate normal stress is 7.2 ksi for member BC, determine the allowable load P if an overall factor of safety of 3.0 is desired. Units: Kips, in.



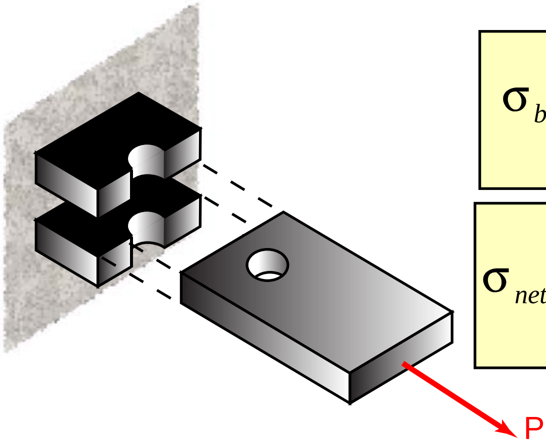
SUMMARY

Normal Stress

$$\sigma = \frac{P}{A}$$



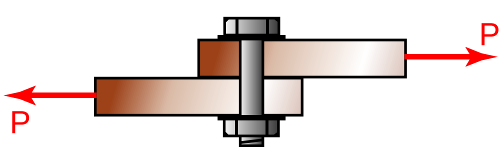
Stresses in Connections

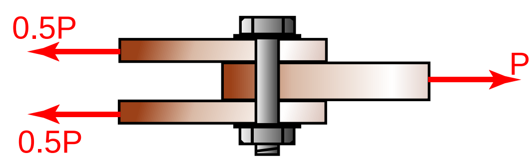


$$\sigma_{bearing} = \frac{Force}{thk_{plate}\, dia_{bolt}}$$

$$\sigma_{net} = \frac{Force}{Area_{net}} = \frac{Force}{thk_{plate}(width_{plate} - dia_{bolt})}$$

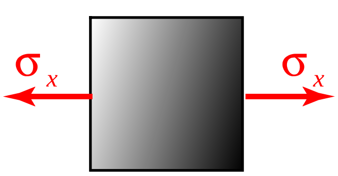
Shear Stresses in Bolts





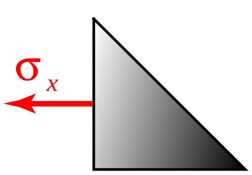
$$\tau_{ave} = \frac{Force_{bolt}}{A_{bolt}}$$

Stresses on Inclined Sections



$$\sigma_\theta = \sigma_x \cos^2\theta$$

$$\sigma_{\max} = \sigma_x \; @\theta = 0°$$



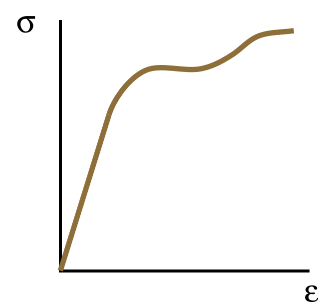
$$\tau_\theta = -\sigma_x \sin\theta \cos\theta$$

$$\tau_{\max} = \frac{\sigma_x}{2} \; @\theta = 45°$$

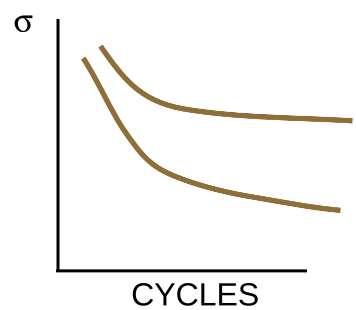
Chapter 2
Stress and Strain- Axial Loading

INTRODUCTION

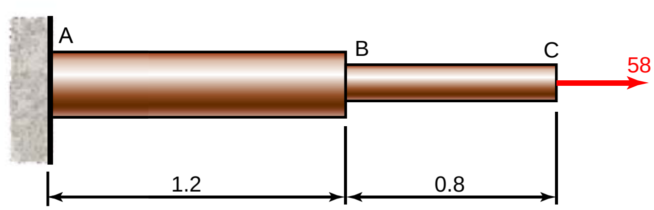
Stress and Strain



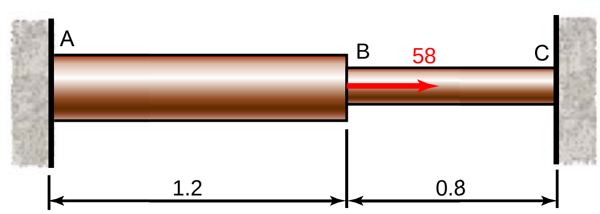
Repeated Loadings; Fatigue



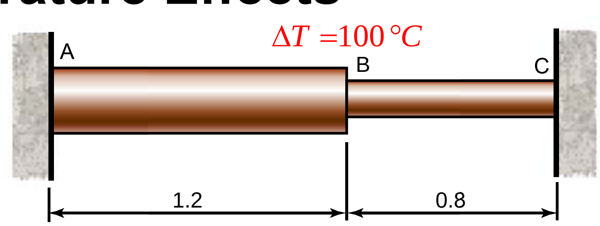
Deformation of Members Under Axial Loading



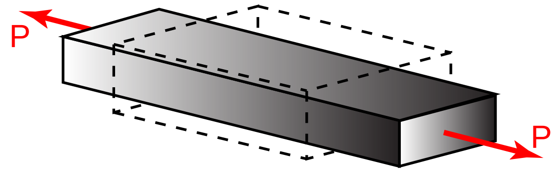
Statically Indeterminate Problems



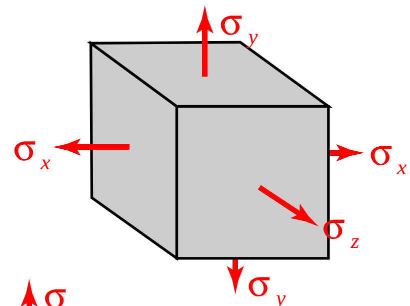
Temperature Effects



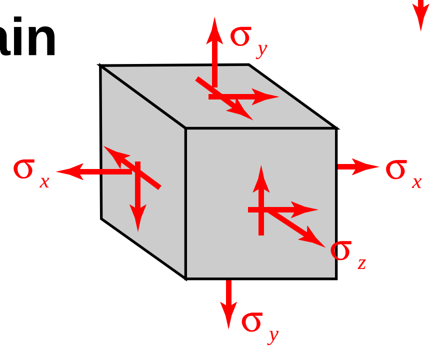
Poisson's Ratio

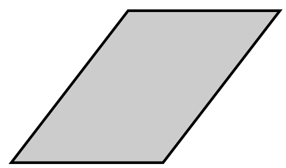


Multiaxial Loading; Generalized Hooke's Law

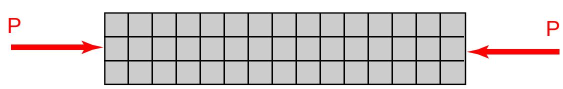


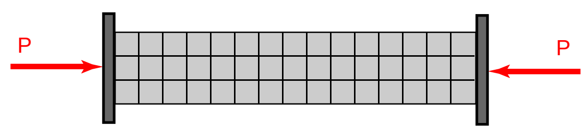
Shearing Strain



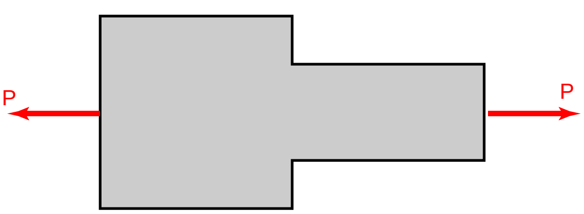


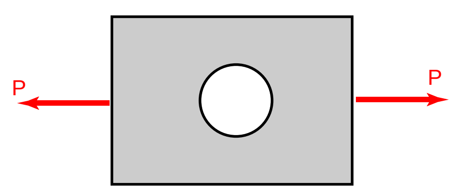
Saint-Venants Principle



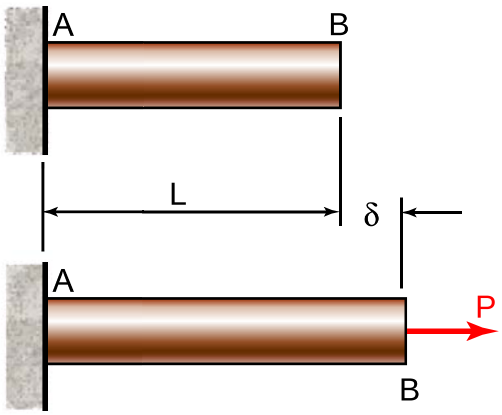


Stress Concentrations



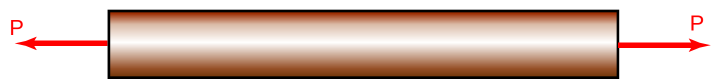


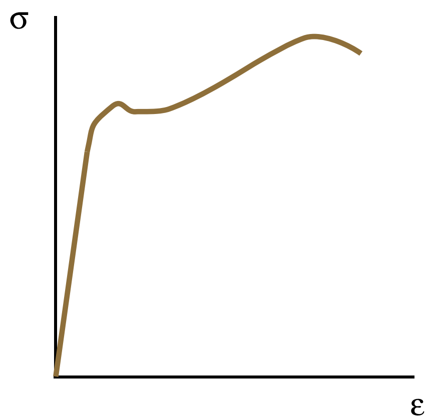
NORMAL STRAIN UNDER AXIAL LOADING



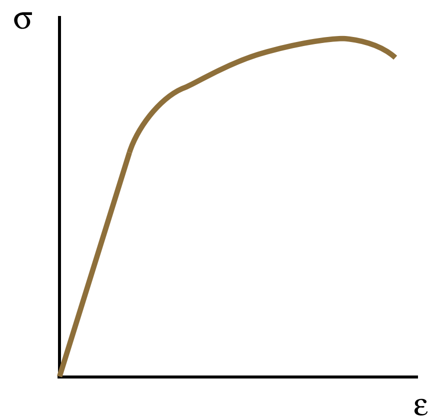
$$\varepsilon = \frac{\delta}{L}$$

STRESS-STRAIN DIAGRAMS

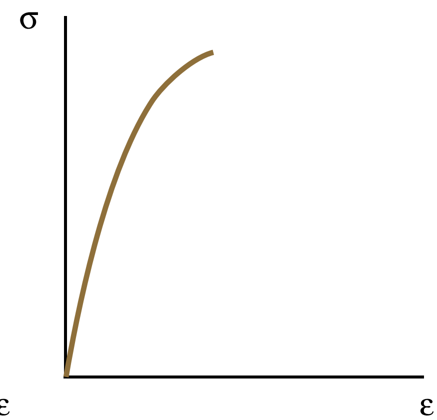




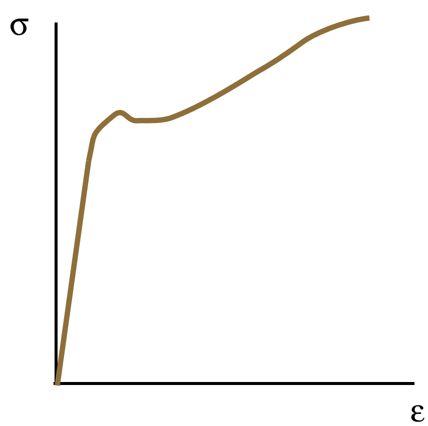
Low-carbon steel



Aluminum Alloy



Brittle material

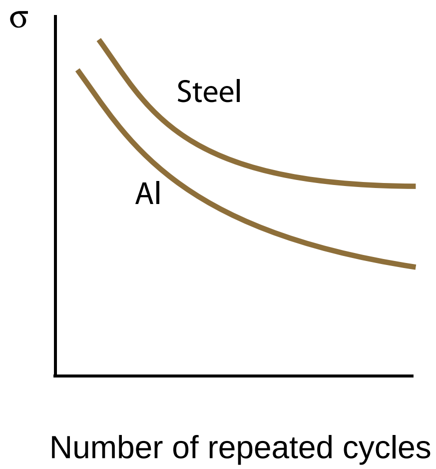


True stess-strain diagram
(ductile material)

Hooke's Law; Modulus of Elasticity

$$\sigma = E\varepsilon$$

REPEATED LOADINGS; FATIGUE



ENDURANCE LIMIT- The stress for which failure does not occur, even for an indefinitely large number of loadings.

FATIGUE LIMIT- The stress corresponding to failure after a specified number of loading cycles, such as 500 million.

EXAMPLE

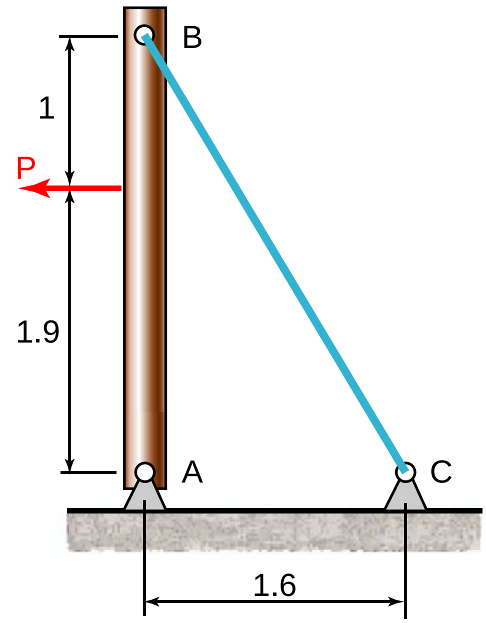
A 5 kN force is applied to a 25 m steel wire. Knowing that E= 200 GPa and the wire stretches 19 mm, determine the (a) diameter of the wire, (b) the corresponding normal stress.

EXAMPLE

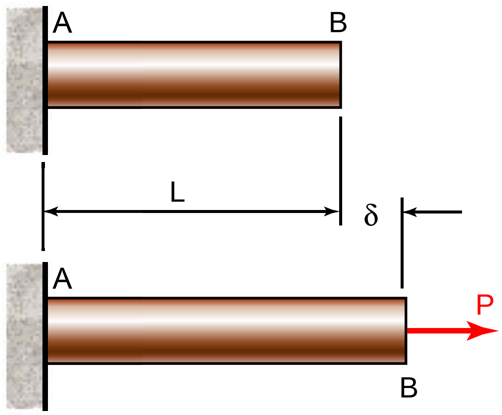
A square aluminum bar should not stretch more than 1.6 mm. Knowing that E= 70 GPa and the allowable tensile strength is 120 MPa, determine (a) the maximum allowable length of the bar, (b) the required dimensions of the cross section if a tensile load of 32 kN is applied.

EXAMPLE

The 5 mm diameter steel wire BC has an E value of 200 GPa. If the maximum normal stress in the wire is not to exceed 185 MPa and an elongation of 6 mm, find the applied load P.



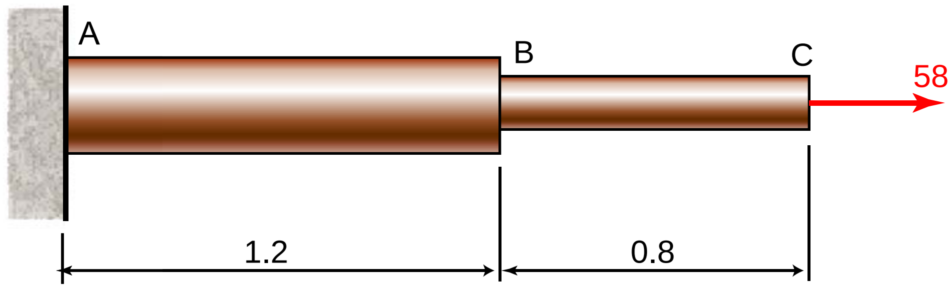
DEFORMATIONS OF MEMBERS UNDER AXIAL LOADINGS



$$\delta = \frac{PL}{AE}$$

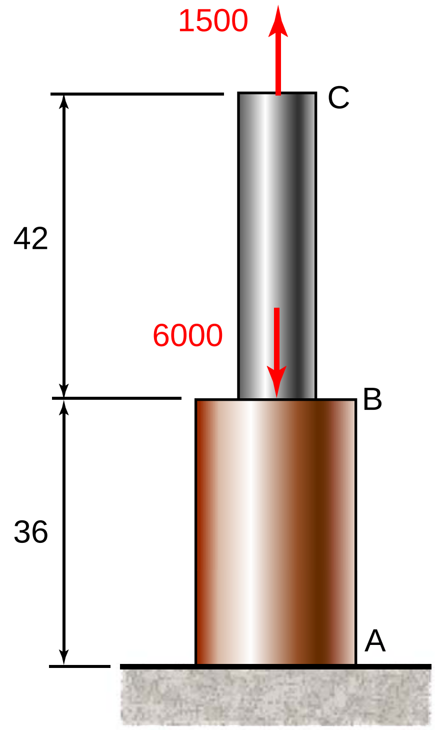
EXAMPLE

Knowing that rod AB has a diameter of 45 mm, determine the diameter for BC for which the displacement of point C will be 3 mm. E= 105 GPa. Units: kN, m.



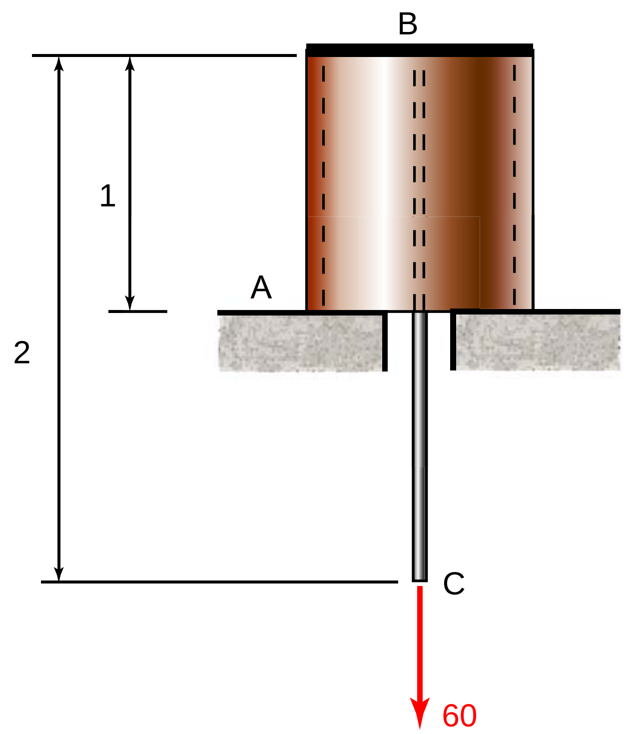
EXAMPLE

The 3" diameter rod AB is made of copper (E= 17,000 ksi) and BC is made with aluminum (E= 10,000 ksi). Determine the diameter of rod BC so that the displacement of C is 0. Units: lbs, in.



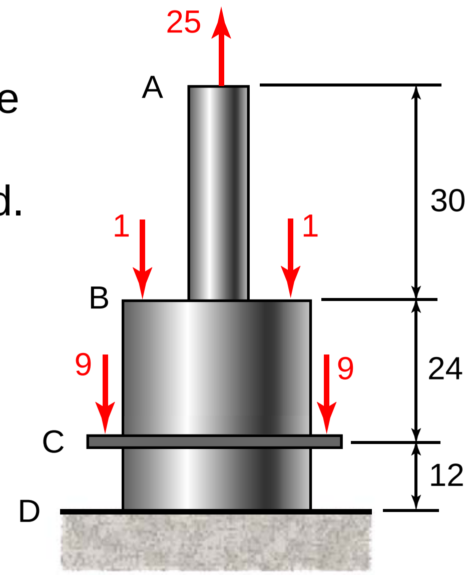
Example

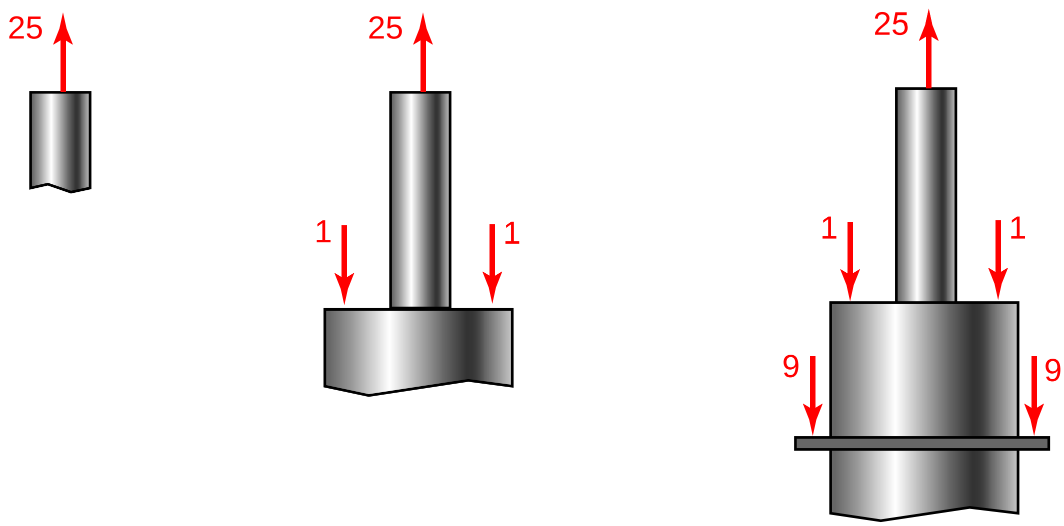
Determine the displacement at the end of the rod at point C. The brass pipe section AB has an outside diameter of 75 mm and thickness of 4 mm. The steel rod is attached to a rigid plate on the top of the pipe. The steel rod BC has a diameter of 10 mm.
E (steel)= 200 GPa and E (brass)= 105 GPa. Units: kN, m.



Example

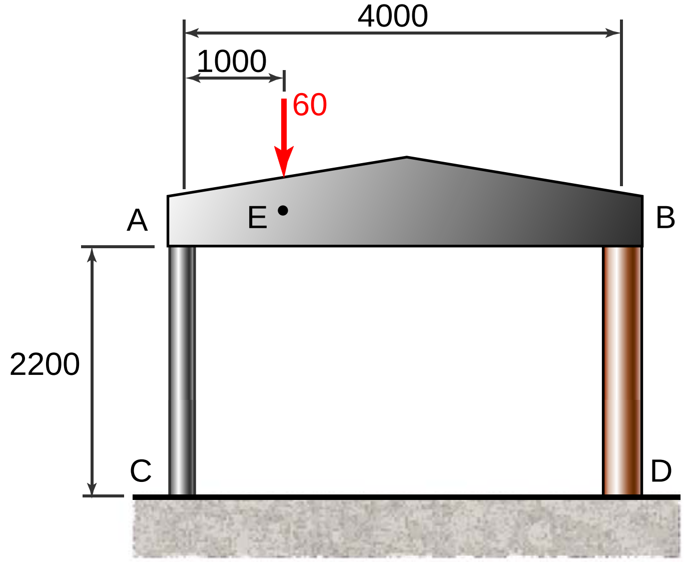
The two steel bar segments, AB and BD, have cross-sectional areas of 2 and 5 in², respectively. At C a rigid thin plate is installed. Determine the vertical displacement of A.
E= 29000 ksi. Units: kips, in.

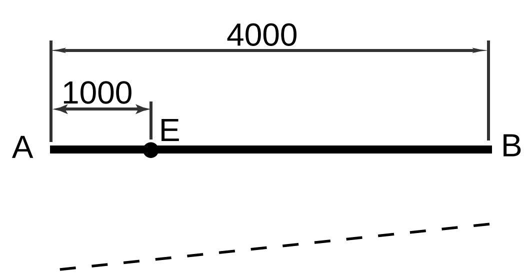




Example

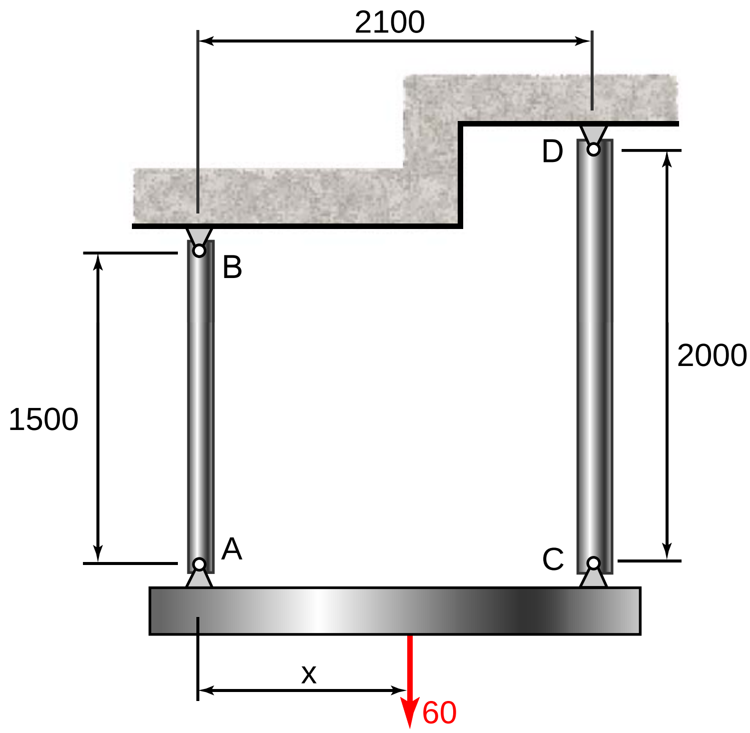
Post AC is made of steel and has a diameter of 18 mm, and BD is made of copper and has a diameter of 42 mm. Determine the displacement of point E on the rigid beam AB. E(steel)= 200 GPa, E(copper)= 120 GPa. Units: mm, kN





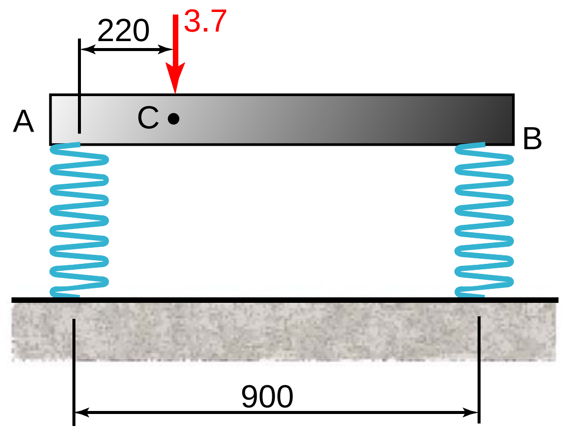
Example

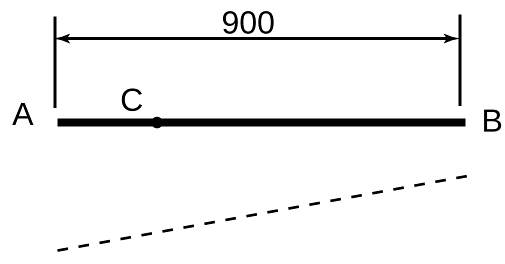
Two steel bars are pin-connected to a rigid member. Determine the location where the 60 kN force should be applied so that the rigid member AC remains horizontal . Bar AB has a cross-sectional area of 15 mm^2, and bar CD has a cross-sectional area of 25 mm^2. E(steel)= 200 GPa. Units: kN, mm.



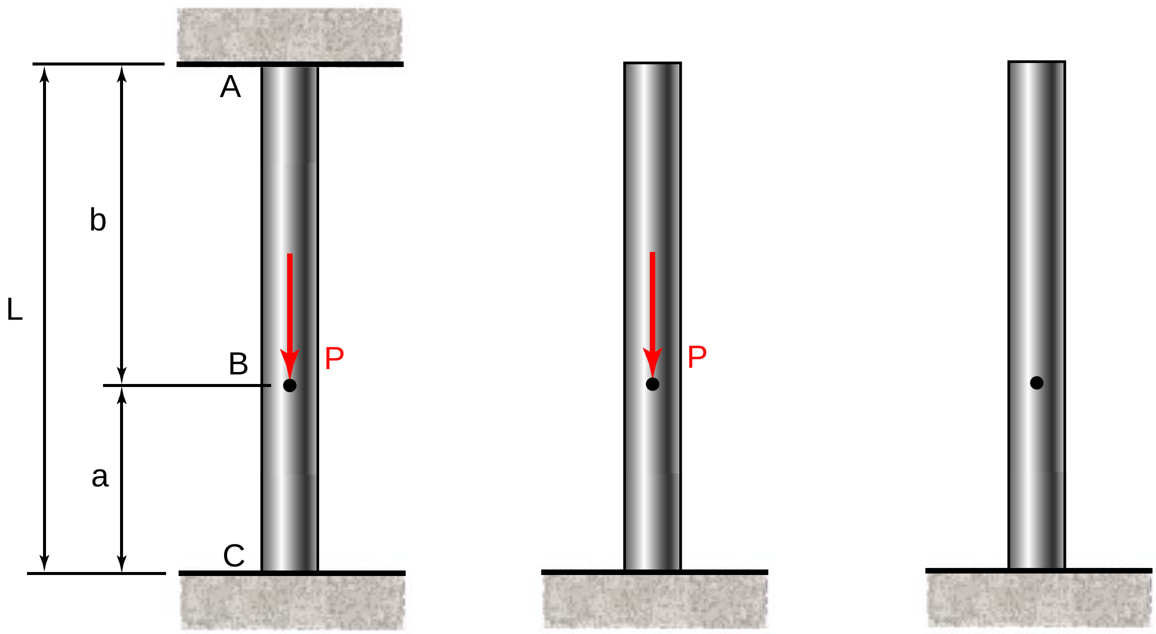
Example

The horizontal rigid beam AB rests on the two short springs with the same length. The spring at A has stiffness of 250 kN/m and the spring at B has a stiffness of 150 kN/m. Determine the displacement under the load.   Units: kN, mm.



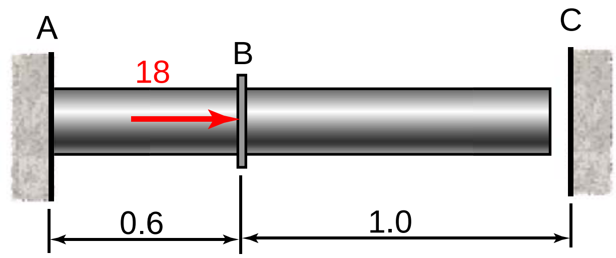


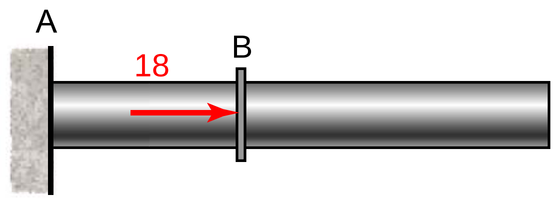
STATICALLY INDETERMINATE PROBLEMS

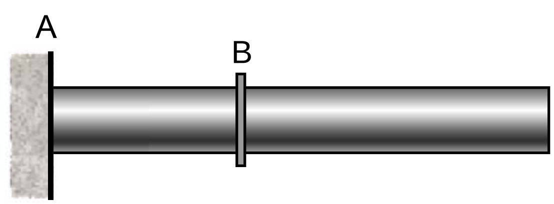


EXAMPLE

The steel rod has a diameter of 7 mm. It is attached to the fixed wall at A, and before it is loaded there is a 1 mm gap between the wall at C and the rod. Neglecting the collar at B, find the reactions at A and C. E (steel)= 200 GPa. Units: kN, m.

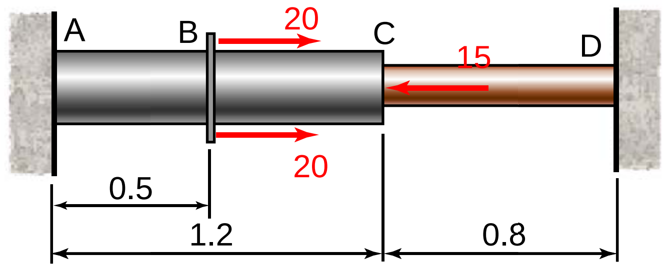


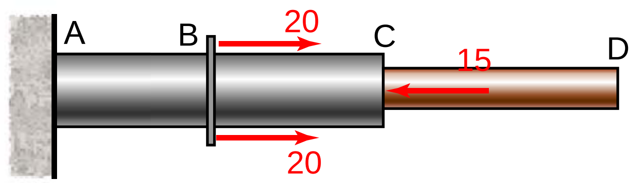


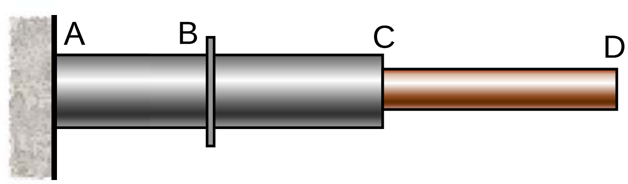


Example

The assembly ABCD is welded to the wall at A and D. The steel rod ABC has a diameter of 11 mm and the copper rod CD has a diameter of 7 mm. A thin rigid flange is placed at B. Determine the displacement of point B. E (steel)= 200 GPa, E (copper)= 120 GPa. Units: kN, m.

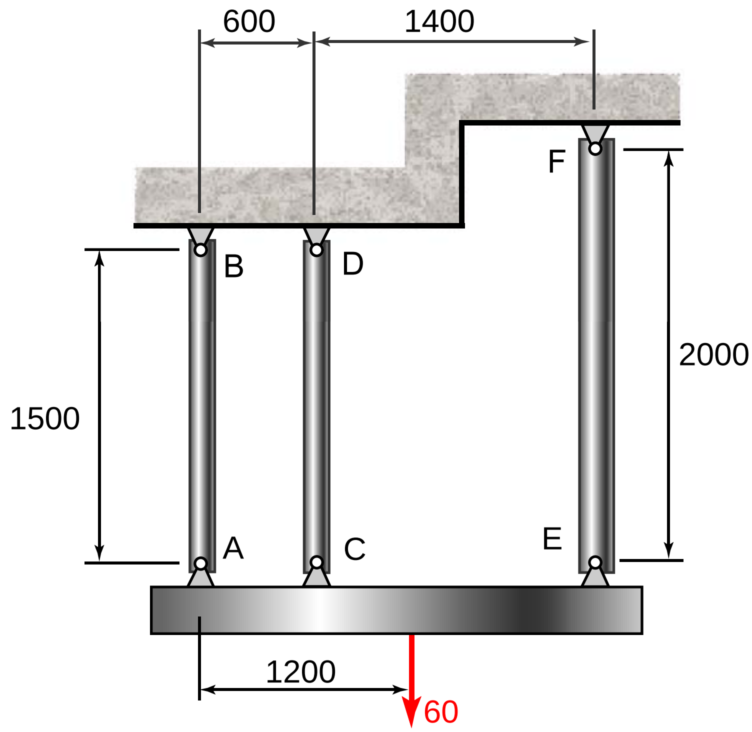


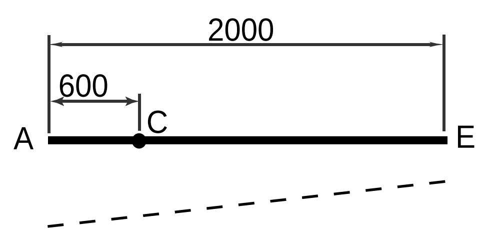




Example

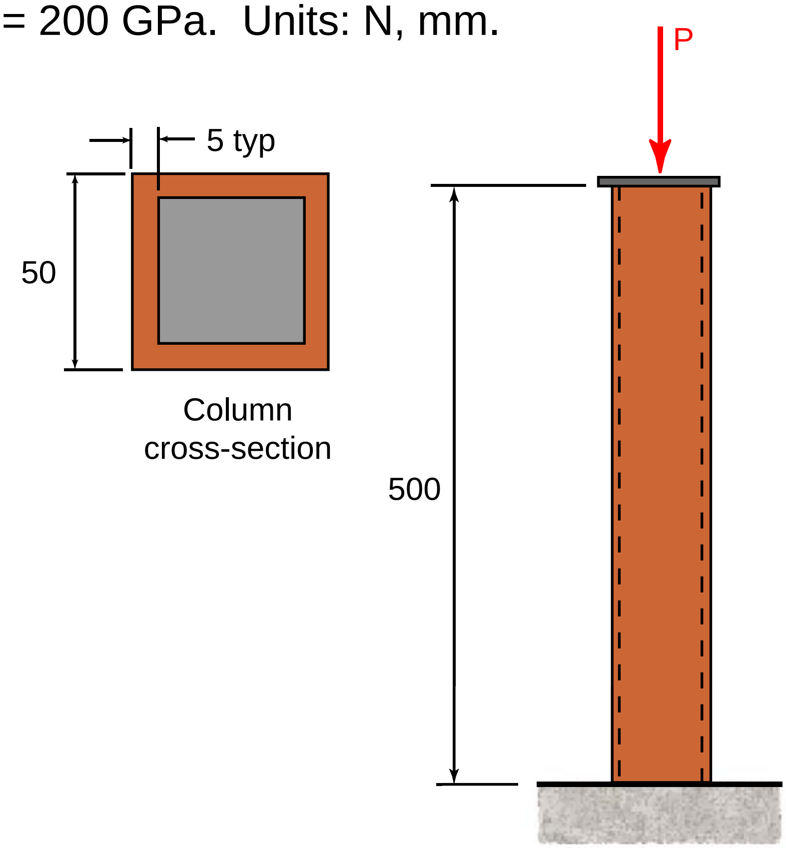
The three steel bars are pin-connected to a rigid member. Determine the force developed in each bar. Bars AB and CD each have a cross-sectional area of 15 mm², and bar EF has a cross-sectional area of 25 mm². E(steel)= 200 GPa. Units: kN, mm.





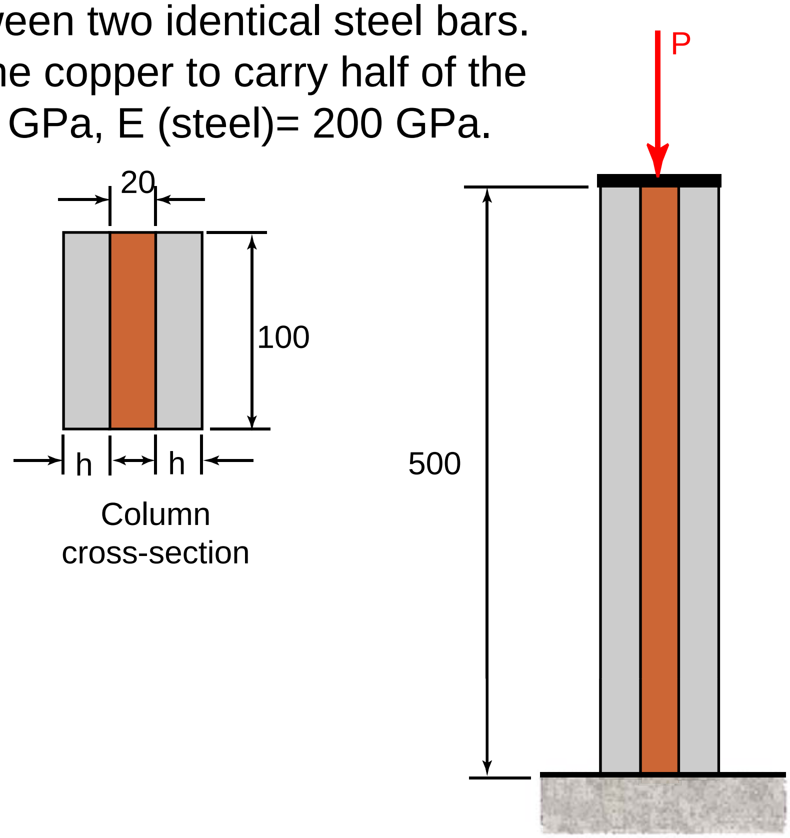
Example

The square column has an outer shell of brass and and interior core of steel. Find the force required to create a shortening of 0.20 mm.
E (brass)= 105 GPa, E (steel)= 200 GPa. Units: N, mm.



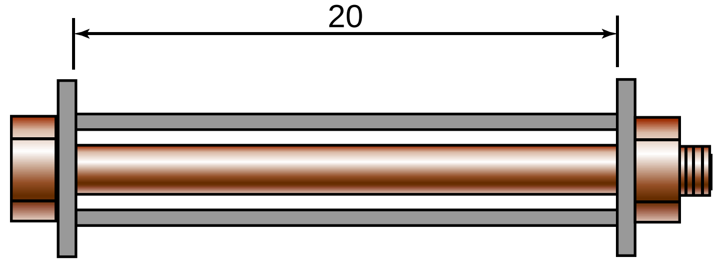
Example

A copper bar is placed between two identical steel bars. Determine "h" in order for the copper to carry half of the total load. E (copper)= 120 GPa, E (steel)= 200 GPa. Units: N, mm.



Example

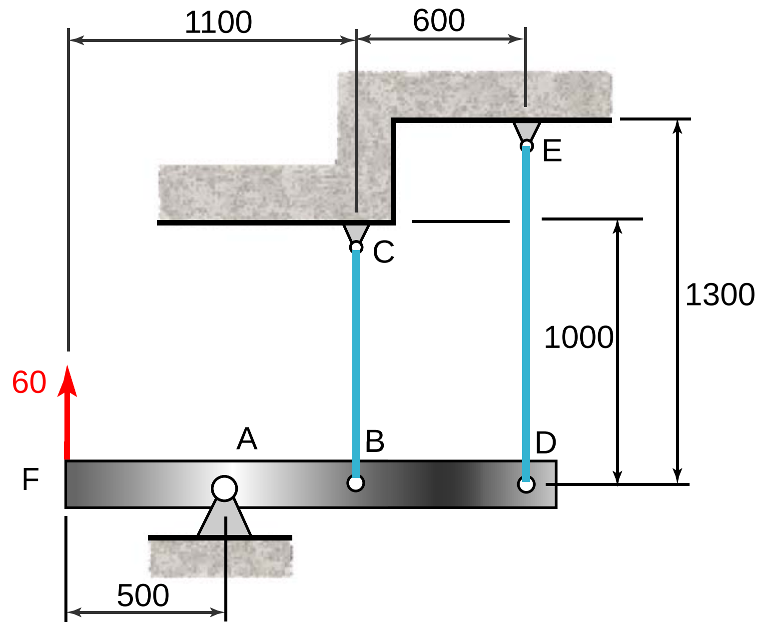
A brass bolt with a diameter of 0.375" is fitted inside a 7/8" diameter steel tube with a wall thickness of 1/8". After the nut has been snugged, it is tightened 1/4 turn. The bolt is single threaded and has a pitch of 0.1". Determine the normal stress in the bolt and the tube. E (brass)= 15,000 ksi and E (steel)= 29,000 ksi. Units: kips (k), in.

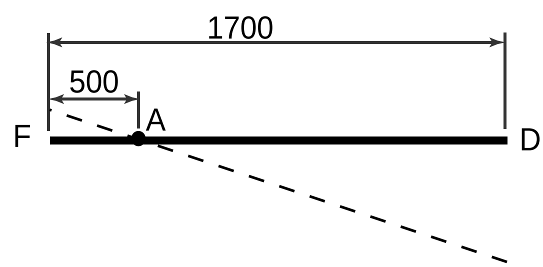


Example

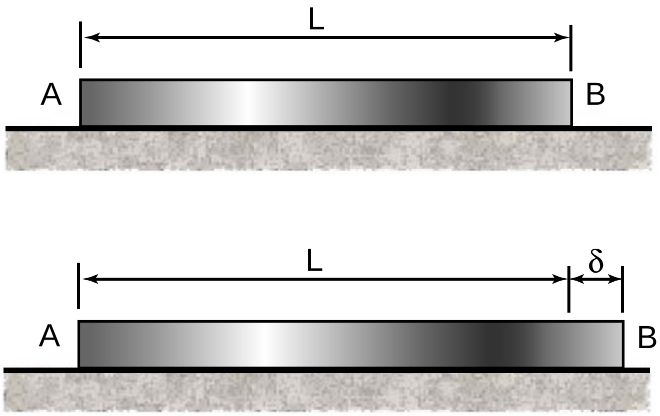
The rigid steel beam is pin-connected at A and to two 6 mm diameter steel wires. Determine the force developed in each wire.

E(steel)= 200 GPa. Units: kN, mm.





PROBLEMS INVOLVING TEMPERATURE CHANGE



$$\delta_T = \alpha(\Delta T)L$$

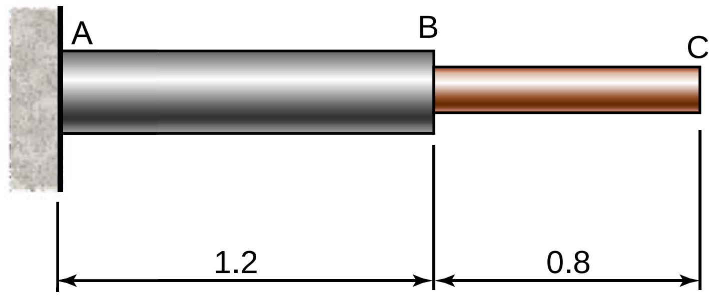
Example

The steel rod AB has a diameter of 11 mm and the copper rod BC has a diameter of 7 mm. Determine the displacement of point C if the assembly is subjected to a temperature increase of 50°C.

Units: m.

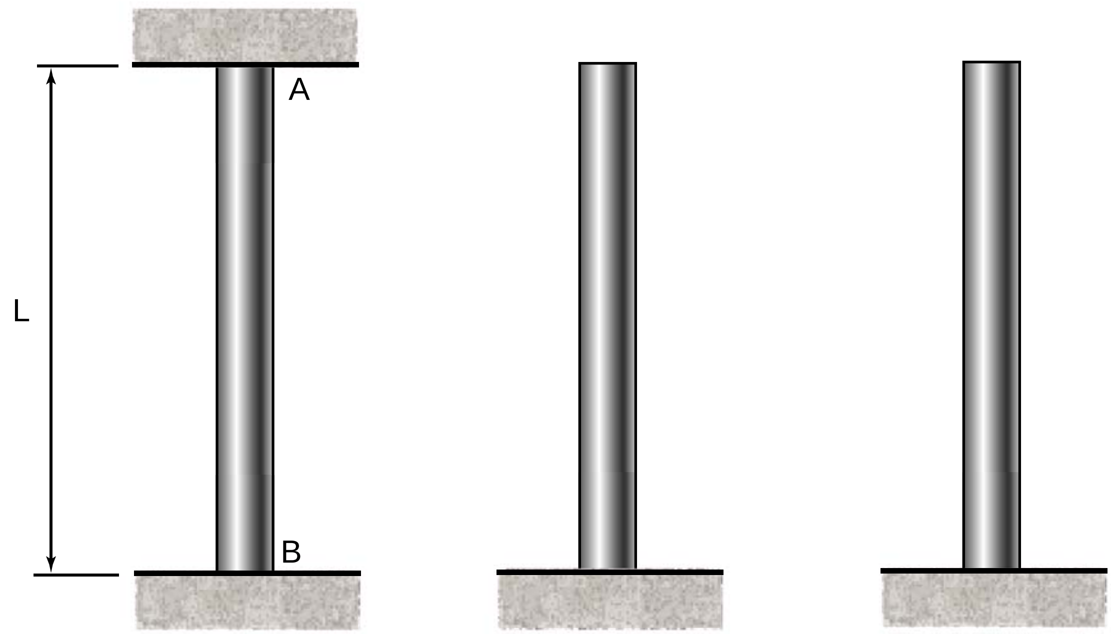
Copper: α= 17E-6/ºC

Steel: α= 11.7E-6º/C



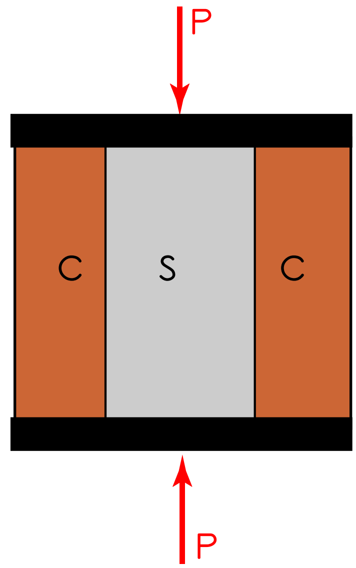
Example

The steel rod shown is subjected to a temperature increase of 60˚F. Calculate the reactions at the supports and the stress in the bar. E(steel)= 29,000 ksi, α= 0.0000065/°F, area= 4 sq. in. Units: k (kips),



Example

A solid steel rod S is placed inside a copper pipe C having the same length. The coefficient of thermal expansion of copper is larger than the coefficient of steel. After being assembled, the cylinder and tube are compressed between two rigid plates by forces P. Obtain a formula for the increase in temperature that will cause all of the load to be carried by the copper tube. Units: k (kips), in.

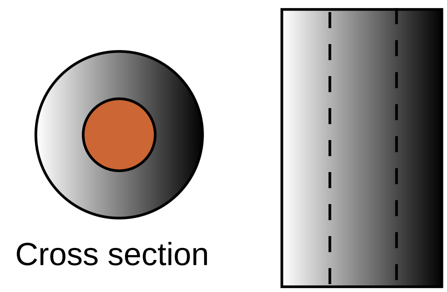


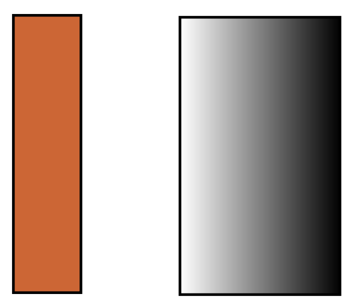
Example

The 2.5" diameter aluminum shell is completely bonded to the 1" diameter brass core and is unstressed at 70°F. Determine the stress in each if the temperature is raised to 170°F.

Brass: E= 15,000 ksi, α= 11.6E-6/°F

Aluminum: E= 10,600 ksi, α= 12.9E-6/°F



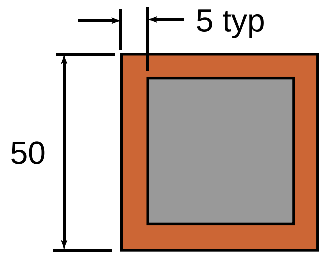


Example

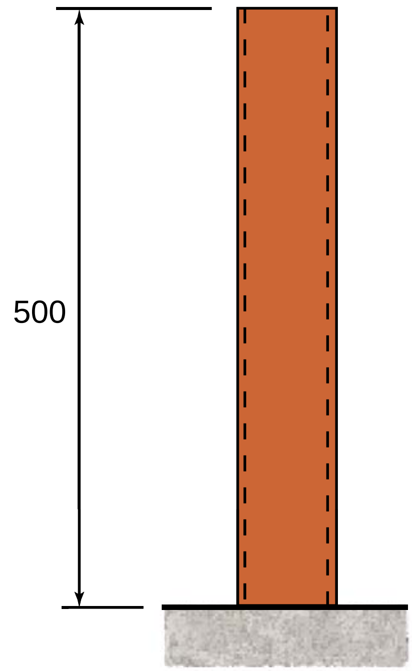
The square column has an outer shell of brass and inner core of steel. Determine the largest allowable temperature increase if the stress in the steel is not to exceed 55 MPa. Units: mm.

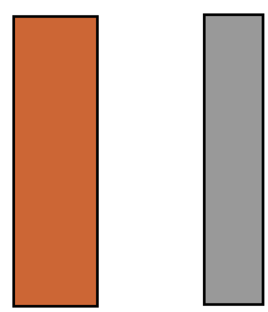
E (brass)= 105 GPa, α= 20.9E-6/°C

E (steel)= 200 GPa, α= 11.7E-6/°C



Column
cross-section





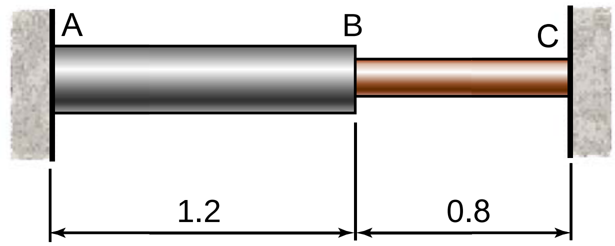
Example

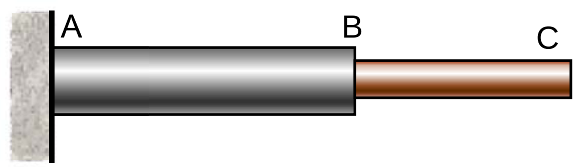
The steel rod AB has a diameter of 11 mm and the copper rod BC has a diameter of 7 mm. Determine the reactions if the assembly is subjected to a temperature increase of 50°C.

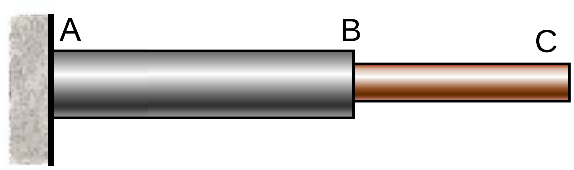
Units: kN, m.

E (copper)= 120 GPa, α= 17E-6/°C

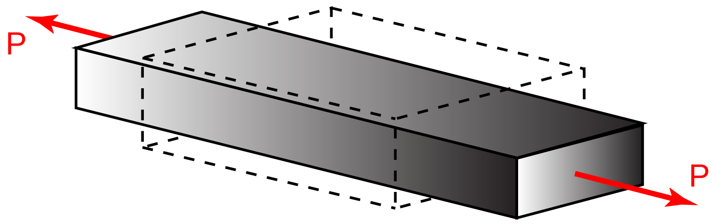
E (steel)= 200 GPa, α= 11.7E-6/°C







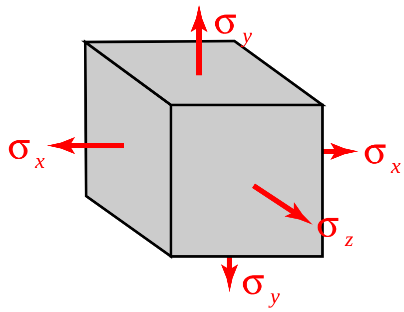
POISSON'S RATIO



$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

$$\varepsilon_y = \varepsilon_z = -\frac{\nu \sigma_x}{E}$$

MULTIAXIAL LOADING; GENERALIZED HOOKE'S LAW

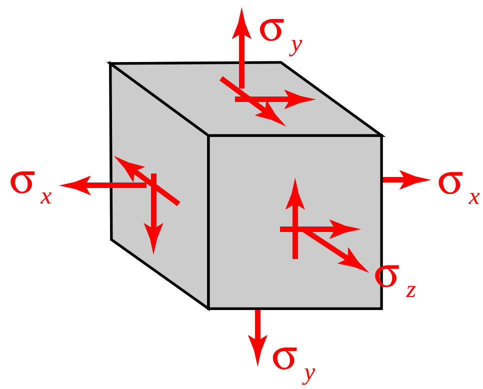


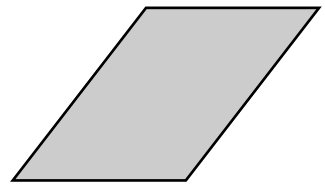
$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

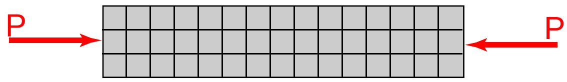
SHEARING STRAIN

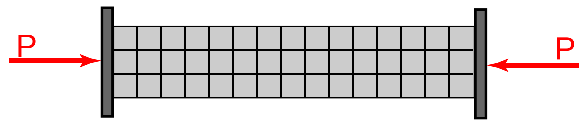


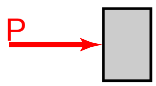


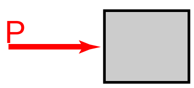
$$\tau_{xy} = G\gamma_{xy}$$
$$\tau_{yz} = G\gamma_{yz}$$
$$\tau_{zx} = G\gamma_{zx}$$

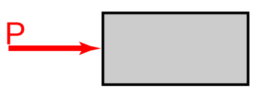
SAINT-VENANT'S PRINCIPLE



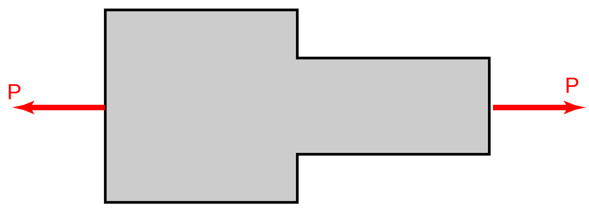


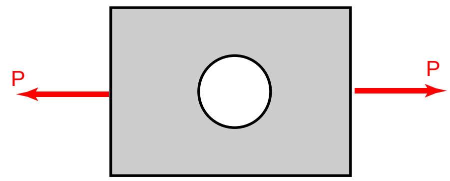


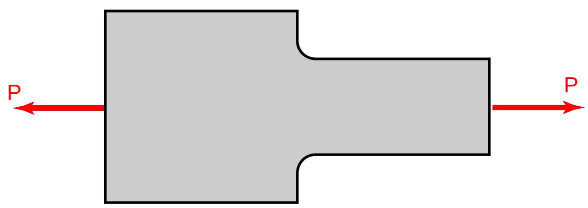


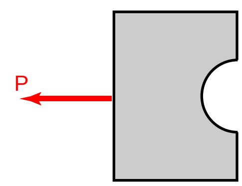


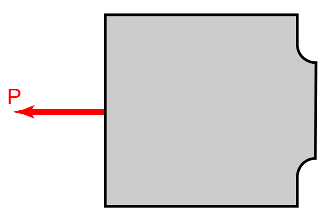
STRESS CONCENTRATIONS



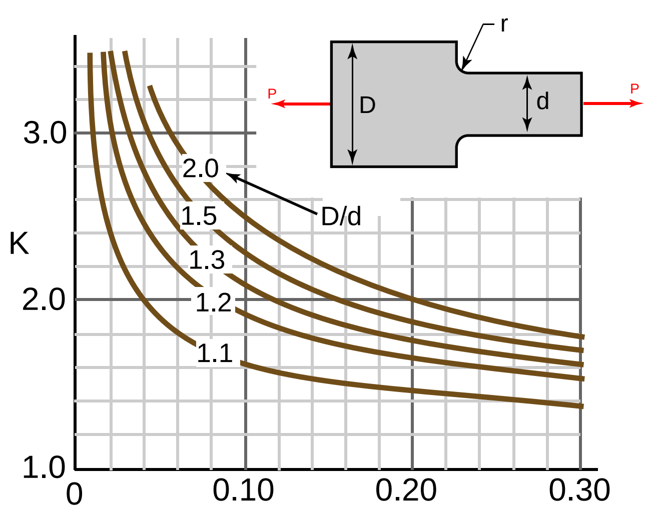


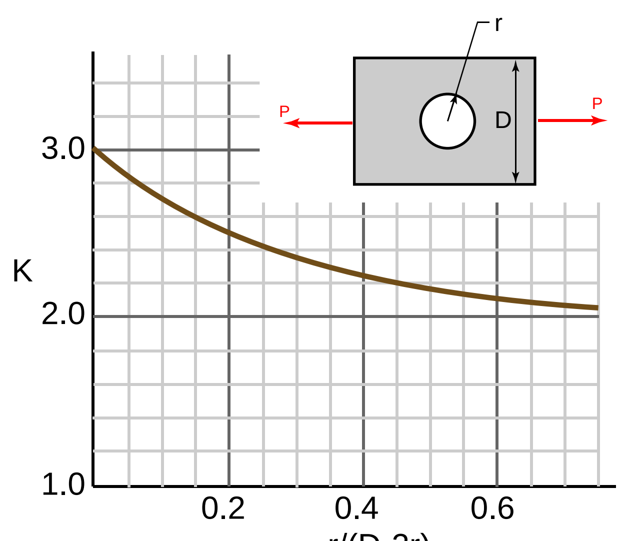






$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

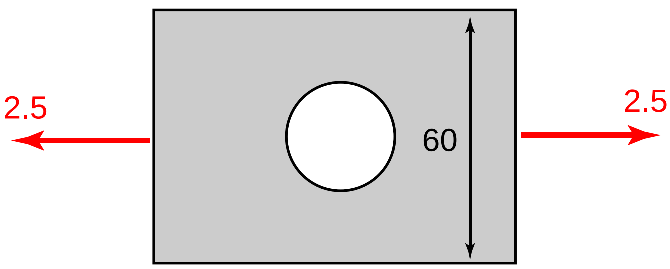


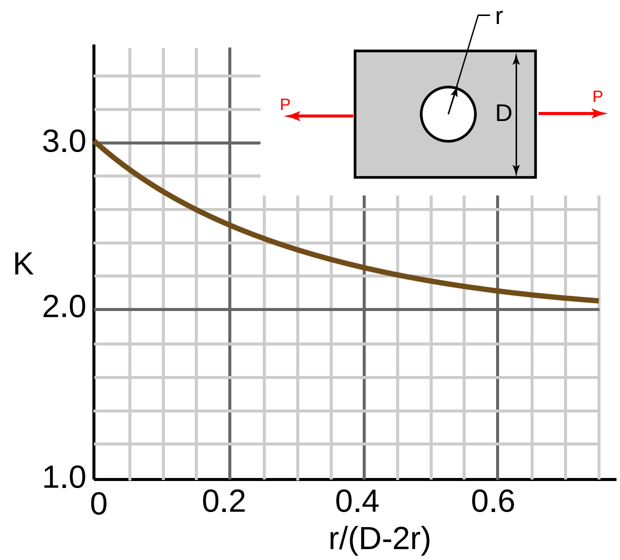


W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

EXAMPLE

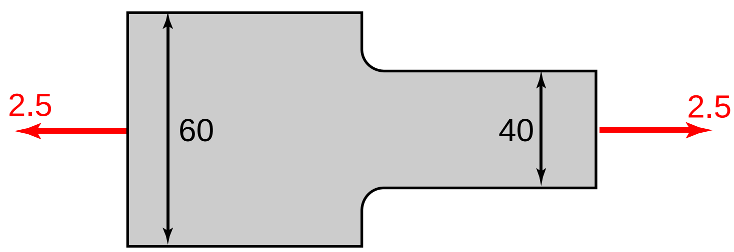
For the 5 mm thick bar, determine the maximum normal stress for hole diameters 12 mm and 20 mm. Units: kN, mm.

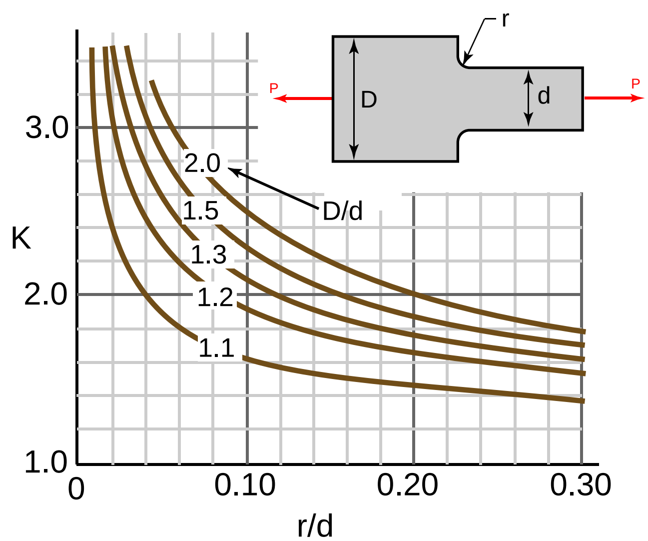




EXAMPLE

For the 5 mm thick bar, determine the maximum normal stress for fillet radii of 6 mm and 10 mm. Units: kN, mm.

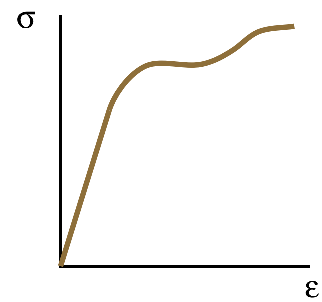




SUMMARY

Stress and Strain

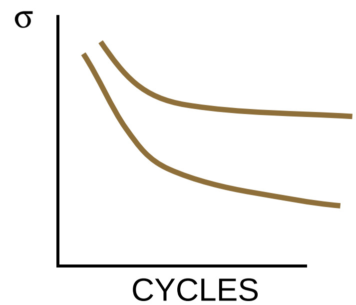
$$\sigma = E\varepsilon$$



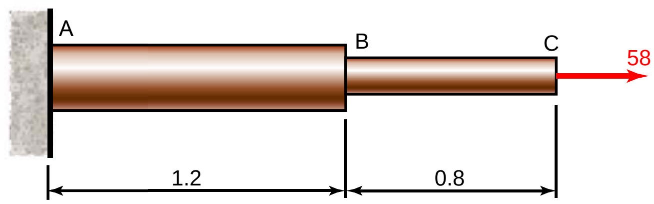
Repeated Loadings; Fatigue

ENDURANCE LIMIT- The stress for which failure does not occur, even for an indefinitely large number of loadings.

FATIGUE LIMIT- The stress corresponding to failure after a specified number of loading cycles, such as 500 million.

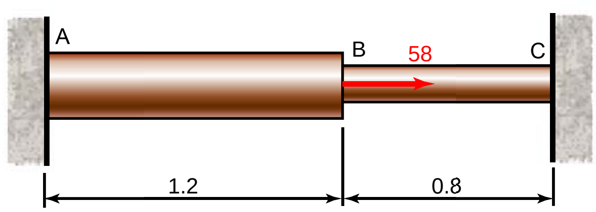


Deformation of Members Under Axial Loading

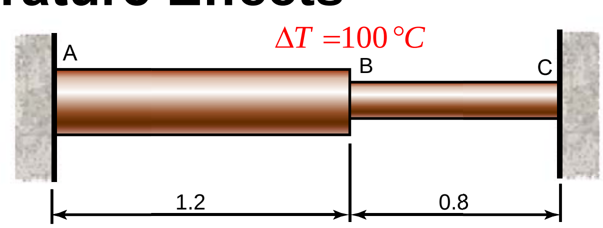


$$\delta = \frac{PL}{AE}$$

Statically Indeterminate Problems

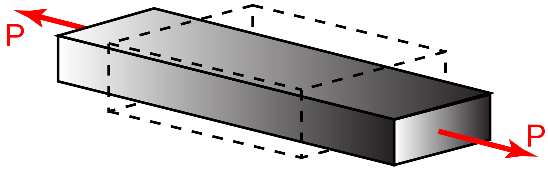


Temperature Effects



$$\delta_T = \alpha(\Delta T)L$$

Poisson's Ratio



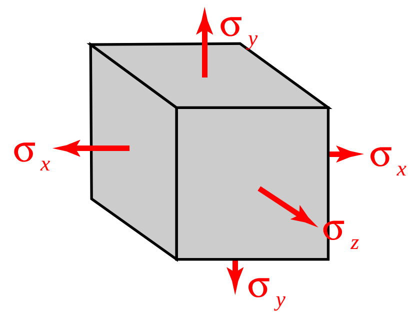
$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

Multiaxial Loading; Generalized Hooke's Law

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

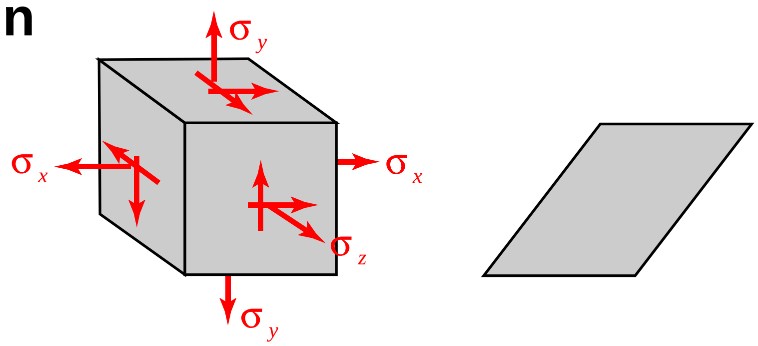


Shearing Strain

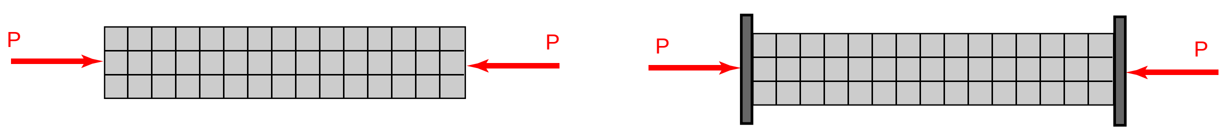
$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz}$$

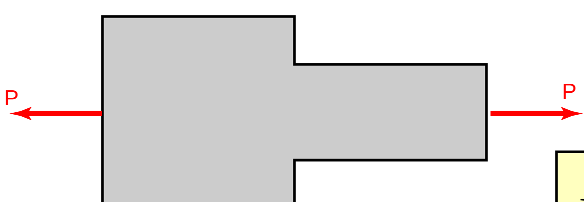
$$\tau_{zx} = G\gamma_{zx}$$



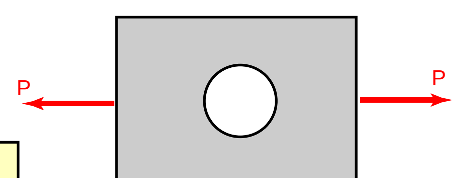
Saint-Venants Principle



Stress Concentrations



$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

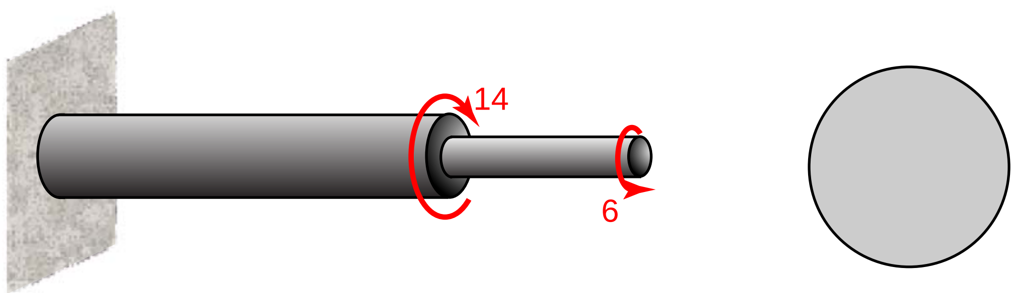


Chapter 3
Torsion

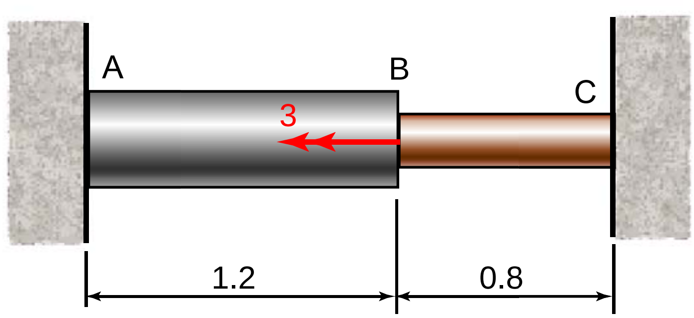
INTRODUCTION

Stresses in the Elastic Range

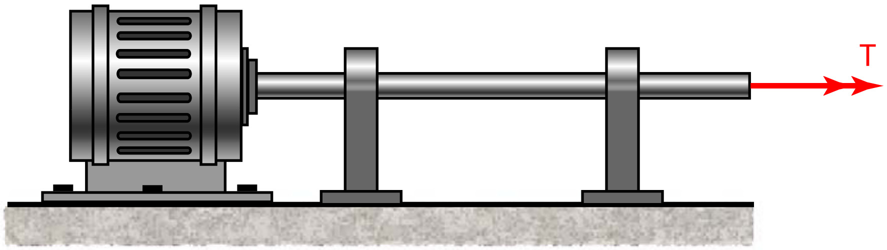
Angle of Twist in the Elastic Range



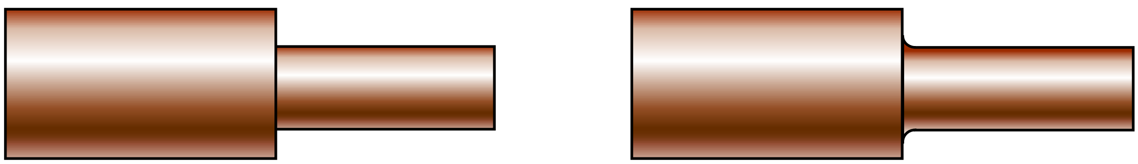
Statically Indeterminate Shafts



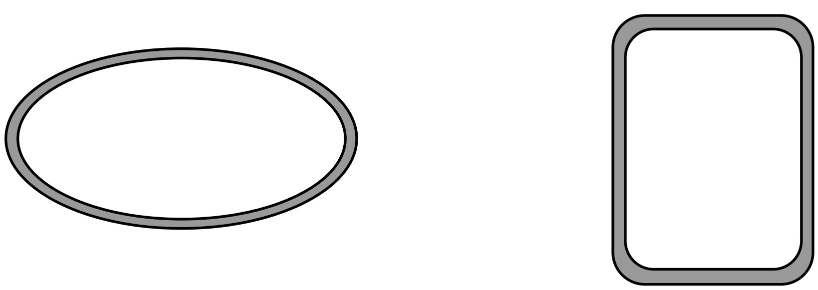
Design of Transmission Shafts



Stress Concentrations in Circular Shafts

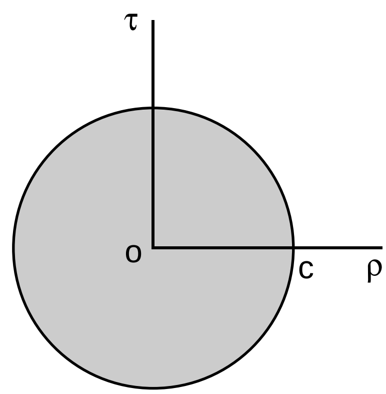


Thin-Walled Hollow Shafts

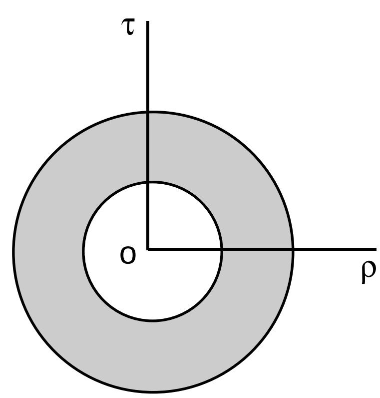


STRESSES IN THE ELASTIC RANGE

$$\tau_{\max} = \frac{Tr}{I_p} = \frac{Tc}{J}$$



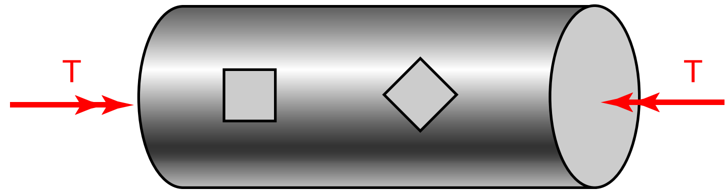
$$\tau = \frac{T\rho}{I_p} = \frac{T\rho}{J}$$



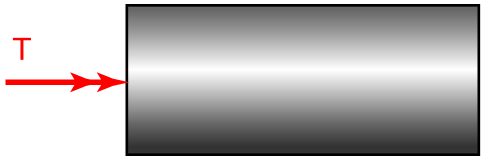
$$J = I_p = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

$$J = I_p = \frac{\pi}{32}(d_o^4 - d_i^4)$$

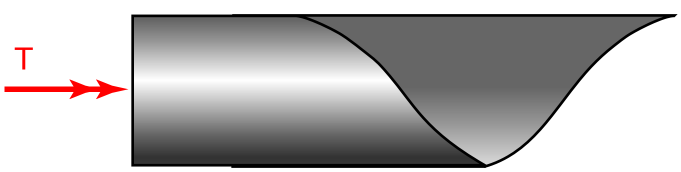
STRESSES ON INCLINED SECTIONS



$$\sigma = \tau_{max} = \frac{Tc}{J}$$



Failure due to shear stress.

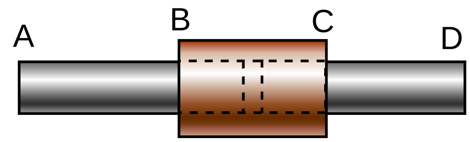


Failure due to normal stress.

Example

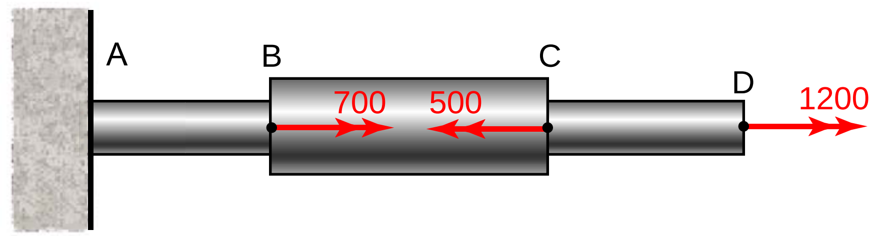
A copper coupling (BC) is used to connect two steel shafts (AB and CD). The diameter of the steel shaft is 25 mm, determine the outside diameter of the coupling so that the shear stress in it is half that of the steel shaft.

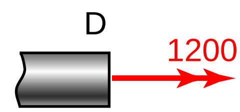
Units: kN, m.

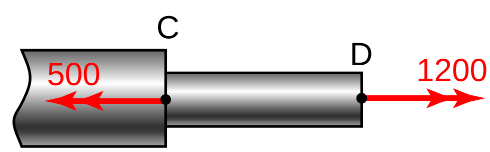


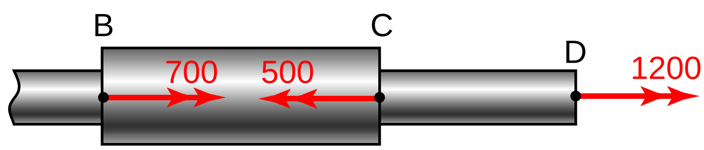
Example

Shafts AB and CD have an outside diameter of 0.75". Shaft BC has an outside diameter of 1.25". Determine the largest stress in ABCD.
Units: in-lb.



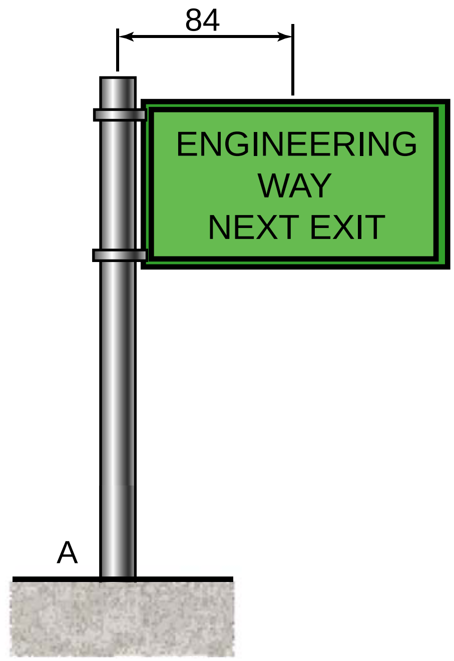






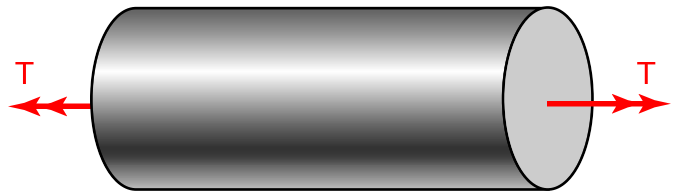
Example

The sign is subjected to a wind force of 5000 lb at it’s centroid 84" from the center of the column. The column has an outside diameter of 10" and a wall thickness of 0.125". Considering only this force, determine the torsional shear stress at A. Units: in.



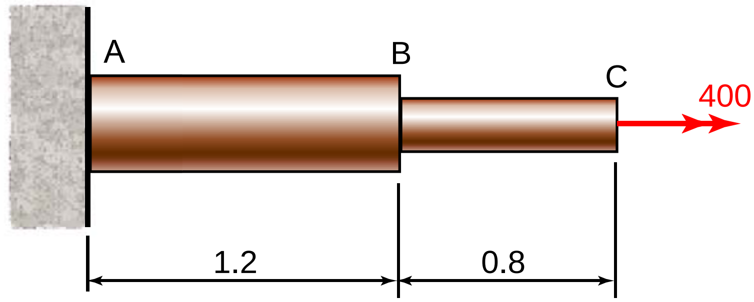
ANGLE OF TWIST IN THE ELASTIC RANGE

$$\phi = \frac{TL}{GJ}$$



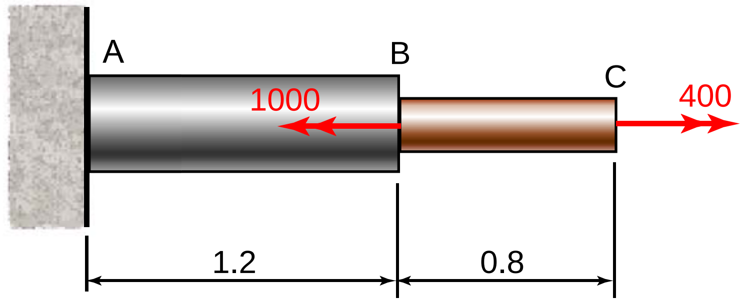
Example

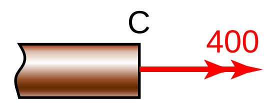
Shaft AB has a diameter of 60 mm. Determine the diameter of BC for which the displacement of point C will be 1.5°. G= 38 GPa.
Units: N•m, m.

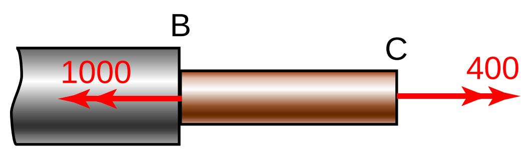


Example

The steel shaft AB has a diameter of 60 mm and the copper shaft BC has a diameter of 45 mm. Determine the rotation of points B and C.
Units: N•m, m.
G (steel)= 77.2 GPa
G (copper)= 44 GPa.





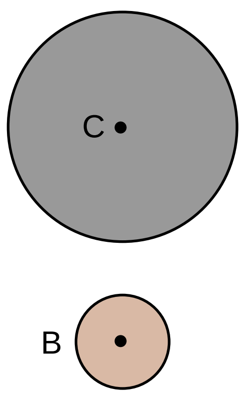


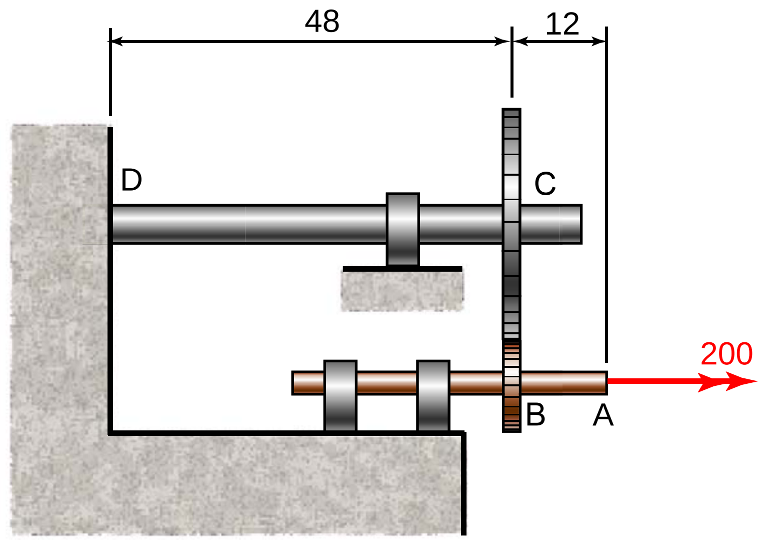
Example

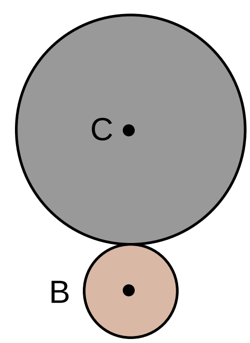
The copper shaft AB has a diameter of 1" and the steel shaft CD has a diameter of 1.75". The two shafts are connected by a 4" diameter gear at B and a 10" diameter gear at C. Point D is welded to the wall. Determine the rotation of point A. Units: lb•ft, in.

G (steel)= 11.2E6 psi

G (copper)= 6.4E6 psi



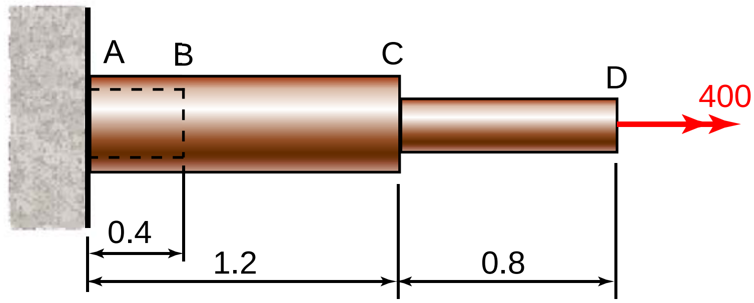




Example

Shaft ABC has a diameter of 60 mm and shaft CD has a diameter of 45 mm. The first 0.4 m of AB is hollow with a wall thickness of 4 mm. Determine the rotation of point D. G= 38 GPa.

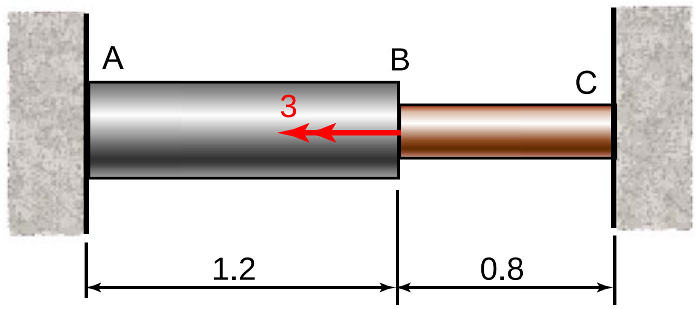
Units: N•m, m.

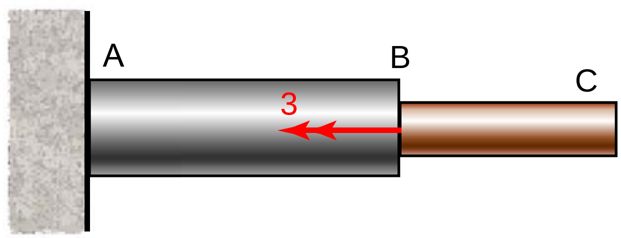


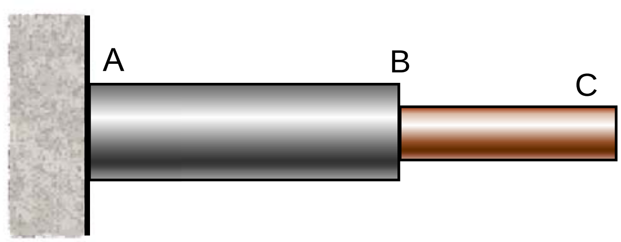
STATICALLY INDETERMINATE SHAFTS

Example

The steel shaft AB has a diameter of 60 mm and the copper shaft BC has a diameter of 45 mm. Determine the reactions at A and C.
Units: kN•m, m.
G (steel)= 77.2 GPa
G (copper)= 44 GPa.





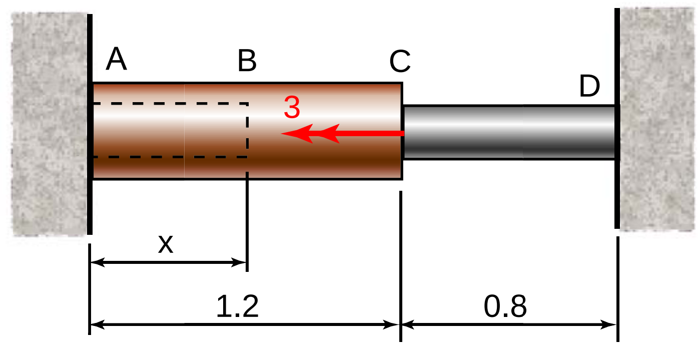


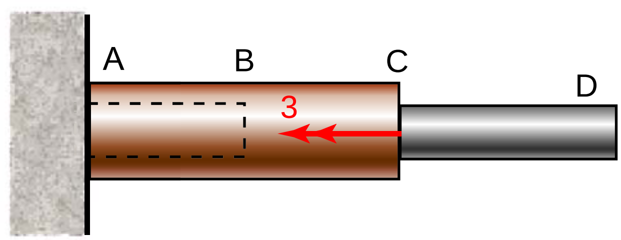
Example

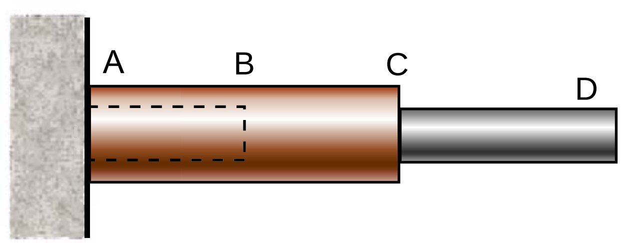
The copper shaft ABC is hollow between A and B, and solid between B and C. Shaft ABC has an outer diameter of 60 and an inside diameter between A and B of 52 mm. The solid steel shaft CD has a diameter of 45. Determine the length x so that the reactions at A and D are equal. Units: kN•m, m.

G (steel)= 77.2 GPa

G (copper)= 44 GPa.







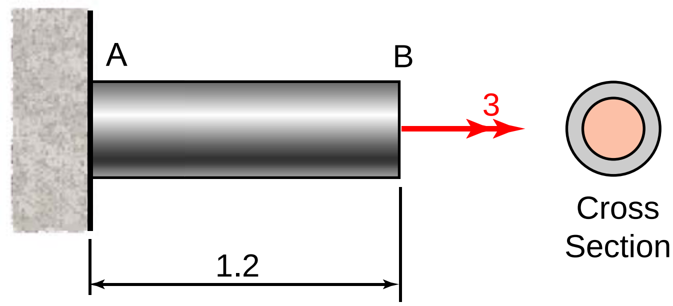
Example

The shaft AB has an outer shell of steel and an inner core of copper. The outside diameter of the steel shaft is 30 mm and the copper core has a diameter of 20 mm. The two materials are firmly connected along their lengths. Determine the torque in each material.

Units: kN•m, m.

G (steel)= 77.2 GPa

G (copper)= 44 GPa.



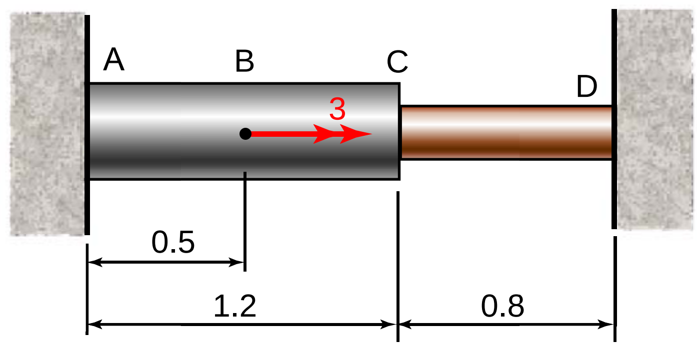
Example

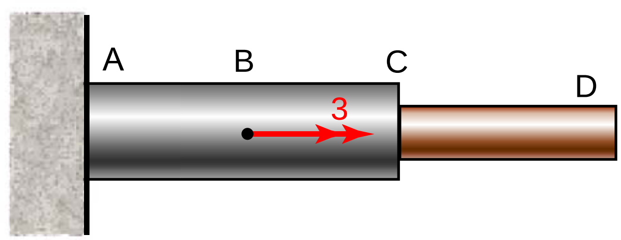
The steel shaft ABC has a diameter of 60 and the copper shaft CD has a diameter of 45 mm. Determine the reactions at A and D.

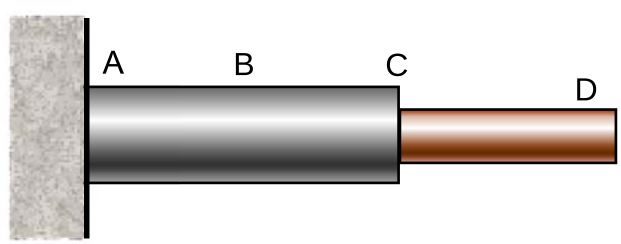
Units: kN•m, m.

G (steel)= 77.2 GPa

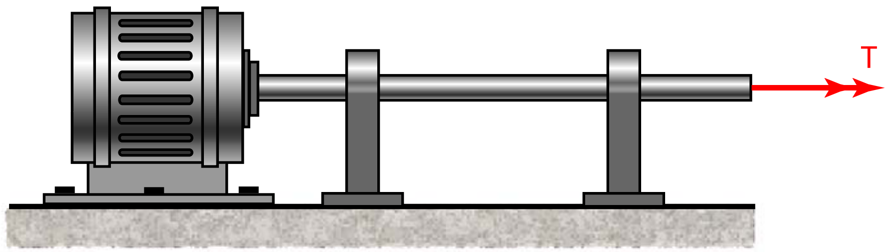
G (copper)= 44 GPa.







DESIGN OF TRANSMISSION SHAFTS



$$P = T\omega$$

$$P = 2\pi fT$$

$$T = \frac{P}{2\pi f}$$

$$T = \frac{60P}{2\pi n} = \frac{30P}{\pi n}$$

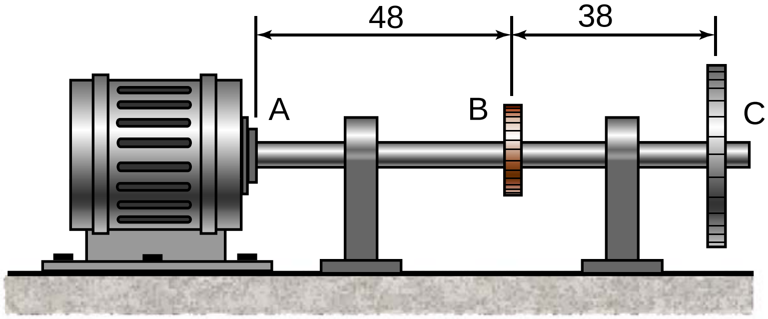
$$H = Horsepower = \frac{2\pi nT}{33,000}$$

Example

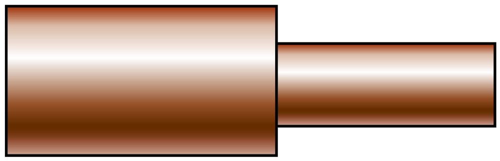
A pipe is designed to transmit 90 kW at 15 Hz. The inside diameter of the shaft is to be three-fourths of the outer diameter. a) Calculate the minimum required diameter d if the maximum shear stress is 50 MPa. b) Find the diameter if the allowable normal stress is 65 MPa.

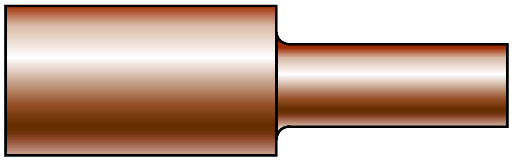
Example

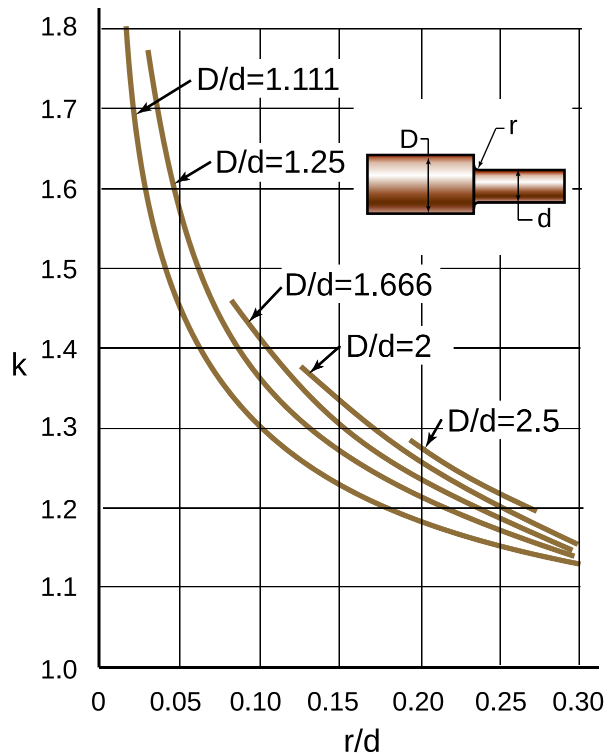
The motor generates 150 hp at 100 rpm and gears B and C consume 100 and 50 hp respectively. Determine the diameter of the solid uniform shaft if the maximum shear stress is limited to 14,000 psi and the maximum rotation at C is 1.75°. G= 11.2E6 psi. Units: lbs, in.



STRESS CONCENTRATIONS IN CIRCULAR SHAFTS





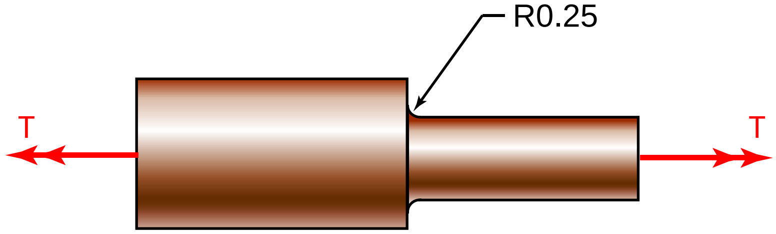


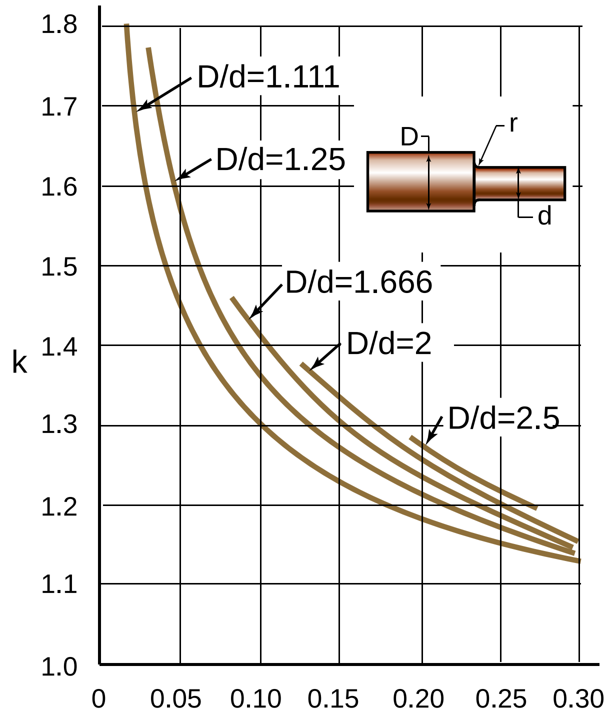
Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

Determine the maximum torque that can be applied if the allowable shear stress is 9500 psi. The diameters of the shafts are 2.06 and 1.25 in. Units: lbs, in.



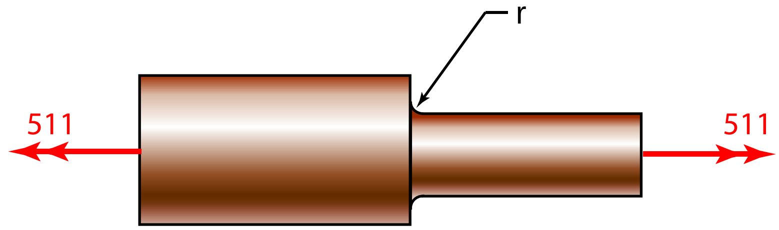


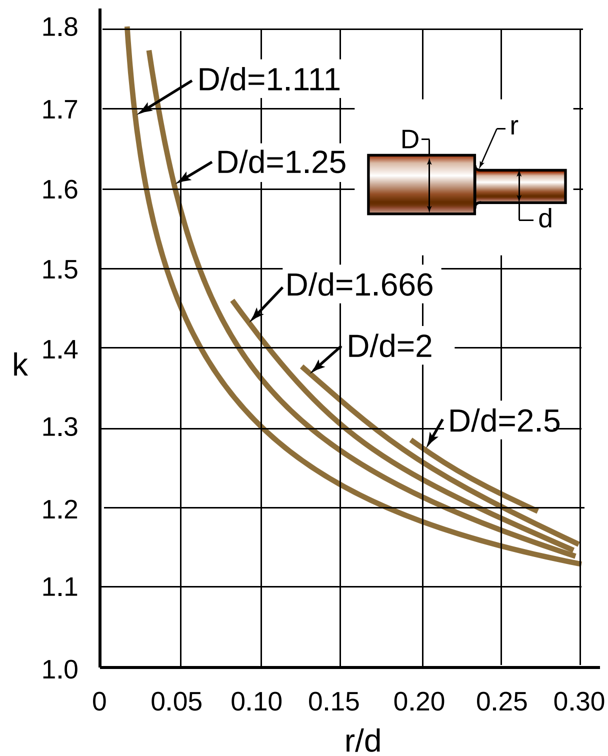
Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

Determine the smallest fillet size if the allowable shear stress is 135 MPa. The diameters of the shafts are 50 and 30 mm. Units: N•m.



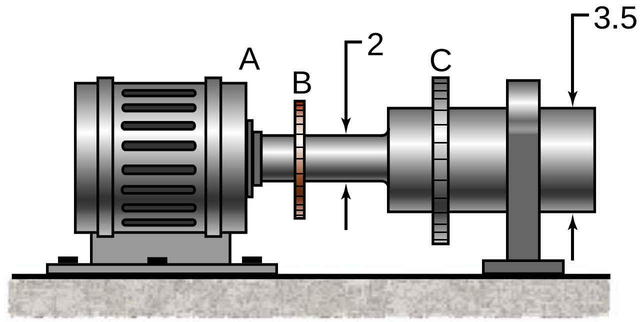


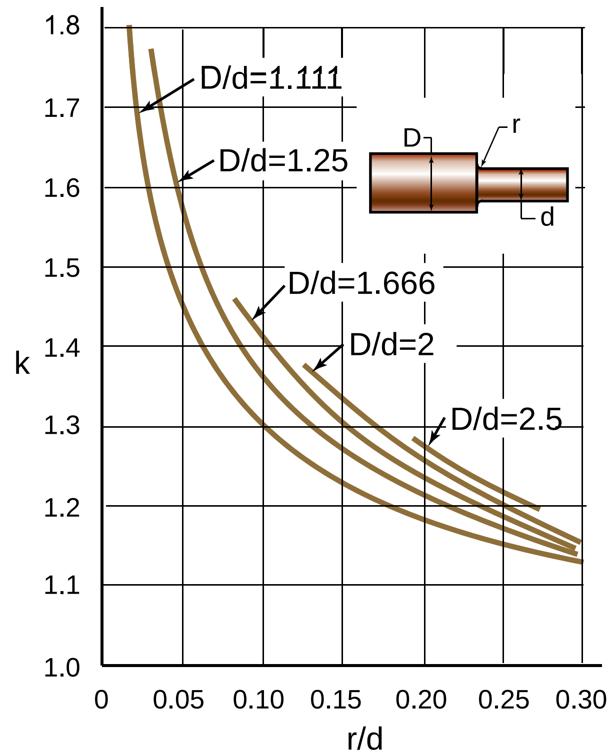
Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

The motor generates 2,000 ft-lb of torque to the shaft at A. The gears at B, and C consume 500, and 1,500 ft-lb respectively. Determine the maximum shear stress at the .25" fillet between B and C.
Units: in.

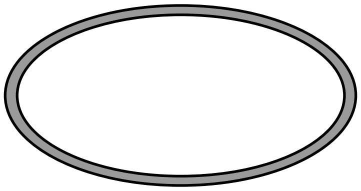


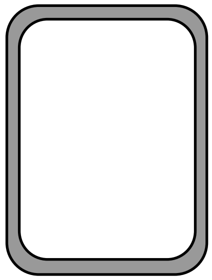


Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

THIN-WALLED HOLLOW SHAFTS





$$\tau = \frac{T}{2tA_m}$$

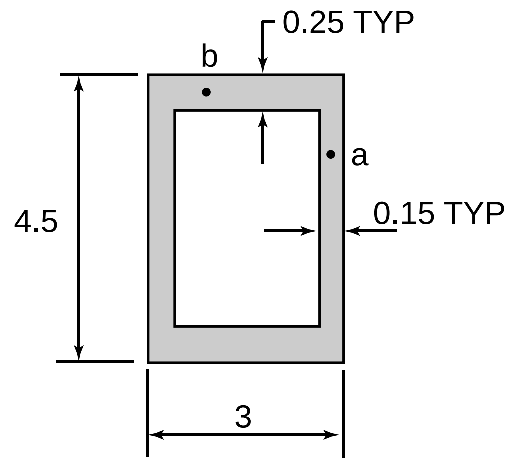
$$\phi = \frac{TL}{4A^2G}\oint\frac{ds}{t} = \frac{TL}{GJ}$$

Where

$$J = \frac{4A_m^2}{\sum\frac{L_m}{t}}$$

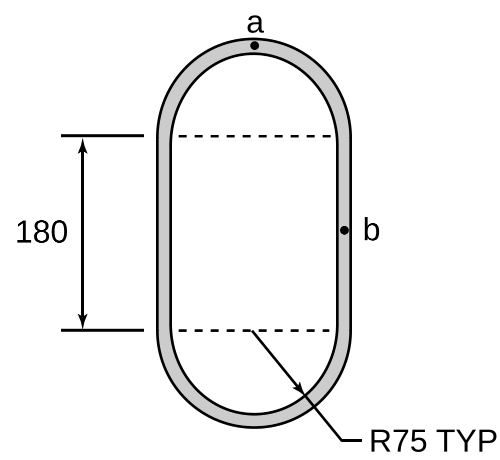
Example

A 45 kip-in torque is applied to the hollow shaft. Neglecting stress concentrations, determine the shear stress at points a and b.
Units: in.



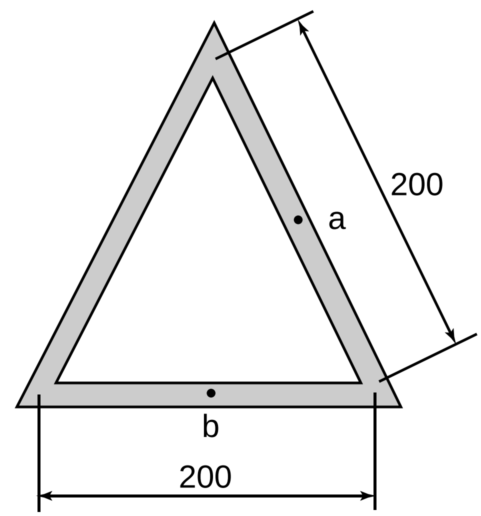
Example

A 70 kN•m torque is applied to the hollow 10 mm uniform shaft. Neglecting stress concentrations, determine the shear stress at points a and b. Units: mm.



Example

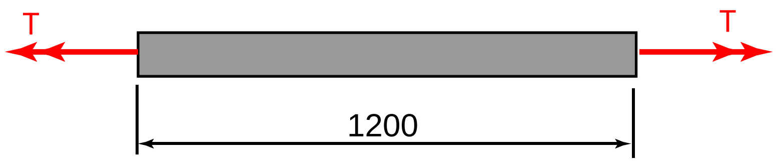
A 17 kN•m torque is applied to the hollow 8 mm uniform shaft. Neglecting stress concentrations, determine the shear stress at points a and b. Units: mm.

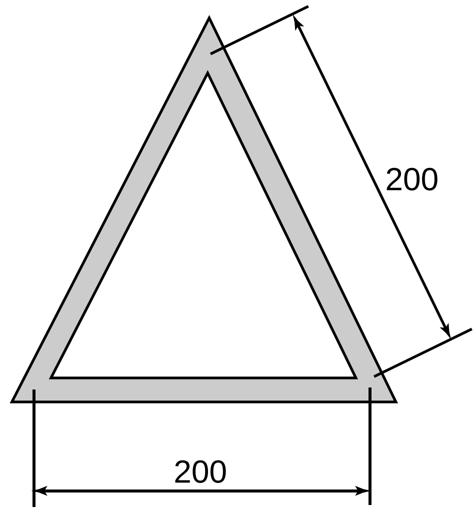


Example

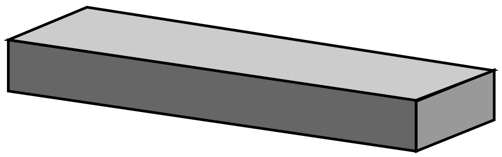
A 17 kN•m torque is applied to the hollow 8 mm uniform shaft. Determine the rotation of the 1200 mm shaft.

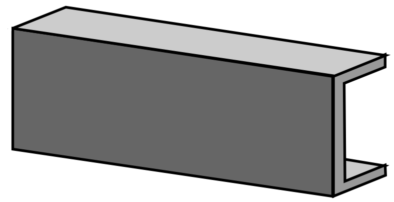
G= 77.2 GPa. Units: mm.

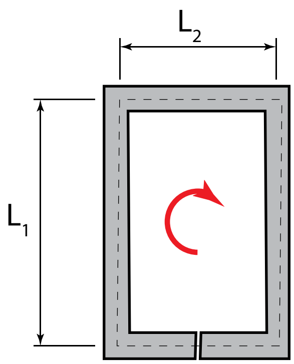




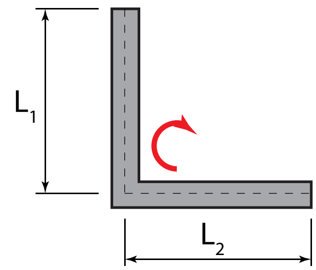
TORSION OF NONCIRCULAR MEMBERS (OPEN SHAPES)



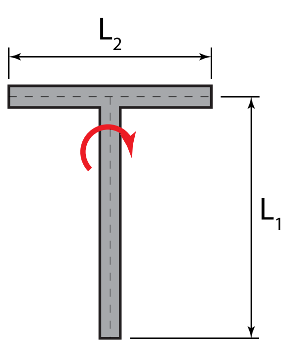




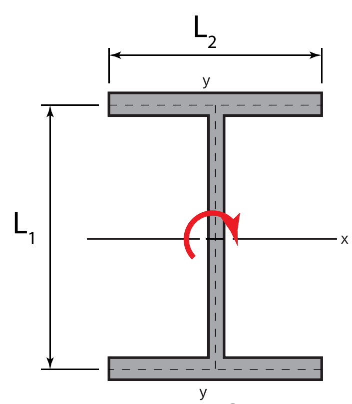
a= $2(L_1 + L_2)$
b= t (if uniform thk)



a= $L_1 + L_2$
b= t (if uniform thk)



a= $L_1 + L_2$
b= t (if uniform thk)



a= $L_1 + 2L_2$
b= t (if uniform thk)

$$\tau = \frac{T}{C_1 ab^2} \qquad \phi = \frac{TL}{\sum (C_2 ab^3 G)}$$

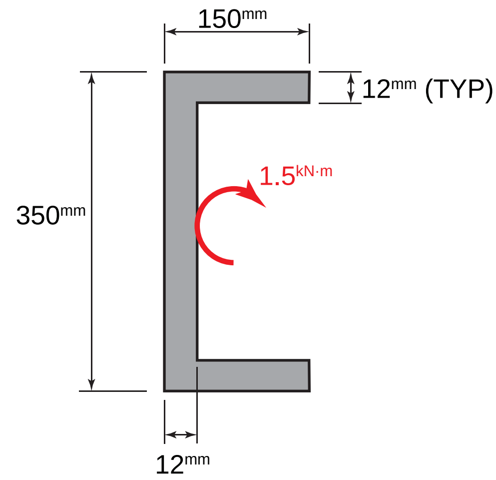
Where:

a= sum of the lengths
b= thickness of any part

a/b	C_1	C_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10	0.312	0.312
∞	0.333	0.333

Example

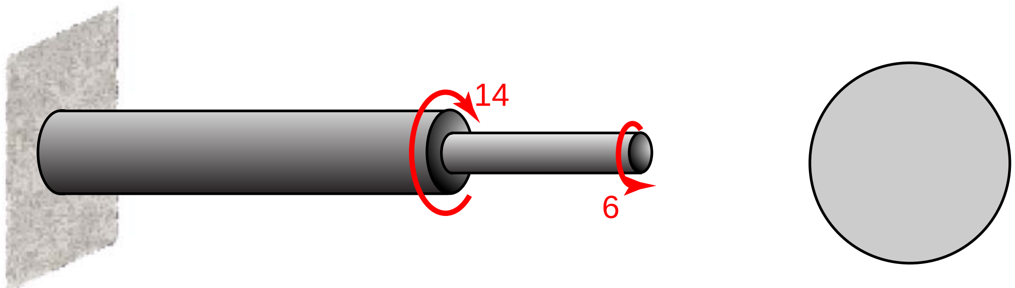
Determine the maximum shear stress for the shape below.



SUMMARY

Stresses in the Elastic Range

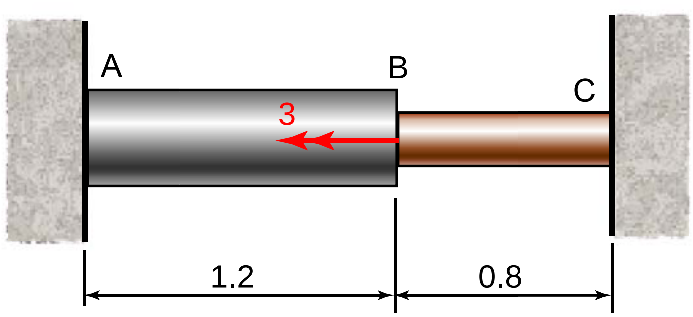
Angle of Twist in the Elastic Range



$$\tau_{max} = \frac{Tr}{I_p} = \frac{Tc}{J}$$

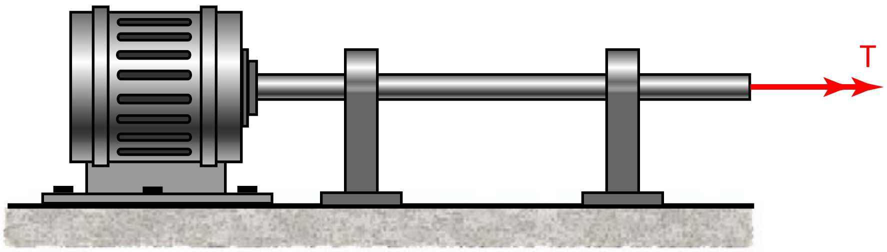
$$\phi = \frac{TL}{GJ}$$

Statically Indeterminate Shafts



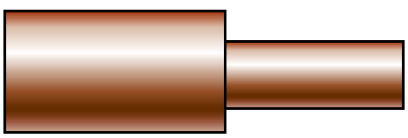
SUMMARY

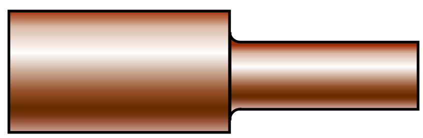
Design of Transmission Shafts

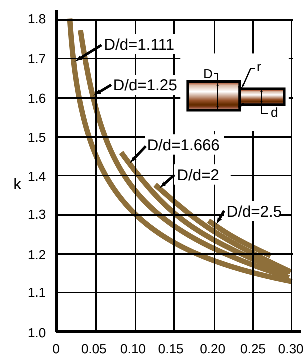


$$T = \frac{P}{2\pi f}$$

Stress Concentrations in Circular Shafts



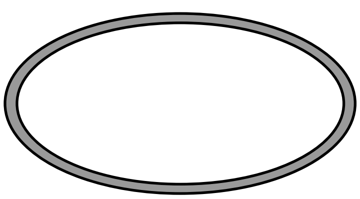


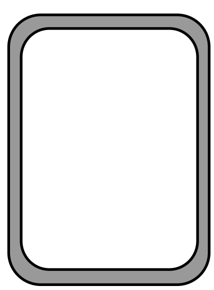


Stress-concentration factors for fillets in circular shafts

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Thin-Walled Hollow Shafts





$$\tau = \frac{T}{2tA_m}$$

$$\phi = \frac{TL}{4A^2G}\oint\frac{ds}{t} = \frac{TL}{GJ}$$

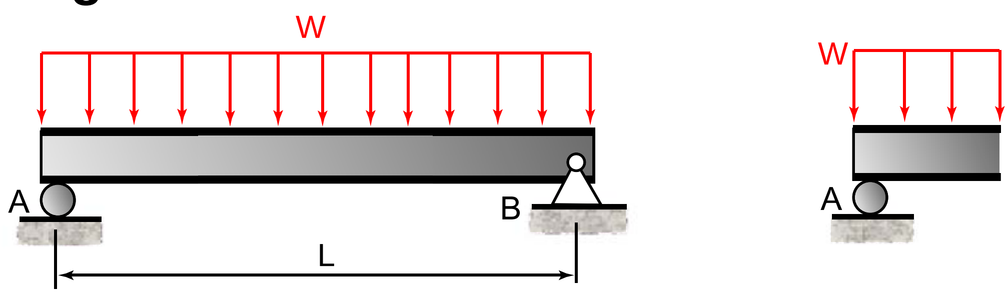
Where

$$J = \frac{4A_m^2}{\sum \frac{L_m}{t}}$$

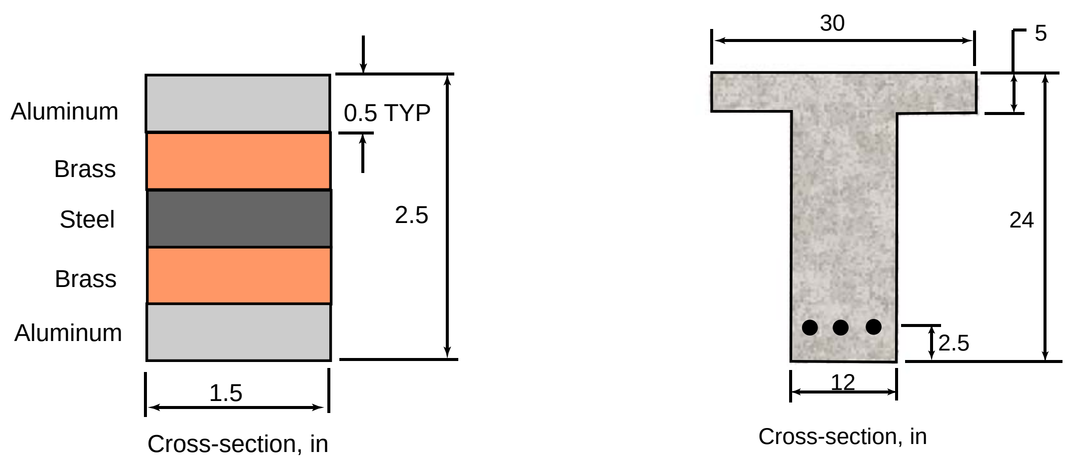
Chapter 4
Pure Bending

INTRODUCTION

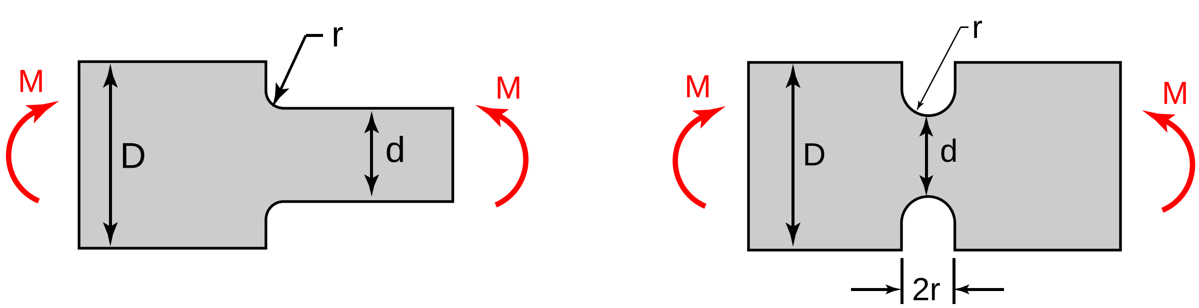
Bending Stress



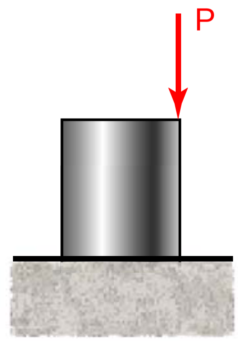
Bending of Members made of Several Materials



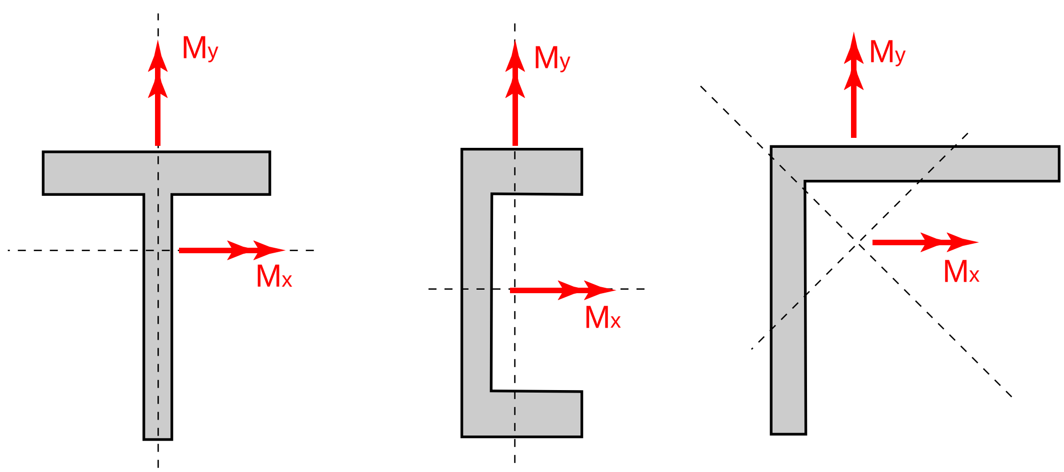
Stress Concentrations



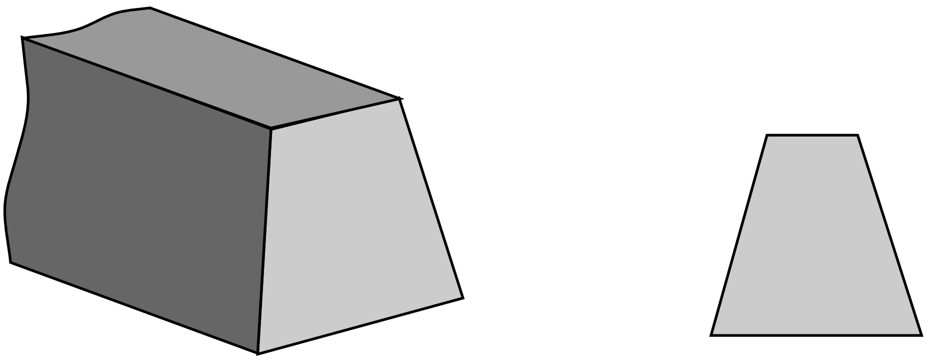
Eccentric Axial Loading in a Plane of Symmetry

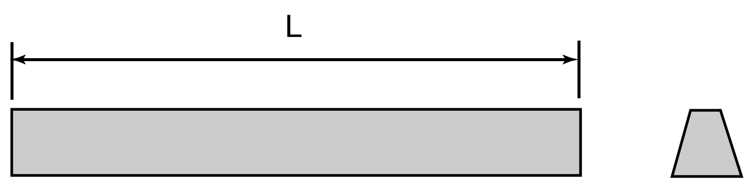


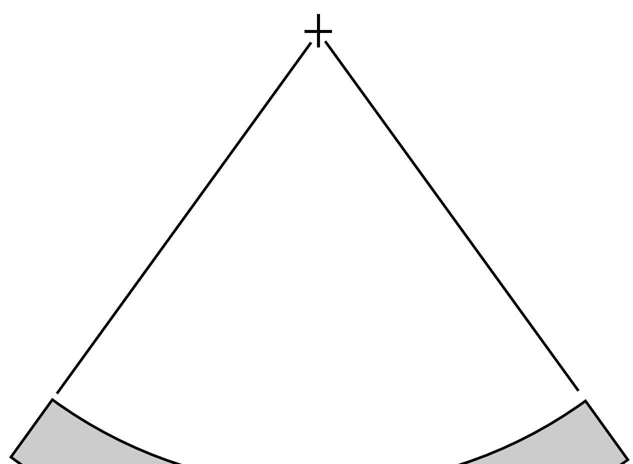
Unsymmetric Bending



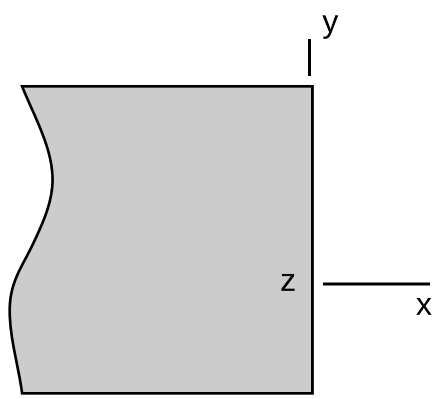
BENDING STRESS

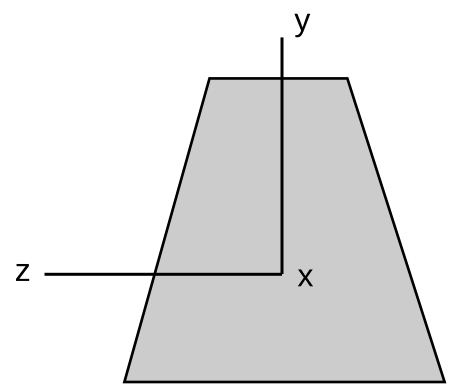






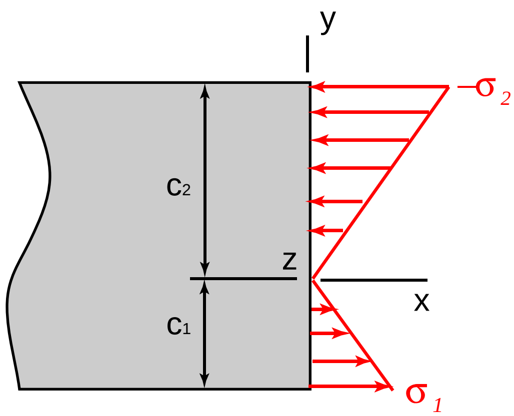
BENDING STRESS- continued

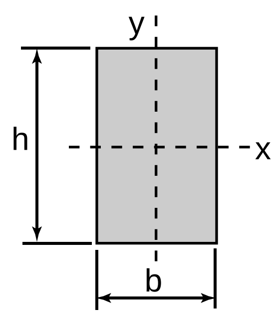




$$\sigma_x = -\frac{My}{I}$$

SECTION MODULUS

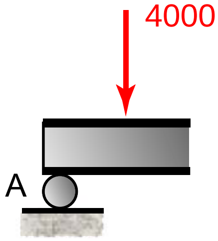


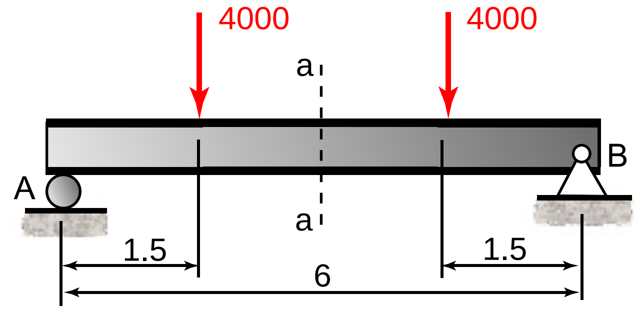


$$\sigma_x = -\frac{My}{I} = -\frac{M}{S}$$

Example

Find the maximum bending stress at section a-a (3 m from A) of the W150x29.8 beam. Units: N, m.





W150x29.8

$\text{Area, A} = 3790^{mm^2}$

$\text{Depth, d} = 157^{mm}$

$\text{Flange Width, } b_f = 153^{mm}$

$\text{Flange Thickness, } t_f = 9.3^{mm}$

$\text{Web Thickness, } t_w = 6.6^{mm}$

$I_x = 17.2x10^{6mm^4}$

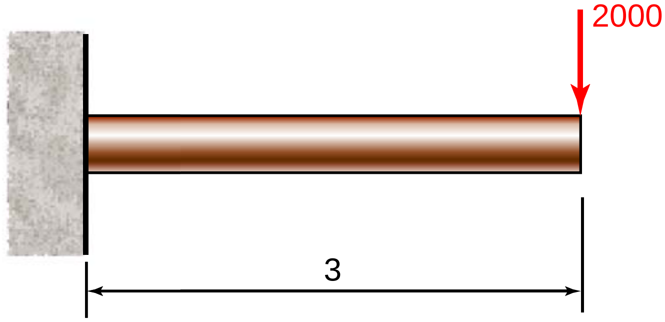
$I_y = 5.56x10^{6mm^4}$

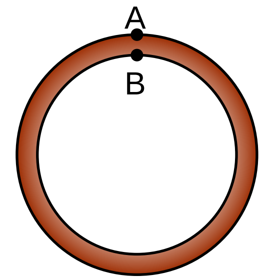
$S_x = 219x10^{3mm^3}$

$S_y = 72.7x10^{3mm^3}$

Example

Find the bending stresses at the wall at points A and B for a 6" pipe with a wall thickness of 0.125". Units: lb, ft.

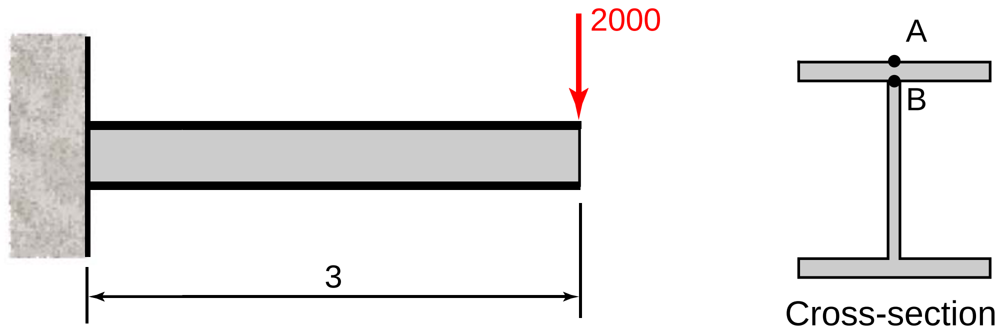




Cross-section

Example

Find the bending stresses at the wall at points A and B for the W6x20 beam. Units: lb, ft.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

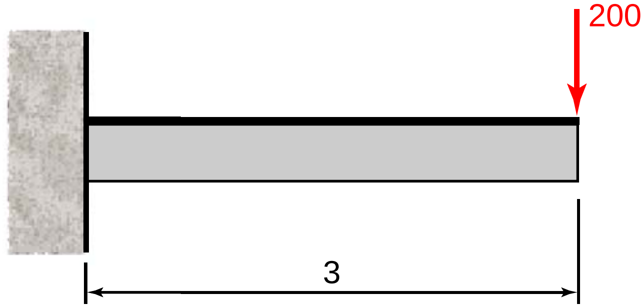
$I_y = 13.3^{in^4}$

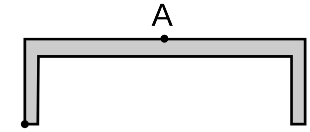
$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

Find the bending stresses at the wall at points A and B for the C6x13 beam. Units: lb, ft.





Cross-section

C6x13

Area, $A = 3.83^{in^2}$

Depth, $d = 6.00^{in}$

Flange Width, $b_f = 2.16^{in}$

Flange Thickness, $t_f = 0.343^{in}$

Web Thickness, $t_w = 0.437^{in}$

$I_x = 17.4^{in^4}$

$I_y = 1.05^{in^4}$

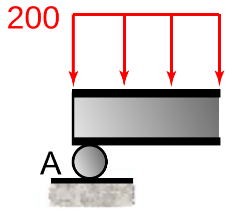
$S_x = 5.80^{in^3}$

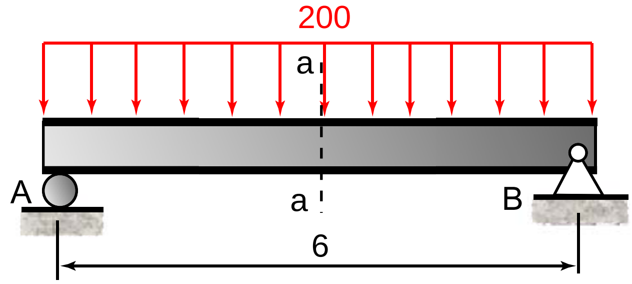
$S_y = 0.642^{in^3}$

$\overline{x} = 0.514^{in}$

Example

Find the maximum bending stress at section a-a (3 m from A) of the W150x29.8 beam. Units: N/m, m.





W150x29.8

Area, A $= 3790^{mm^2}$

Depth, d $= 157^{mm}$

Flange Width, $b_f = 153^{mm}$

Flange Thickness, $t_f = 9.3^{mm}$

Web Thickness, $t_w = 6.6^{mm}$

$I_x = 17.2x10^{6 mm^4}$

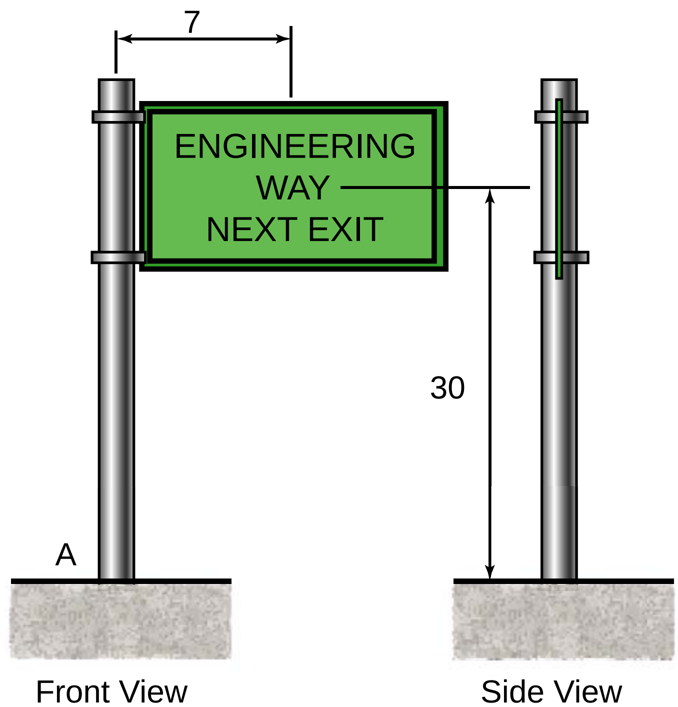
$I_y = 5.56x10^{6 mm^4}$

$S_x = 219x10^{3 mm^3}$

$S_y = 72.7x10^{3 mm^3}$

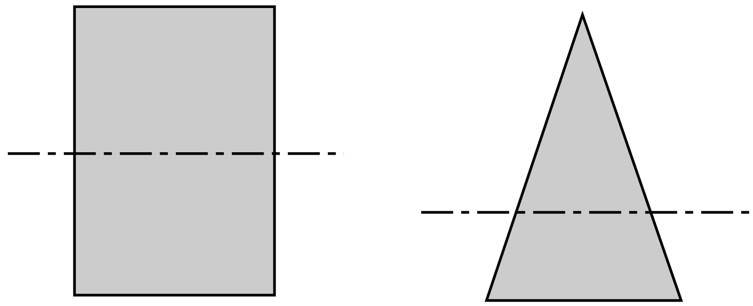
Example

The sign is subjected to a wind force of 250 lb at it’s centroid 7' from the center of the column. The column has an outside diameter of 10" and a wall thickness of 0.25". Considering only this force, determine the maximum bending stress. Units: ft.

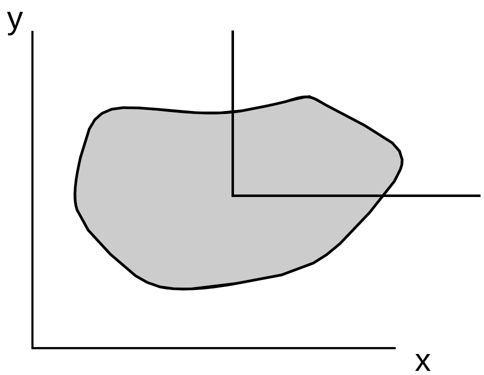


PARALLEL-AXIS THEOREM

MOMENTS OF INERTIA

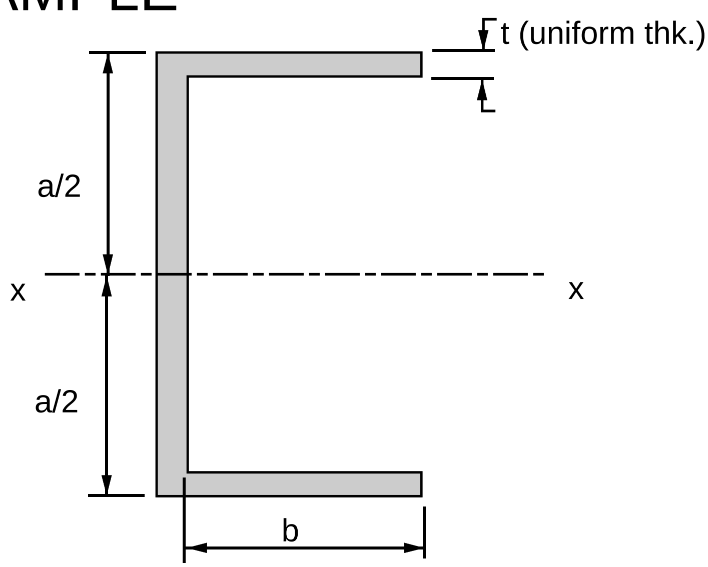


PARALLEL-AXIS THEOREM



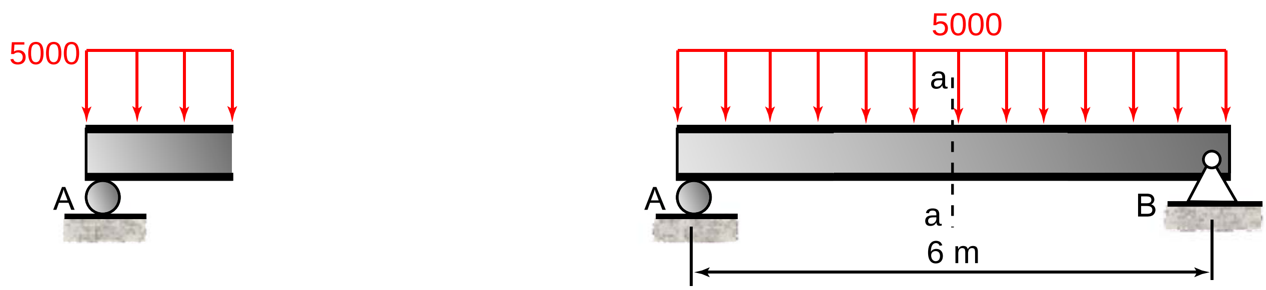
$$I_x = \sum (I_{x'} + A d_y^2)$$
$$I_y = \sum (I_{y'} + A d_x^2)$$

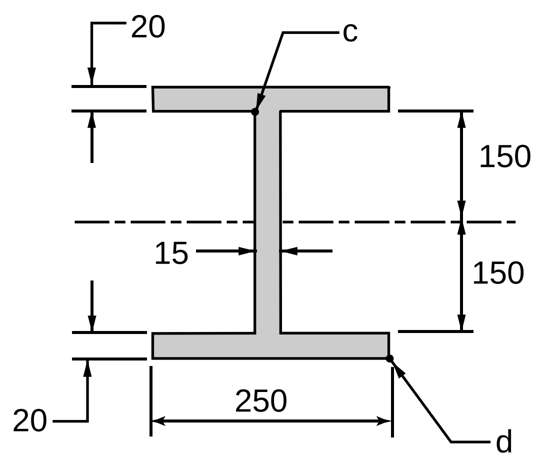
EXAMPLE



Example

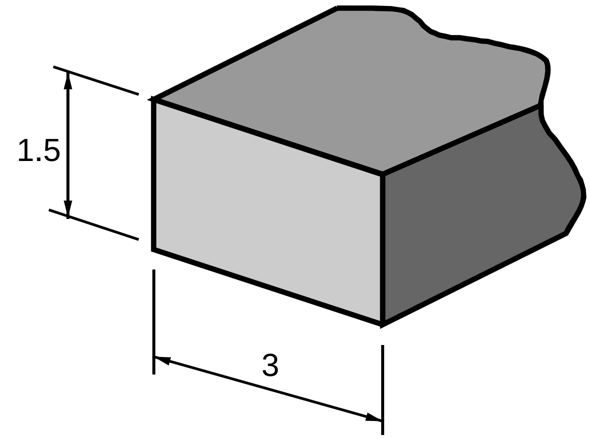
The simply-supported beam below has a cross-sectional area as shown. Determine the bending stress that acts at points c and d, located at section a-a (3 m from A). Units: N/m, mm (uno).

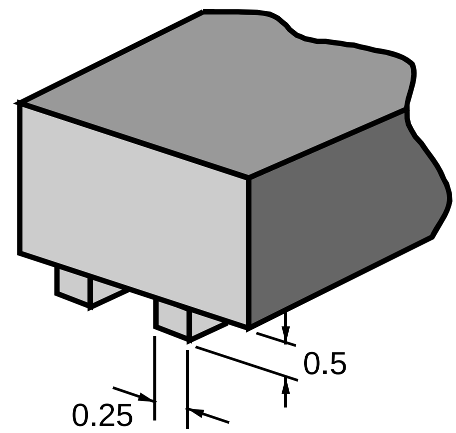


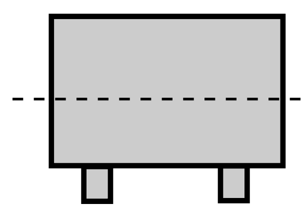


Example

The member is designed to resist a moment of 5 kip•in about the horizontal axis. Determine the maximum normal stress in the member for the two similar cross-sections. Units: in.

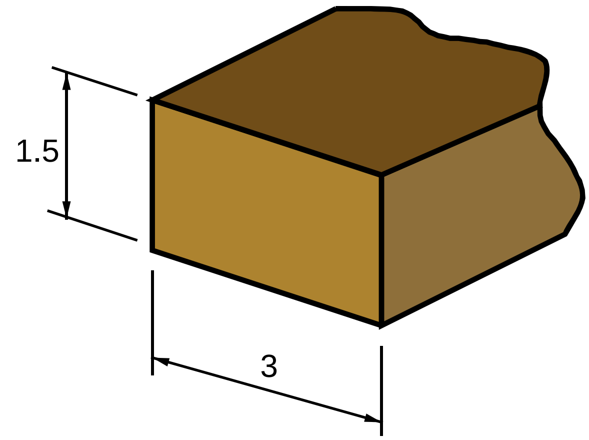




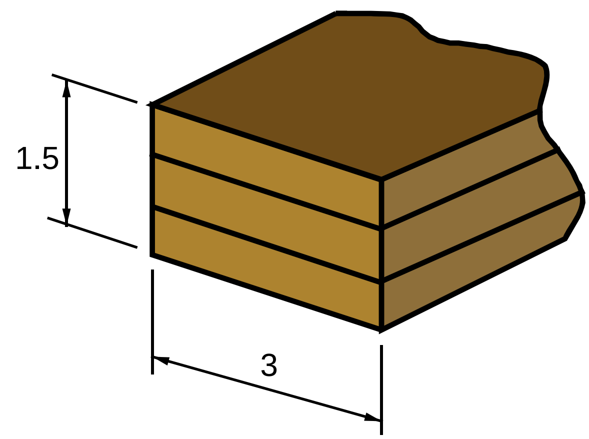


Example

Compare the bending stresses between the two cases for a moment about the horizontal axis. Case 1 is a simple solid cross-section, whereas case 2 is made up of 3 identical boards. The 3 boards aren't connected together and simply rest on one another. Units: in.



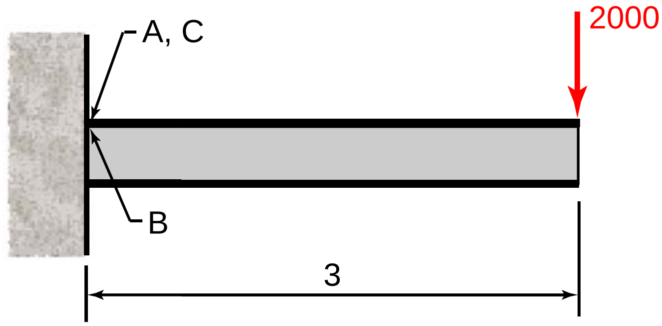
Case 1

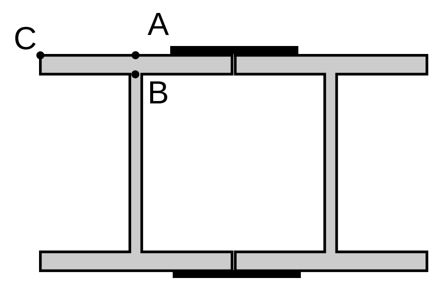


Case 2

Example

The two beams are connected by a thin rigid plate on the top and bottom side of the flanges. Find the bending stresses at the wall at points A, B and C for the W6x20 beam. Units: lb, ft





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

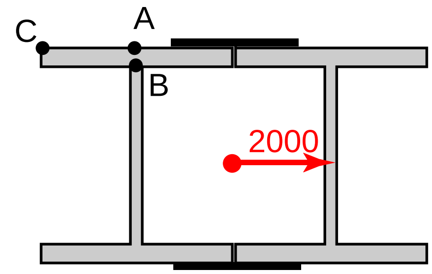
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

The two beams are connected by a thin rigid plate on the top and bottom side of the flanges. Find the bending stresses at the wall at points A, B and C for the W6x20 beam. Units: lb, ft



Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

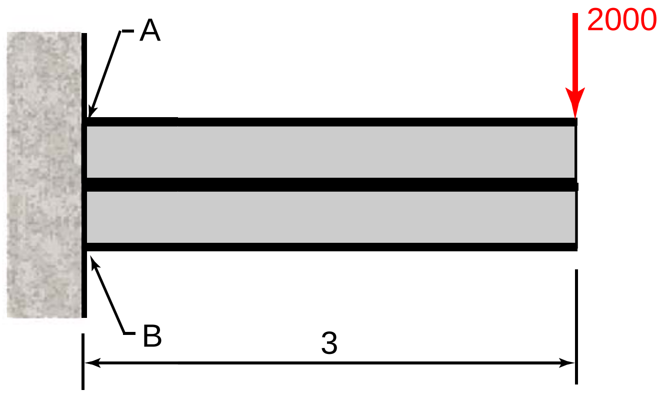
$I_y = 13.3^{in^4}$

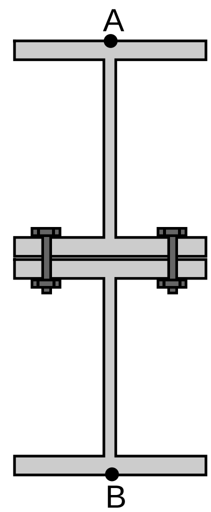
$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

The two beams are connected by bolts through the flanges. Find the bending stresses at the wall at points A and B for the W6x20 beam.
Units: lb, ft





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

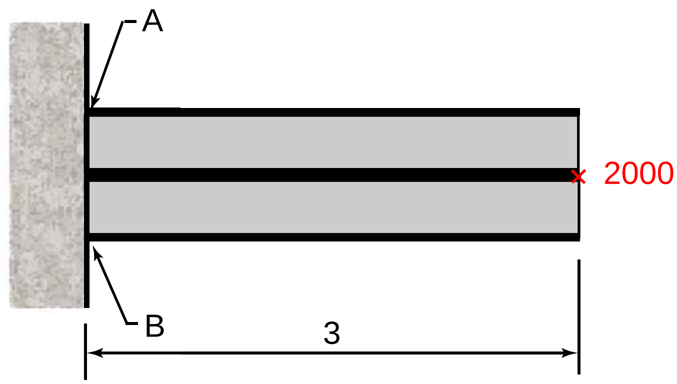
$I_y = 13.3^{in^4}$

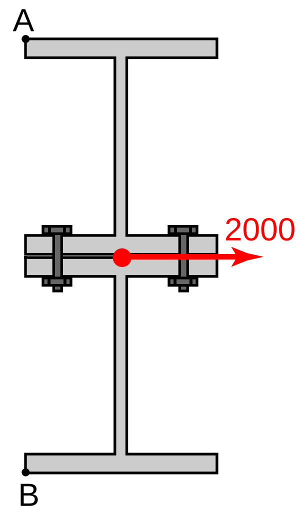
$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

The two beams are connected by bolts through the flanges. Find the bending stresses at the wall at points A and B for the W6x20 beam. Units: lb, ft





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

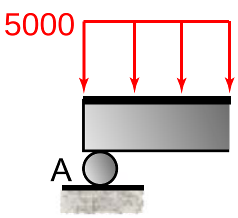
$I_y = 13.3^{in^4}$

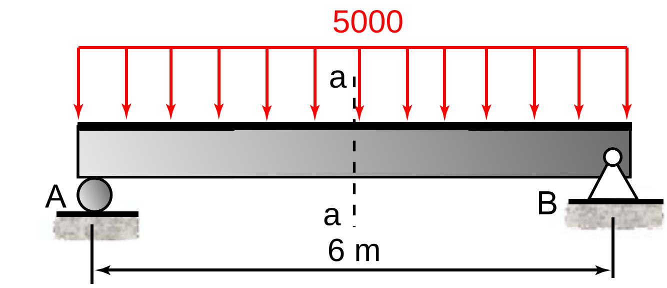
$S_x = 13.4^{in^3}$

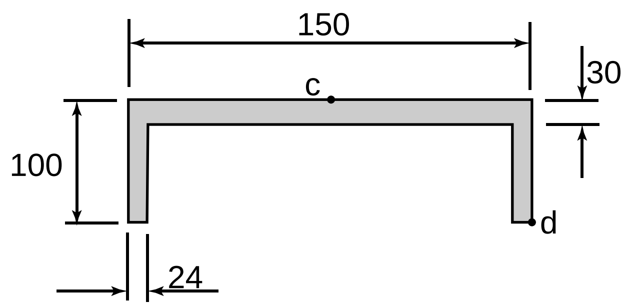
$S_y = 4.41^{in^3}$

Example

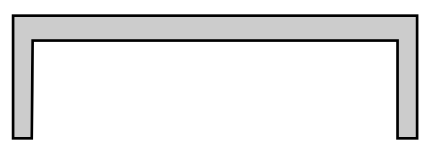
The simply-supported beam below has a cross-sectional area as shown. Determine the bending stress that acts at points c and d, located at section a-a (3 m from A). Units: N/m, mm (UNO).



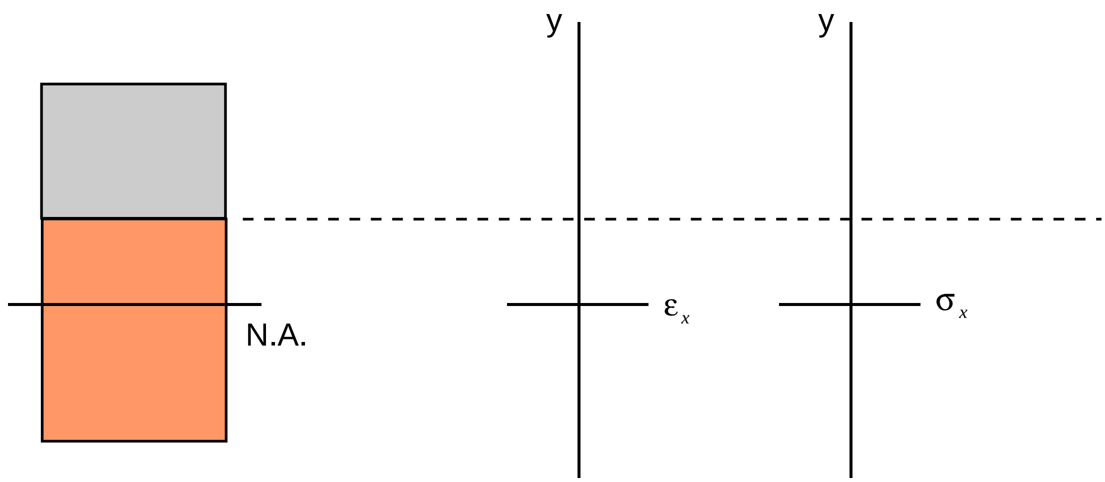


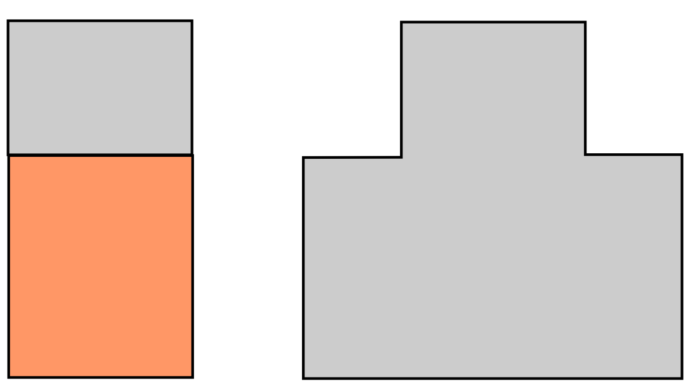


Cross-section



BENDING OF MEMBERS MADE OF SEVERAL MATERIALS



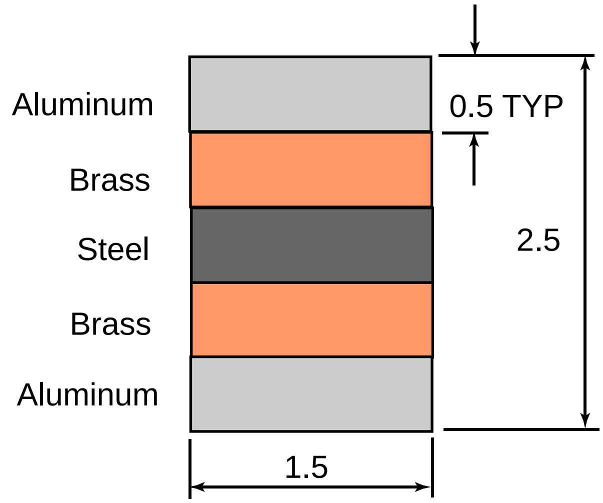


$$\sigma_x = -n\frac{My}{I}$$

Example

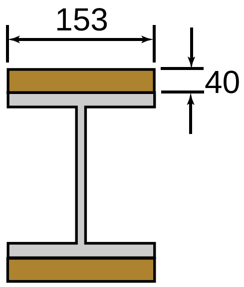
Find the stress in each of the three metals if a moment of 12 k-in is applied about the horizontal axis.

E (aluminum)= 10E6, E(steel)= 30E6, E (brass)= 15E6 psi. Units: in.

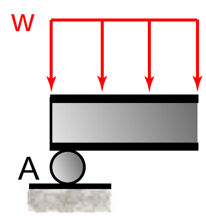


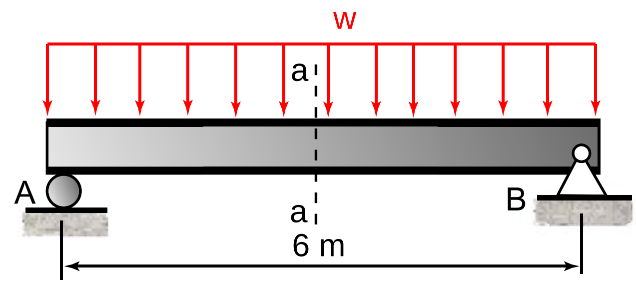
Example

A W150x29.8 wide flange beam is reinforced with wood planks that are securely connected to the flanges. E_{steel}/E_{wood}= 20. If the allowable stresses in the wood and steel are 4.5 MPa and 52 MPa, respectively, determine the allowable distributed load w based on section a-a (3 m from A). Units: N/m, mm (UNO).



Cross-section





W150x29.8

$\text{Area, A} = 3790^{mm^2}$

$\text{Depth, d} = 157^{mm}$

$\text{Flange Width, } b_f = 153^{mm}$

$\text{Flange Thickness, } t_f = 9.3^{mm}$

$\text{Web Thickness, } t_w = 6.6^{mm}$

$I_x = 17.2x10^{6mm^4}$

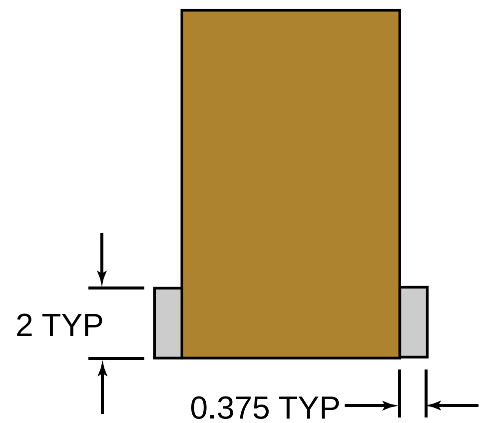
$I_y = 5.56x10^{6mm^4}$

$S_x = 219x10^{3mm^3}$

$S_y = 72.7x10^{3mm^3}$

Example

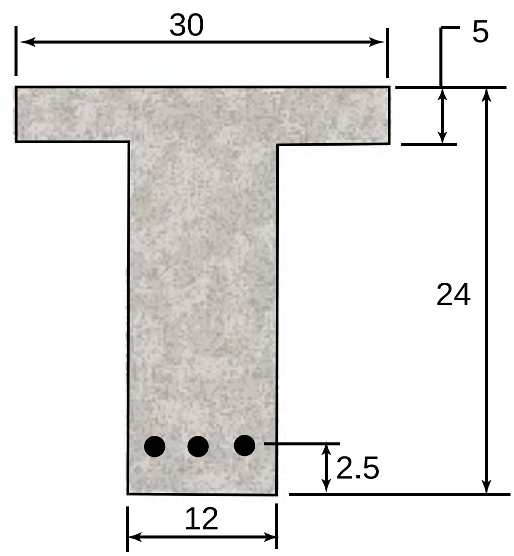
Two steel plates are securely fastened to a 6"x10" wood beam. E_{steel}/E_{wood}= 20. Knowing that the beam is bent about the horizontal axis by a 125 kip-in moment, determine the maximum stress in (a) the wood, (b) the steel. Units: in.



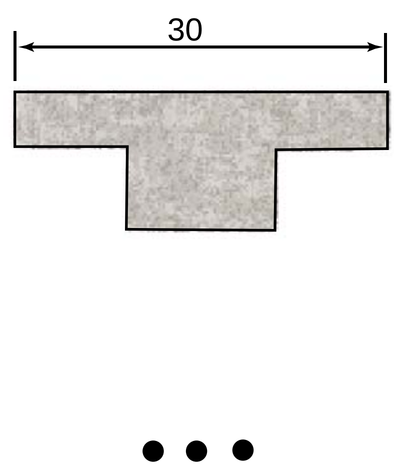
Cross-section

Example

Determine the stress in the concrete and steel if a moment of 1500 kip-in is applied about the horizontal axis. Area of steel= 3.14 sq. in. E (steel)= 30E6 psi, E (concrete)= 3.75E6 psi. Units: in.



Cross-section

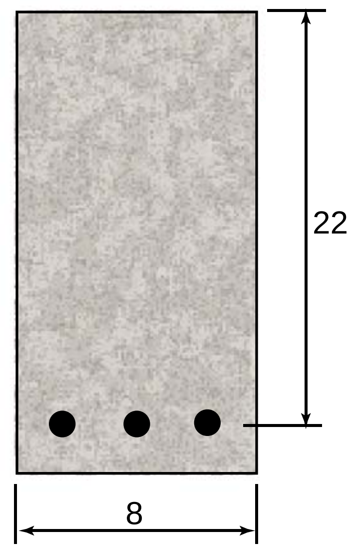


Cross-section

Example

Determine the required steel area for the beam to be balanced. Allowable stress in the steel and concrete are 33,000 and 3,000 psi respectively.

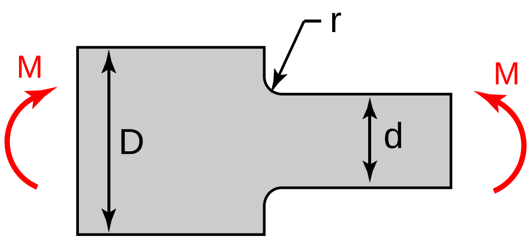
E (steel)= 29E6 psi, E (concrete)= 3.5E6 psi. Units: in.

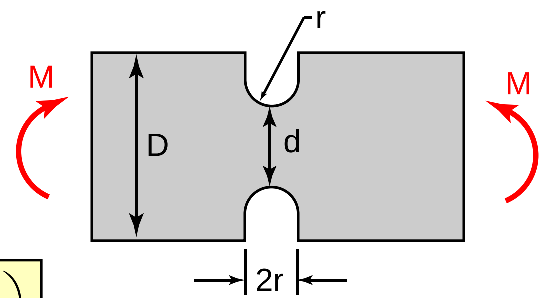


Cross-section

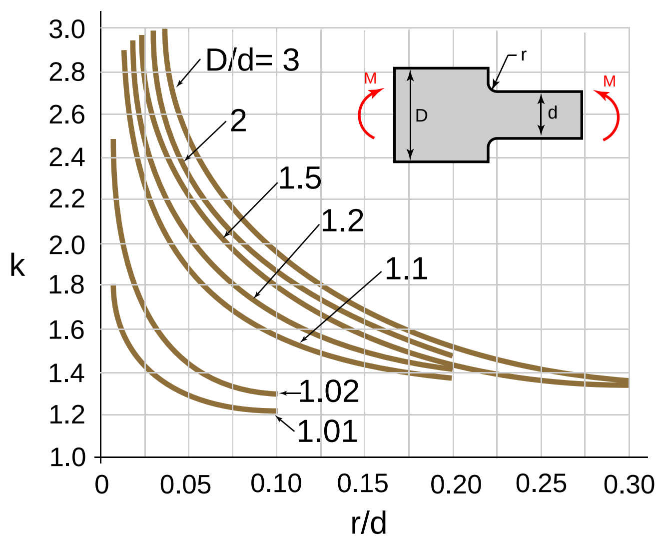
Cross-section

STRESS CONCENTRATIONS

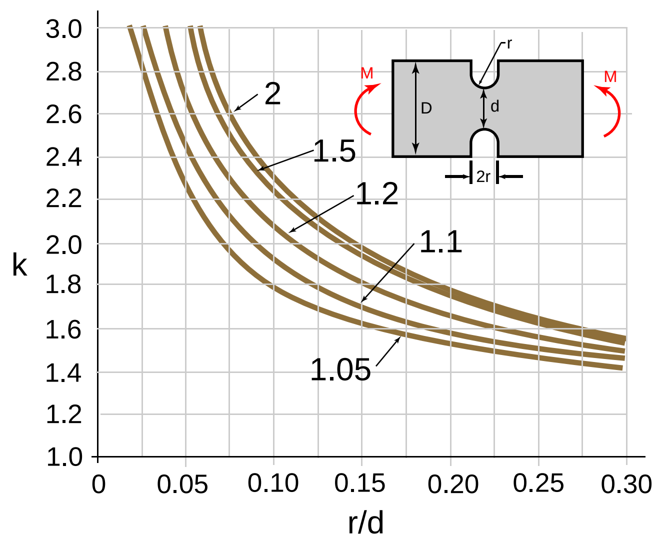




$$\sigma_{max} = k\left(\frac{My}{I}\right)$$



Stress-concentration factors for flat bars with fillets under pure bending

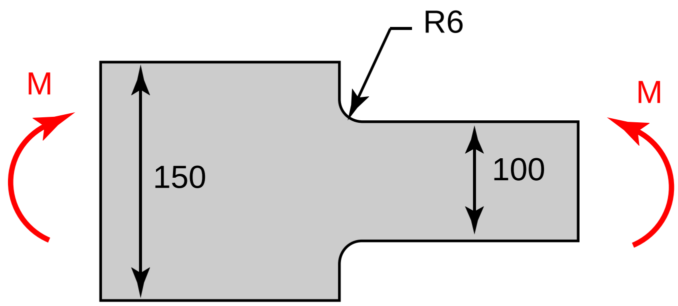


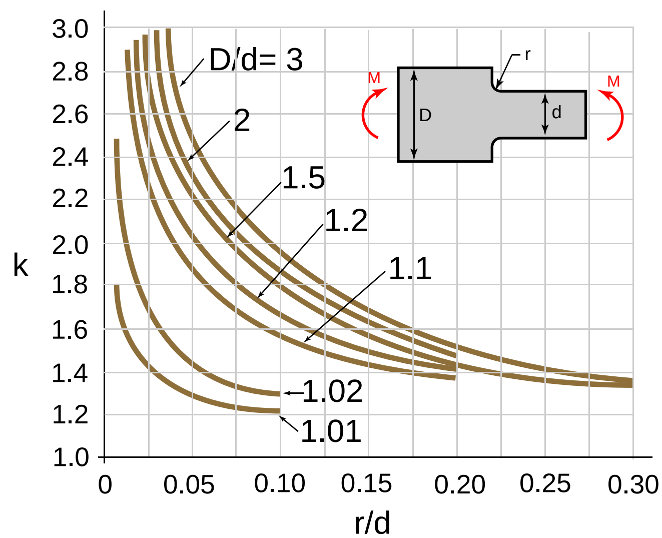
Stress-concentration factors for flat bars with groves under pure bending

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

For the 13 mm thick plate, determine the largest bending moment that can be applied if the allowable bending stress is 90 MPa. Units: mm.



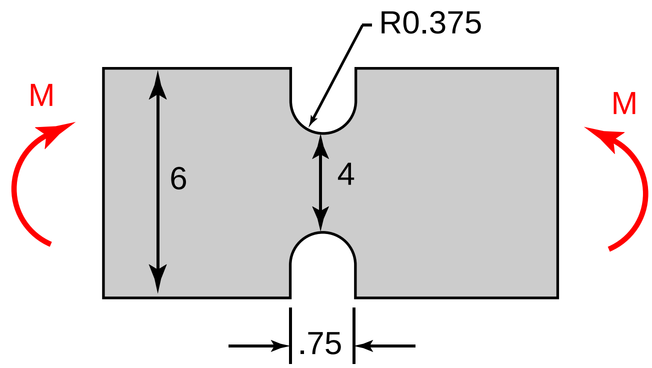


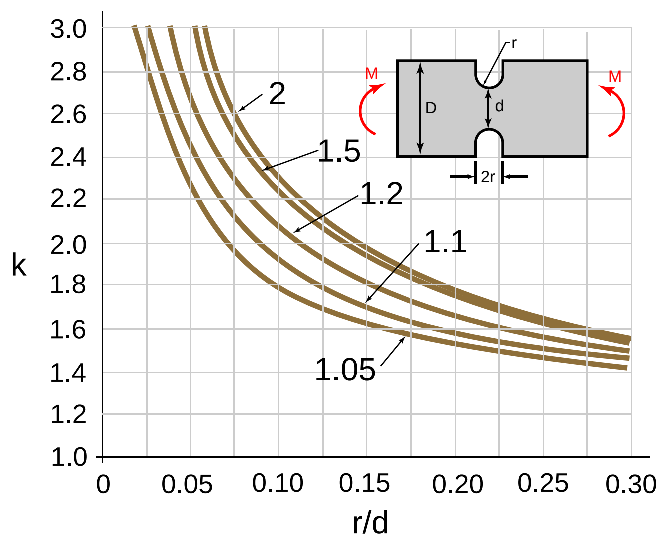
Stress-concentration factors for flat bars with fillets under pure bending

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

Example

For the 7/8" thick plate, determine the largest bending moment that can be applied if the allowable bending stress is 24,000 psi. Units: in.

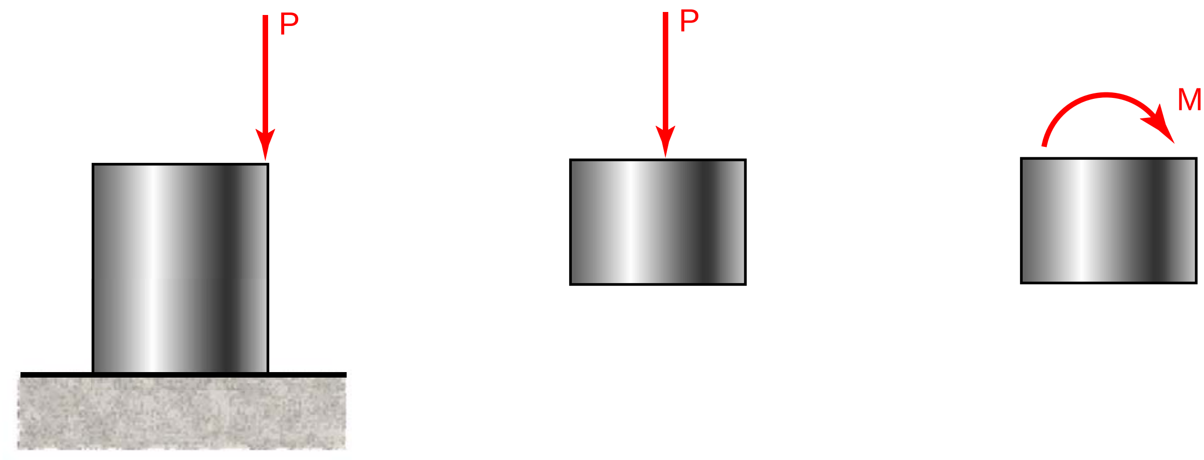




Stress-concentration factors for flat bars with groves under pure bending

Ref.: W.D. Pilkey, *Peterson's Stress Concentration Factors*, 2nd ed., John Wiley and Sons, New York, 1997

ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

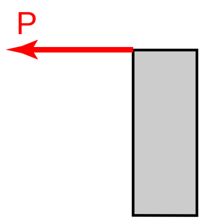


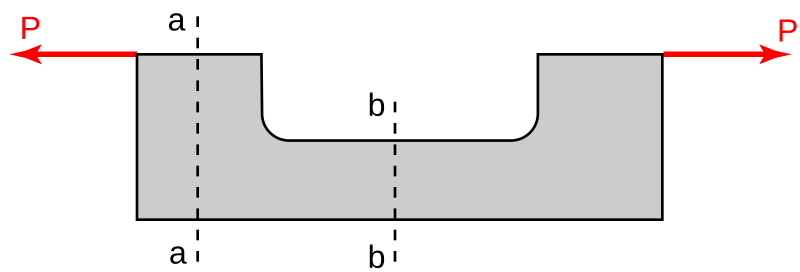
In general,

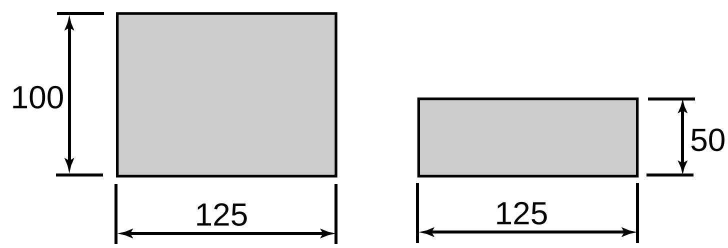
$$\sigma_x = \frac{P}{A} + \frac{My}{I}$$

Example

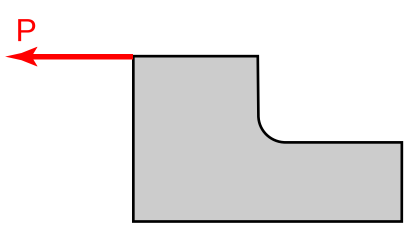
For the solid rectangular bar, determine the largest load P that can be applied based on a maximum normal stress of 130 MPa. Ignore any stress concentrations. Units: mm.





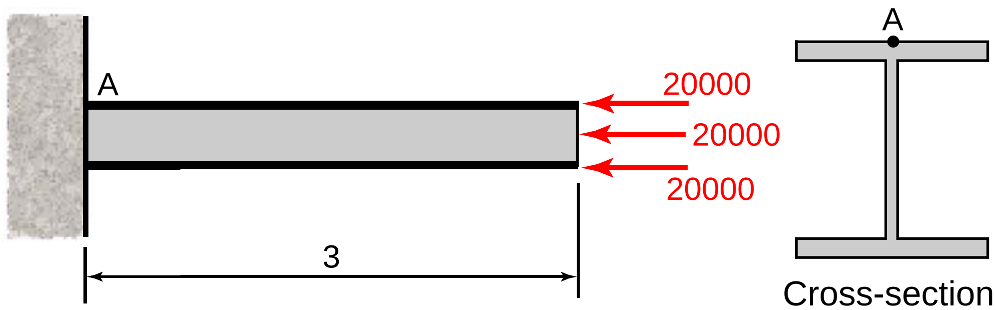


Section a-a   Section b-b



Example

The three loads are applied at the end of the W6x20 beam. Find the normal stress at the wall at point A for the beam, (a) if all three loads are applied, (b) the bottom load is removed. Units: lb, ft.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

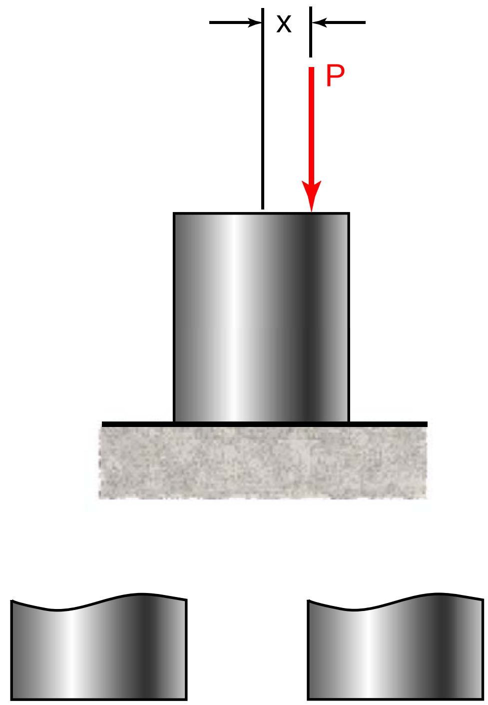
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

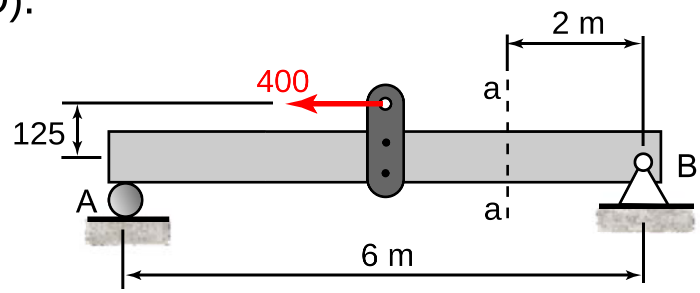
Example

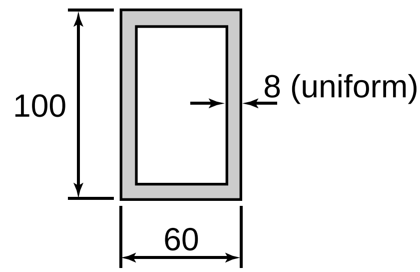
The 100 mm diameter solid circular bar has an eccentric load P applied. Determine the maximum location x that the load can be placed without inducing any tensile stresses. Units: mm.



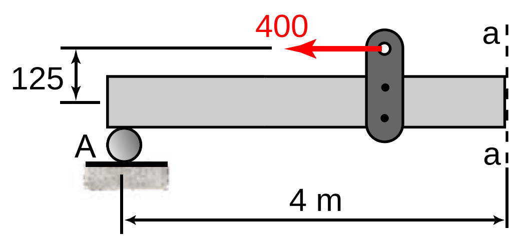
Example

Compute the maximum tension and compression stresses located at section a-a.   Units: N, mm (UNO).

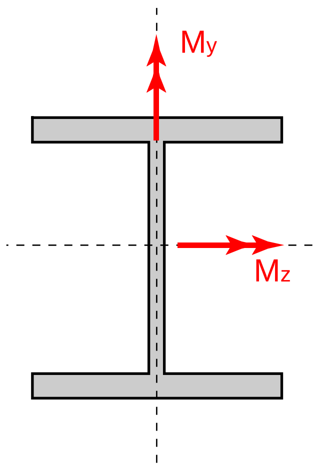




Section a-a

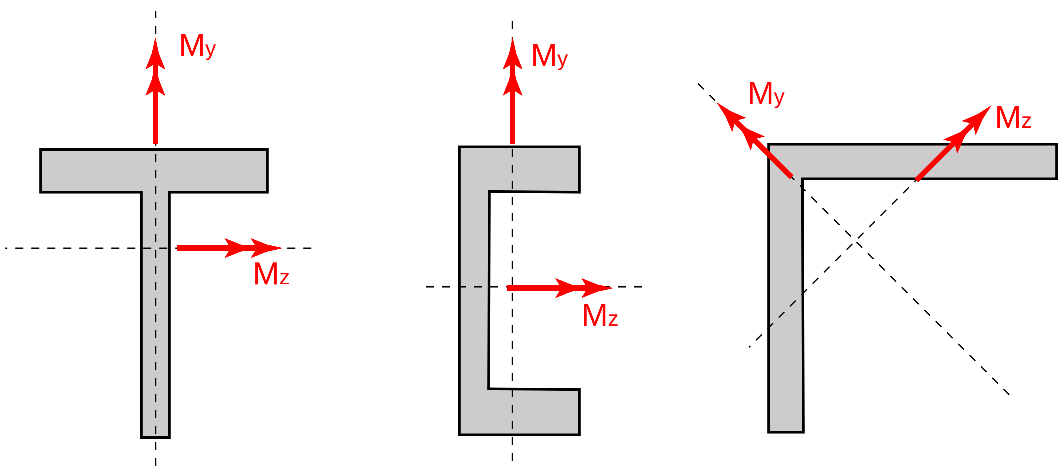


UNSYMMETRIC BENDING



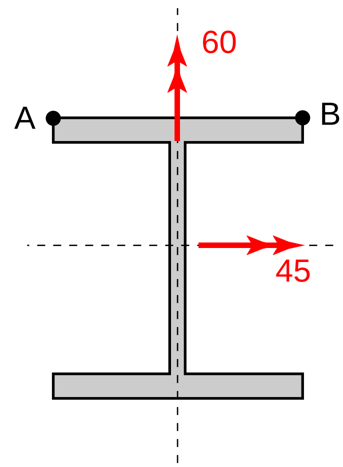
$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



Example

For the W6x20 section, determine the normal stresses at A and B.
Units: kip•in.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

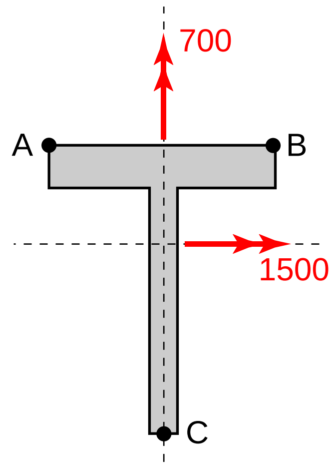
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

For the WT18x150 section, determine the normal stresses at A, B and C. Units: k•in.



WT18x150

$\text{Area, A} = 44.10^{in^2}$

$\text{Depth, d} = 18.4^{in}$

$\text{Flange Width, } b_f = 16.7^{in}$

$\text{Flange Thickness, } t_f = 1.68^{in}$

$\text{Web Thickness, } t_w = 0.945^{in}$

$I_x = 1230^{in^4}$

$I_y = 648^{in^4}$

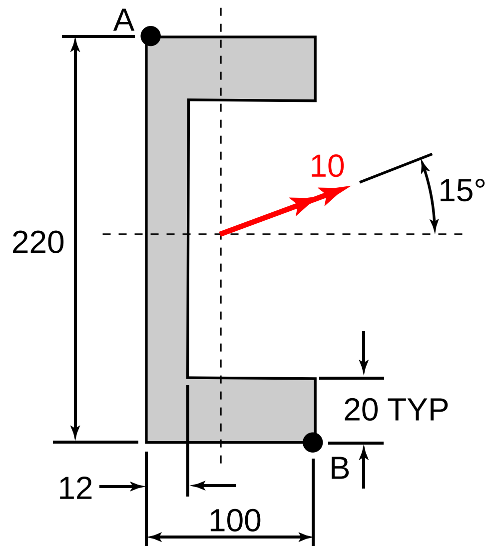
$S_x = 86.1^{in^3}$

$S_y = 77.8^{in^3}$

$\bar{y} = 4.13^{in}$

Example

For the channel section, determine the normal stresses at A and B. Units: kN•m, mm.



Example

For the L76x76x12.7 angle section, determine the normal stresses at A and B. Units: N•m.

$\text{Area, A} = 1770^{mm^2}$

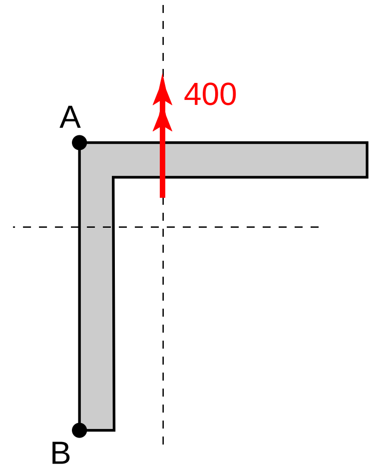
$d = b = 76^{mm}$

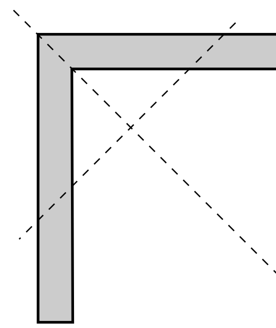
$\overline{x} = \overline{y} = 23.6^{mm}$

$\text{Thickness, t} = 12.7^{mm}$

$I_x = I_y = 0.915x10^{6 mm^4}$

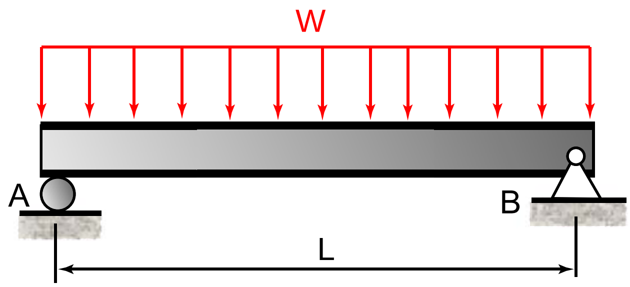
$r_z = 14.8^{mm}$





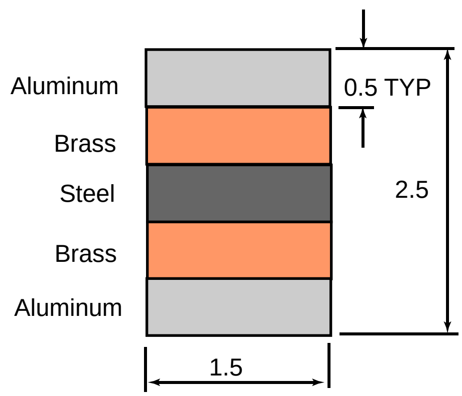
SUMMARY

Bending Stress



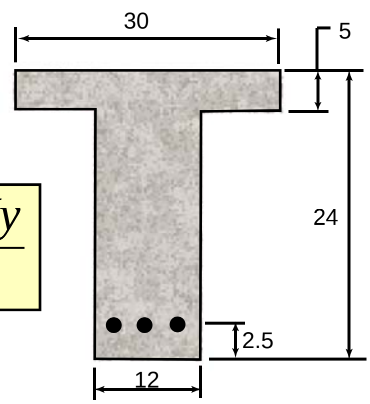
$$\sigma_x = -\frac{My}{I} = -\frac{M}{S}$$

Bending of Members made of Several Materials



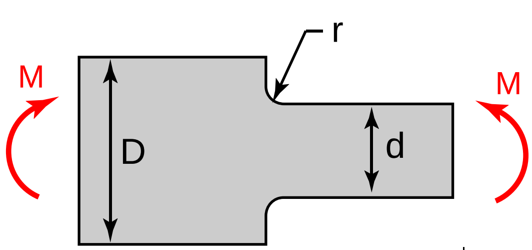
Cross-section, in

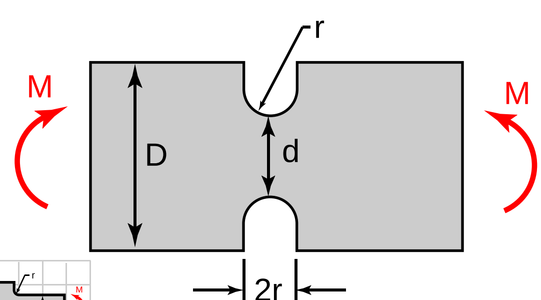
$$\sigma_x = -n\frac{My}{I}$$

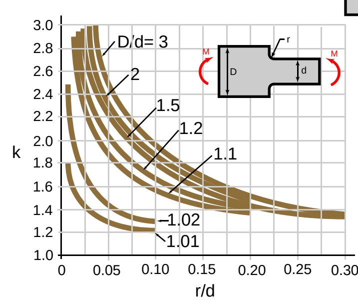


Cross-section, in

Stress Concentrations



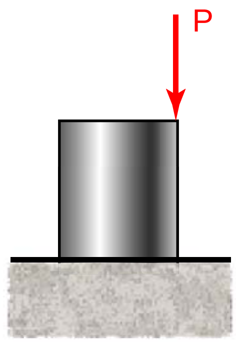




Stress-concentration factors for flat bars with fillets under pure bending

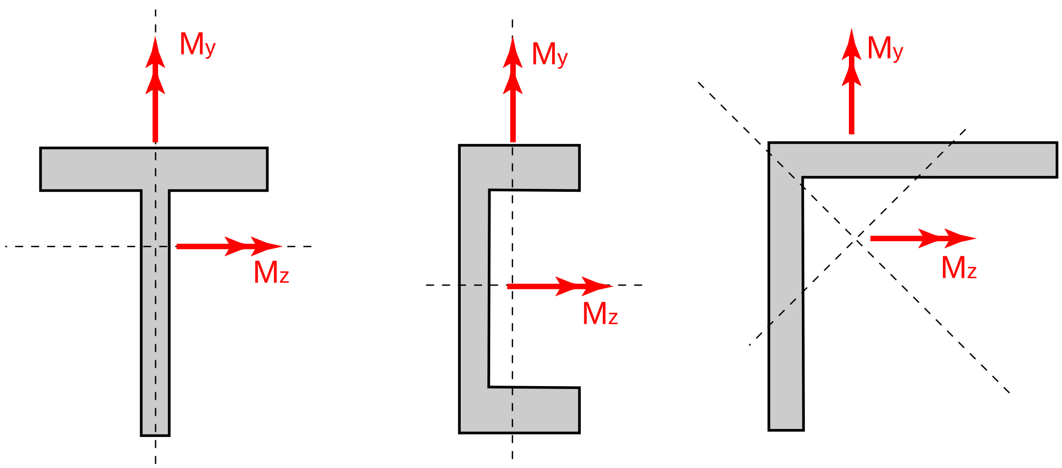
SUMMARY

Eccentric Axial Loading in a Plane of Symmetry



$$\sigma_x = \frac{P}{A} - \frac{My}{I}$$

Unsymmetric Bending

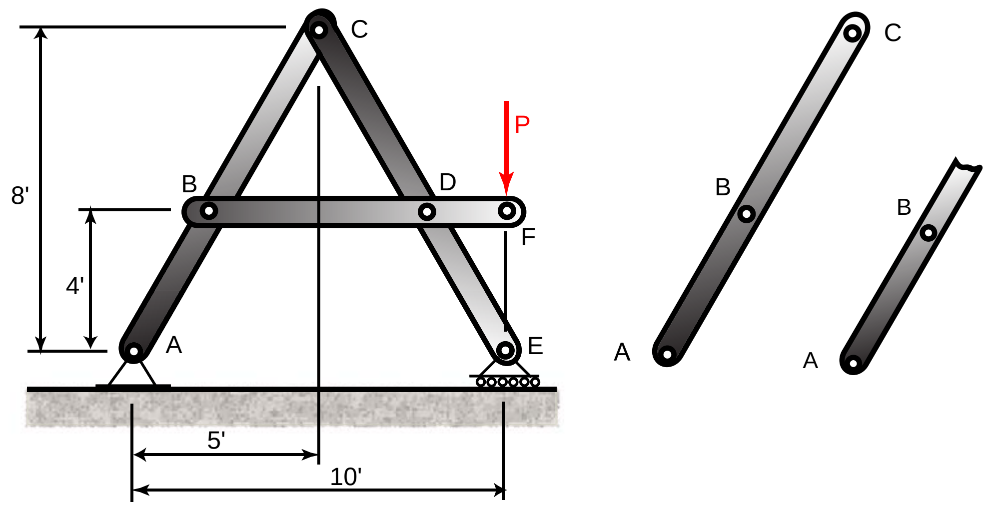


$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

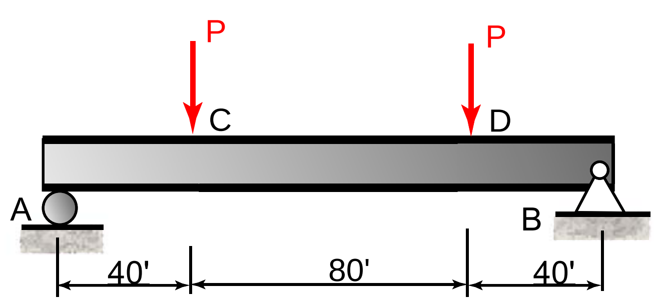
Chapter 5
Analysis and Design of Beams for Bending

INTRODUCTION

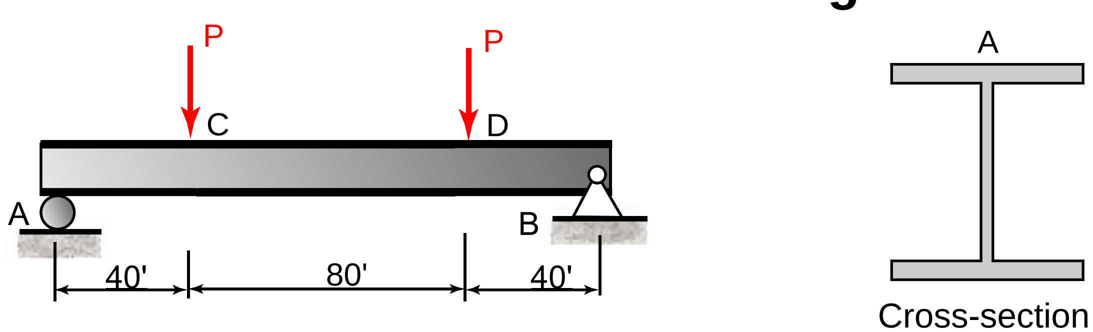
Internal Forces in Members



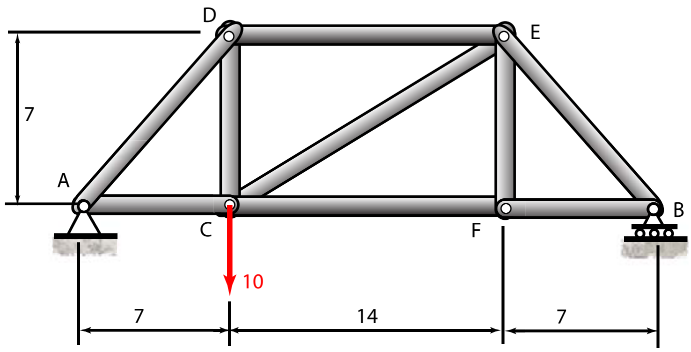
Shear and Bending-Moment Diagrams

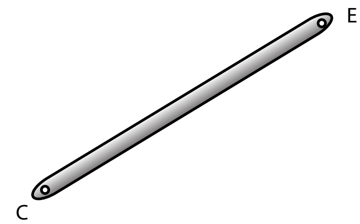


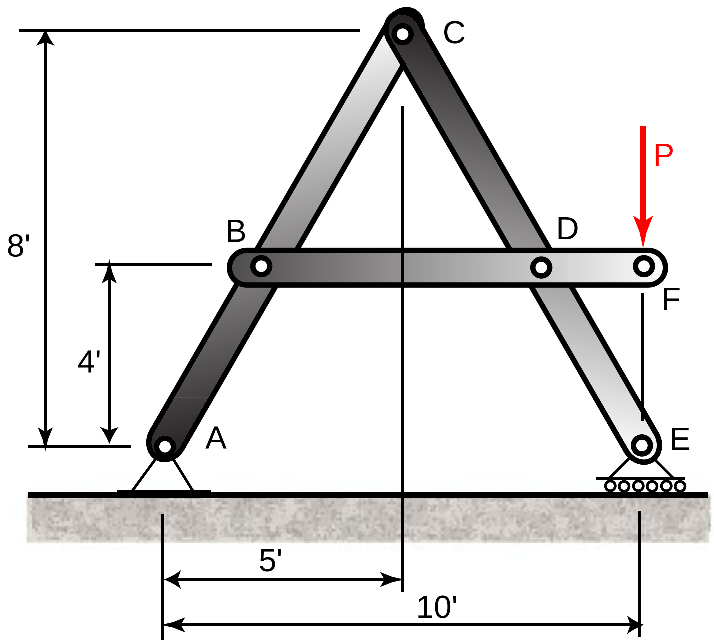
Design of Prismatic Beams for Bending

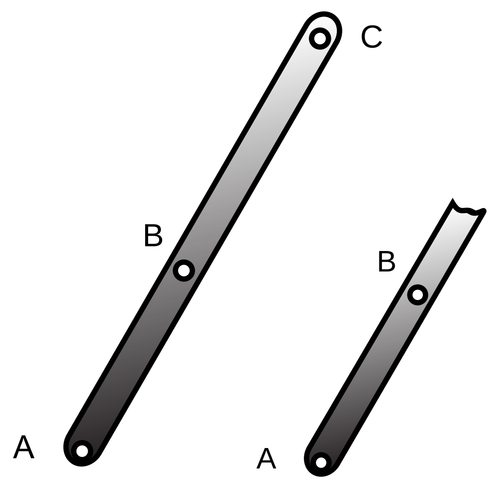


INTERNAL FORCES IN MEMBERS



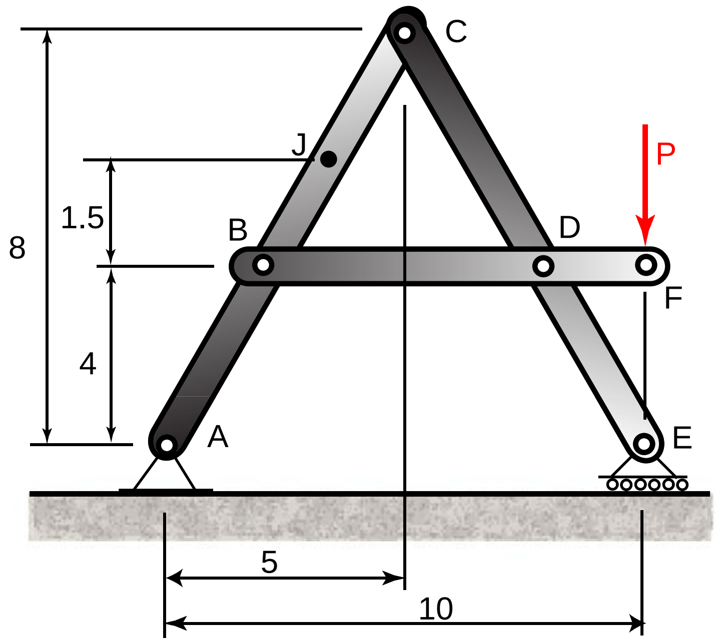


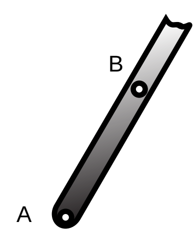




Example

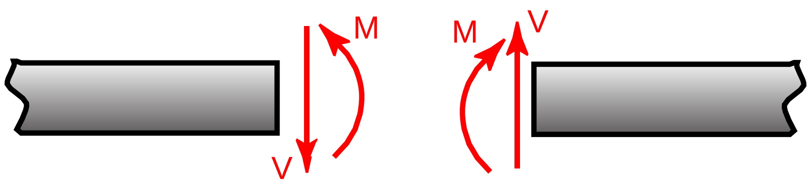
Determine the internal forces at point J. P= 5000 lb.
Units: Lb, ft.



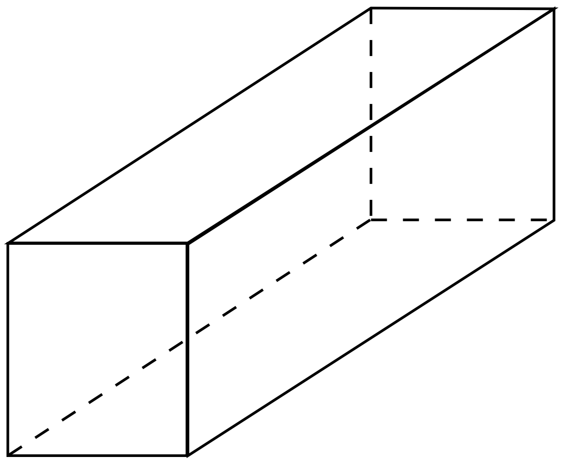


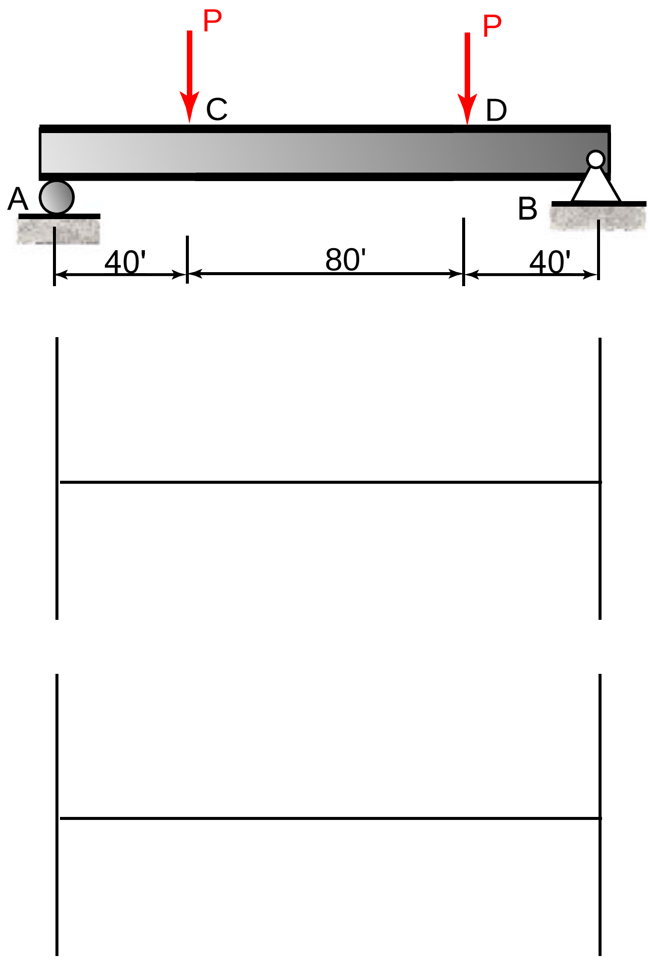
Shear and Bending Moment Diagrams

Sign Convention



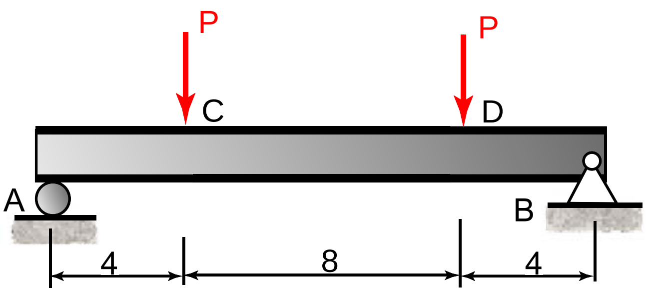
Why do we Need to Draw Shear and Bending Diagrams?



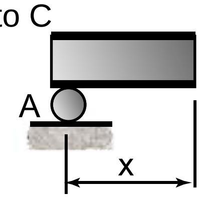
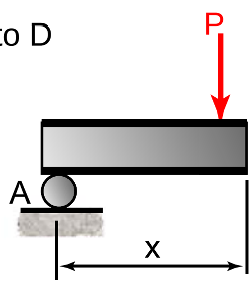
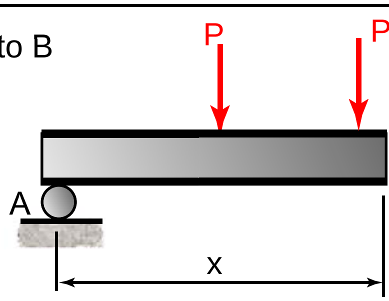


Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.

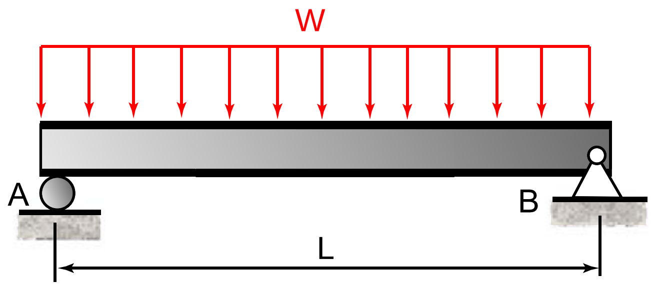


Support Reactions

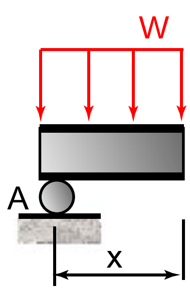
Segment	Shear	Moment
A to C 		
C to D 		
D to B 		

Example

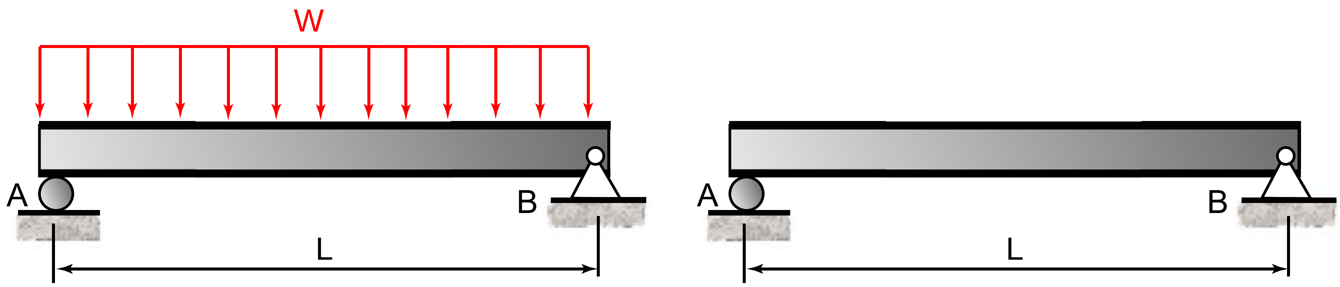
Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes.



Support Reactions

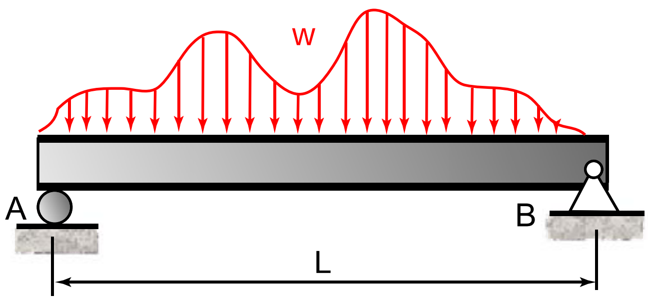
Segment	Shear	Moment
		

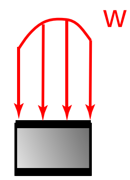
Caution:



When drawing FBDs, always use the original loading and not the equivalent.

RELATIONS AMONG LOAD, SHEAR, AND BENDING-MOMENT





$$\Delta V = -w\Delta x$$

The change in shear is equal to the area under the load curve.

$$\frac{dV}{dx} = -w$$

The slope of the shear diagram is equal to the value of the w load.

$$\frac{dM}{dx} = V$$

The slope of the moment diagram is equal to the value of the shear.

$$\Delta M = V\Delta x$$

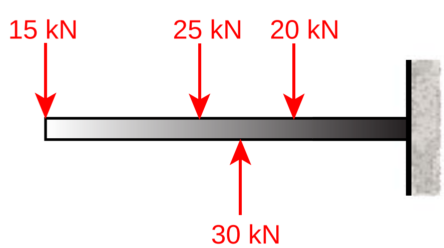
The change in moment is equal to the area under the shear curve.

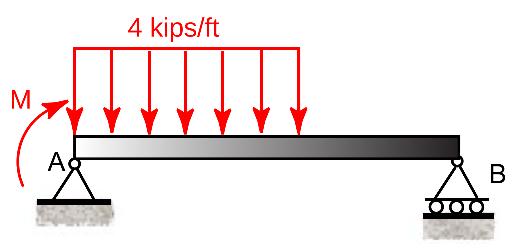
Observations about the Shape of Shear/ Moment Diagrams

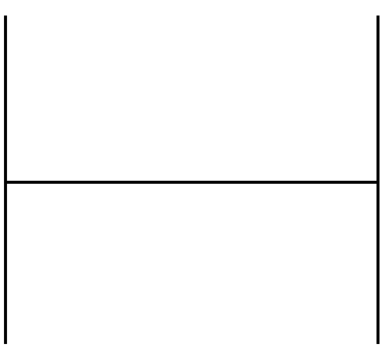
Shear Diagrams:
- -Are a plot of forces (note the units).
- -Discontinuities occur at concentrated forces.

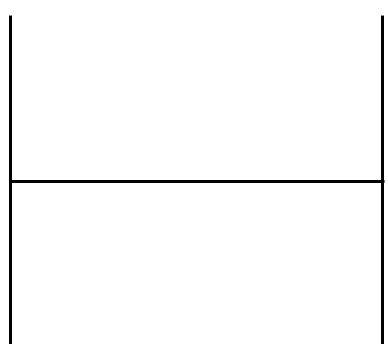
Moment Diagrams:
- -Are a plot of moments (note the units).
- -Discontinuities occur at concentrated moments.

Miscellaneous:
- -Check your work by noting that you always start and end at zero.
- -Always use the original FBD and not the equivalent.





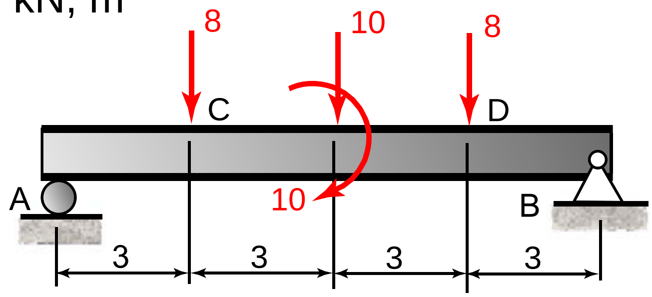


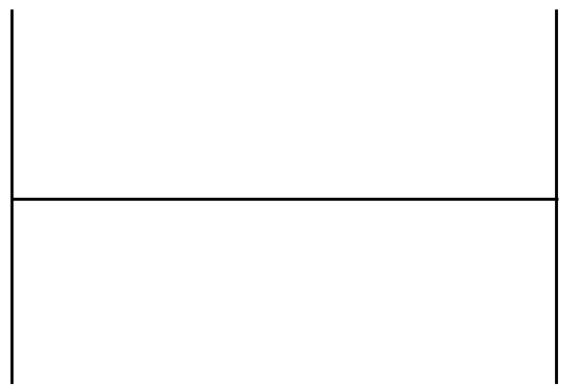


Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: kN, m

Support Reactions

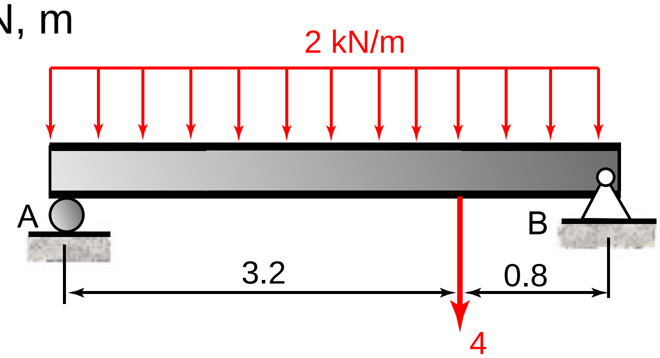


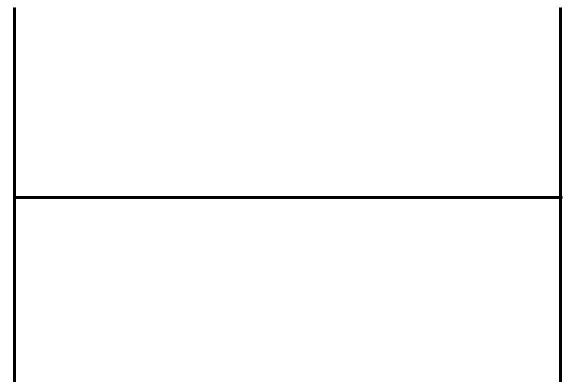


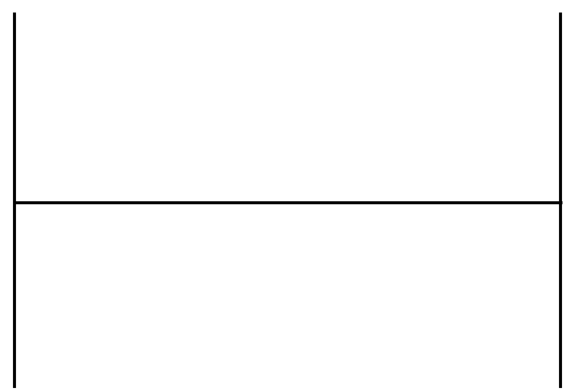
Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: kN, m

Support Reactions

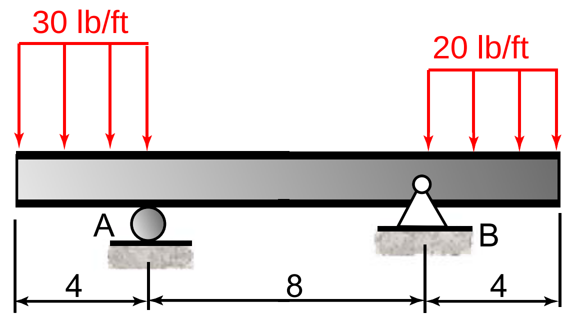


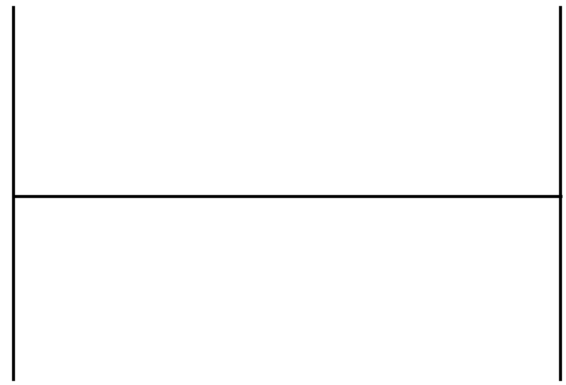


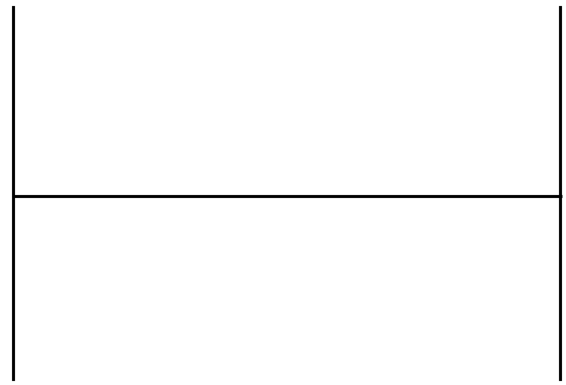


Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.

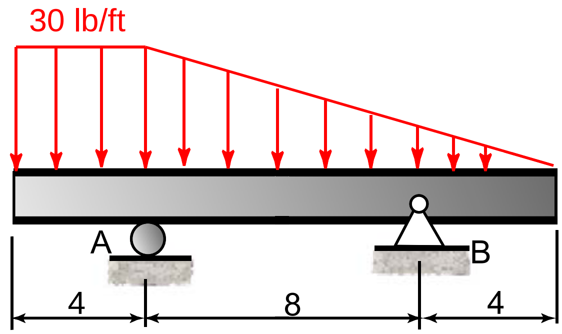


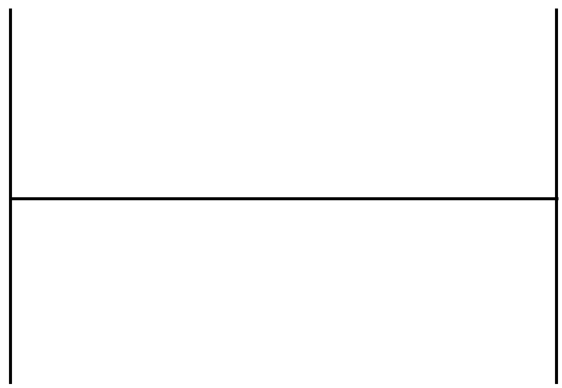


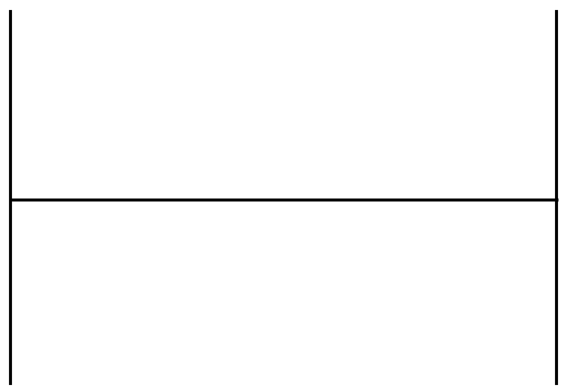


Example

Sketch the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.

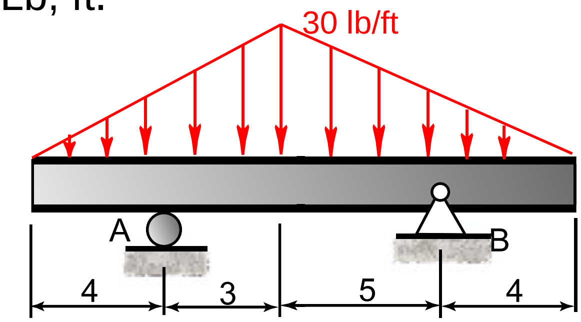






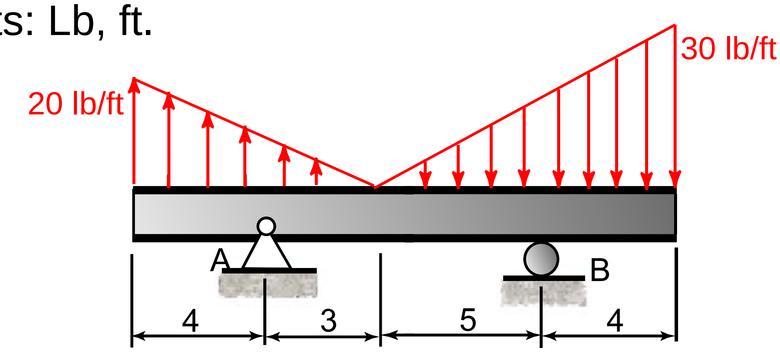
Example

Sketch the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.



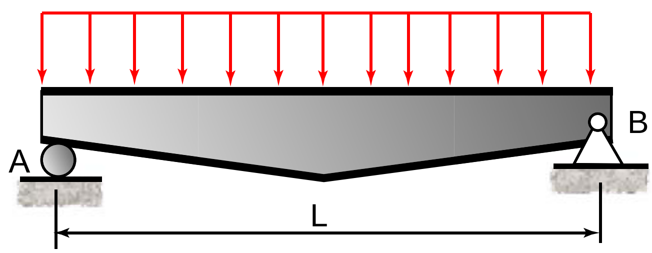
Example

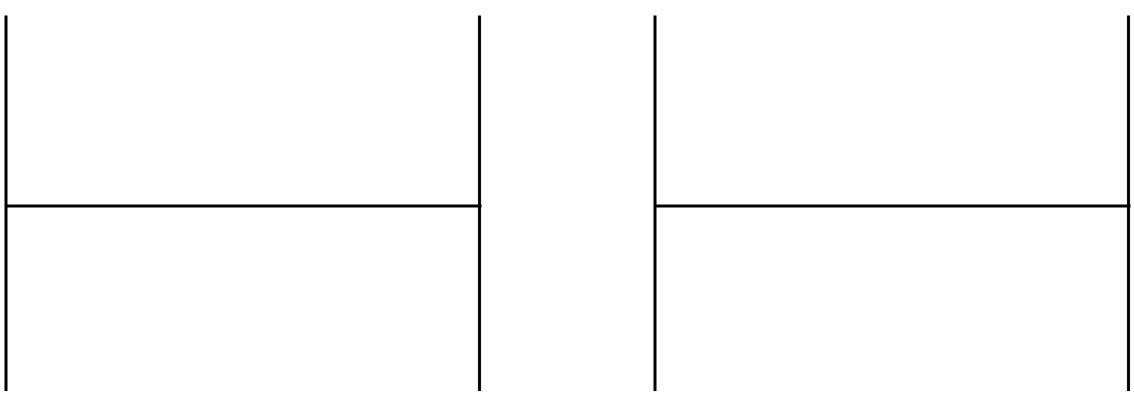
Sketch the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.



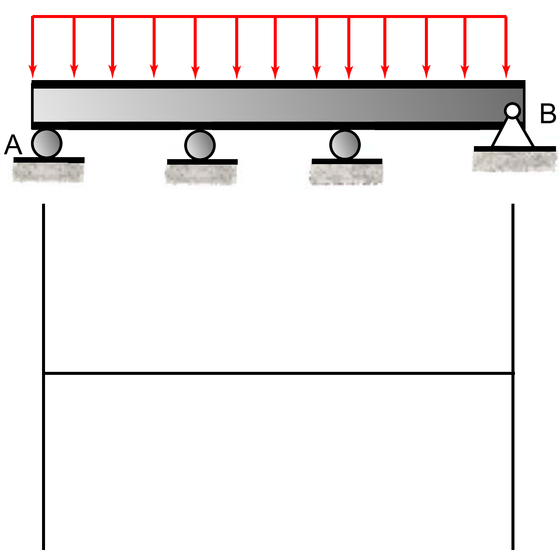
Example

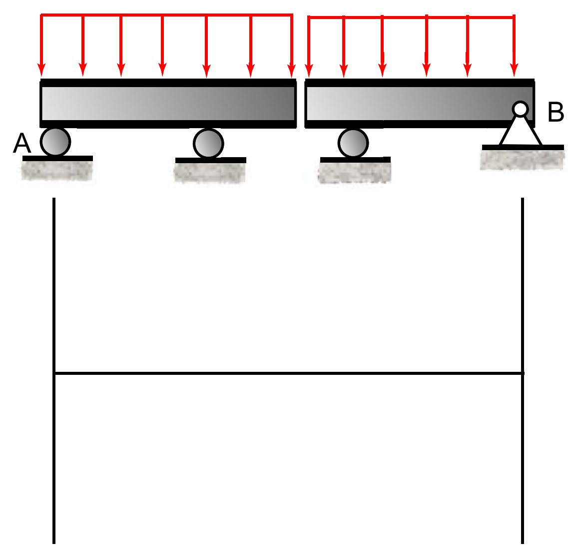
Here is an example of how the shape of the girder reflects the shear and bending diagrams.





So why did they put that gap in the bridge?

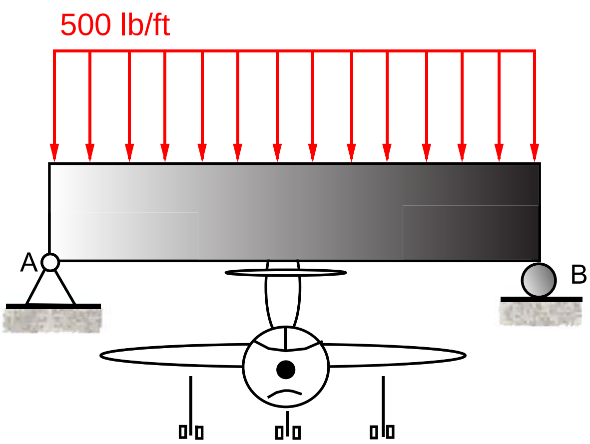


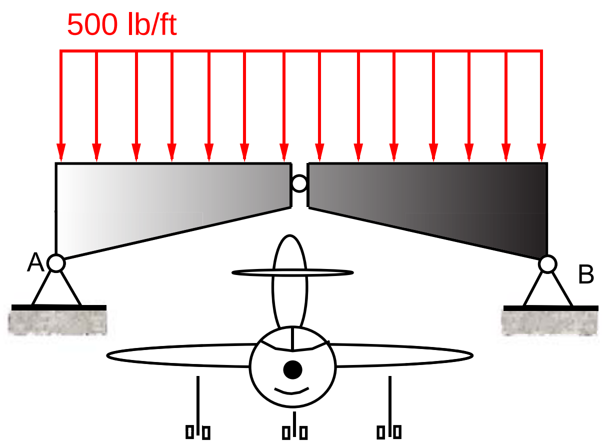


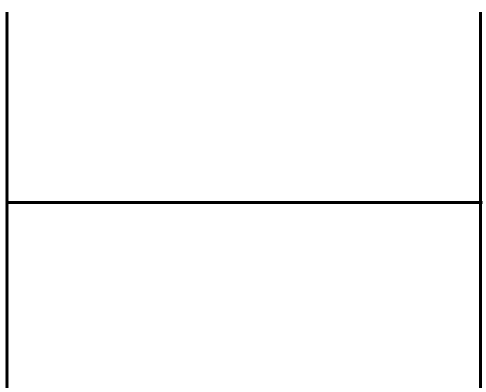
Pins

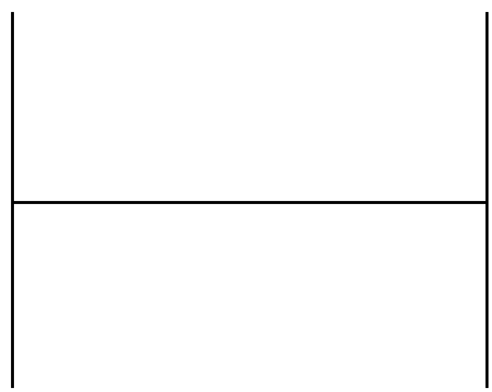
Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. The addition of the internal pin at the center of the beam allows additional head room because rather than the moment being a maximum in the center it becomes zero. This design is used at Wings Air West in SLO. Total span= 75 ft.



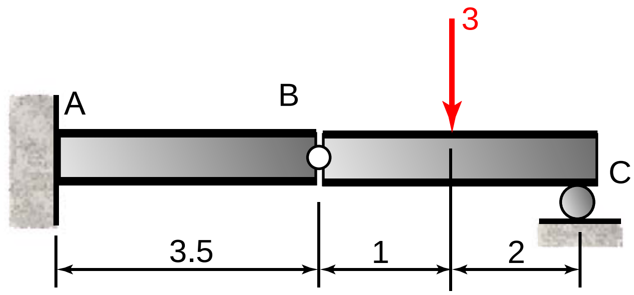




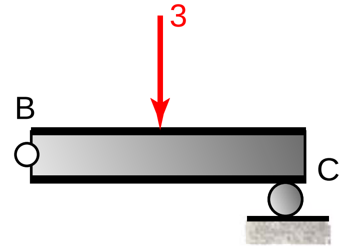


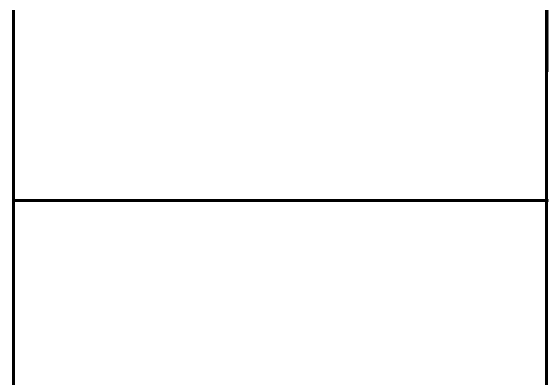
Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. This example demonstrates that with the addition of an internal pin we get an additional equation, otherwise we would have too many unknowns.



Support Reactions

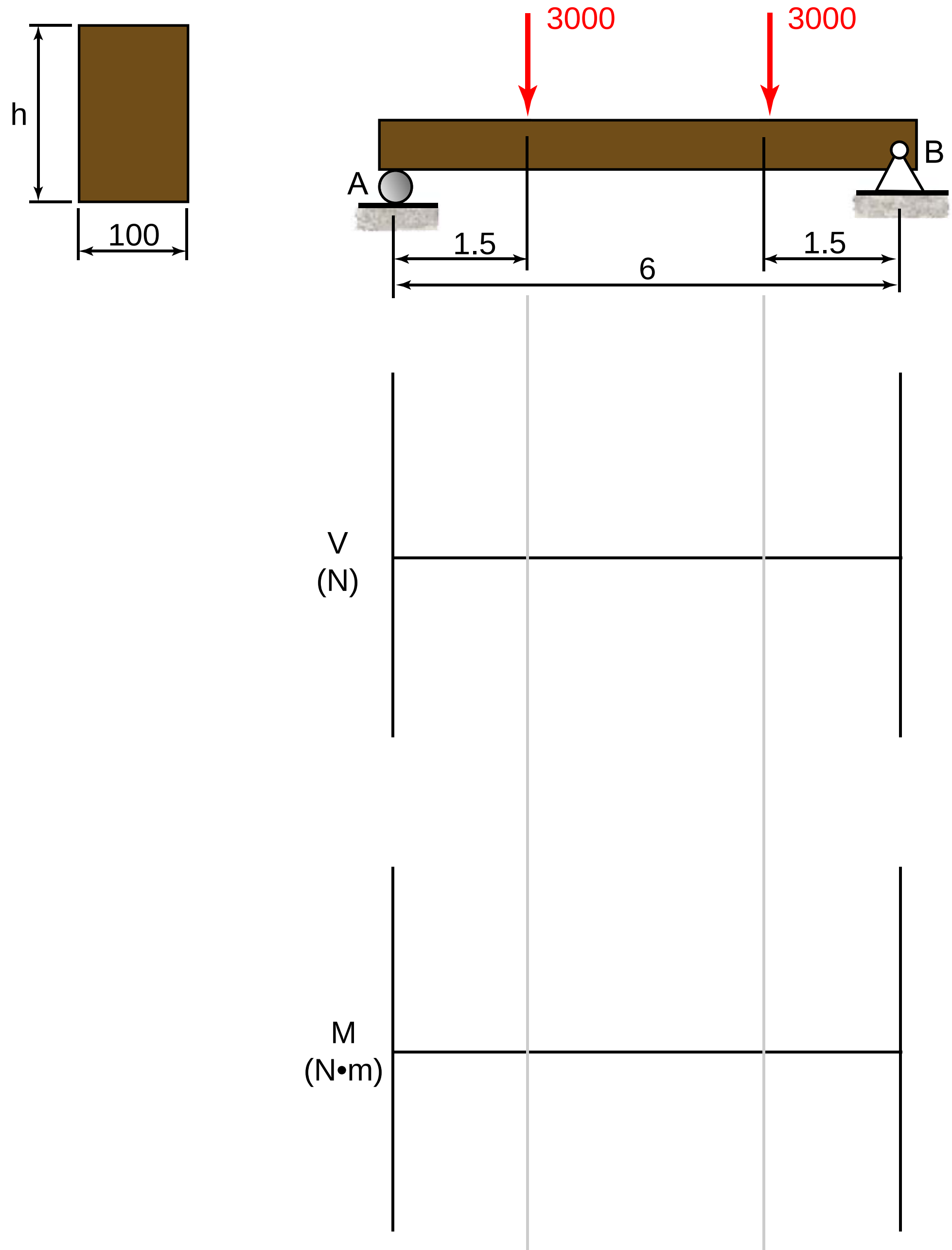




DESIGN OF PRISMATIC BEAMS FOR BENDING

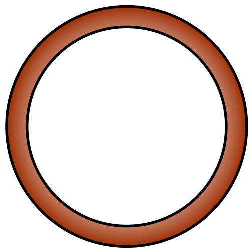
Example

Design the cross section's minimum height of the beam, knowing that the grade of wood used has an allowable bending stress of 12 MPa. Units: N, m.

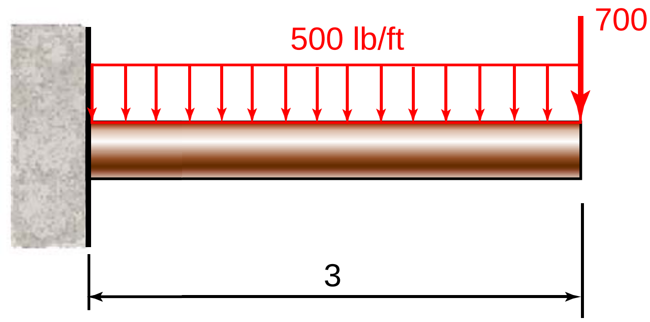


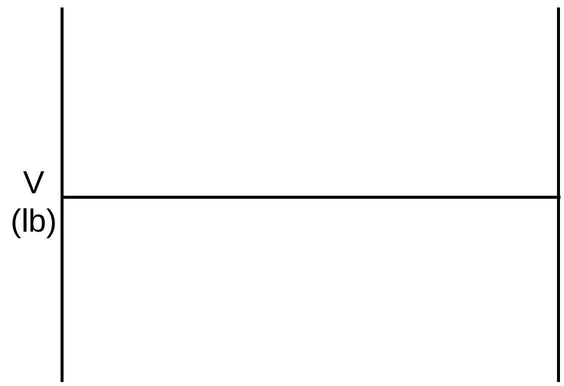
Example

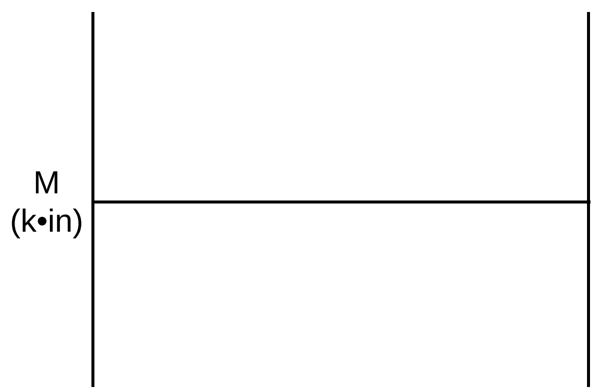
Knowing that the grade of copper used has an allowable bending stress of 15 ksi, determine the minimum wall thickness for the 6" diameter pipe. Units: lb, ft.



Cross-section

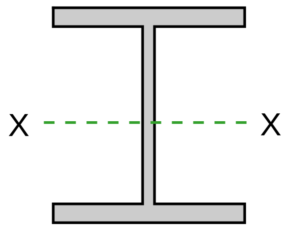




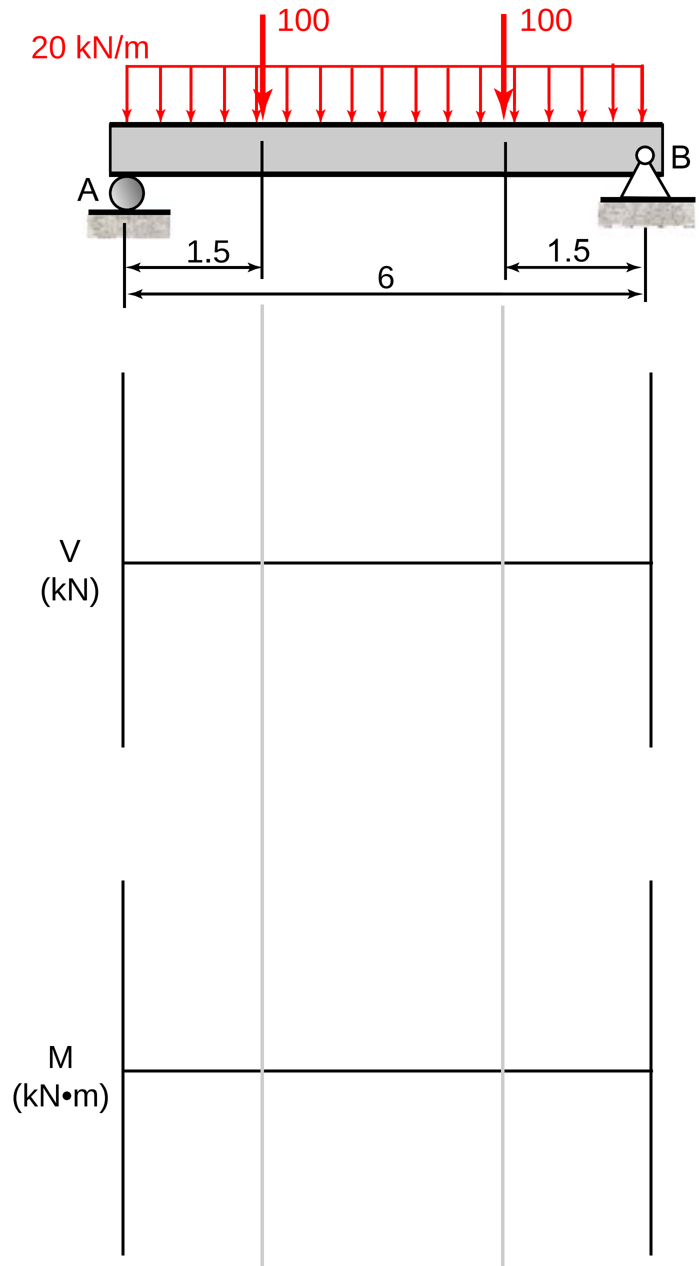


Example

Knowing that the allowable bending stress for steel is 160 MPa, determine the most economical W410-shape to support the load. Units: kN, m.

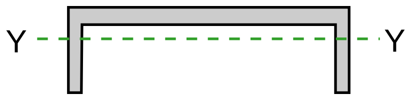


Cross-section

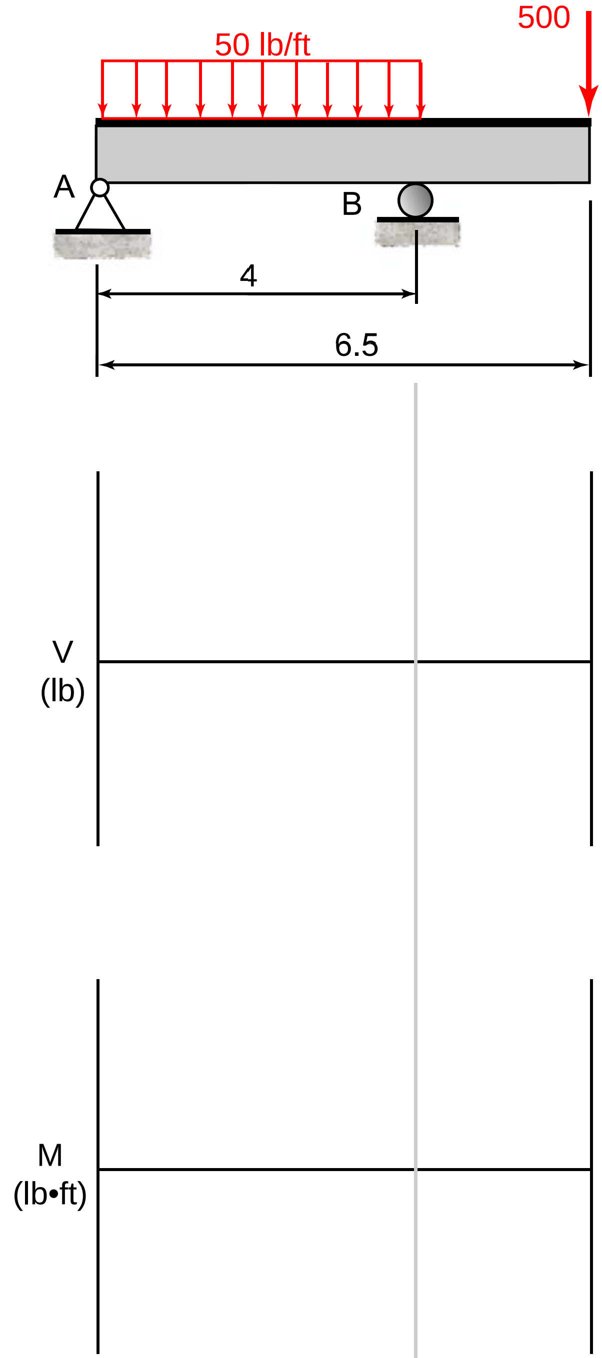


Example

Knowing that the allowable bending stress for steel is 24 ksi, determine the most economical C7-shape to support the load.
Units: lb, ft.

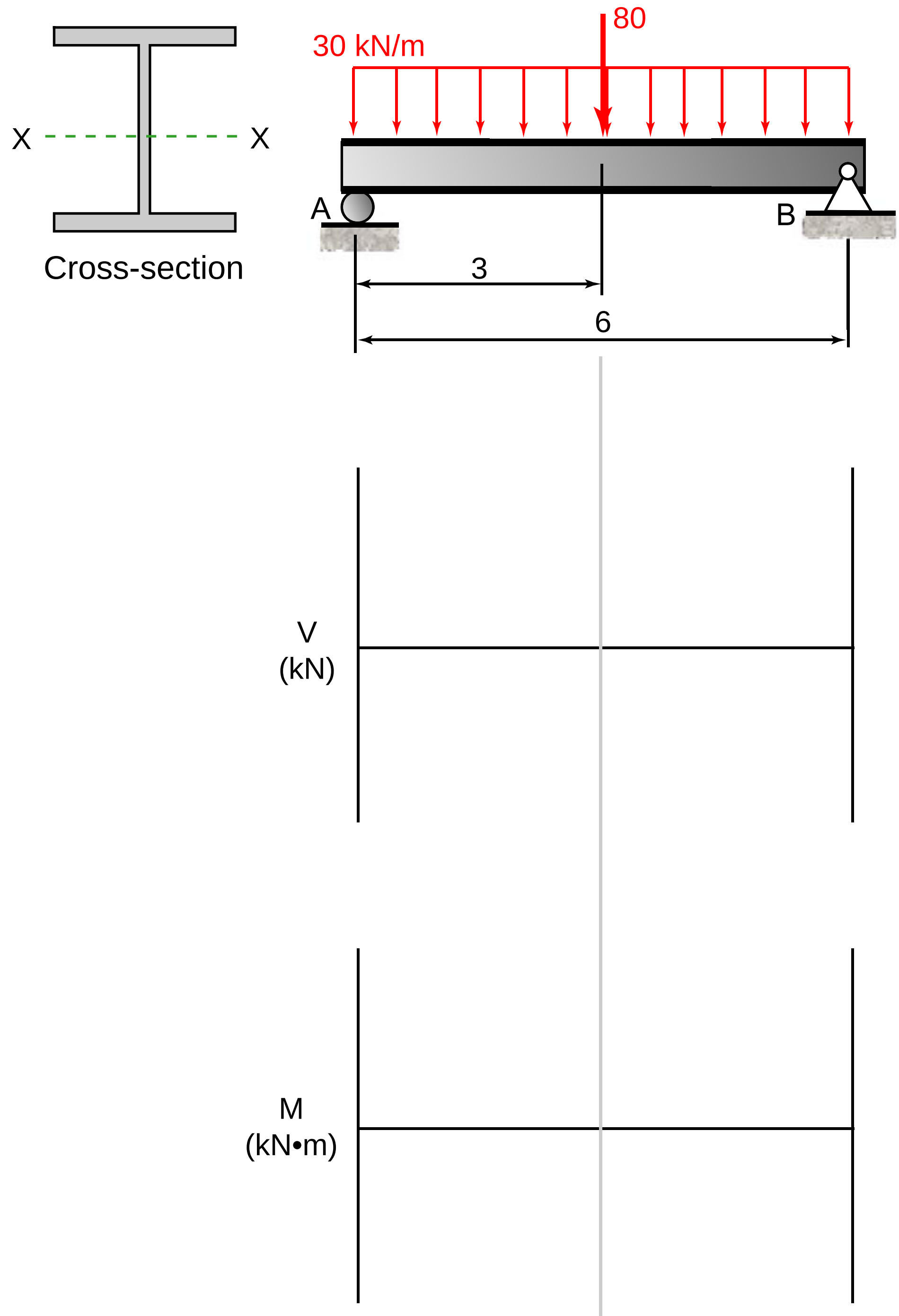


Cross-section



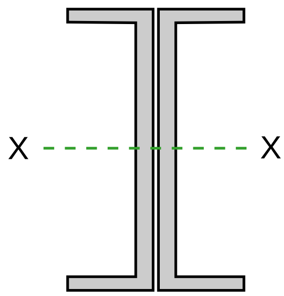
Example

Knowing that the allowable bending stress for steel is 160 MPa, determine the most economical W310-shape to support the load. Units: kN, m.

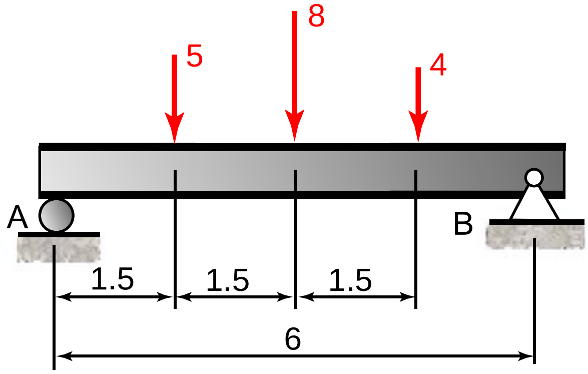


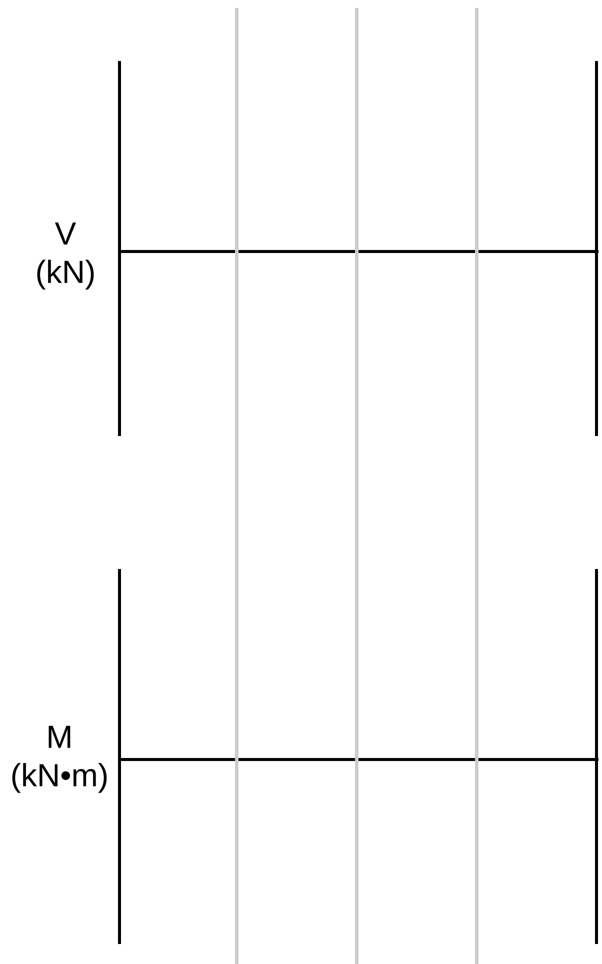
Example

Two rolled-steel C150 channels are welded back to back. Knowing that the allowable bending stress for steel is 160 MPa, determine the most economical channels to support the load.
Units: kN, m.



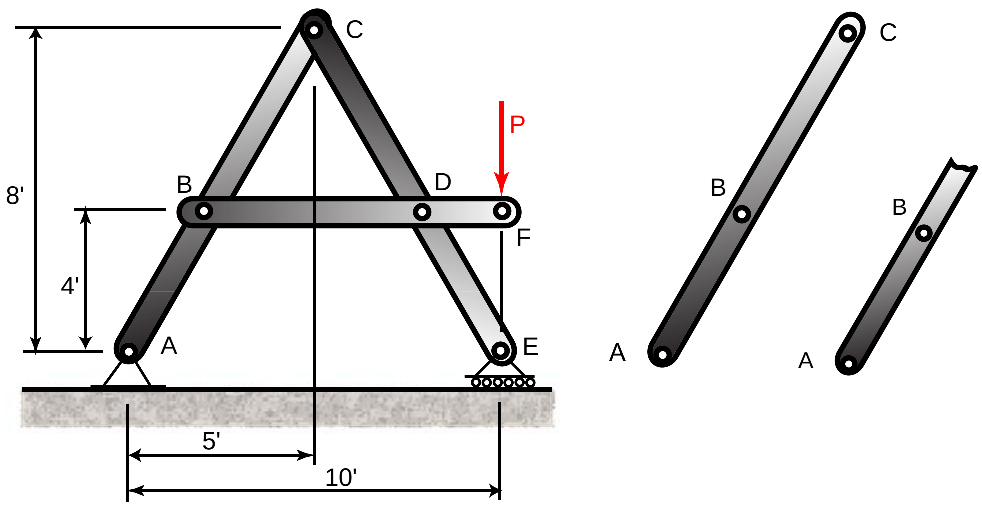
Cross-section



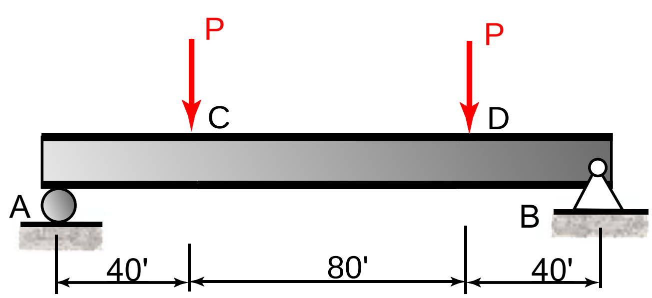


SUMMARY

Internal Forces in Members



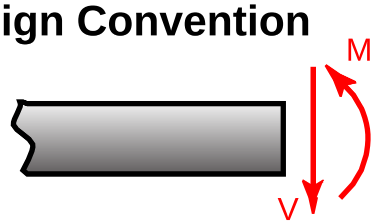
Shear and Bending-Moment Diagrams



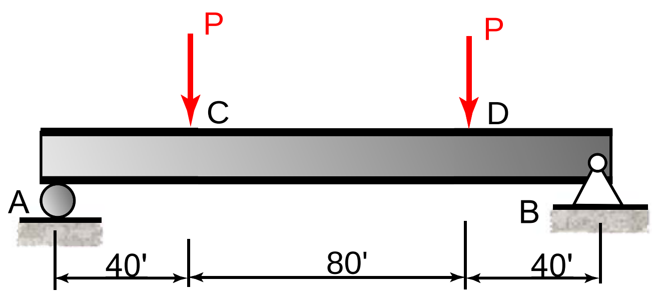
$$\Delta V = -w\Delta x$$

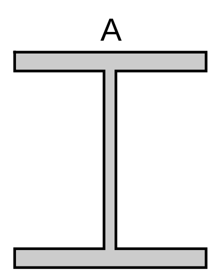
$$\Delta M = V\Delta x$$

Sign Convention



Design of Prismatic Beams for Bending



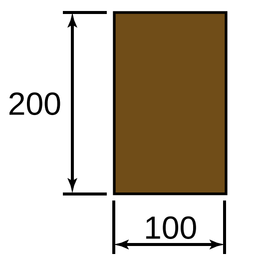


Cross-section

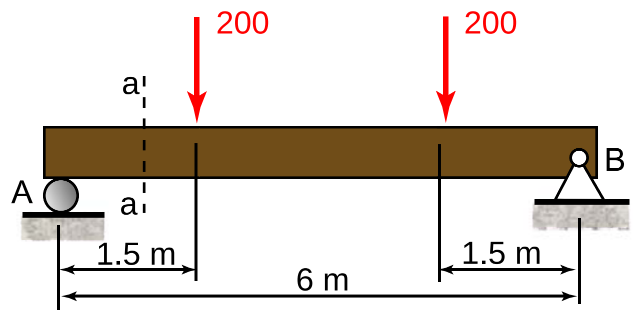
Chapter 6
Shearing Stresses in Beams and Thin-Walled Members

INTRODUCTION

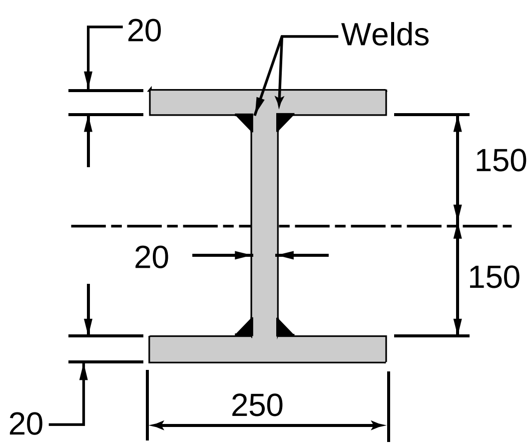
Shearing Stresses in Beams

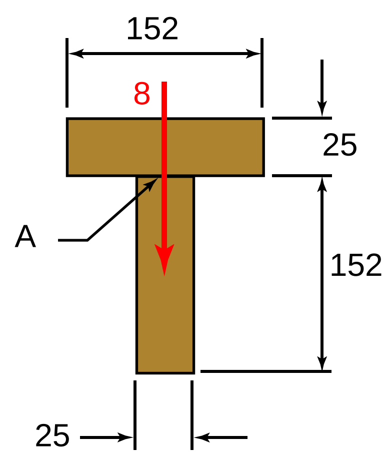


Cross-section

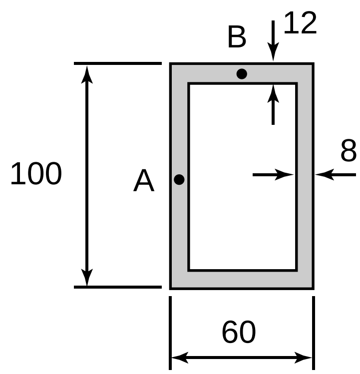


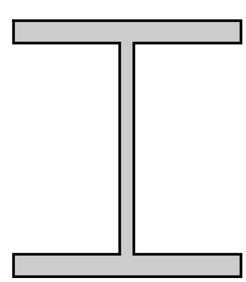
Shearing Forces and Stresses in Built-Up Members



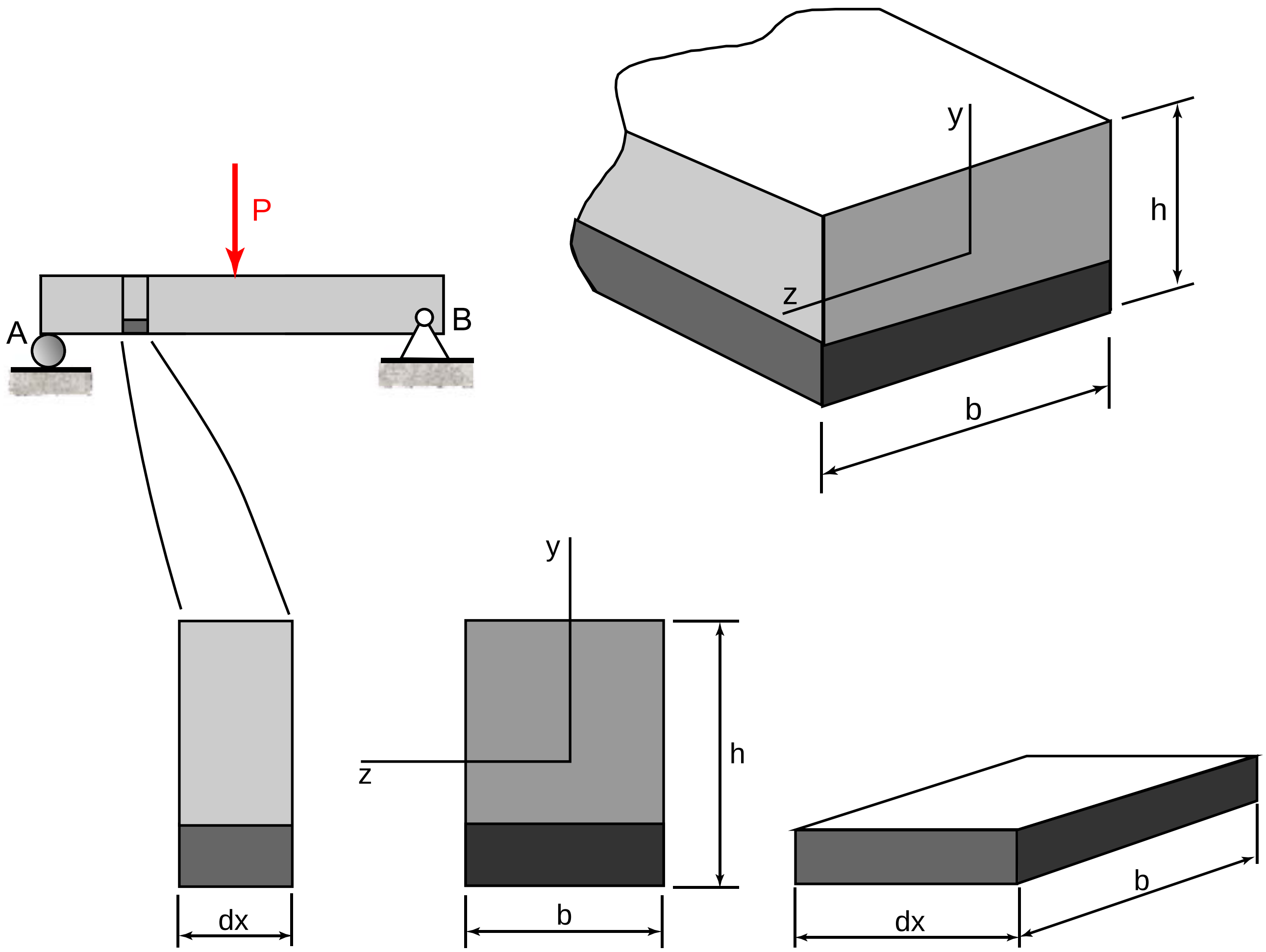


Shearing Stresses in Thin-Walled Members

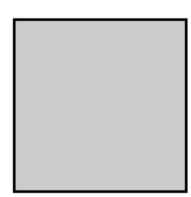




SHEARING STRESSES IN A BEAM



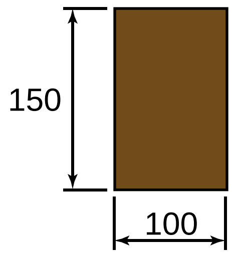
Cross-section



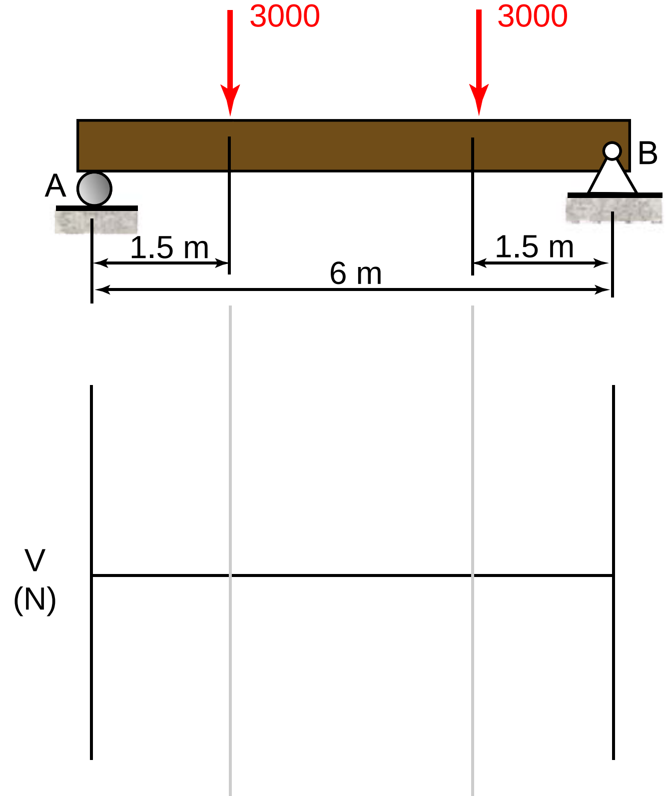
$$\tau = \frac{VQ}{Ib} = \frac{VQ}{It}$$

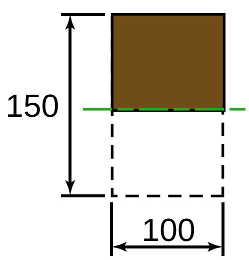
Example

Determine the maximum shearing stress.
Units: N, mm (UNO).



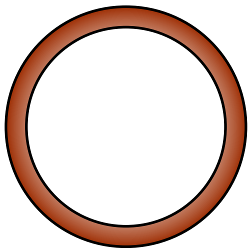
Cross-section



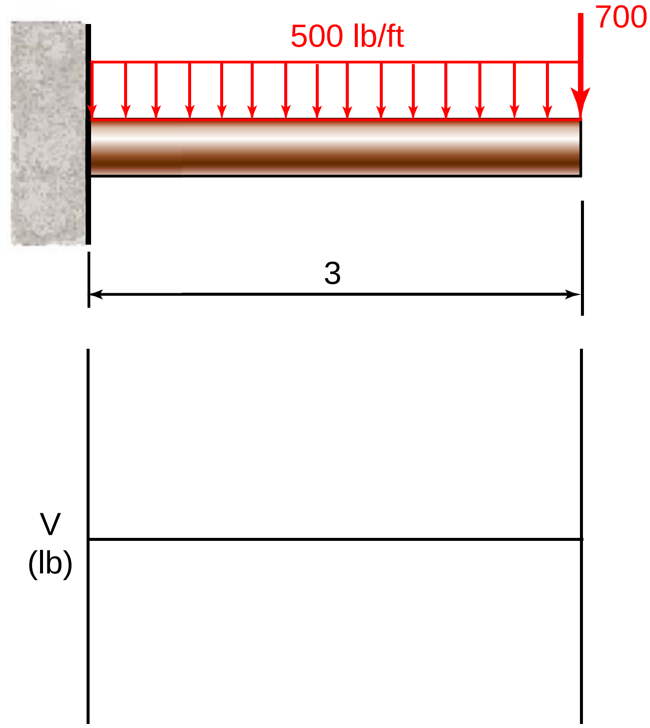


Example

Determine the maximum shear stress for the 6" diameter pipe. The pipe has a wall thickness of 0.28".
Units: lb, ft.

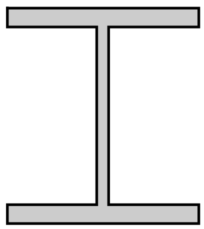


Cross-section

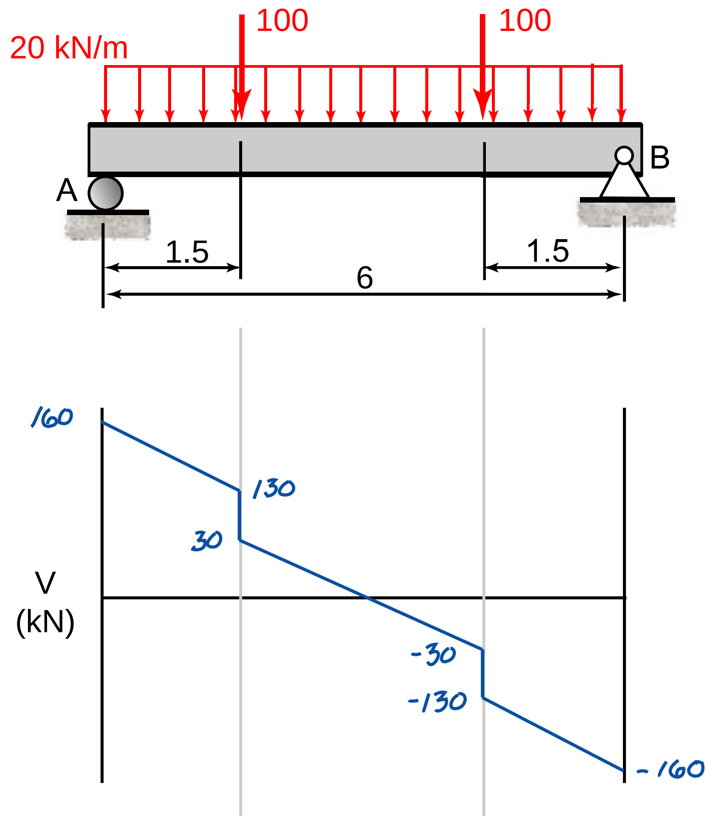


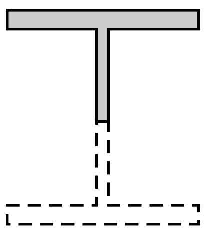
Example

For the W410x85 section, determine the maximum shear stress.
Units: kN, m.



Cross-section





W410x85

Area, $A = 10800^{mm^2}$

Depth, $d = 417^{mm}$

Flange Width, $b_f = 181^{mm}$

Flange Thickness, $t_f = 18.2^{mm}$

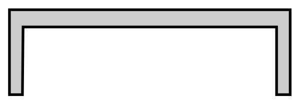
Web Thickness, $t_w = 10.9^{mm}$

$I_x = 315x10^{6mm^4}$

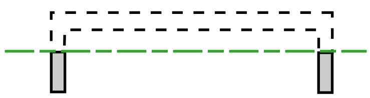
$I_y = 18.0x10^{6mm^4}$

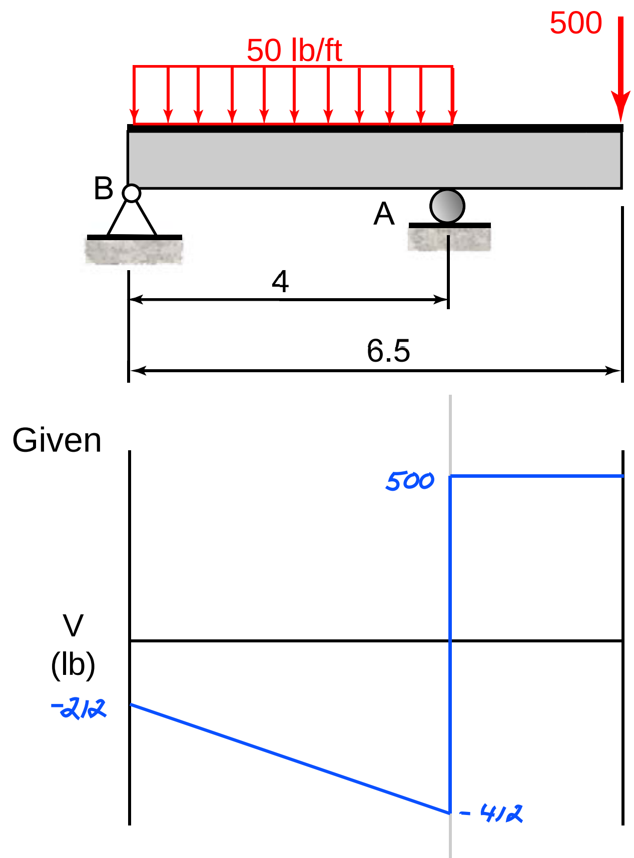
Example

For the C7x9.8 channel section, determine the maximum shear stress.
Units: lb, ft.



Cross-section





C7x9.8

$\text{Area, } A = 2.87^{in^2}$

$\text{Depth, } d = 7.00^{in}$

$\text{Flange Width, } b_f = 2.09^{in}$

$\text{Flange Thickness, } t_f = 0.366^{in}$

$\text{Web Thickness, } t_w = 0.210^{in}$

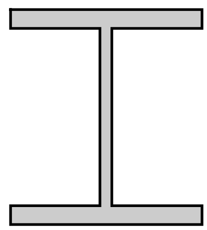
$I_x = 21.3^{in^4}$

$I_y = 0.968^{in^4}$

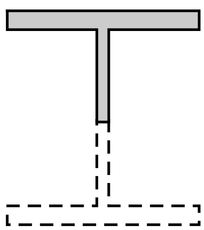
$\overline{x} = 0.540^{in}$

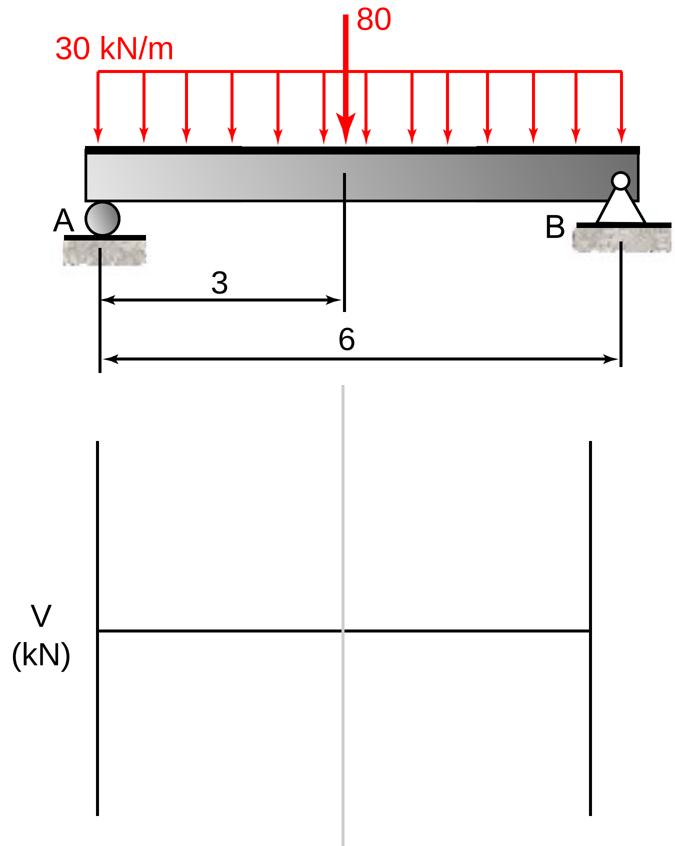
Example

For the W310x107 section, determine the maximum shear stress.
Units: kN, m.



Cross-section





W310x107

Area, A = 13600^{mm^2}

Depth, d = 311^{mm}

Flange Width, $b_f = 306^{mm}$

Flange Thickness, $t_f = 17.0^{mm}$

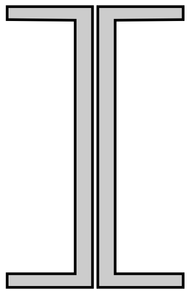
Web Thickness, $t_w = 10.9^{mm}$

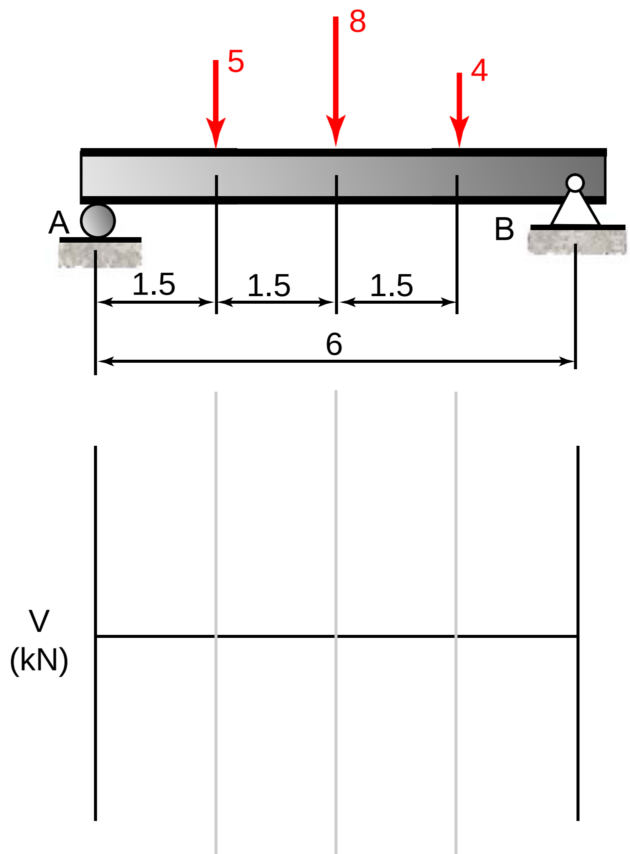
$I_x = 248x10^{6 mm^4}$

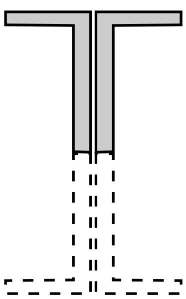
$I_y = 81.2x10^{6 mm^4}$

Example

Two rolled-steel C150x12.2 channels are welded back to back. Determine the maximum shear stress.
Units: kN, m.







C150x12.2

Area, $A = 1540^{mm^2}$

Depth, $d = 152^{mm}$

Flange Width, $b_f = 48^{mm}$

Flange Thickness, $t_f = 8.7^{mm}$

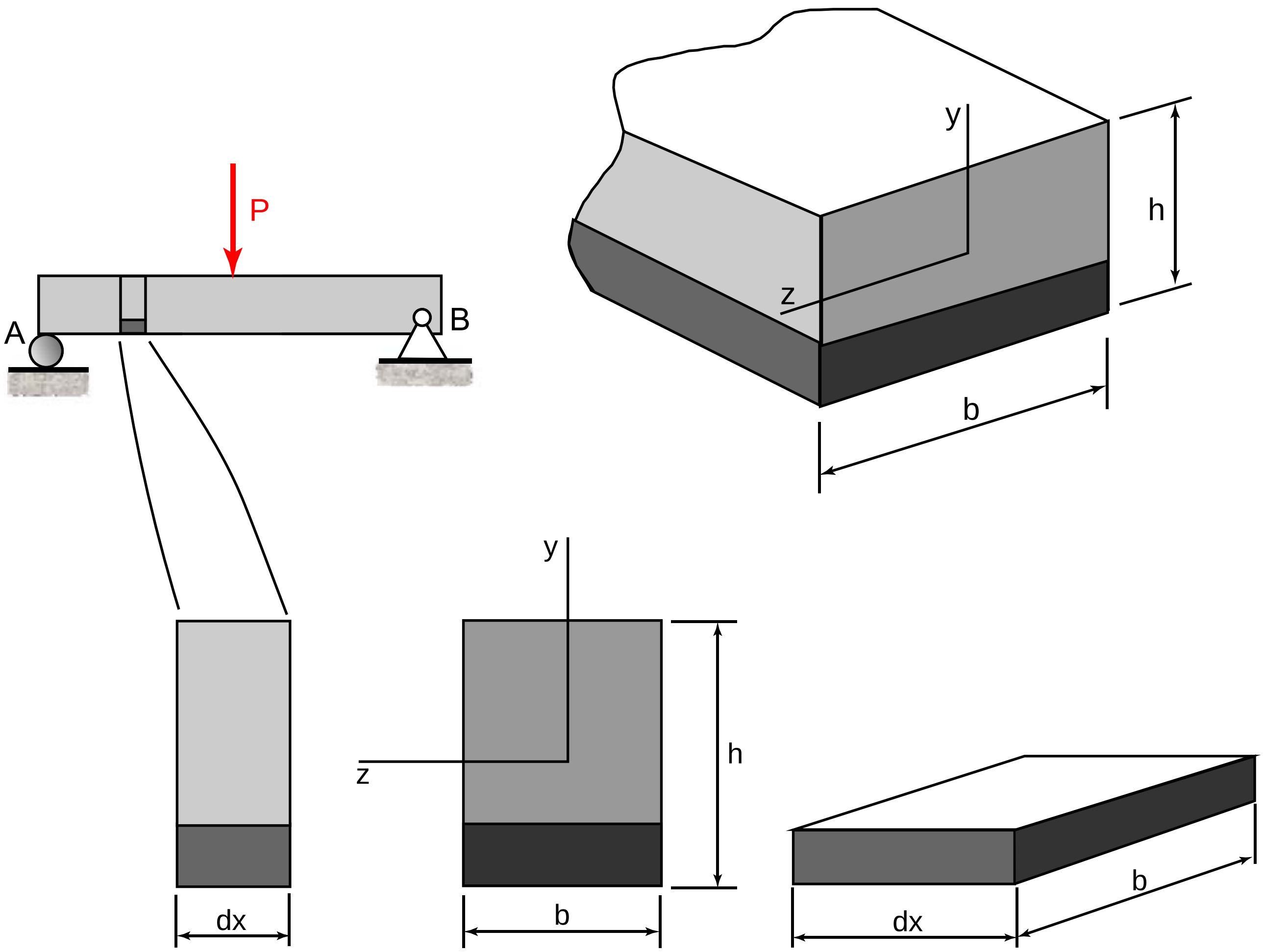
Web Thickness, $t_w = 5.1^{mm}$

$I_x = 5.35x10^{6mm^4}$

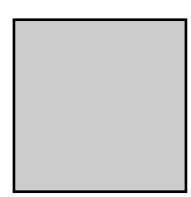
$I_y = 0.276x10^{6mm^4}$

$\overline{x} = 12.7^{mm}$

SHEARING STRESSES IN A BUILT-UP BEAM



Cross-section



$$\tau = \frac{VQ}{Ib} = \frac{VQ}{It}$$

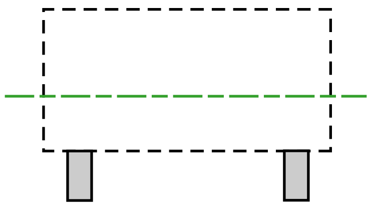
Example

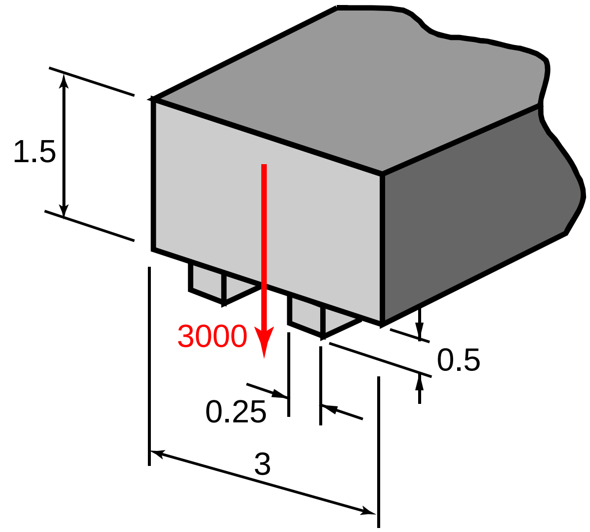
The two 0.25"x0.5" strips are glued to the 3"x1.5" main member. Determine the maximum shear stress in the glue between them. Units: lb, in.

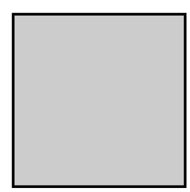
From a previous solution:

$\overline{y} = 0.802'' \downarrow$

$I = 1.09^{in^4}$

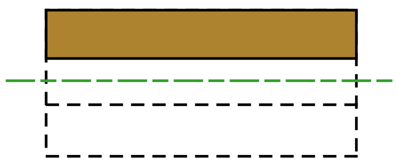


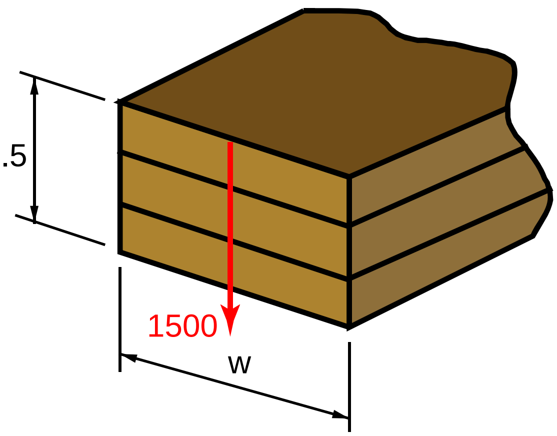


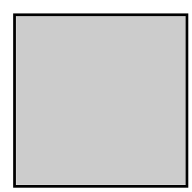


Example

The three 0.50" thick boards are glued together using a glue with a shear capacity of 350 psi. Based on the glue capacity, compute the minimum width of the boards to resist a vertical shear force of 1500 lb. Units: lb, in.





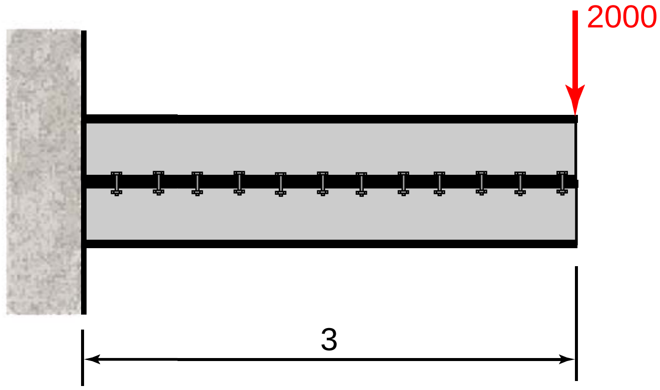


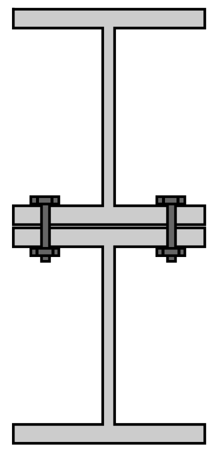
Example

The two beams are connected every 6" by bolts through the flanges. Determine the force in each bolt for the W6x20 built-up beam.
Units: lb, ft

From a previous solution:

$I = 196^{in^4}$





W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

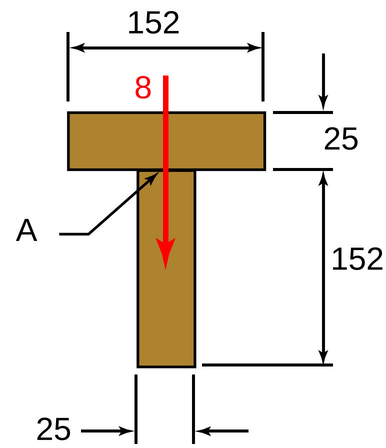
$I_y = 13.3^{in^4}$

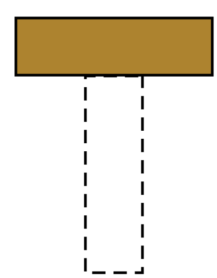
$S_x = 13.4^{in^3}$

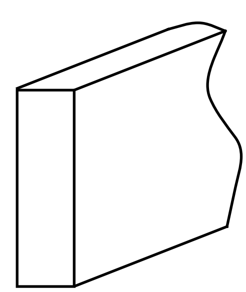
$S_y = 4.41^{in^3}$

Example

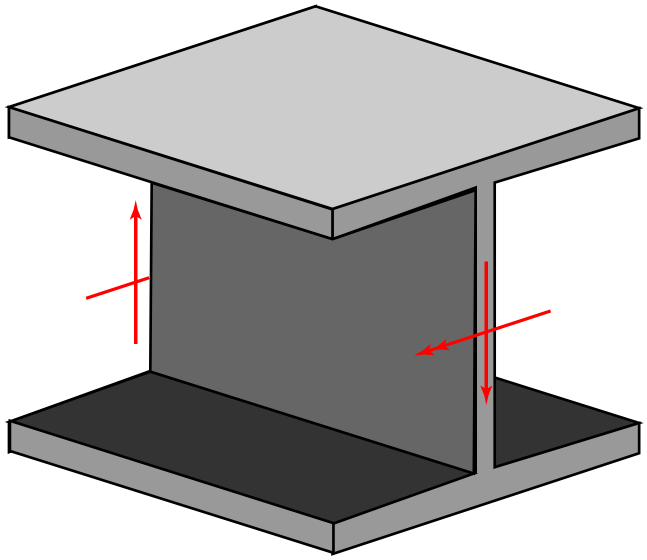
The two boards are glued at A and is subjected to a vertical shear force of 8 kN. Determine the shear stress in the glue.
Units: kN, mm.

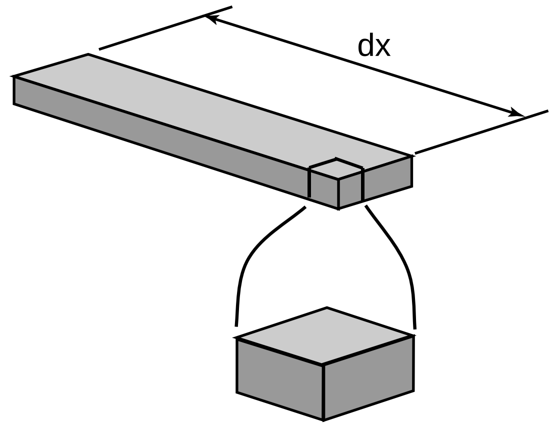


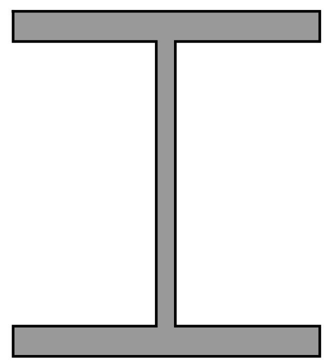


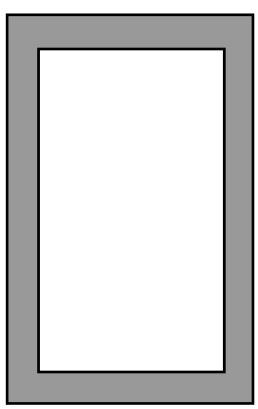


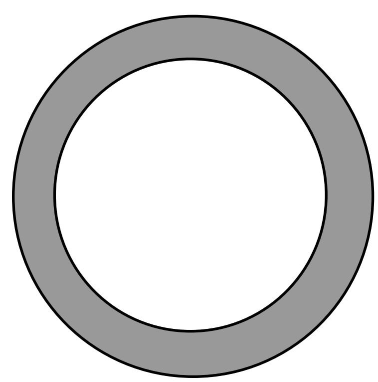
SHEARING STRESSES IN THIN-WALLED MEMBERS





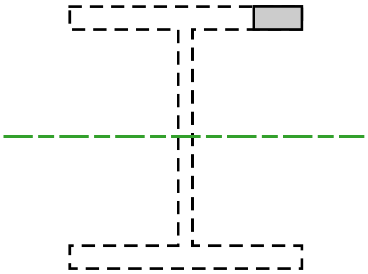


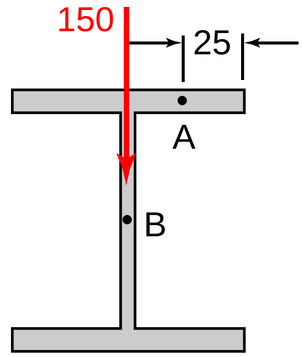




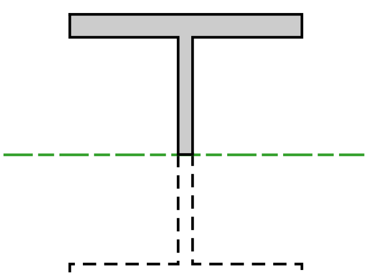
Example

Knowing that the vertical shear in the W150x29.8 beam is 150 kN, determine the shearing stress at (a) point A, (b) point B.
Units: kN, mm.





Cross-section



W150x29.8

Area, $A = 3790^{mm^2}$

Depth, $d = 157^{mm}$

Flange Width, $b_f = 153^{mm}$

Flange Thickness, $t_f = 9.3^{mm}$

Web Thickness, $t_w = 6.6^{mm}$

$I_x = 17.2x10^{6mm^4}$

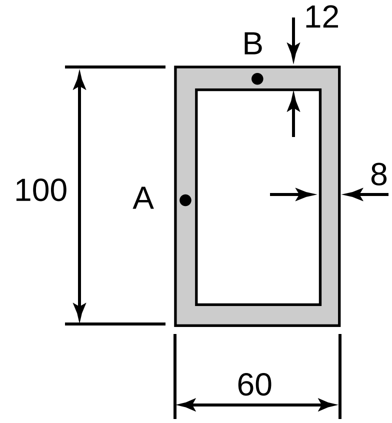
$I_y = 5.56x10^{6mm^4}$

$S_x = 219x10^{3mm^3}$

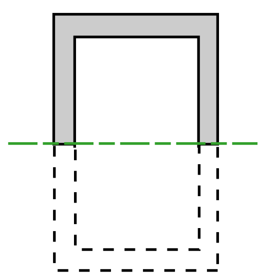
$S_y = 72.7x10^{3mm^3}$

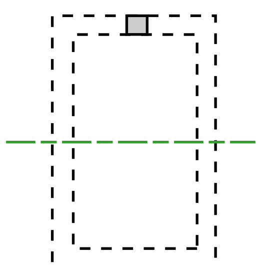
Example

Knowing that the vertical shear in the rectangular tube is 90 kN, determine the shearing stress at (a) point A, (b) point B.
Units: kN, mm.



Cross-section





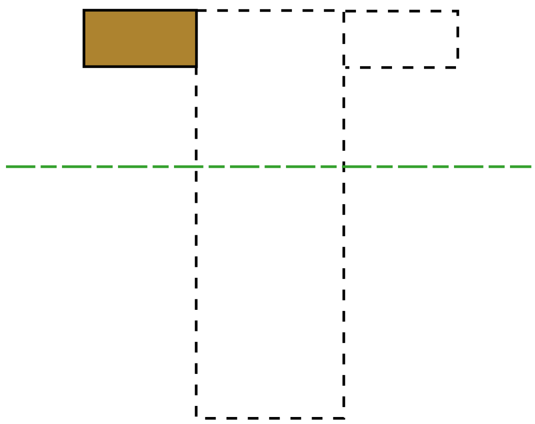
Example

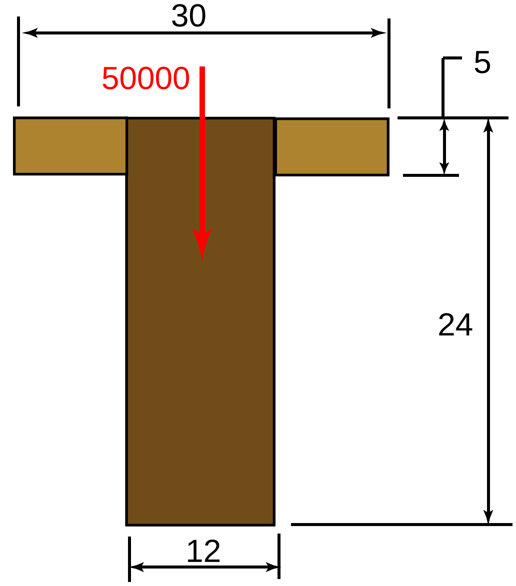
The three boards are glued together and the built-up member is subjected to a vertical shear force of 50000 lb. Determine the shear stress in the glue. Repeat the problem if the two horizontal boards are replaced with a single 30"x5" board. Units: lb, in.

Given:

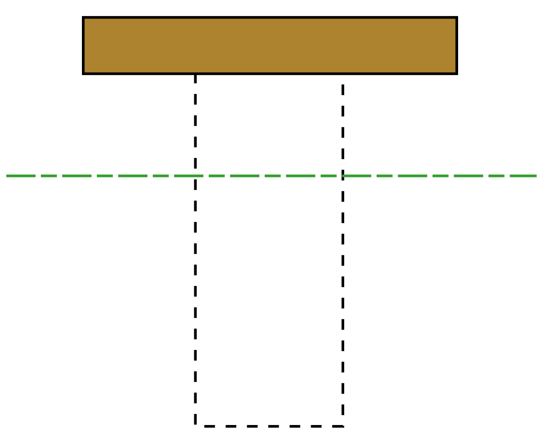
$\bar{y} = 9.74" \downarrow$ (measured from the top)

$I = 20,200^{in^4}$



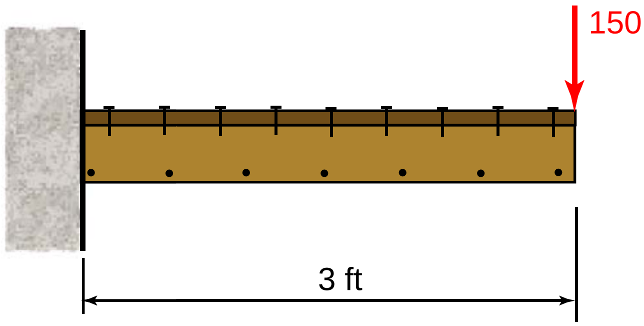


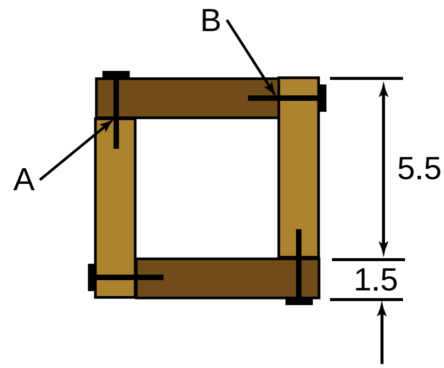
Cross-section



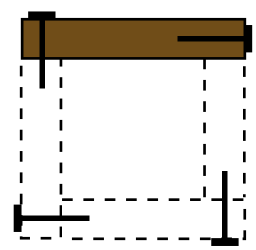
Example

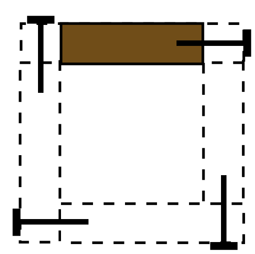
The built-up box beam is constructed by nailing four 2"x6" (nominal size) boards together. If each nail can support a shear force of 70 lb, determine the maximum spacing s of nails at A and B. Units: lb, in.





Cross-section





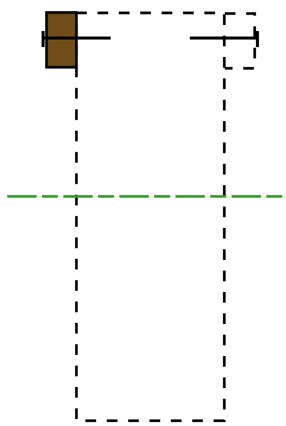
Example

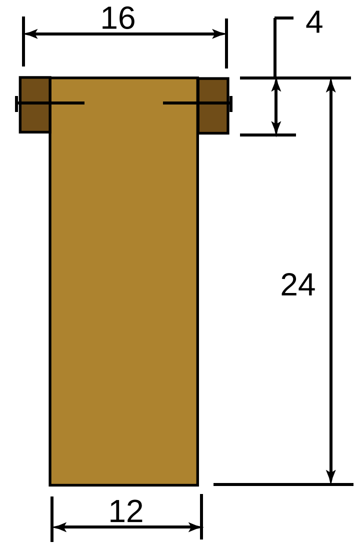
Compute the shear force in each nail to insure that the beams are securely bonded to each other. Assume a shear force of 5000 lbs and that each nail is spaced every 6". Units: lb, in.

Given:

$\bar{y} = 11.5"\downarrow$

$I = 15,300^{in^4}$





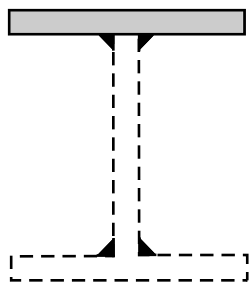
Cross-section

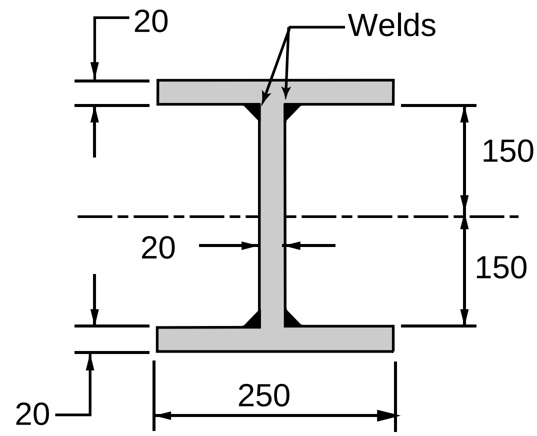
Example

If each of the four welds can support 80 kN/m, determine the required length of weld. Assume a shear force of 20 kN. Units: kN, mm.

From a previous solution:

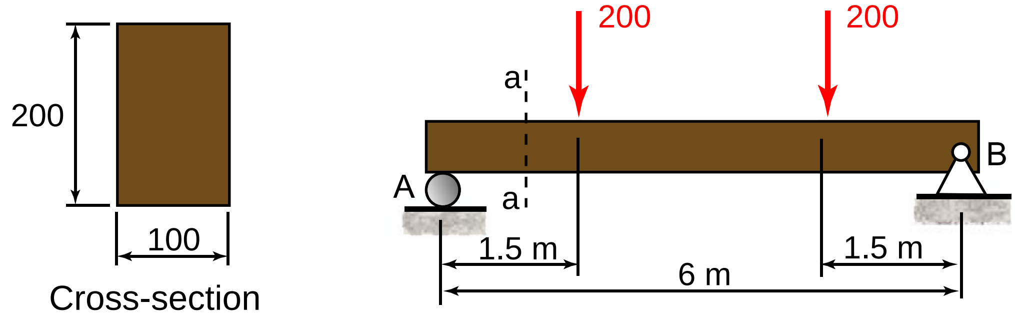
$I = 301x10^{-6in^4}$



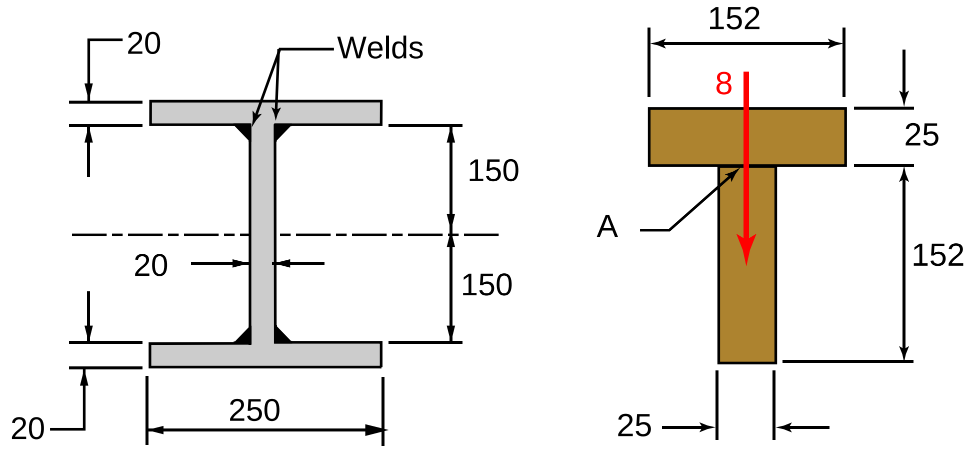


SUMMARY

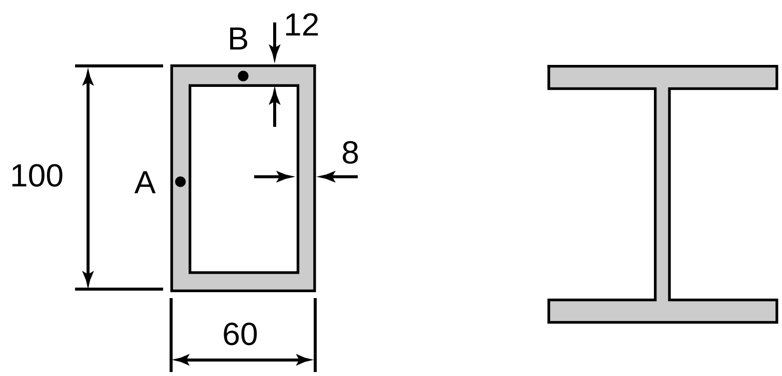
Shearing Stresses in Beams



Shearing Forces and Stresses in Built-Up Members



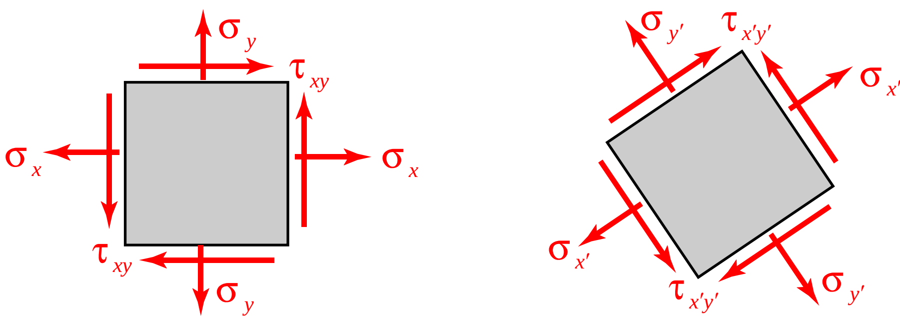
Shearing Stresses in Thin-Walled Members



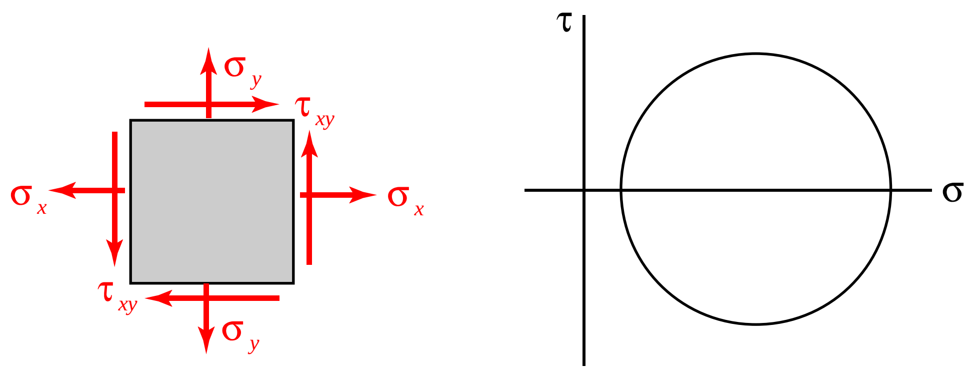
Chapter 7
Transformations of Stress and Strain

INTRODUCTION

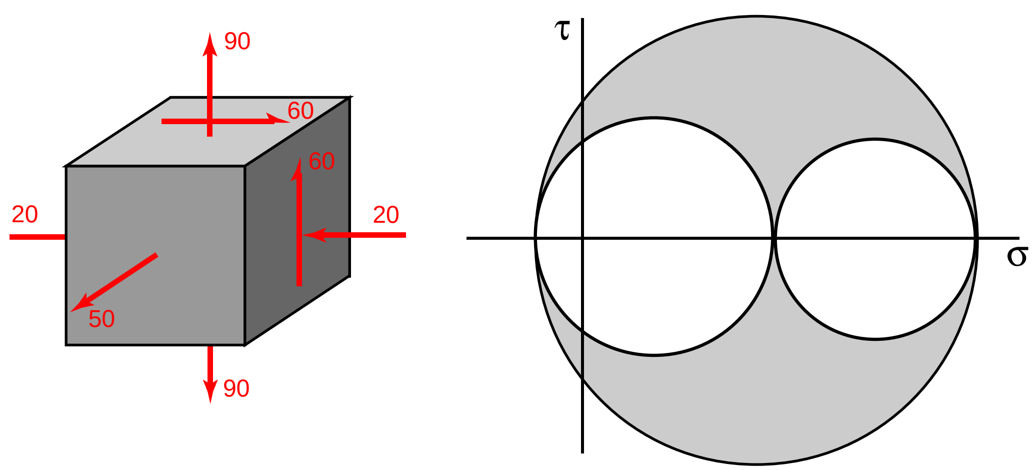
Transformation of Plane Stress



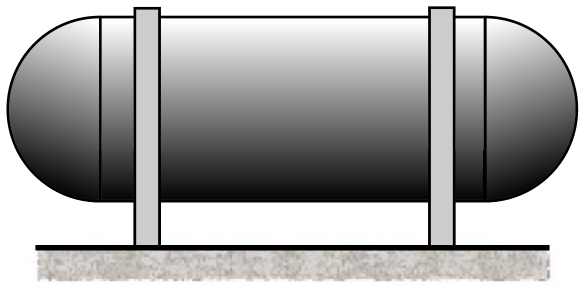
Mohr’s Circle for Plane Stress



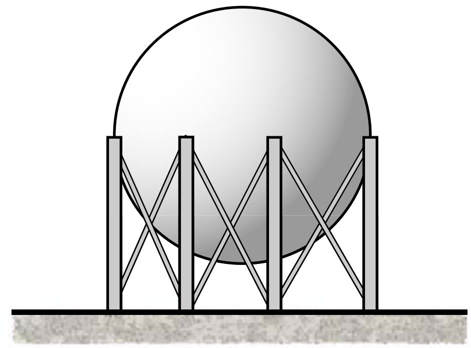
Application of Mohr’s Circle to 3D Analysis



Stresses in Thin-Walled Pressure Vessels

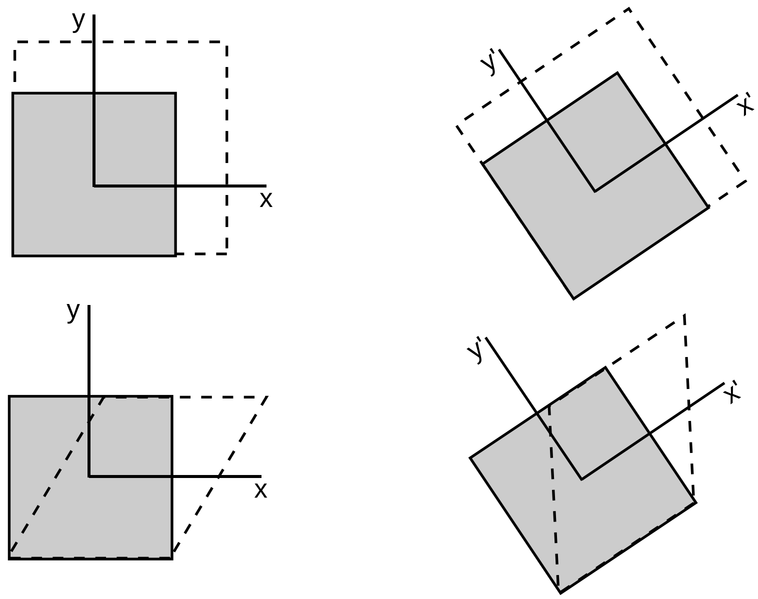


Cylindrical Pressure Vessel

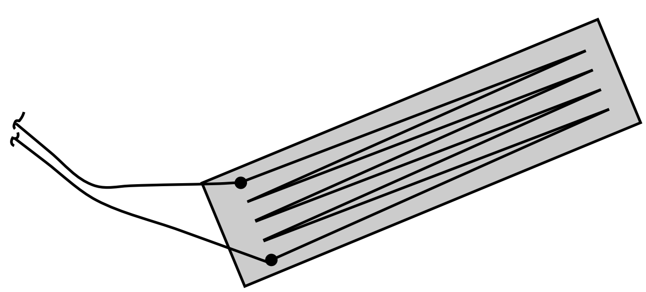


Spherical Pressure Vessel

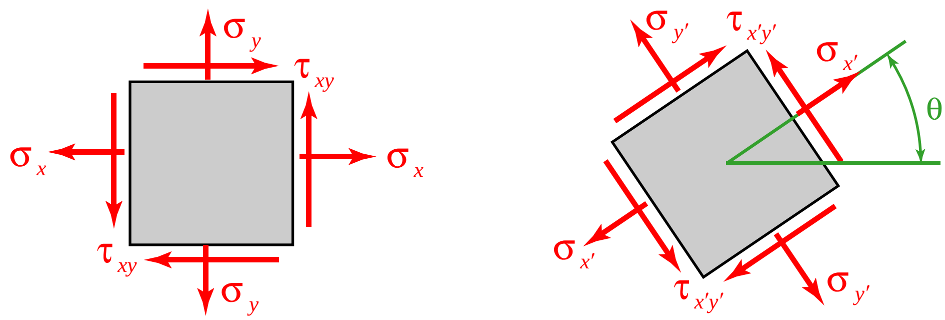
Transformation of Plane Strain



Measurements of Strain; Strain Rosette



TRANSFORMATION OF PLANE STRESS



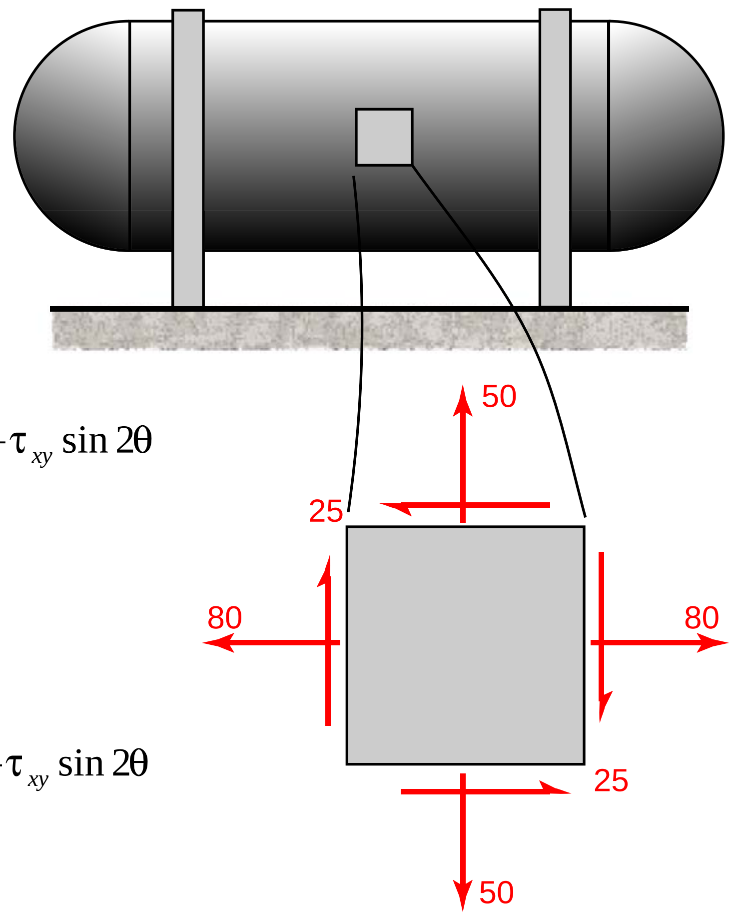
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Example

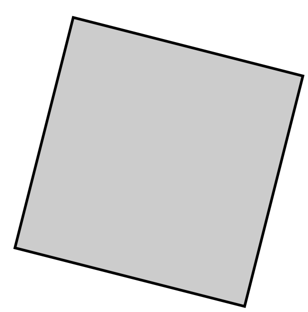
The state of stress at a point on the surface of a pressure vessel is represented on the element shown. Represent the state of stress at the point on another element that is orientated 30° clockwise from the position shown. Units: MPa.



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

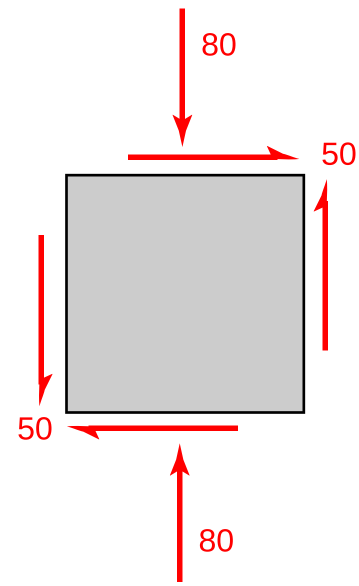
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



Example

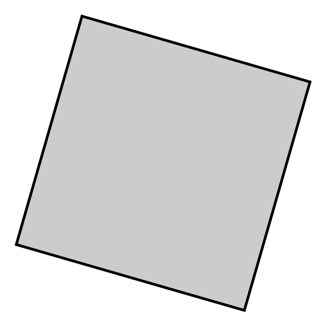
Determine the stresses on a surface that is rotated (a) 30° clockwise, (b) 15° counterclockwise. Units: MPa



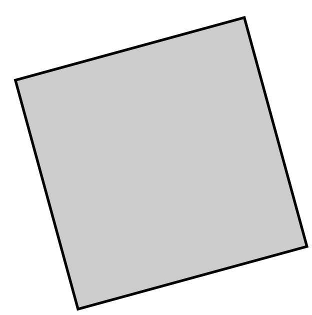
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

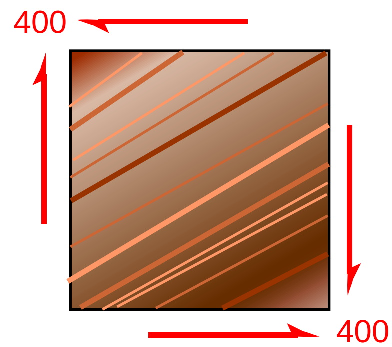


$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

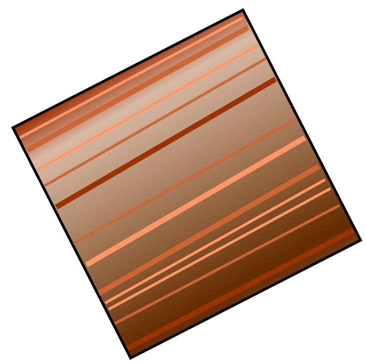
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Example

For the piece of wood, determine the in-plane shear stress parallel to the grain, (b) the normal stress perpendicular to the grain. The grain is rotated 30° from the horizontal. Units: psi

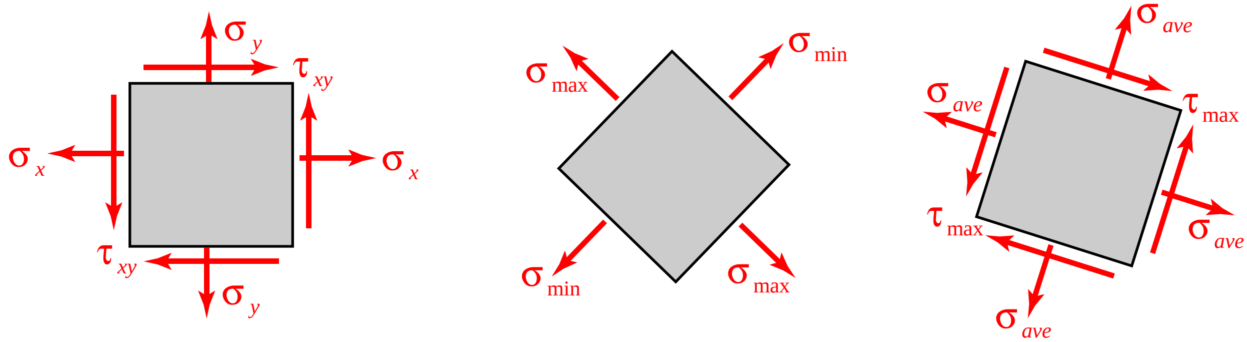


$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

PRINCIPAL STRESSES: MAXIMUM SHEARING STRESS



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

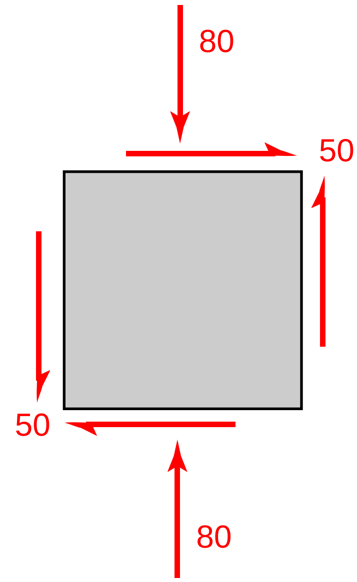
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\theta_s = \theta_p \pm 45°$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa



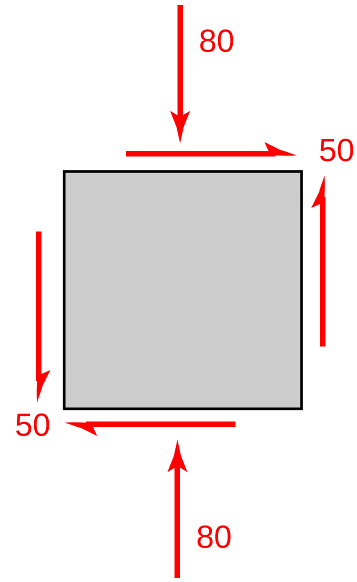
$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

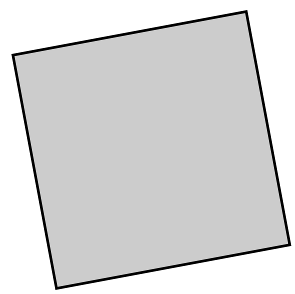
Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Sketch the resulting stresses on the element and the corresponding orientation. Units: MPa

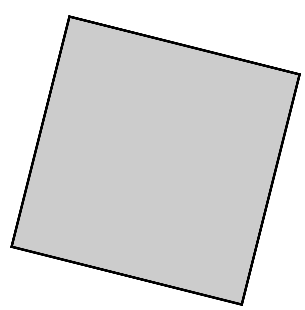


$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$



$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

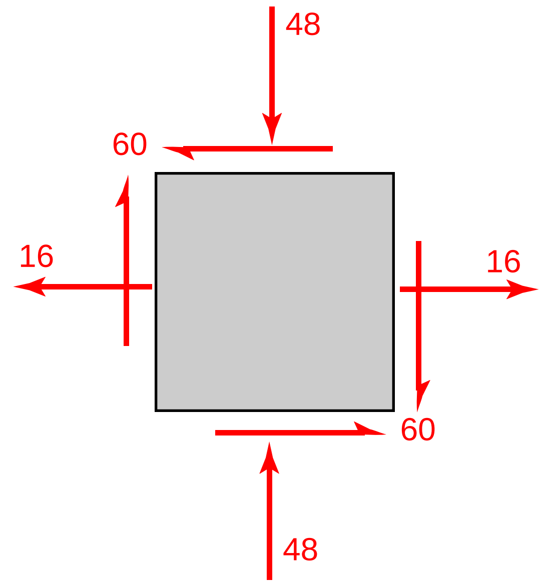


$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa



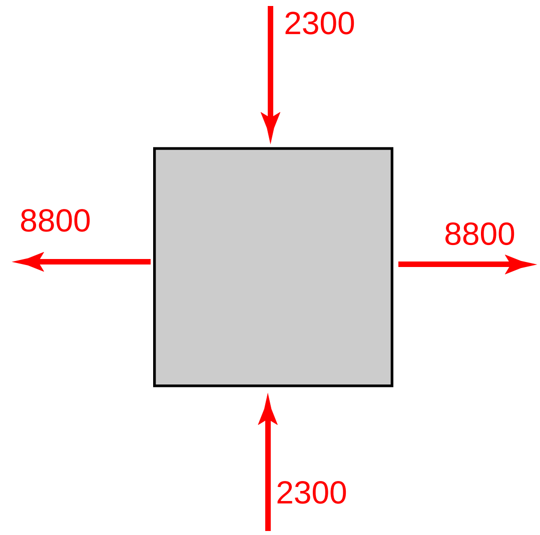
$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: psi



$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

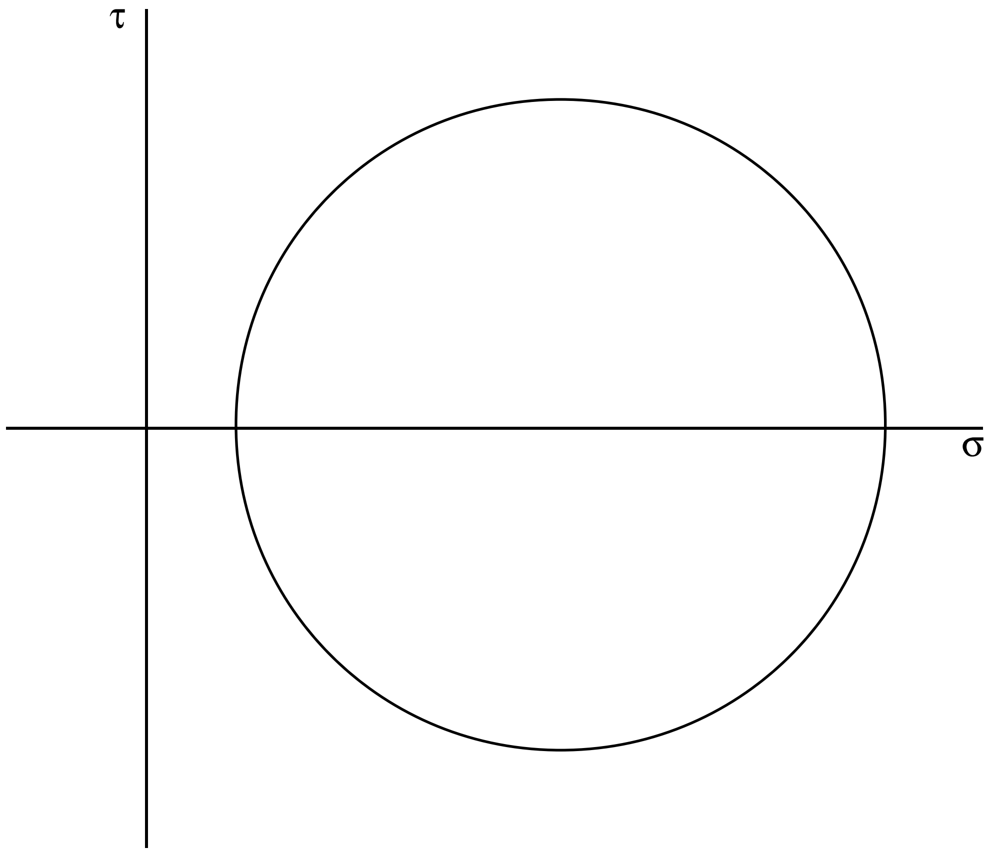
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

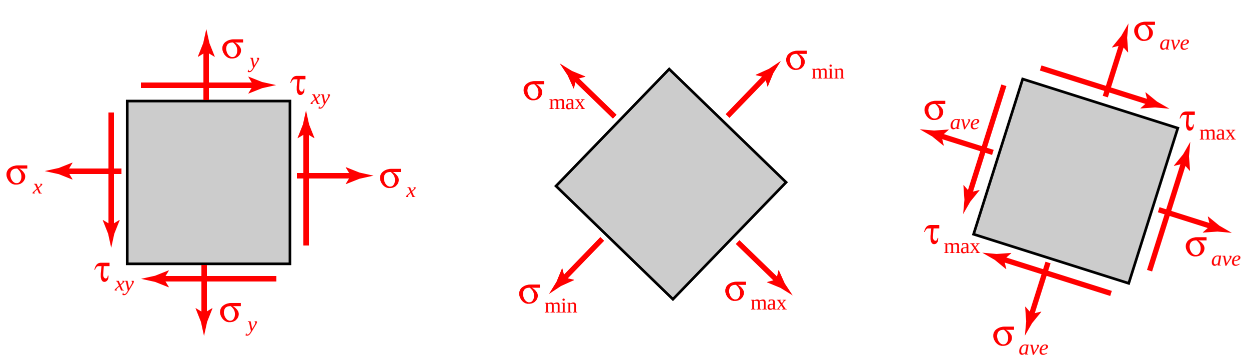
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

MOHR'S CIRCLE FOR PLANE STRESS

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$



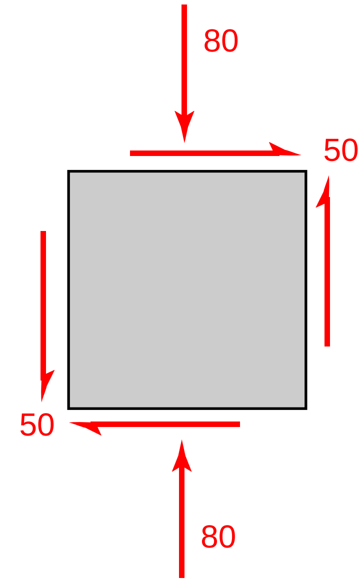


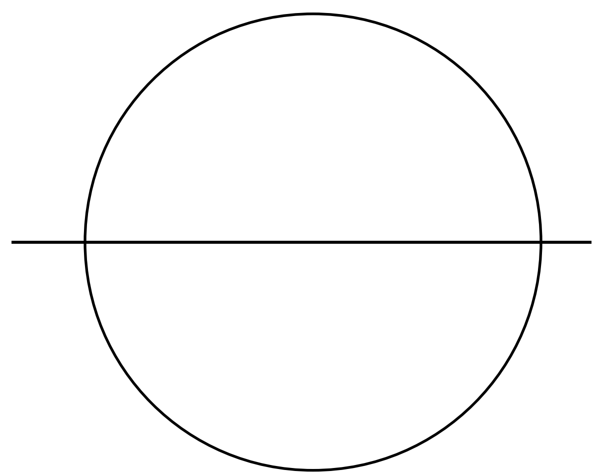
Example

Using Mohr's circle, determine the stresses on a surface that is rotated 30° clockwise. Units: MPa

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

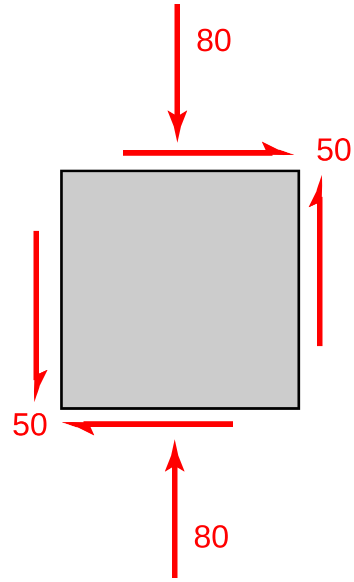
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$





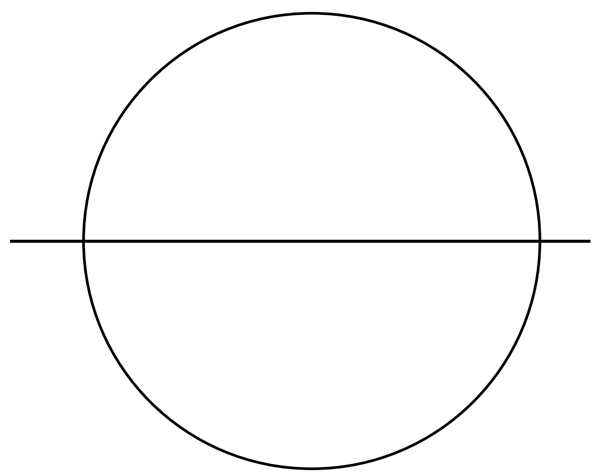
Example

Using Mohr's circle, determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa



$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

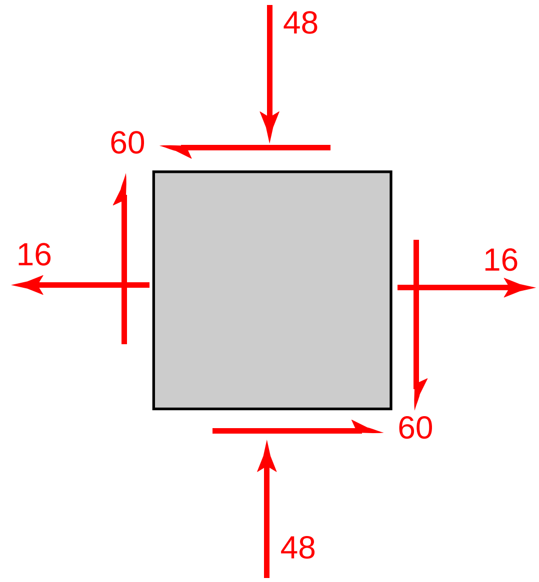


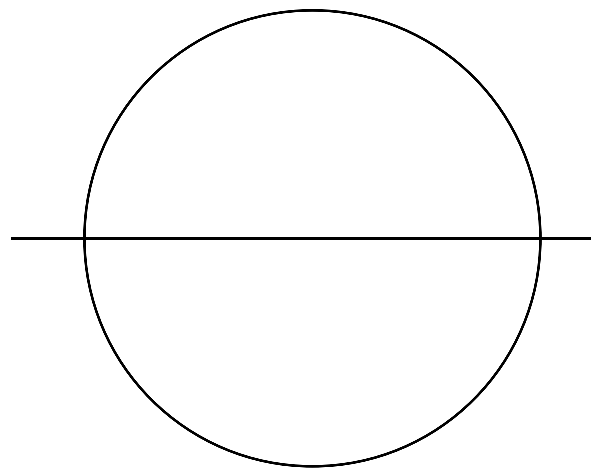
Example

Using Mohr's circle, determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

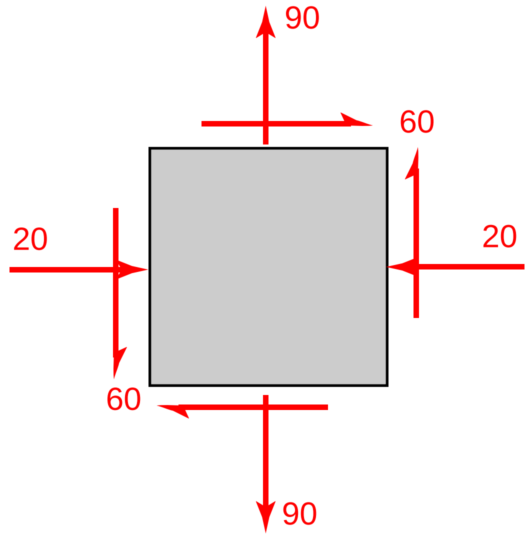




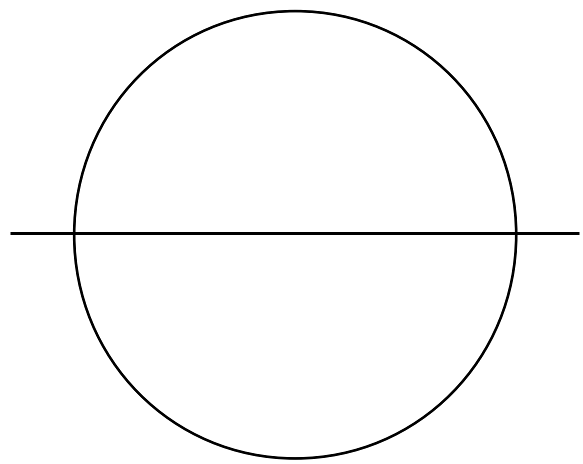
Example

Using Mohr's circle, determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$



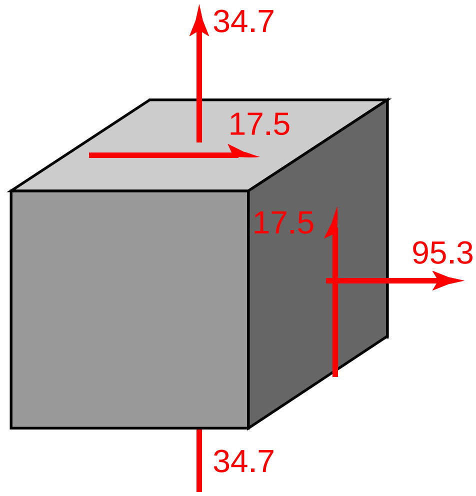
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

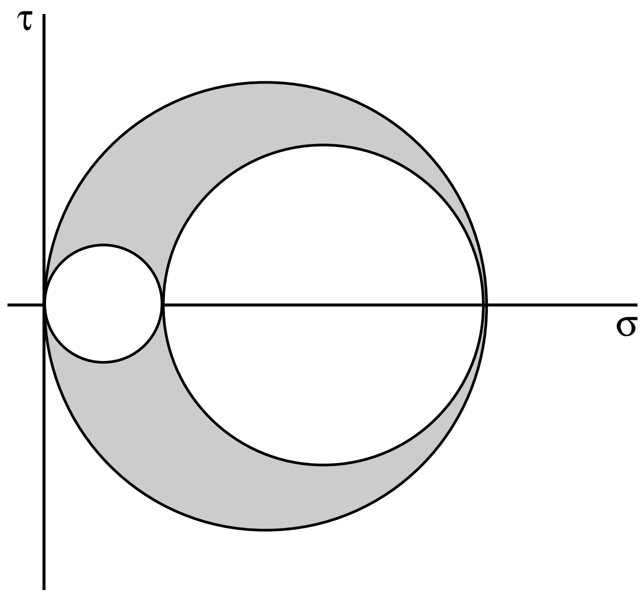


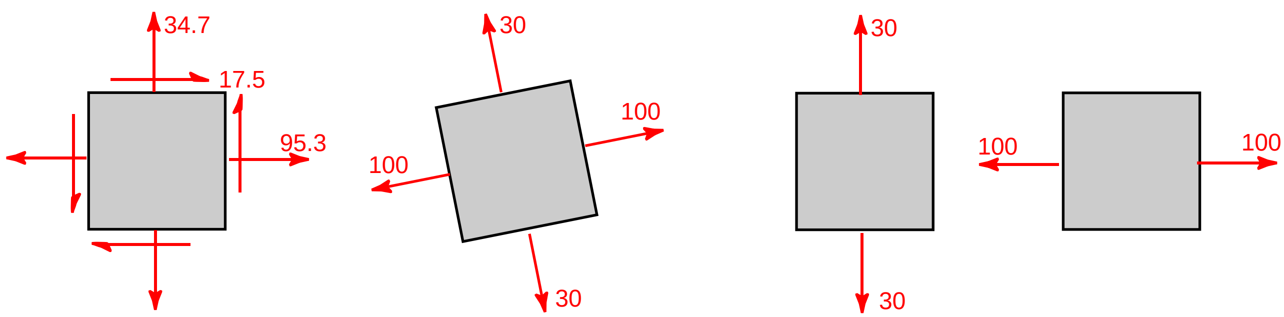
3D APPLICATIONS OF MOHR'S CIRCLE

Example

Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa

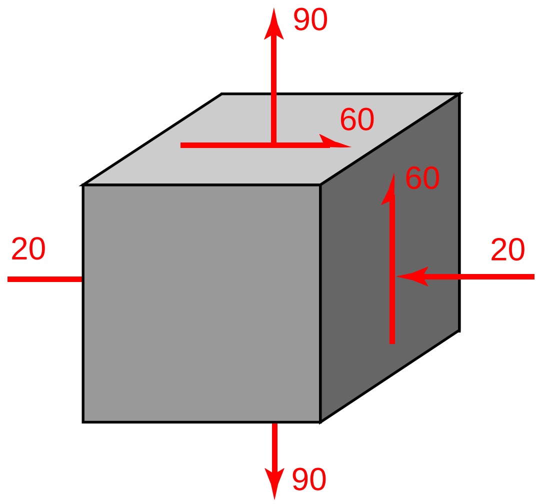


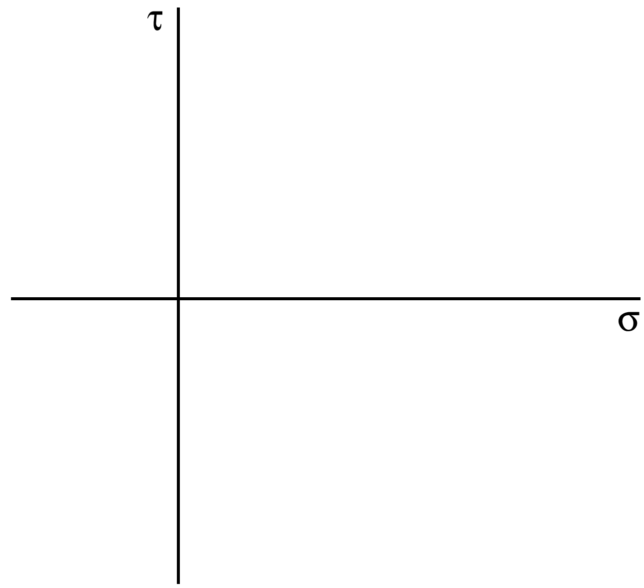


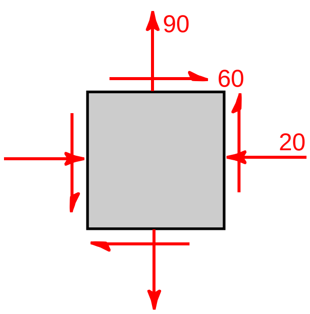


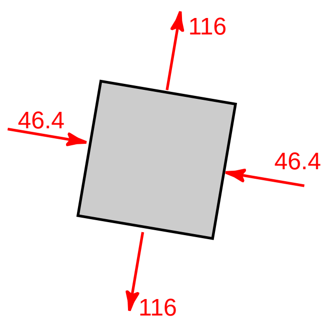
Example

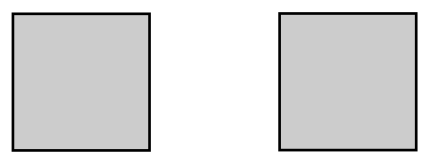
Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa





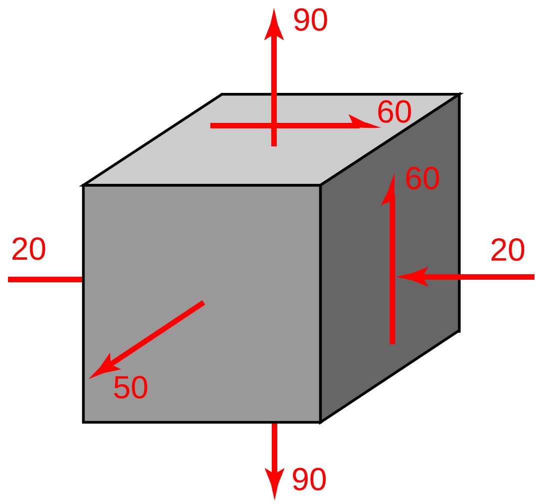


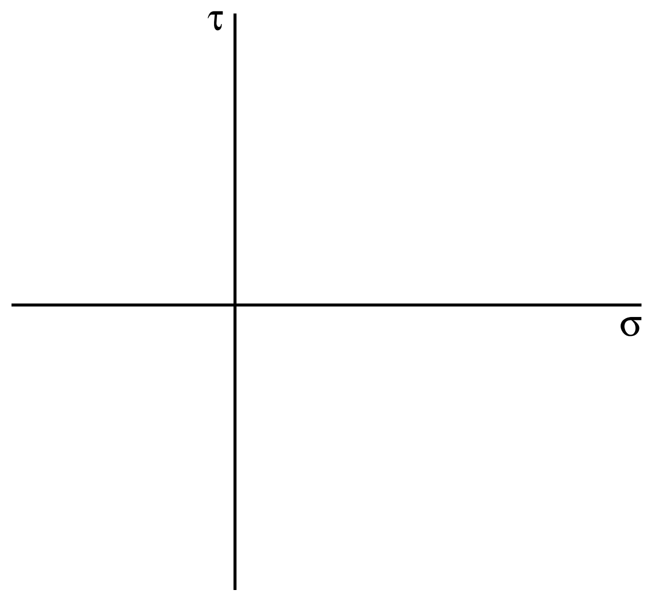


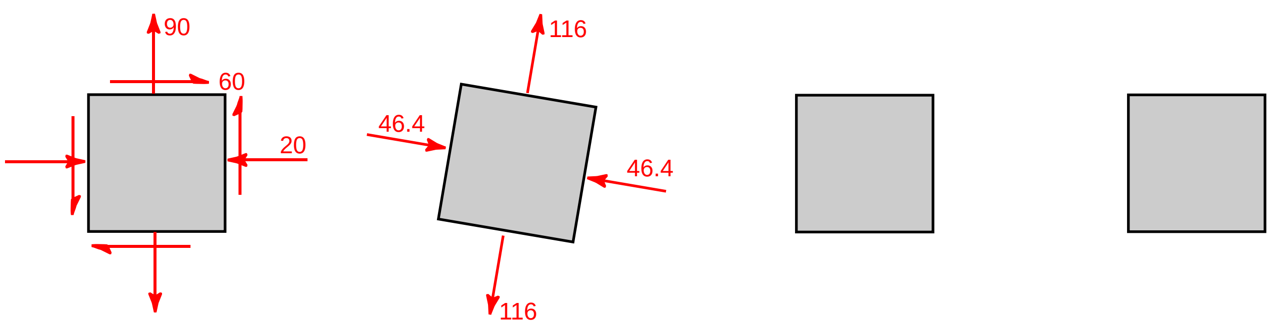


Example

Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa

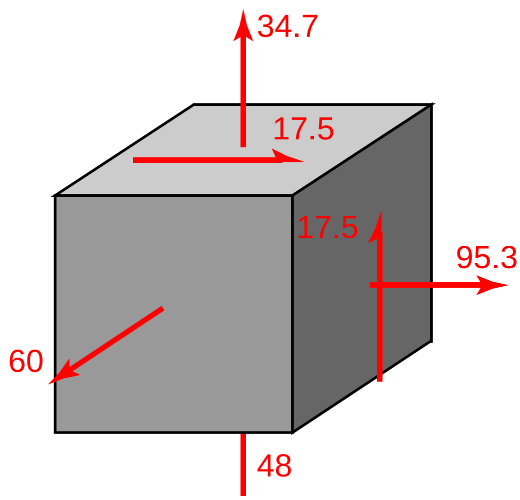


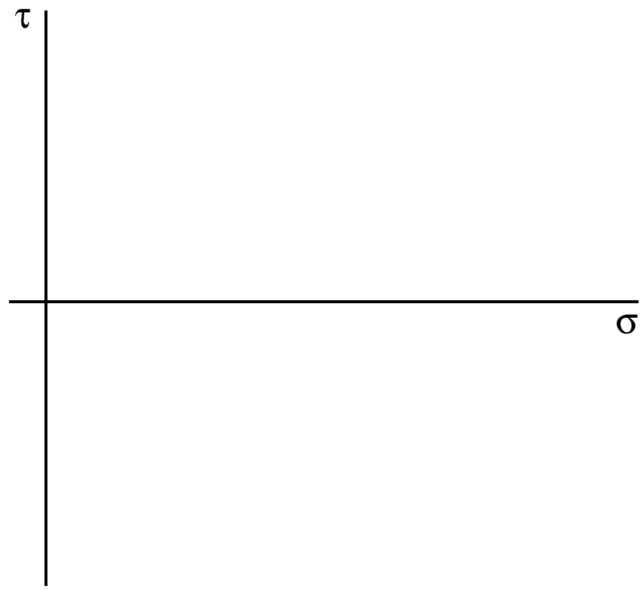


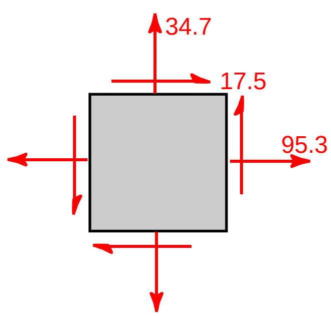


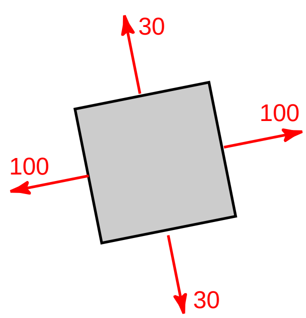
Example

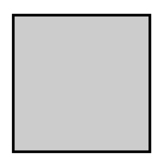
Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa

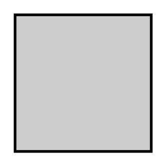






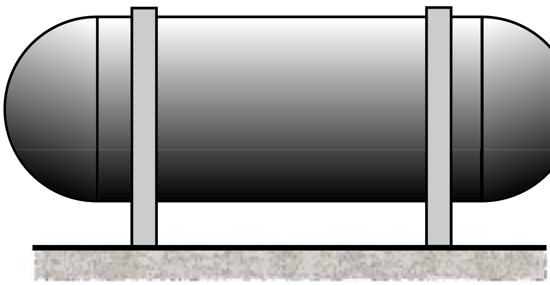




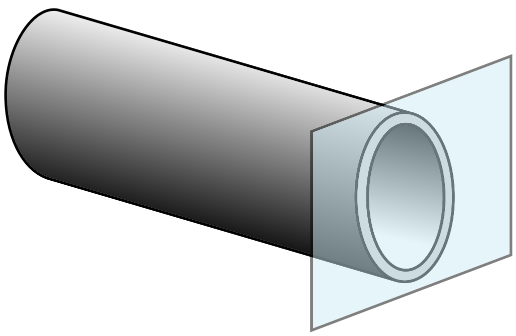


STRESSES IN THIN-WALLED PRESURE VESSELS

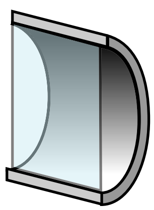
Cylindrical Pressure Vessels



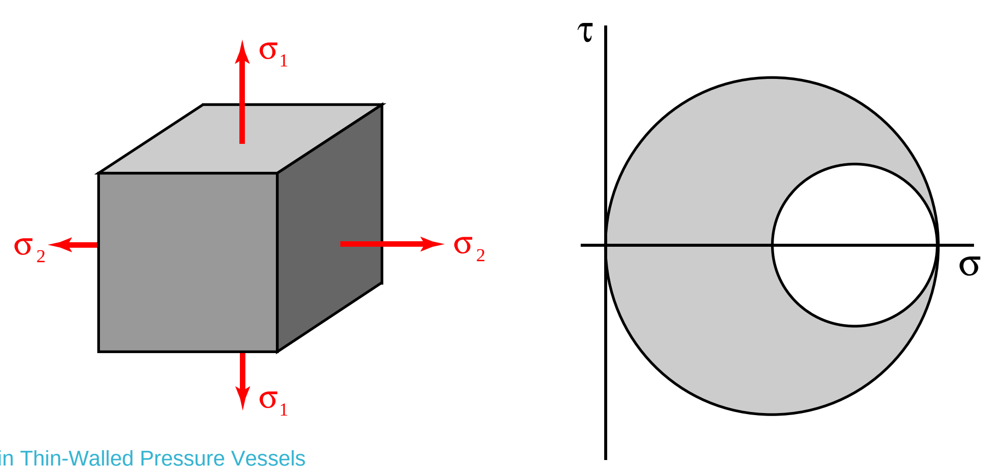
Cylindrical Pressure Vessel



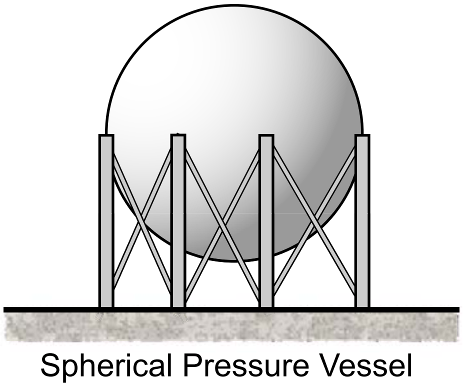
$$\sigma_2 = \frac{pr}{2t}$$

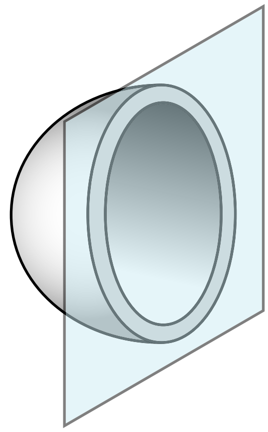


$$\sigma_1 = \frac{pr}{t}$$



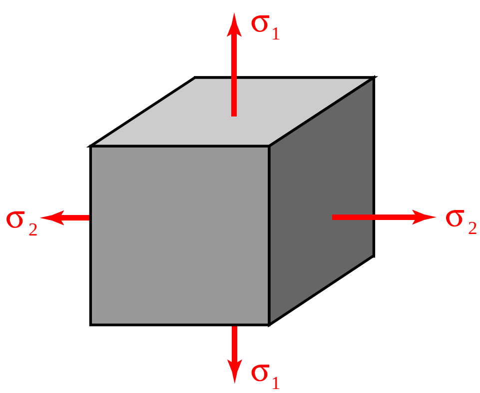
Spherical Pressure Vessels

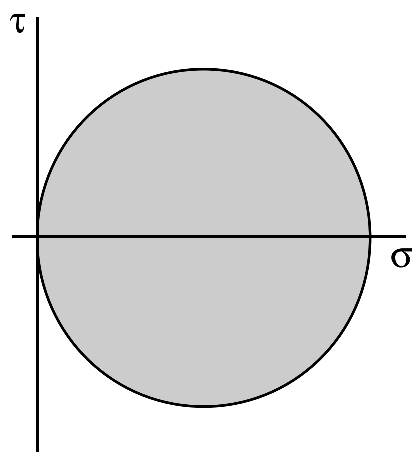




$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

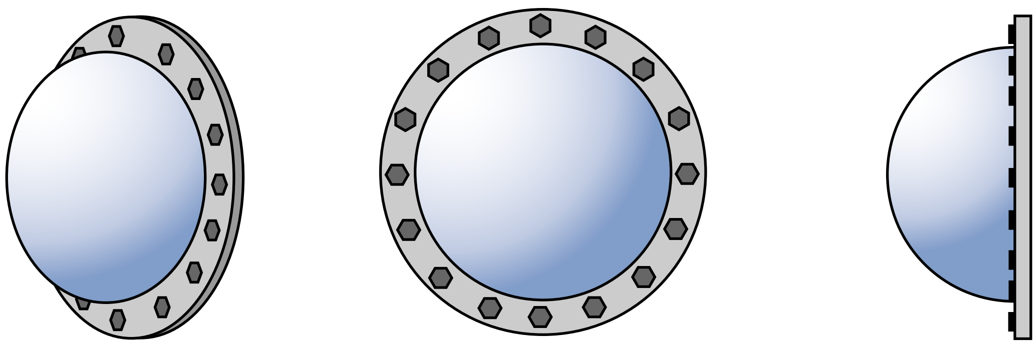
$$\tau_{max} = \frac{pr}{4t}$$





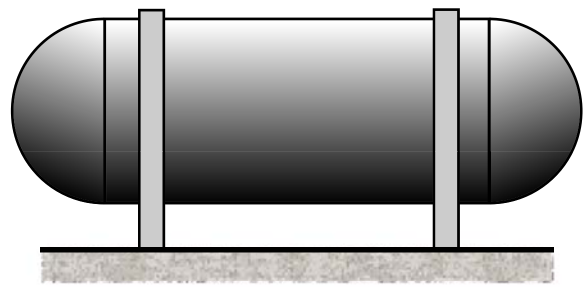
Example

The viewport is attached to the submersible with 16 bolts and has an internal air pressure of 95 psi. The viewport material used has an allowable maximum tensile and shear stress of 700 and 400 psi respectively. The inside diameter of the viewport is 18". Determine the force in each bolt and the wall thickness of the viewport.
Units: in



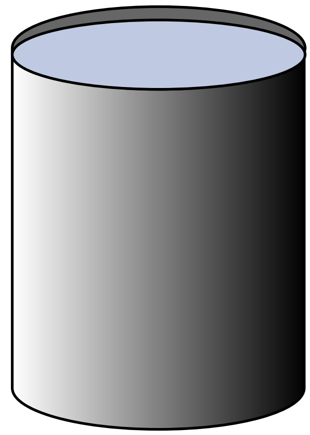
Example

The pressure vessel has an inside diameter of 2 meters and an internal pressure of 3 MPa. If the spherical ends have a wall thickness of 10 mm and the cylindrical portion has a wall thickness of 30 mm, determine the maximum normal and shear stress in each section.



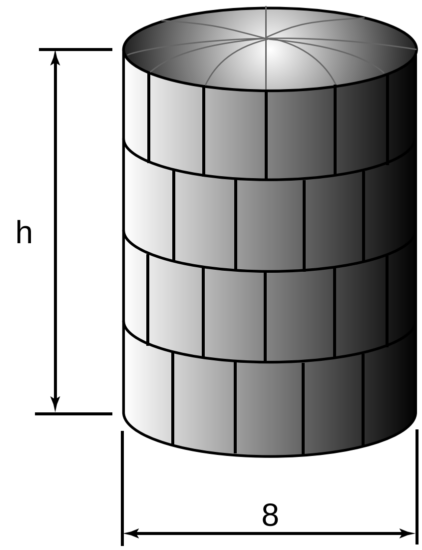
Example

The open water tank has an inside diameter of 50 ft and is filled to a height of 60 ft. Determine the minimum wall thickness due to the water pressure only if the allowable tensile stress is 24 ksi.



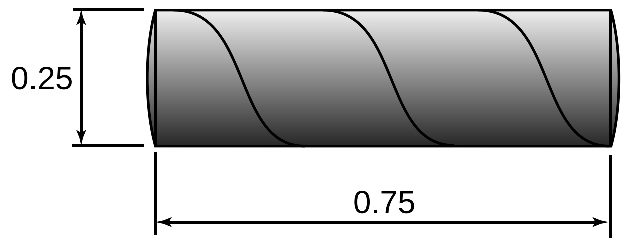
Example

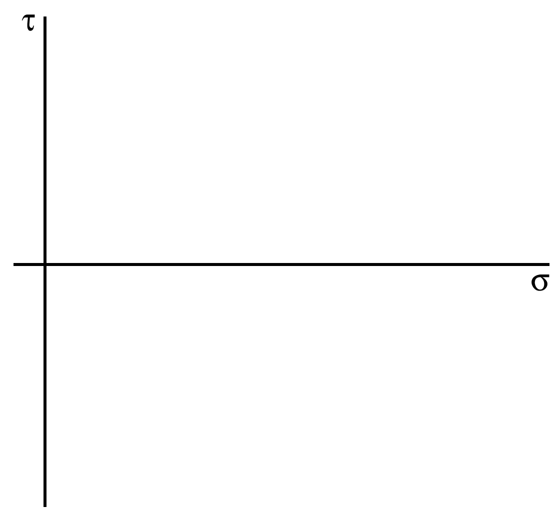
18 mm thick plates are welded as shown to form the cylindrical pressure tank. Knowing that the allowable normal stress perpendiculer to the weld is 60 MPa, determine the maximum allowable internal pressure and the height of the tank. Units: m.



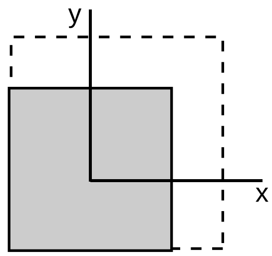
Example

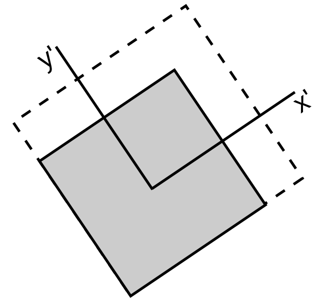
The cylindrical portion of the compressed air tank is made of 10 mm thick plate welded along a helix forming an angle of 45°. Knowing that the allowable stress normal to the weld is 80 MPa, determine the largest gage pressure that can be used in the tank. Units: m

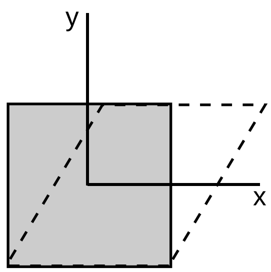


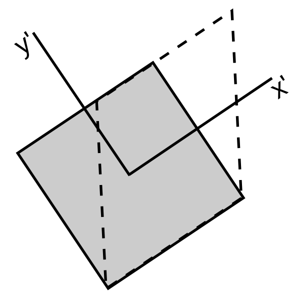


TRANSFORMATION OF PLANE STRAIN









$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

PRINCIPAL STRAINS

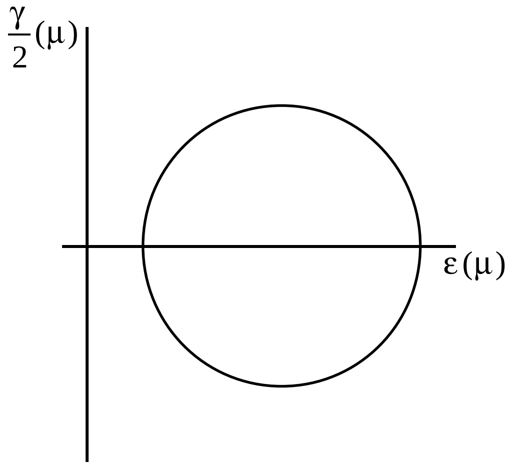
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_{\max,\min} = \varepsilon_{a,b} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_c = -\frac{\upsilon}{1-\upsilon}(\varepsilon_{\max} + \varepsilon_{\min})$$

$$\gamma_{\max} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

MOHR'S CIRCLE FOR PLANE STRAIN



$$\varepsilon_{ave} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_{\max,\min} = \varepsilon_{a,b} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

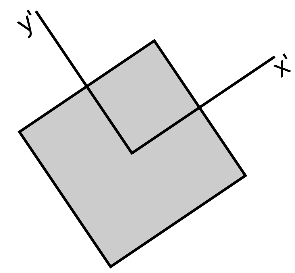
$$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$$

Example

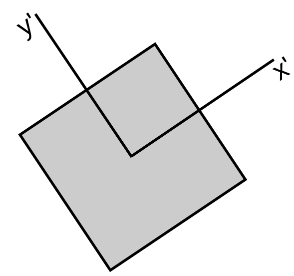
Given the strains below, determine the strains if the element is rotated 30° counterclockwise.

$$\varepsilon_x = -300\mu$$
$$\varepsilon_y = -200\mu$$
$$\gamma_{xy} = +175\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$



$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$



$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

Example

Given the strains below, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum strain. Assume plane stress.

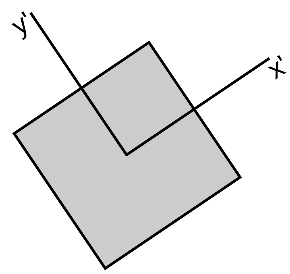
$$\upsilon = 1/3$$

$$\varepsilon_x = -300\mu$$

$$\varepsilon_y = -200\mu$$

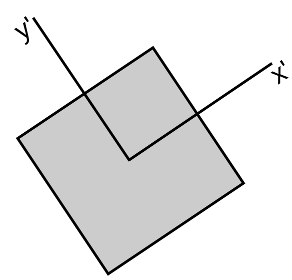
$$\gamma_{xy} = +175\mu$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$



$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

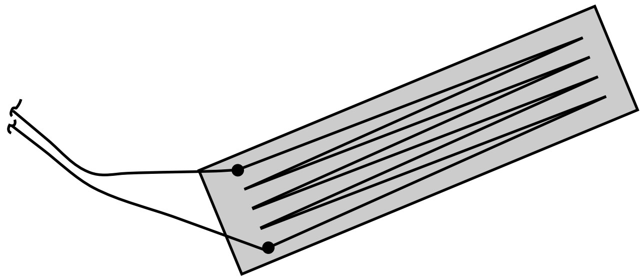
$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

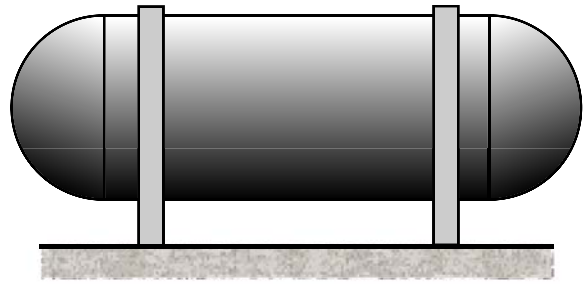


$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_c = -\frac{\upsilon}{1-\upsilon}(\varepsilon_1 + \varepsilon_2)$$

MEASUREMENTS OF STRAIN; STRAIN ROSETTE



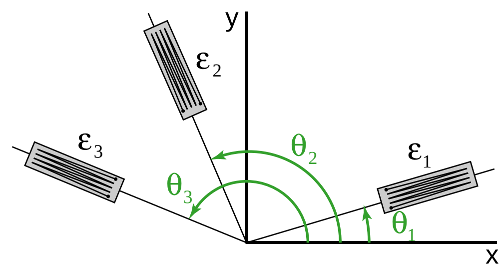


Cylindrical Pressure Vessel

$$\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1$$

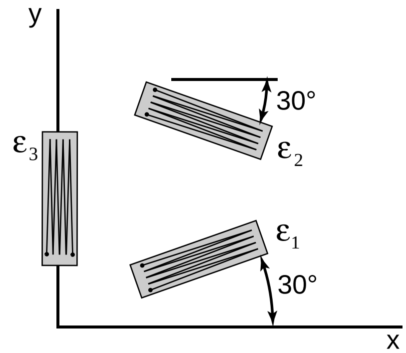
$$\varepsilon_2 = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2$$

$$\varepsilon_3 = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3$$



Example

Given the following strains, determine (a) the in-plane principal strains, (b) the in-plane maximum shearing strain.



$$\varepsilon_1 = +600\mu$$
$$\varepsilon_2 = +450\mu$$
$$\varepsilon_3 = -175\mu$$

$$\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1$$

$$\varepsilon_2 = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2$$

$$\varepsilon_3 = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3$$

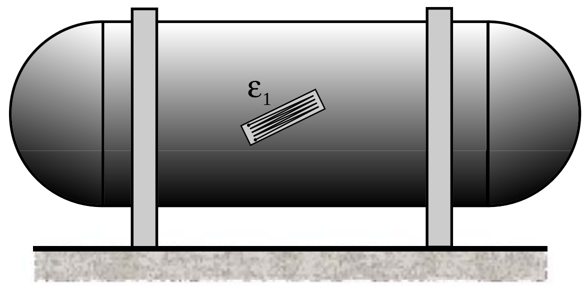
$$\varepsilon_{\max,\min} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

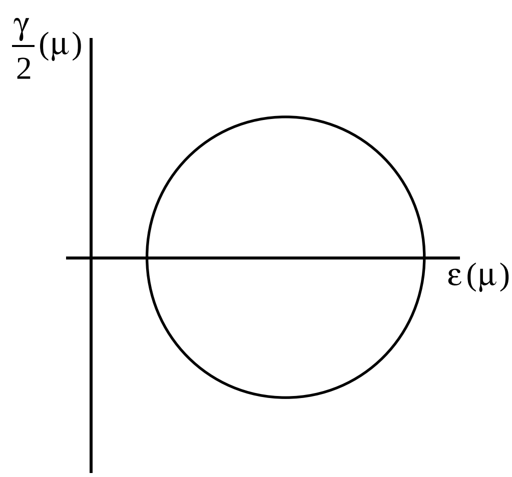
$$\gamma_{\max} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Example

Given the strain measurements below for the 30" diameter, 0.25" thick tank, determine the gage pressure, (b) the principal stresses and the maximum in-plane shearing stress.

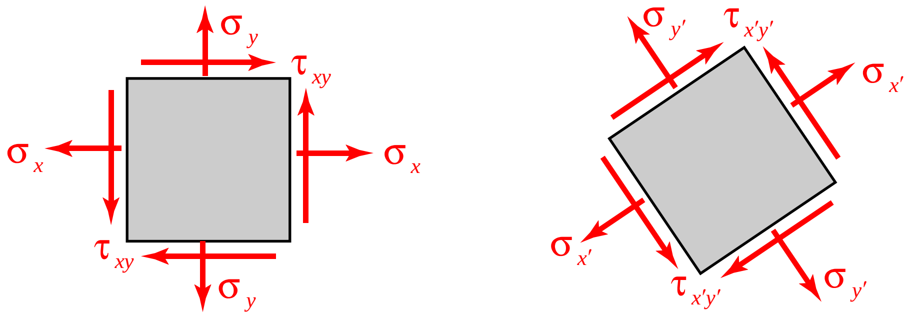
$$\varepsilon_1 = +160\mu$$
$$\upsilon = 0.3$$
$$\theta = 30^\circ$$
$$E = 29x10^{6\,psi}$$



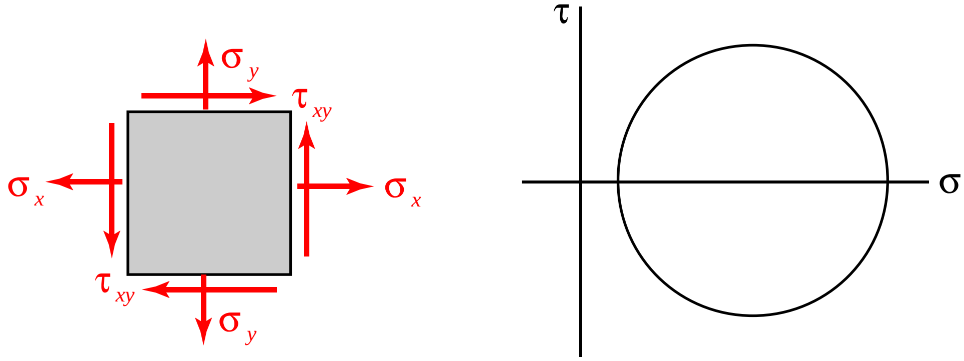


SUMMARY

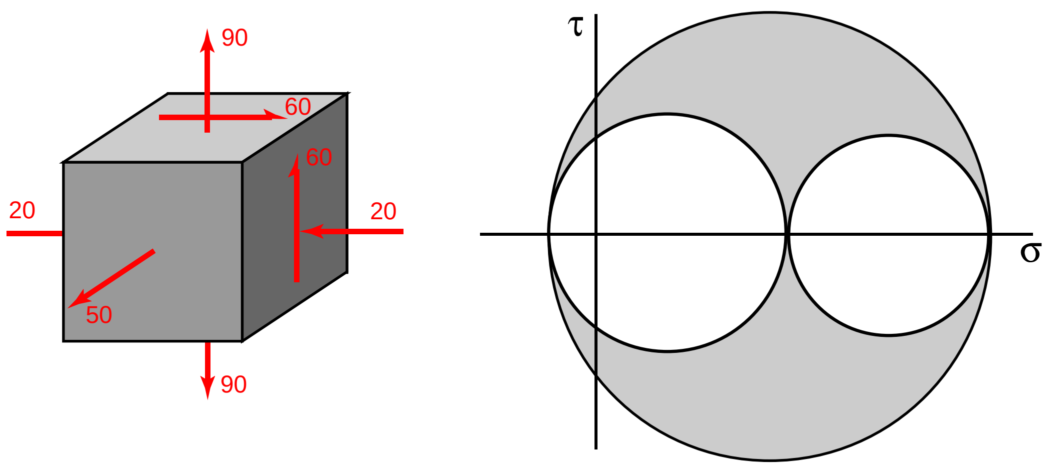
Transformation of Plane Stress



Mohr's Circle for Plane Stress

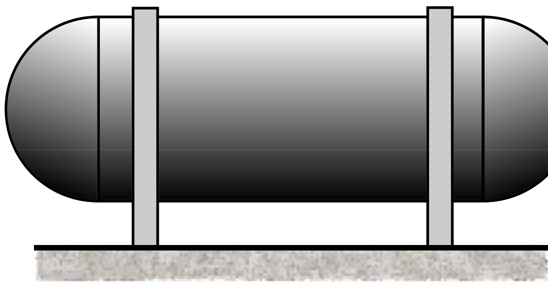


Application of Mohr's Circle to 3D Analysis

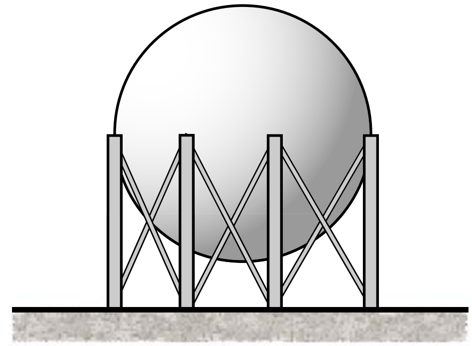


SUMMARY

Stresses in Thin-Walled Pressure Vessels

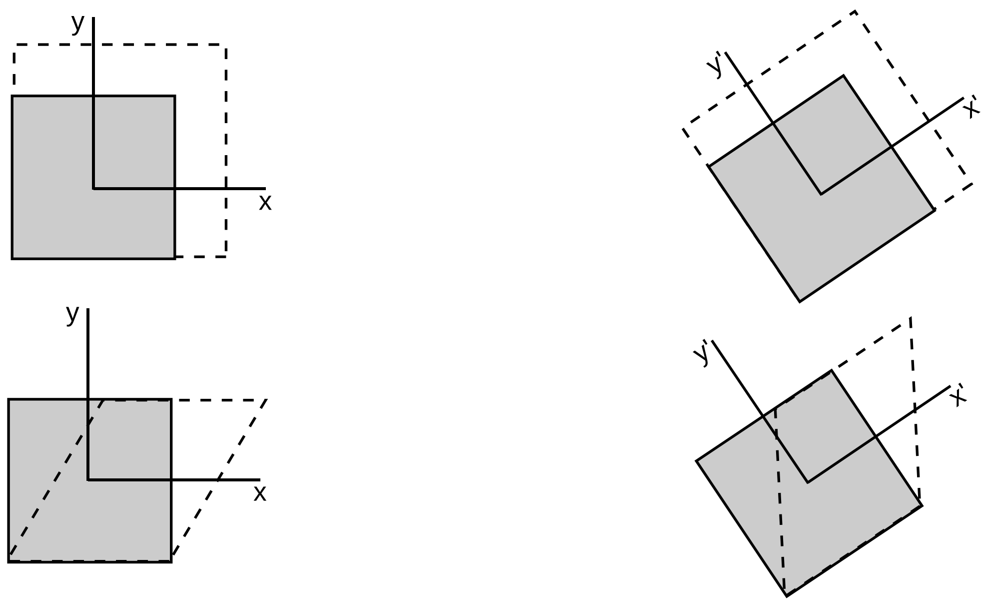


Cylindrical Pressure Vessel

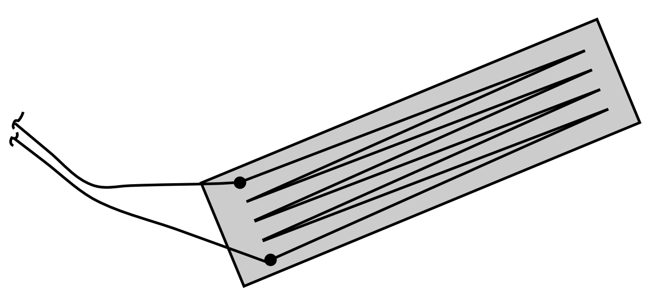


Spherical Pressure Vessel

Transformation of Plane Strain



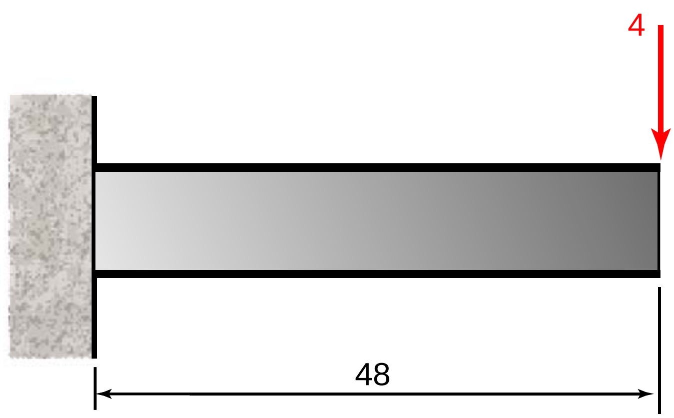
Measurements of Strain; Strain Rosette

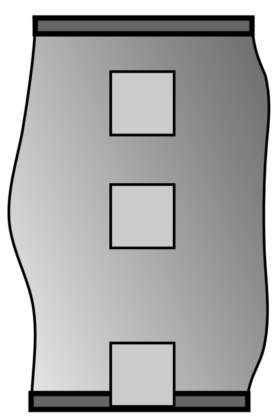


Principal Stresses under a Given Loading

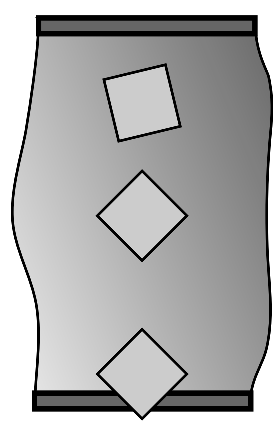
INTRODUCTION

PRINCIPAL STRESSES IN A BEAM

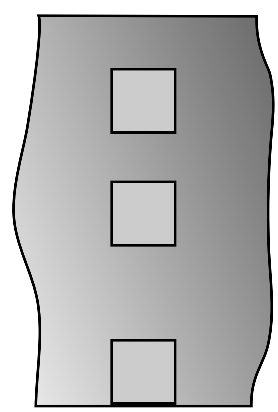




Wide Flange Stresses



Principal Wide Flange Stresses

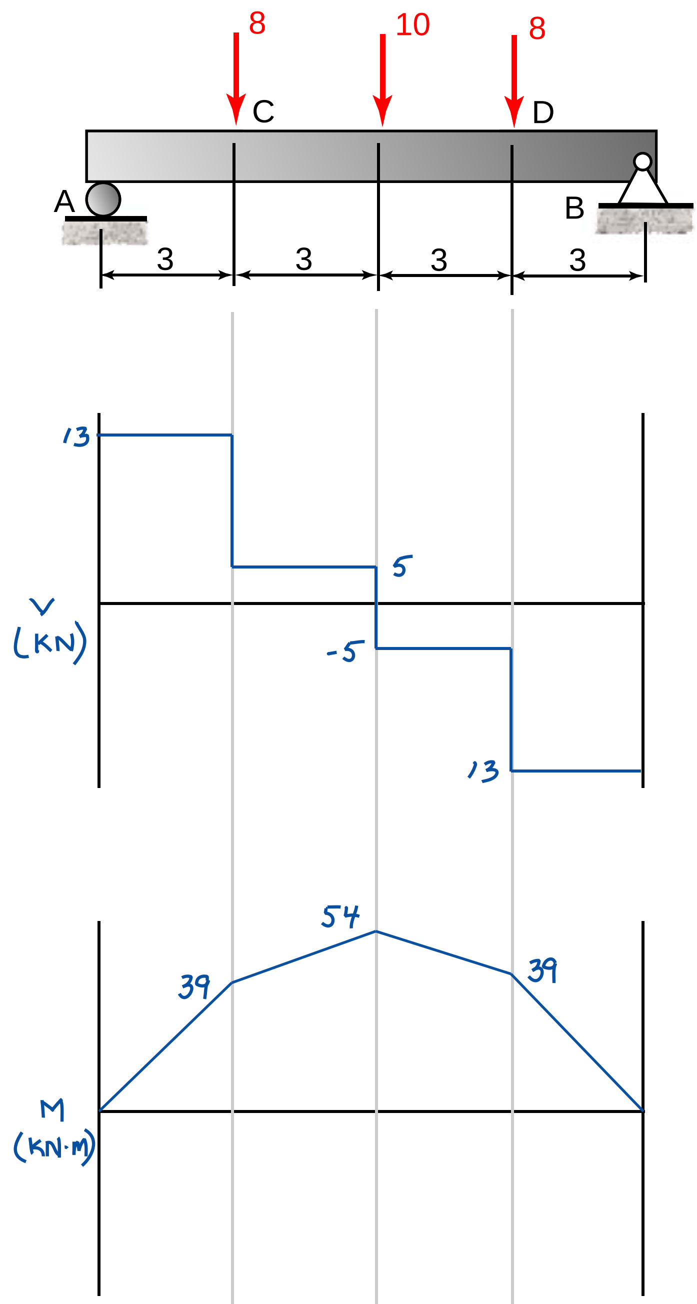


Rectangular Cross-section Stresses

Example

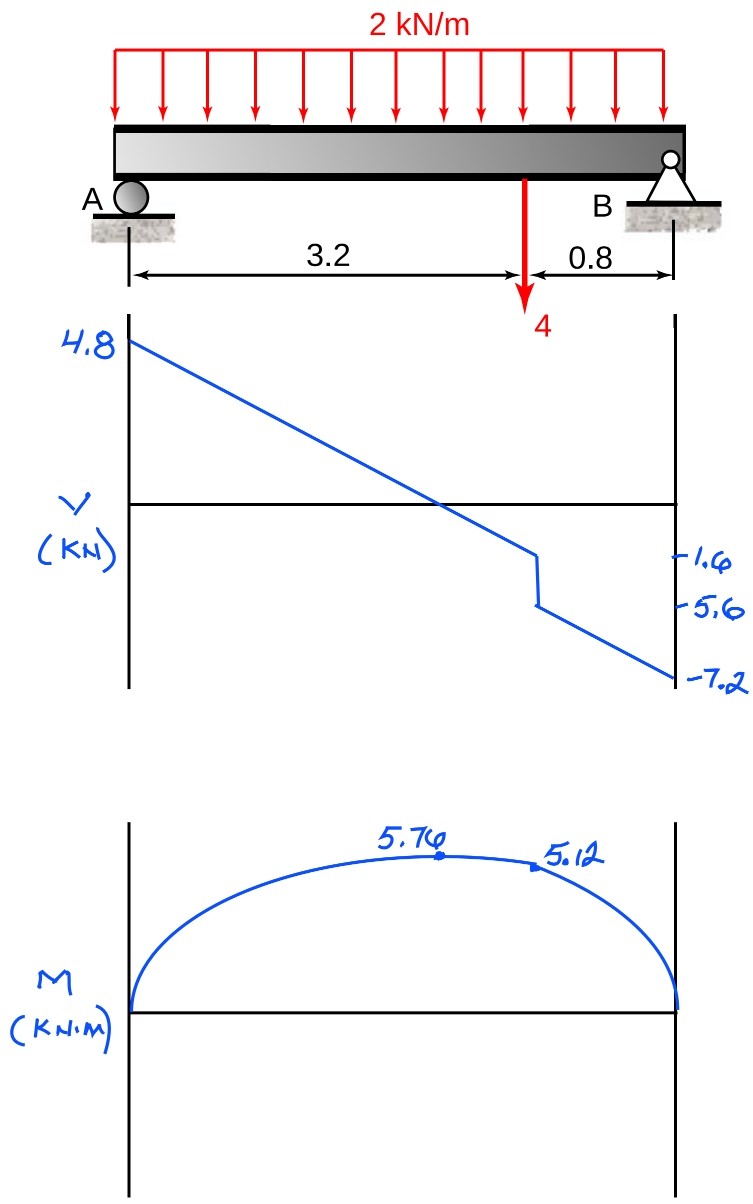
(a) Knowing that the allowable normal stress is 80 MPa and the allowable shear stress is 50 MPa, determine the height of the rectangular section if the width is 100 mm.

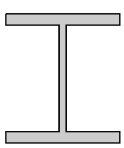
Units: kN, m

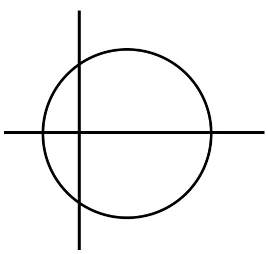


Example

(a) Knowing that the allowable normal stress is 80 MPa and the allowable shear stress is 50 MPa, select the most economical wide-flange shape that should be used to support the loading shown.
(b) Determine the principal stresses at the junction between the flange and web on a section just to the right of the 4 kN load. Units: kN, m

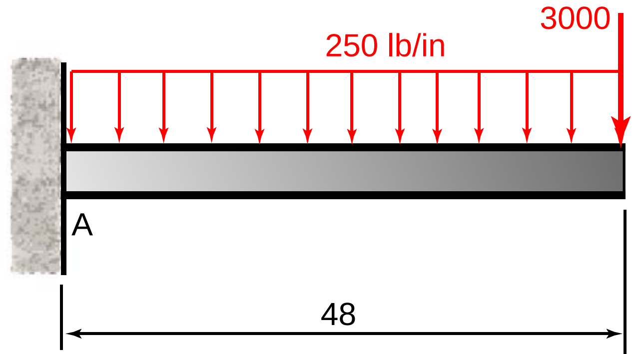


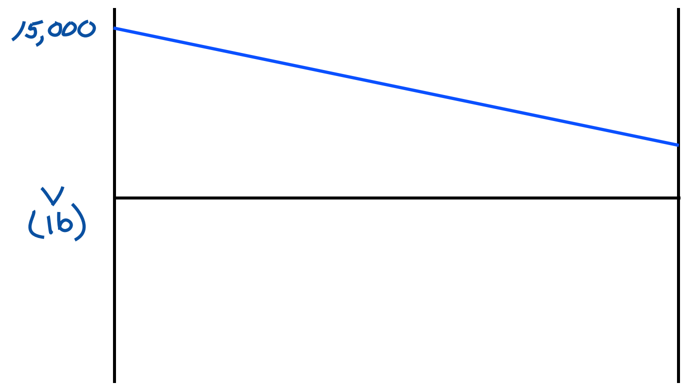


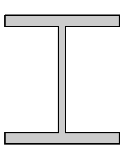


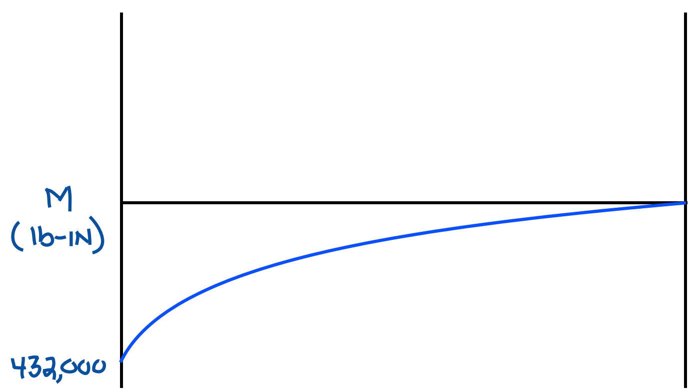
Example

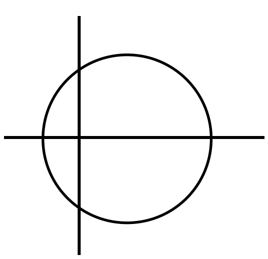
(a) Knowing that the allowable normal stress is 24 ksi and the allowable shear stress is 15 ksi, select the most economical W8 wide-flange shape that should be used to support the loading shown.
(b) Determine the principal stresses at the junction between the flange and web. Units: lb, in.







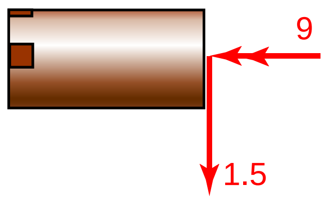


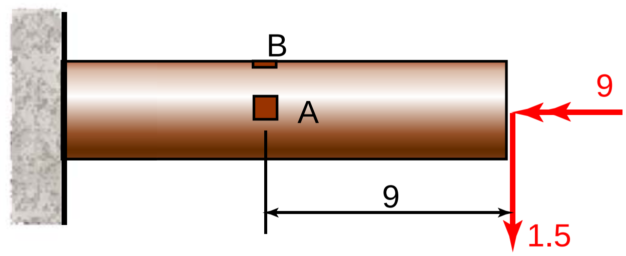


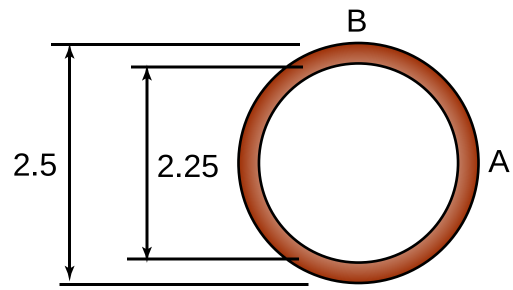
STRESSES UNDER COMBINED LOADINGS

Example

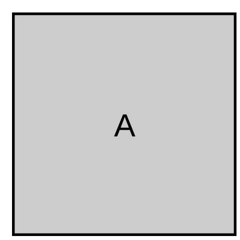
Determine the stresses at A and B. Units: k, k-in, in.

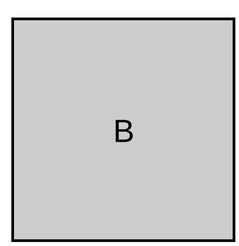






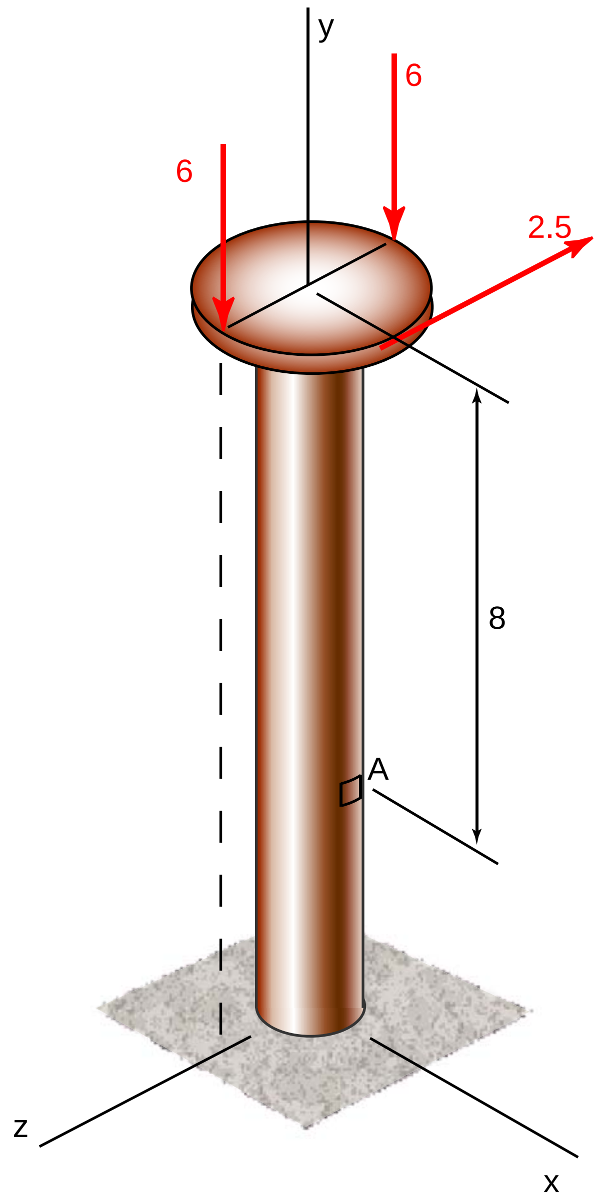
Cross-section

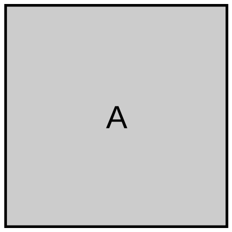




Example

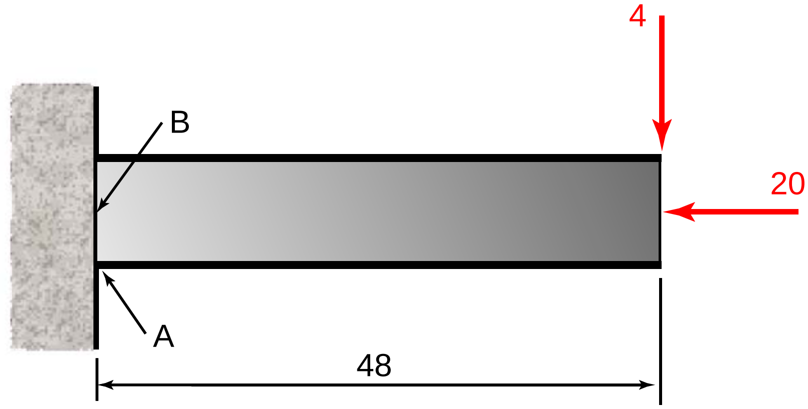
Determine the stresses at A. The disk has a diameter of 4" and the solid shaft has a diameter of 1.8". Units: kips, in.





Example

Determine the stresses at points A and B. The beam is a W6x20.
Units: kips, in.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

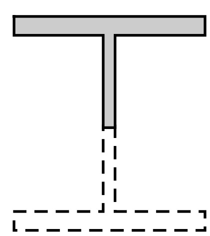
Web Thickness, $t_w = 0.260^{in}$

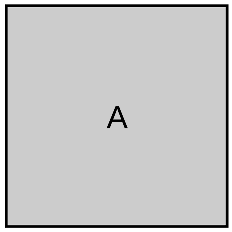
$I_x = 41.4^{in^4}$

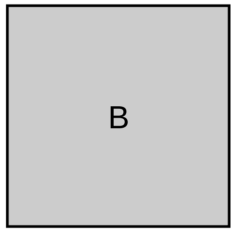
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

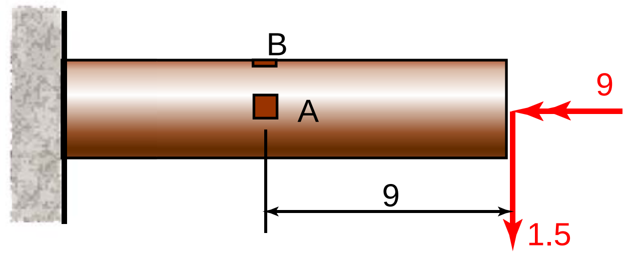




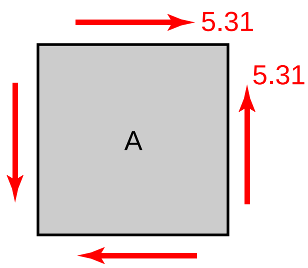


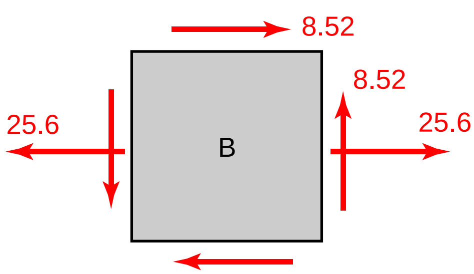
Example

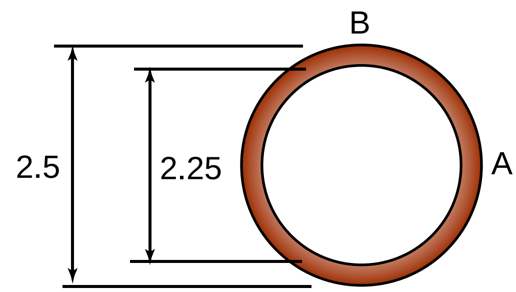
Determine the principal stresses and maximum in-plane shearing stress at A and B. Units: k, k-in.



From a previous solution:







Cross-section

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

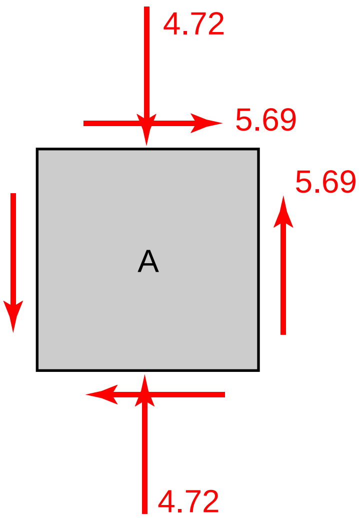
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

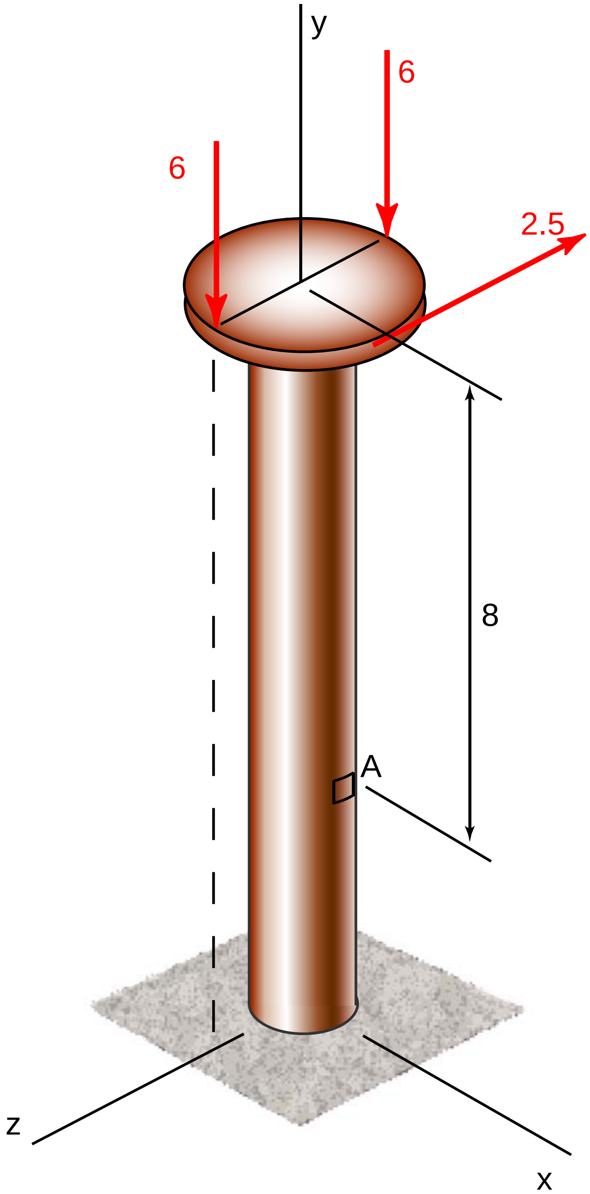
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Example

Determine the principal stresses and maximum in-plane shearing stress at A. The disk has a diameter of 4" and the solid shaft has a diameter of 1.8".

From a previous solution:





$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

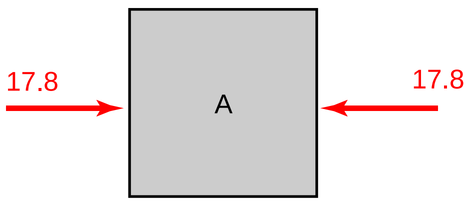
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

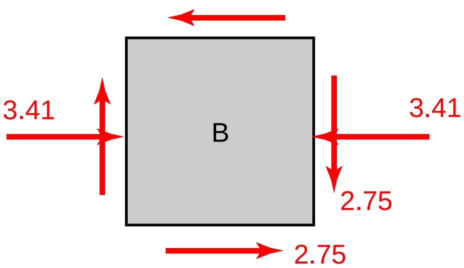
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

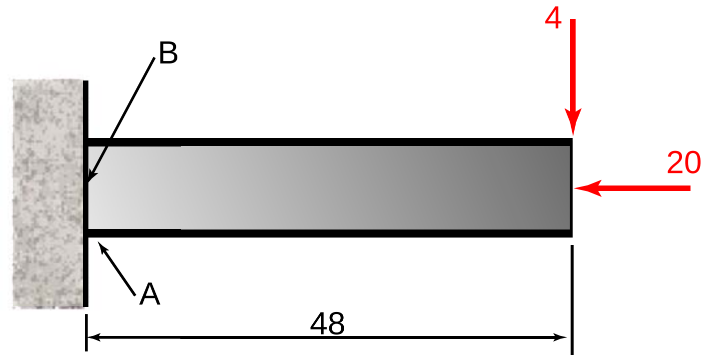
Example

Determine the principal stresses and maximum in-plane shearing stress at A and B. The beam is a W6x20.

From a previous solution:







$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

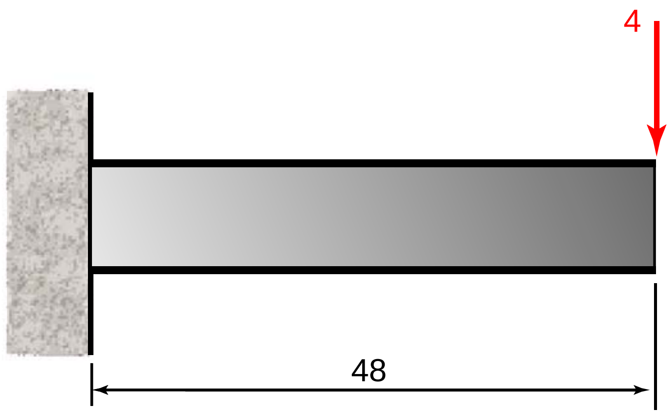
$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

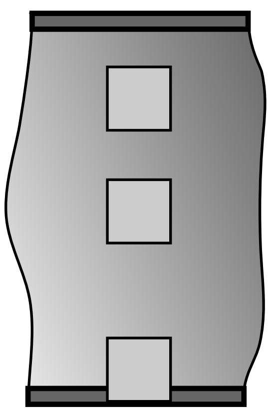
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

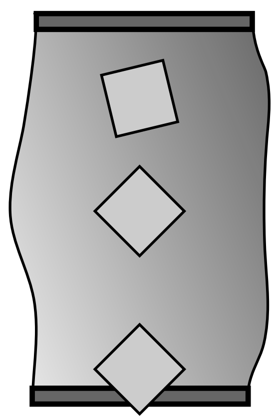
SUMMARY

Principal Stresses in a Beam

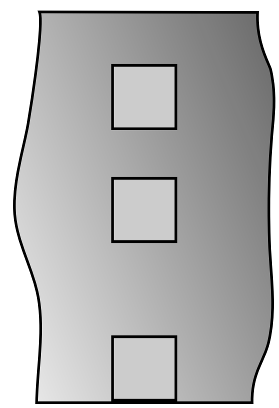




Wide Flange Stresses



Principal Wide Flange Stresses



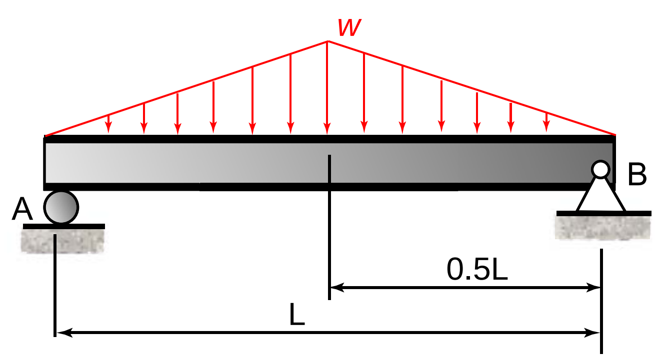
Rectangular Cross-section Stresses

Chapter 9
Deflection of Beams

INTRODUCTION

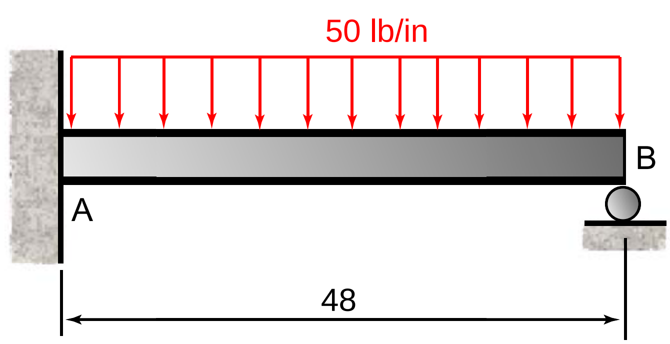
Deflection of Beams using Integration

Deflection of Beams using Superposition

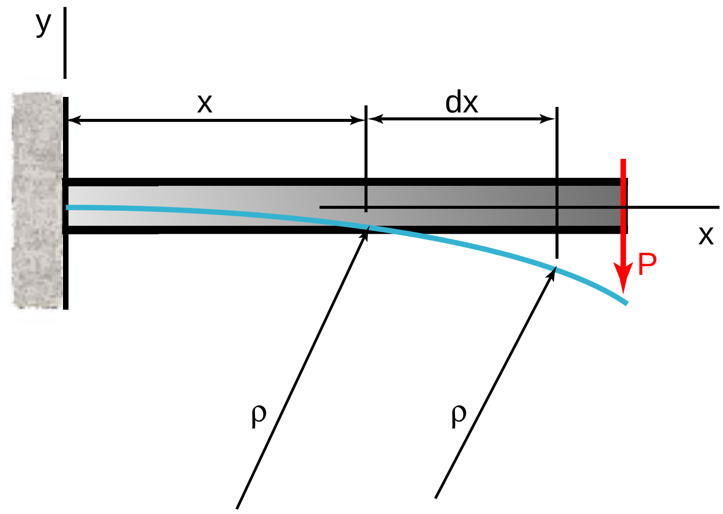


Statically Indeterminate Beams using Integration

Statically Indeterminate Beams using Superposition



EQUATION OF THE ELASTIC CURVE



Review,

$$\frac{dM}{dx} = V \qquad \frac{dV}{dx} = -w$$

$$\frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d\theta}{dx} = \frac{M(x)}{EI}$$

Noting,

$$TAN\theta = \frac{dy}{dx} \cong \theta$$

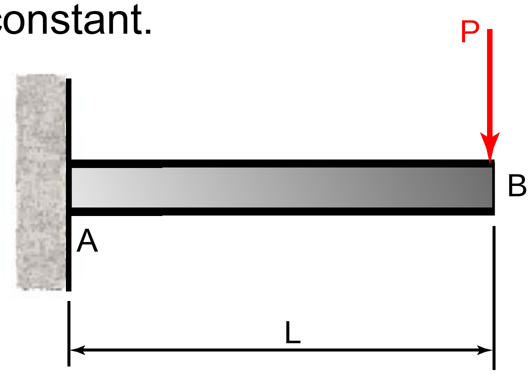
$$\therefore \frac{d\theta}{dx} = \boxed{\frac{d^2y}{dx^2} = \frac{M(x)}{EI}}$$

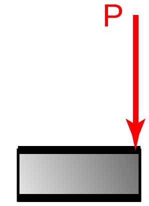
$$\frac{d^3y}{dx^3} = \frac{dM}{EIdx} = \frac{V(x)}{EI}$$

$$\frac{d^4y}{dx^4} = \frac{dV}{dxEI} = -\frac{w(x)}{EI} \qquad \boxed{\frac{d^4y}{dx^4} = -\frac{w(x)}{EI}}$$

Example

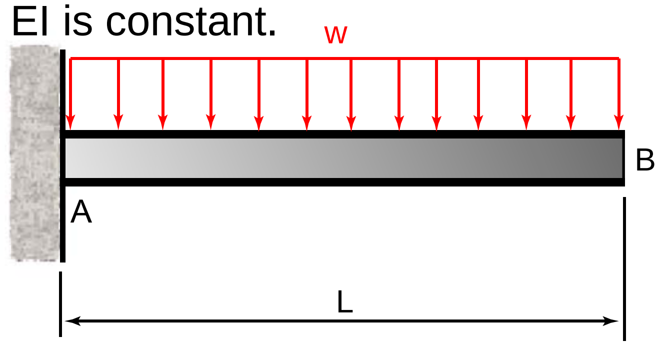
a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the second order differential equation to solve. EI is constant.

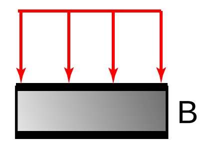




Example

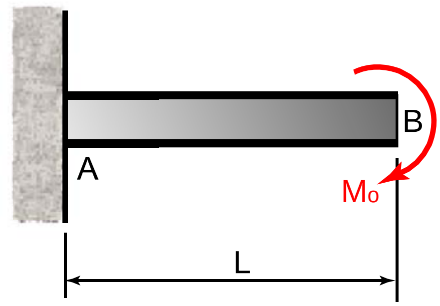
a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the second order differential equation to solve. EI is constant.

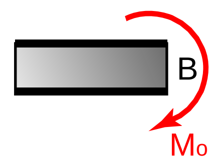




Example

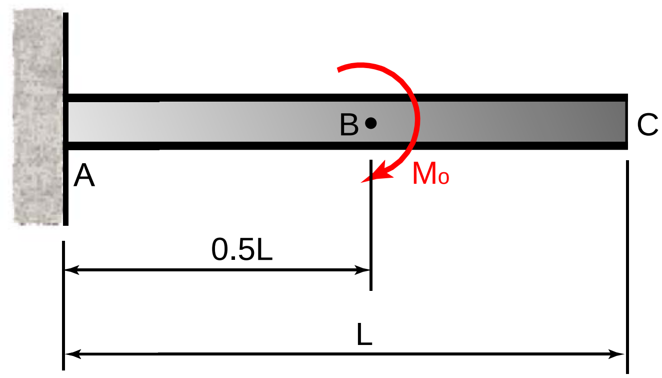
a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the second order differential equation to solve. EI is constant.

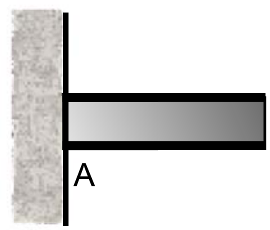


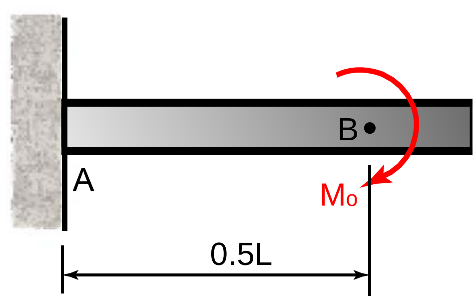


Example

Determine the vertical displacement and slope at point c. Use the second order differential equation to solve. EI is constant.

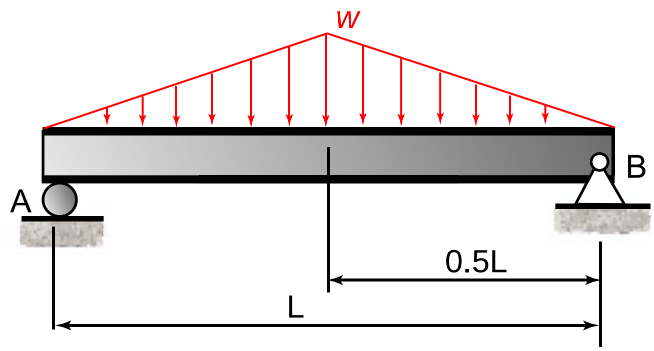


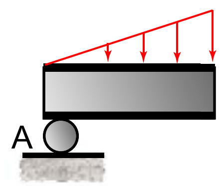




Example

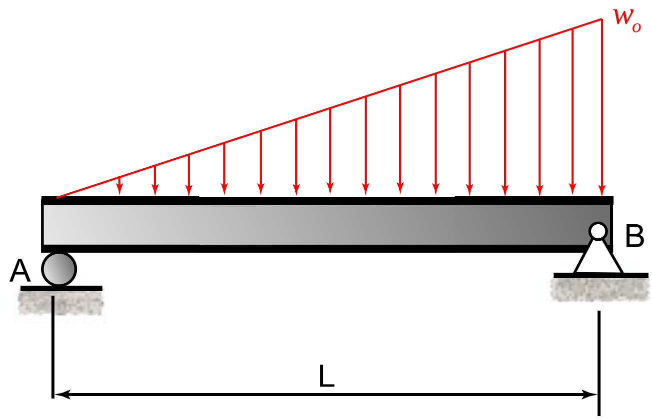
Determine the vertical displacement at the center of the beam. Use the second order differential equation to solve. EI is constant.

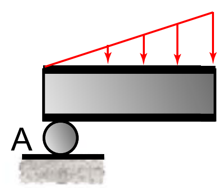




Example

Determine the maximum vertical displacement of the beam. Use the second order differential equation to solve. EI is constant.



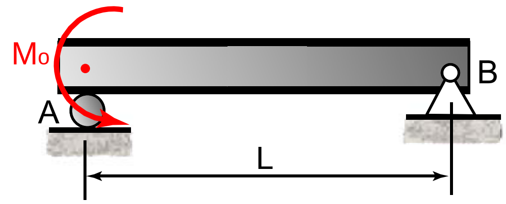


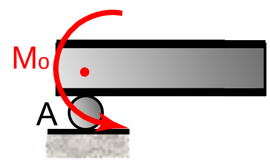
Example

Determine the maximum vertical displacement of the beam. Use the second order differential equation to solve. EI is constant.
Units: kN, m.

$$E = 200GPa$$

$$I = 22.2x10^{6 mm^4}$$



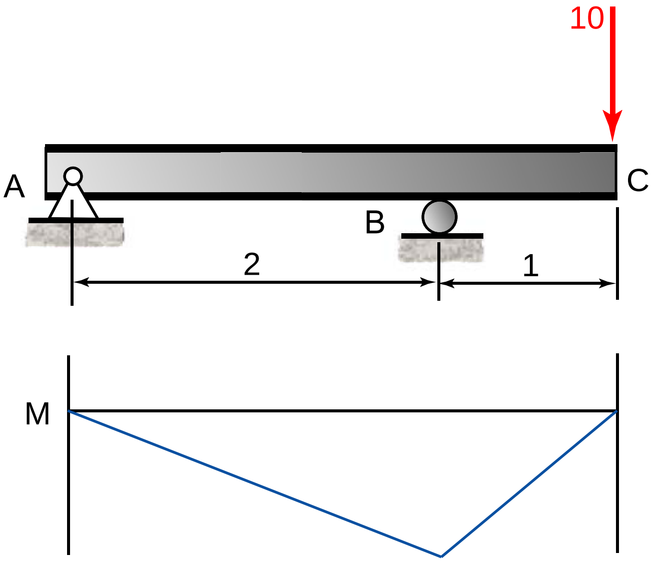


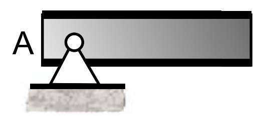
Example

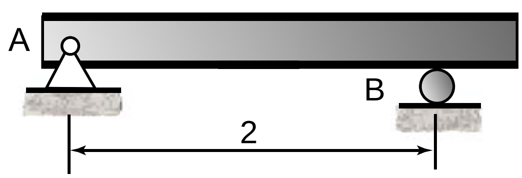
Determine the vertical displacement at C. Use the second order differential equation to solve. EI is constant.
Units: kN, m.

$$E = 200GPa$$

$$I = 22.2x10^{6mm^4}$$

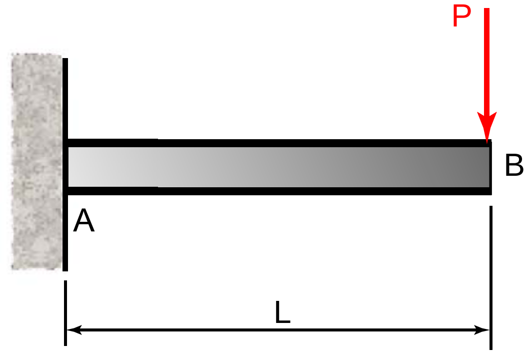


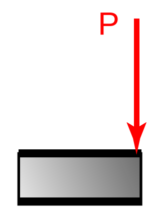




Example

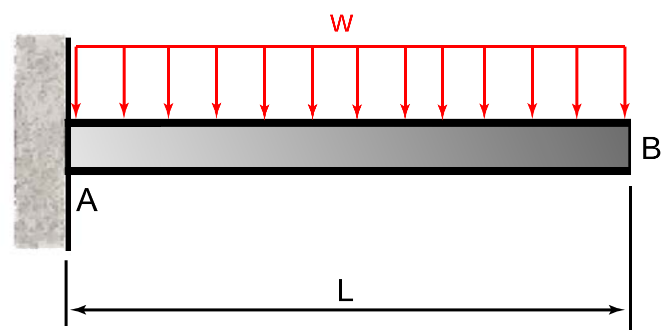
a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the fourth order differential equation to solve. EI is constant.

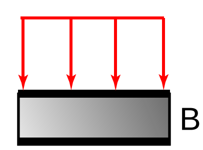




Example

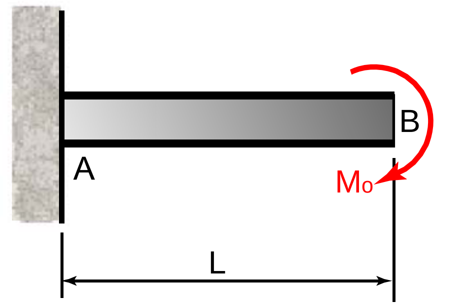
a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the fourth order differential equation to solve. EI is constant.

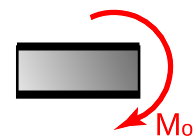




Example

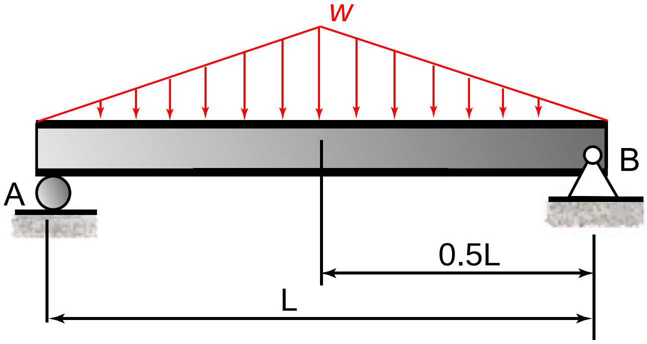
a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement and slope at B. Use the fourth order differential equation to solve. EI is constant.

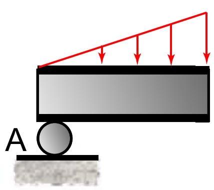




Example

a) Determine the equation for the vertical displacement and slope at any point. b) Find the displacement at L/2 and the slope at A. Use the fourth order differential equation to solve. EI is constant.

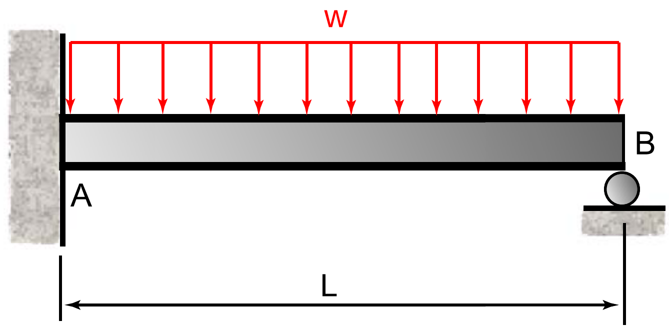




STATICALLY INDETERMINATE BEAMS USING INTEGRATION

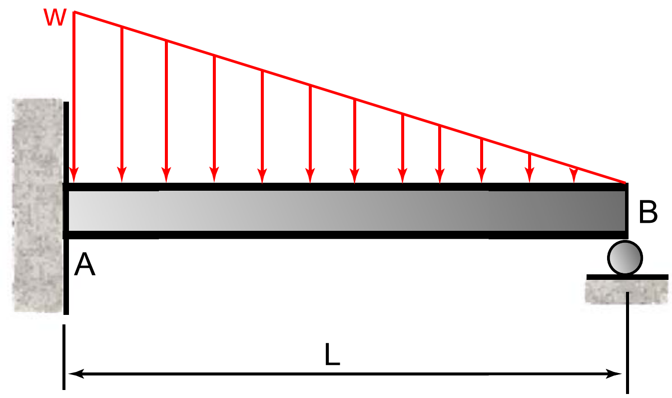
Example

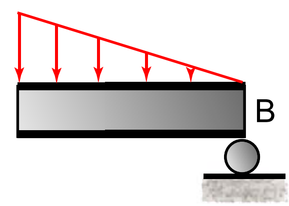
Determine the reactions at A and B. Use the fourth order differential equation to solve. EI is constant. Units: lb, in.



Example

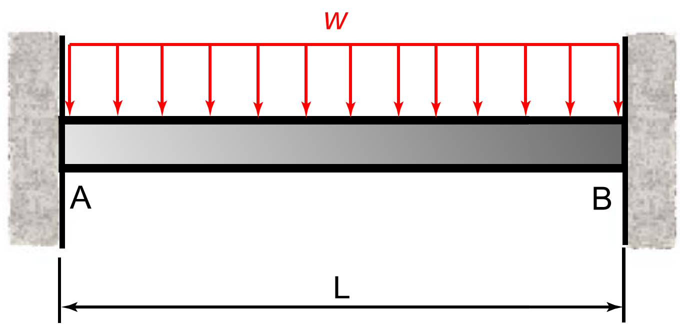
Determine the reactions at A and B. Use the second order differential equation to solve. EI is constant. Units: lb, in.

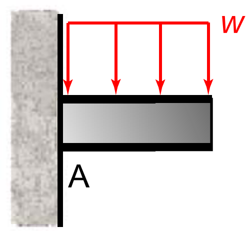




Example

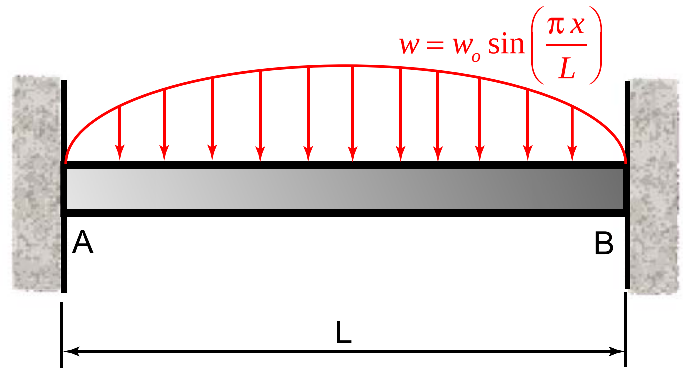
Determine the reactions at A and B. Use the second order differential equation to solve. Neglect the effect of any axial reactions. EI is constant. Units: lb, in.





Example

Determine the reactions at A and B. Use the fourth order differential equation to solve. Neglect the effect of any axial reactions. EI is constant. Units: lb, in.



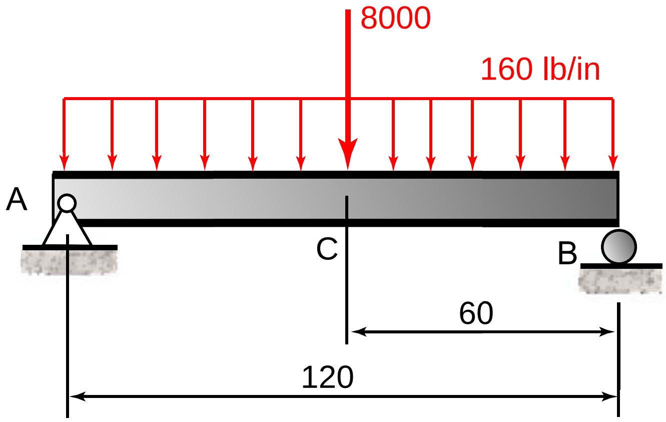
DEFLECTION OF BEAMS USING SUPERPOSITION

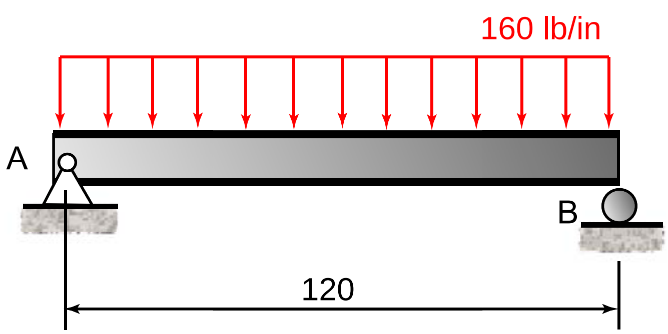
Example

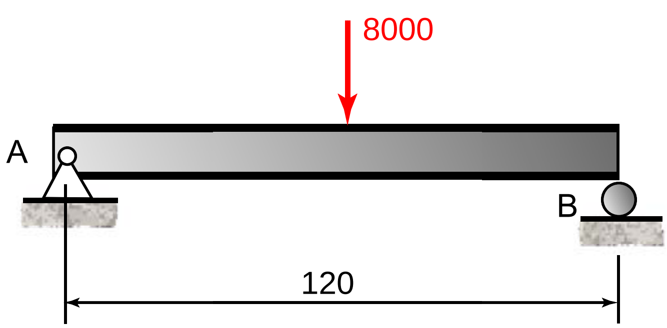
Using superposition, determine the displacement at C. EI is constant.
Units: lb, in.

$$E = 29x10^{6\,psi}$$

$$I = 53.4^{in^4}$$



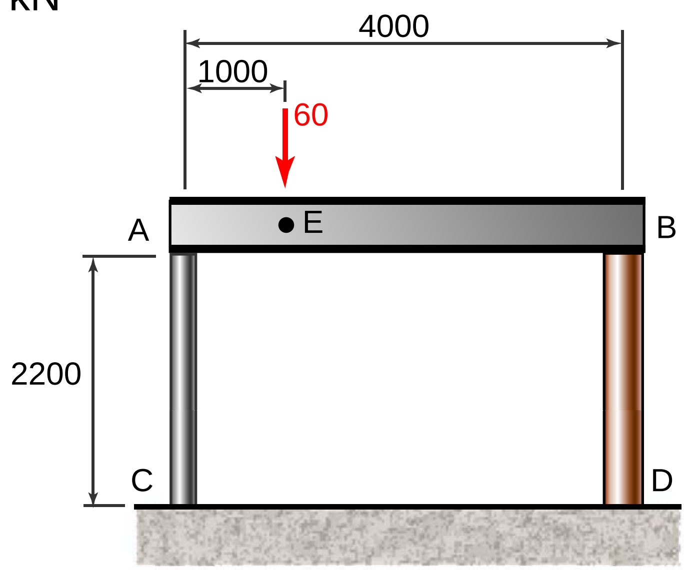


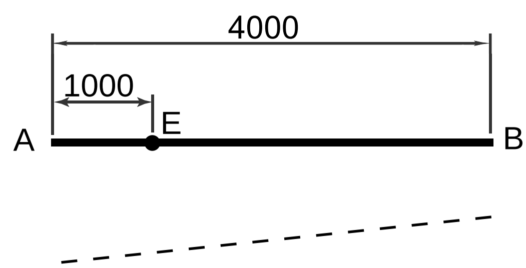


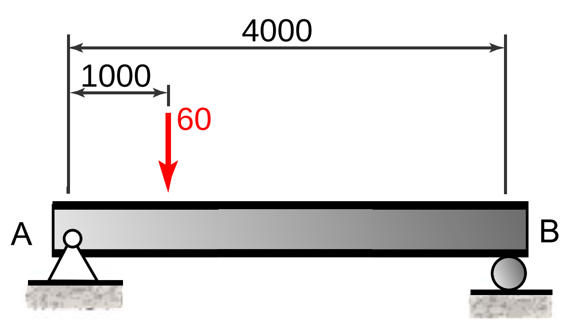
Example

Post AC is made of steel and has a diameter of 18 mm, and BD is made of copper and has a diameter of 42 mm. Determine the displacement of point E on the steel beam AB. E(steel)= 200 GPa, E(copper)= 120 GPa. Units: mm, kN

$I = 45.5x10^{6 mm^4}$

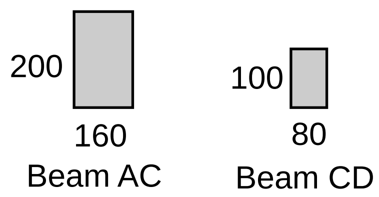


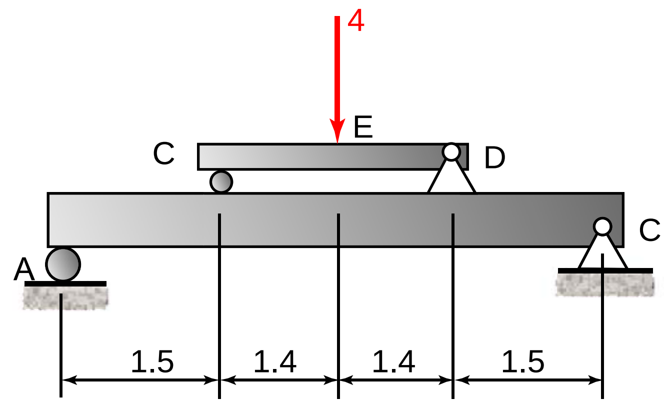


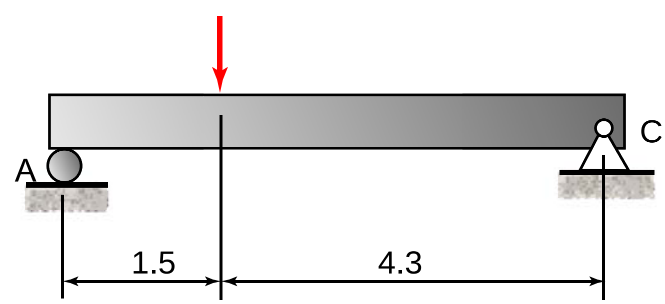


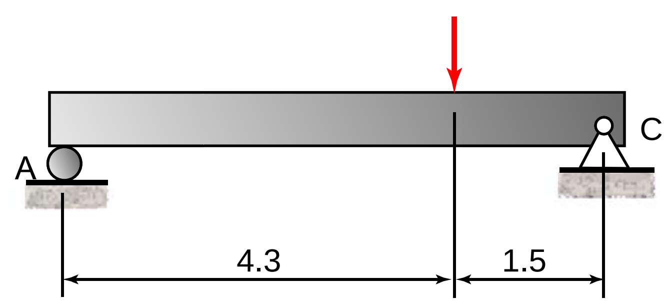
Example

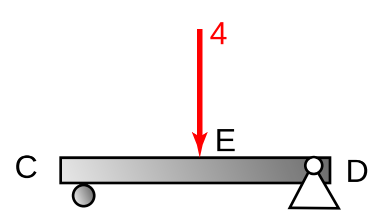
Knowing that each beam has a rectangular cross section as shown, determine the displacement at E. E= 200GPa. EI is constant.
Units: kN, m.







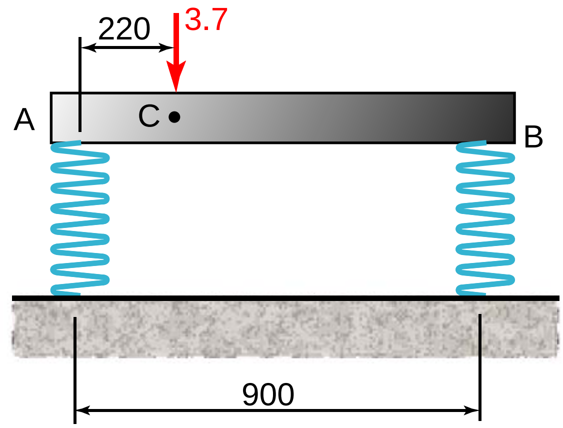


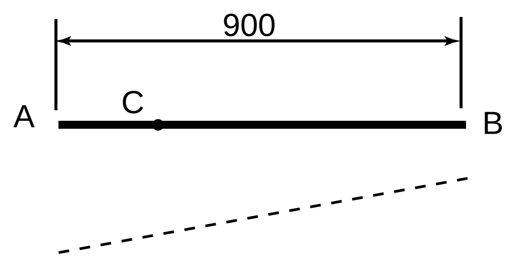


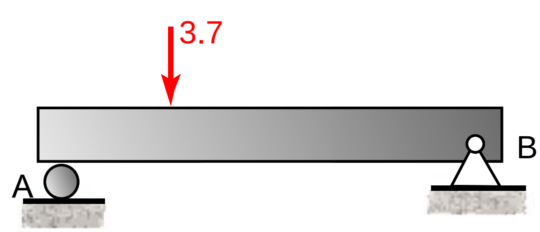
Example

The horizontal beam AB rests on the two short springs with the same length. The spring at A has stiffness of 250 kN/m and the spring at B has a stiffness of 150 kN/m. Determine the displacement under the load. Units: kN, mm.

$EI = 15x10^{3m^2}$



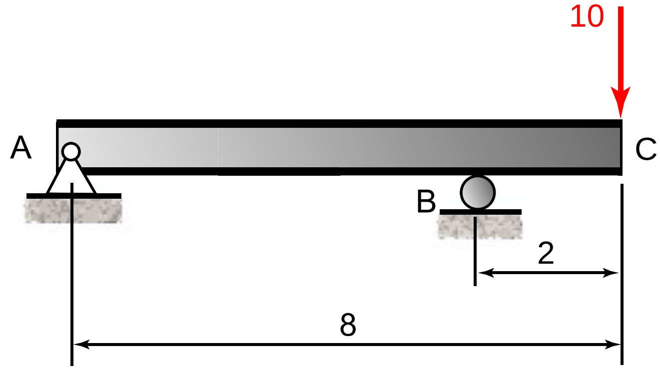


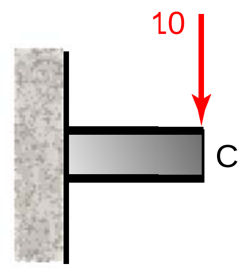


Example

Using superposition, determine the displacement at C. EI is constant. Units: kN, m.

$EI = 21.4x10^{6m^2}$



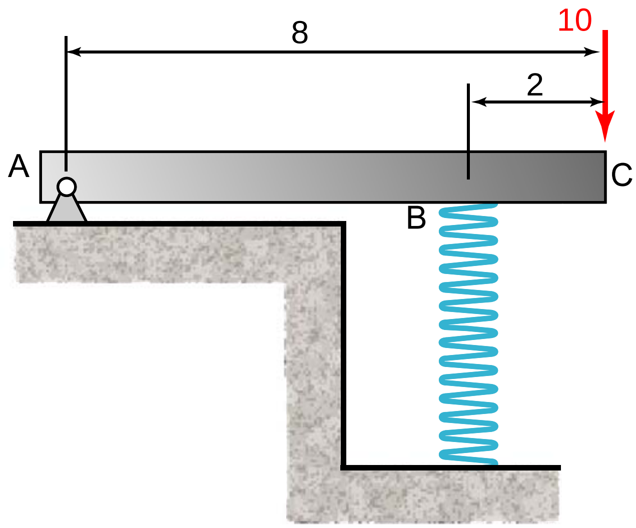


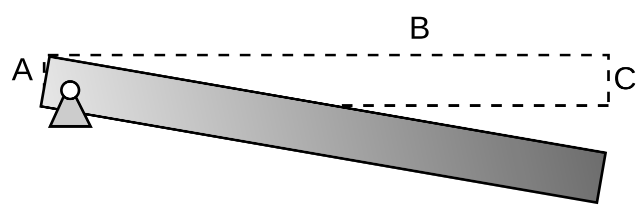
Example

The 160x200 mm rectangular beam ABC rests on a spring at B. The spring at B has stiffness of 2500 kN/m. Determine the displacement at C. Units: kN, m.

$$EI = 21.4x10^{6m^2}$$

From a previous solution with point B being a rigid roller:
y_c= 4.99 mm

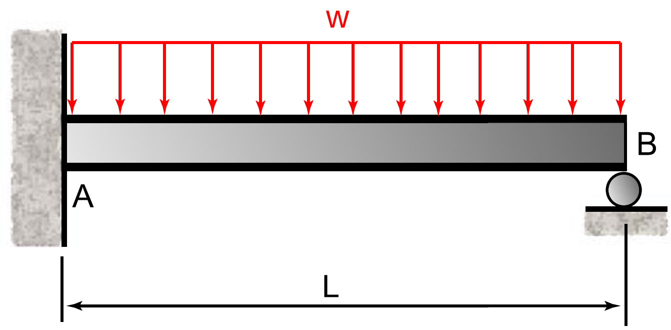


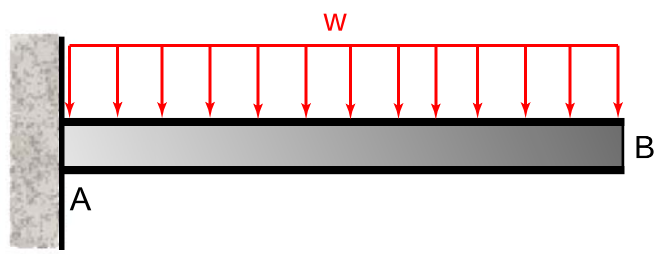


STATICALLY INDETERMINATE BEAMS USING SUPERPOSITION

Example

Using superposition, determine the reactions at A and B.
EI is constant. Units: lb, in.



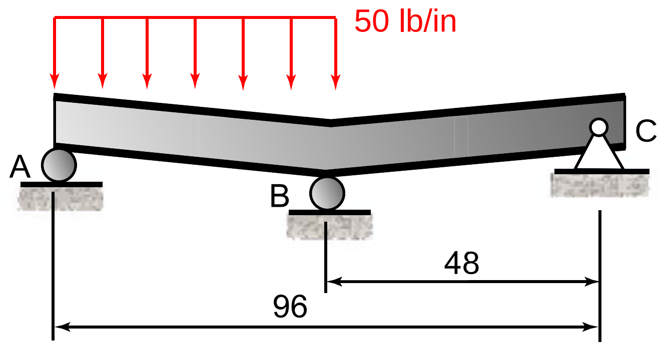


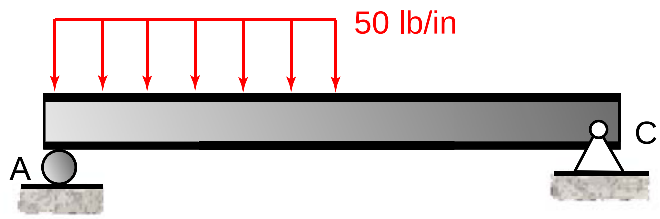
Example

Due to the loading and poor construction, support B settles 1/16".
Using superposition, determine the reactions at A, B, and C.
EI is constant. Units: lb, in.

$E = 29x10^{6\,psi}$

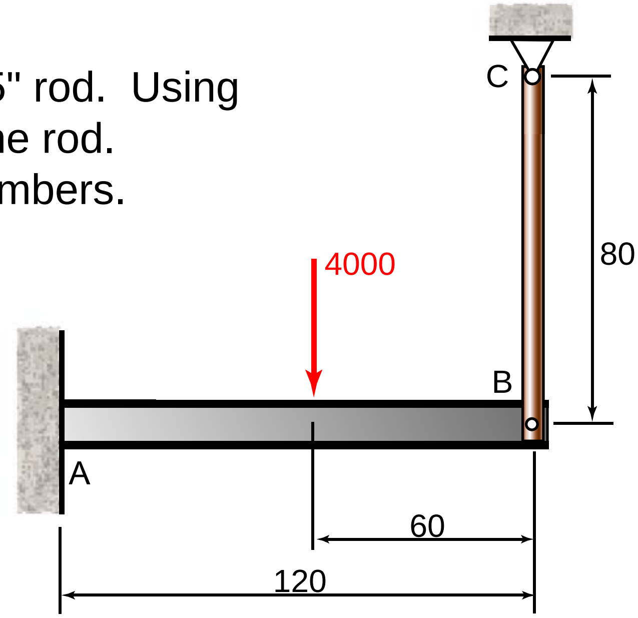
$I = 11.3^{in^4}$

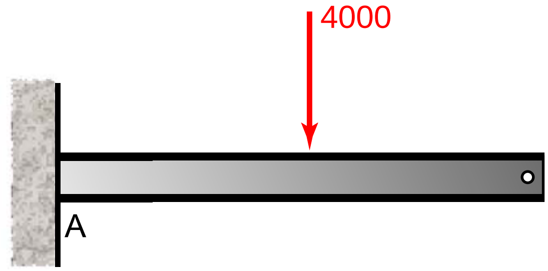




Example

The W6x20 is supported at B by a 0.25" rod. Using superposition, determine the force in the rod.
EI is constant. E=29E6 psi for both members.
Units: lb, in.

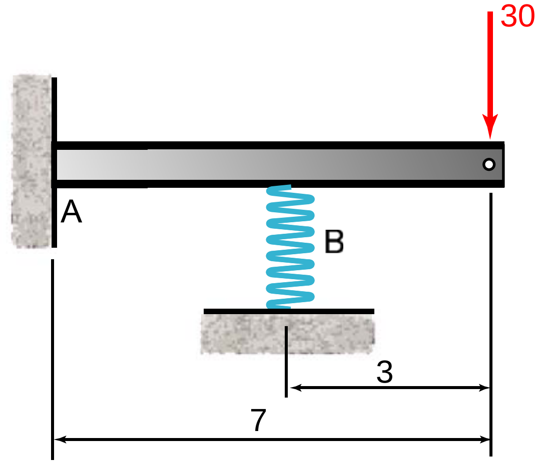




Example

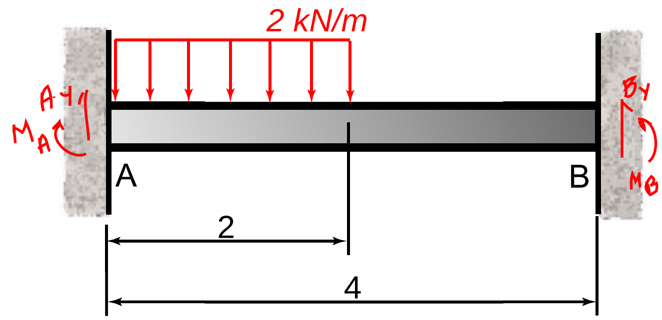
Using superposition, determine the reactions at A and the force in the spring at B. The spring constant is 1 kN/mm. EI is constant.
Units: kN, m.

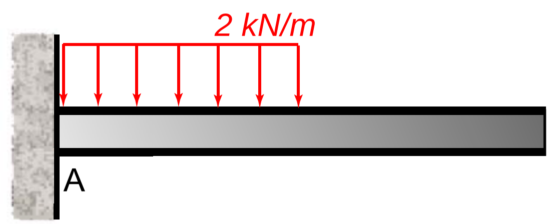
$$EI = 100x10^{6m^2}$$



Example

Using superposition, determine the reactions at A and B. Neglect the effect of any axial reactions. EI is constant. Units: kN, m.

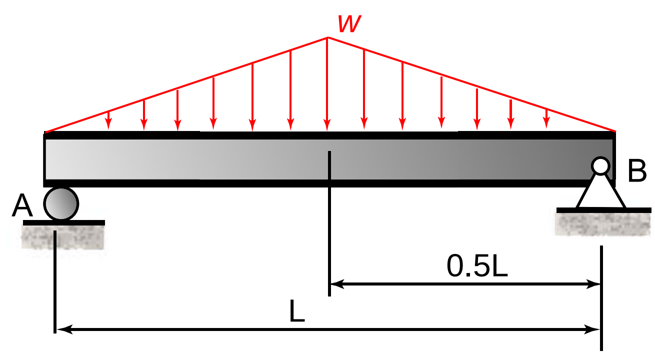




SUMMARY

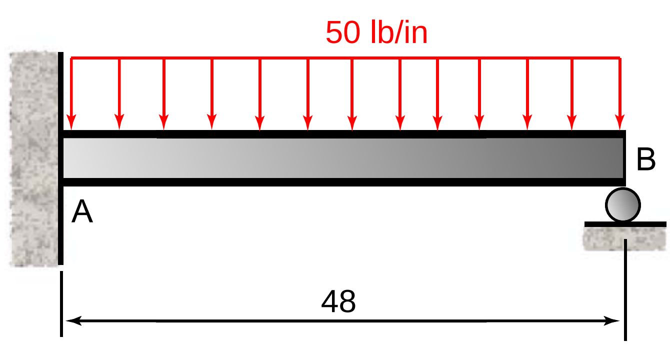
Deflection of Beams using Integration

Deflection of Beams using Superposition



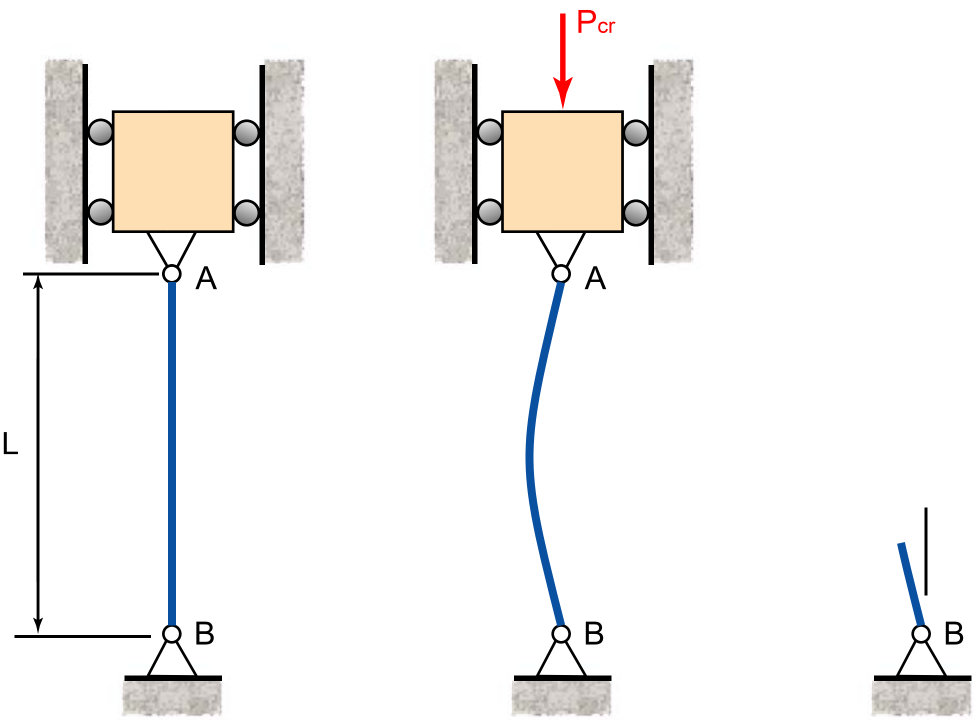
Statically Indeterminate Beams using Integration

Statically Indeterminate Beams using Superposition



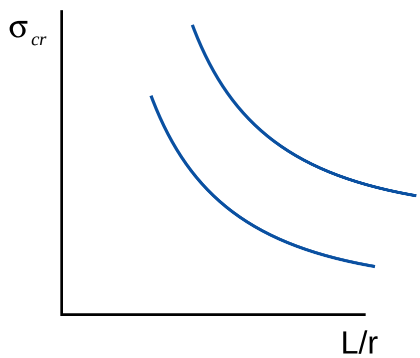
Chapter 10
Columns

COLUMNS WITH PINNED-ENDS



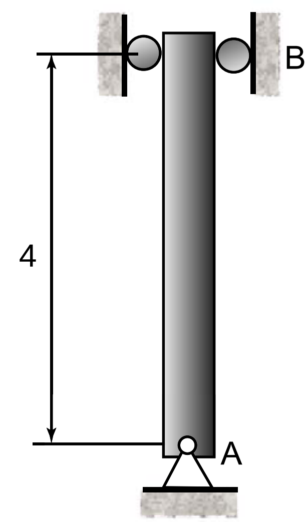
$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}$$



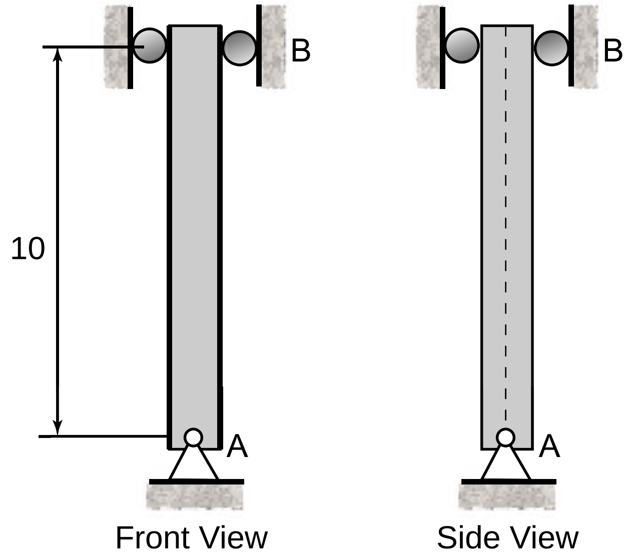
Example

a) Using a factor of safety of 2.5 against buckling, determine the largest load the column can support before it begins to buckle. Consider only in-plane buckling. b) Find the maximum load if the allowable axial stress is 80 MPa. The pipe has an outside diameter of 100 mm and a wall thickness of 6 mm. E=200 GPa. Units: m.



Example

Using a factor of safety of 1.85, determine the largest load the W6x20 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. E= 29E6 psi. Units: ft.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

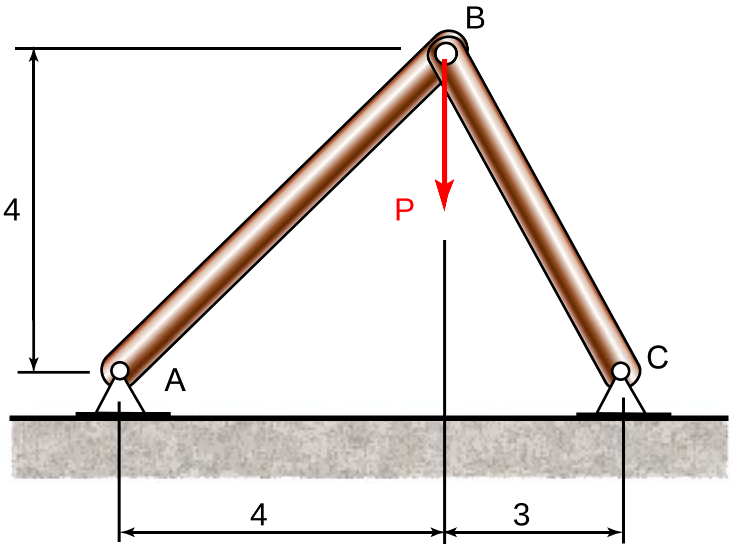
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

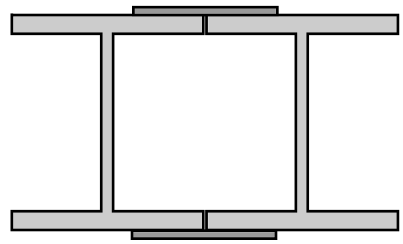
Example

Both members are identical pipe sections with an outside diameter of 100 mm and a wall thickness of 6 mm. Determine the largest load P based on in-plane buckling. E= 200 GPa. Units: m.



Example

Find the critical buckling load for a 28 ft pin-pin columm. The two W6x20 columns are spliced together to insure they work as one. Ignore the properties of the plates used to make the splice.
E= 30E6 psi. Units: ft.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

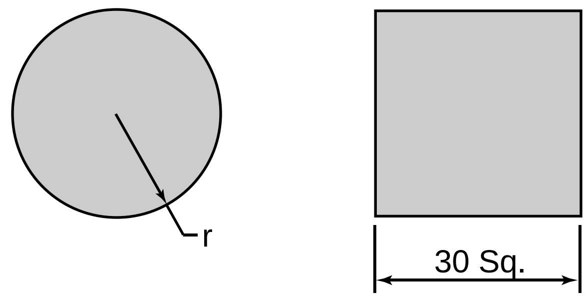
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

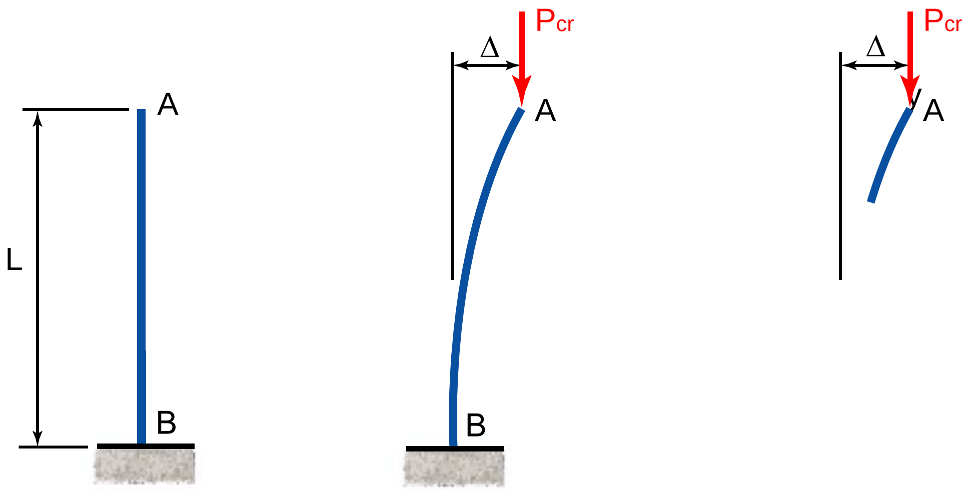
$S_y = 4.41^{in^3}$

Example

Determine the radius of a round column so that it has the same buckling capacity as that of a square 30 mm column. Both columns are identical other than their cross section. Units: mm.



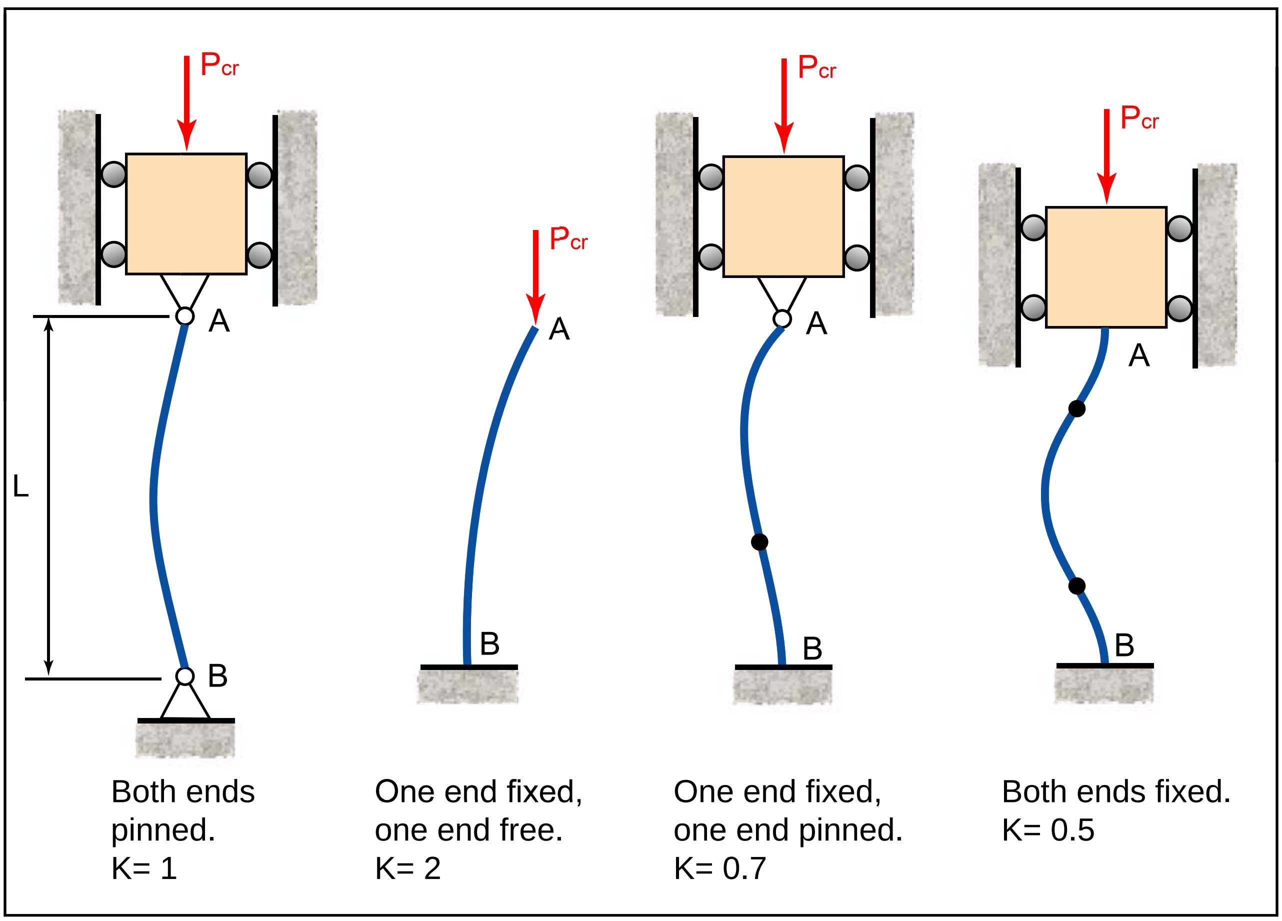
COLUMNS WITH OTHER END CONDITIONS



$$P_{cr} = \frac{\pi^2 EI}{4L^2}$$

Effective Length

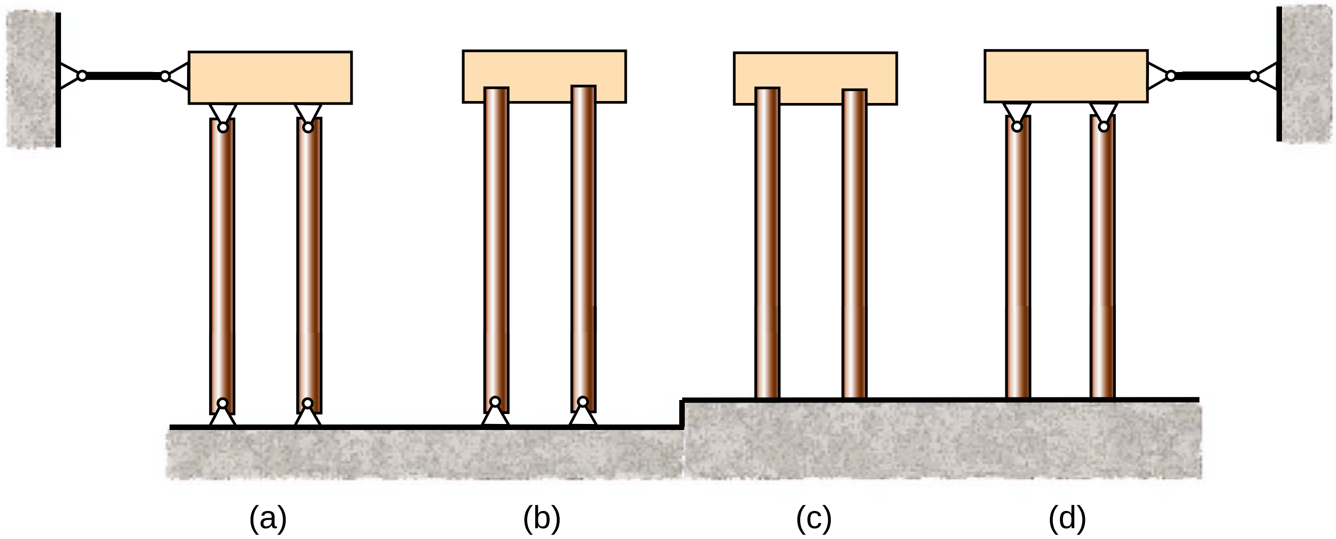
$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 EI}{(L_e)^2}$$



Column K values

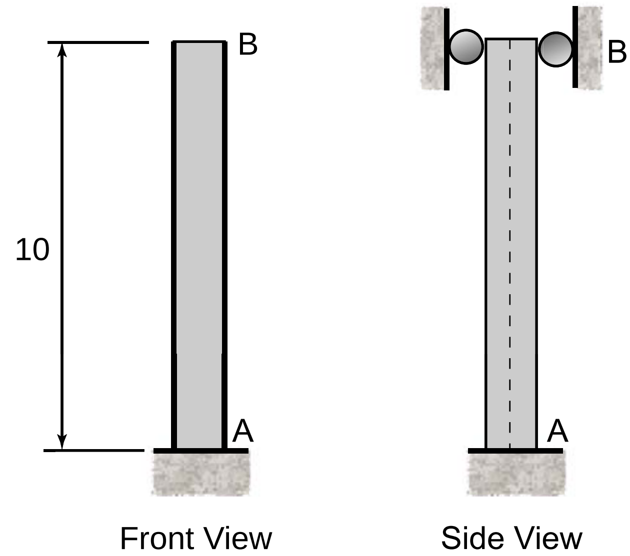
Example

Determine the K value for each of the following conditions:



Example

Determine the largest load the W6x20 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. E= 29E6 psi. Units: ft.



W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

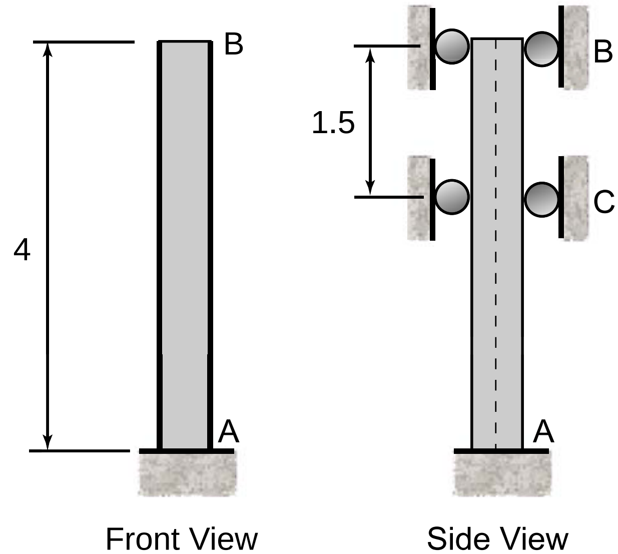
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

Determine the largest load the W150x29.8 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. E= 200 GPa. Units: m.



W150x29.8

Area, $A = 3790^{mm^2}$

Depth, $d = 157^{mm}$

Flange Width, $b_f = 153^{mm}$

Flange Thickness, $t_f = 9.3^{mm}$

Web Thickness, $t_w = 6.6^{mm}$

$I_x = 17.2x10^{6mm^4}$

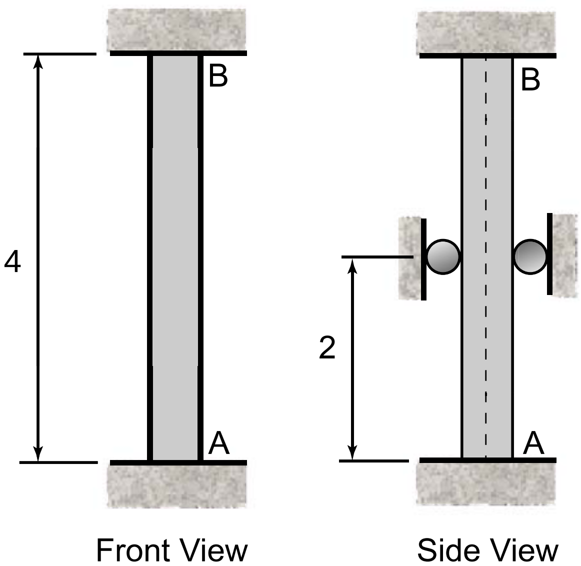
$I_y = 5.56x10^{6mm^4}$

$S_x = 219x10^{3mm^3}$

$S_y = 72.7x10^{3mm^3}$

Example

Determine the largest load the W150x29.8 column can support before it begins to buckle. Consider both in-plane and out of plane buckling. E= 200 GPa. Units: m.



W150x29.8

Area, $A = 3790^{mm^2}$

Depth, $d = 157^{mm}$

Flange Width, $b_f = 153^{mm}$

Flange Thickness, $t_f = 9.3^{mm}$

Web Thickness, $t_w = 6.6^{mm}$

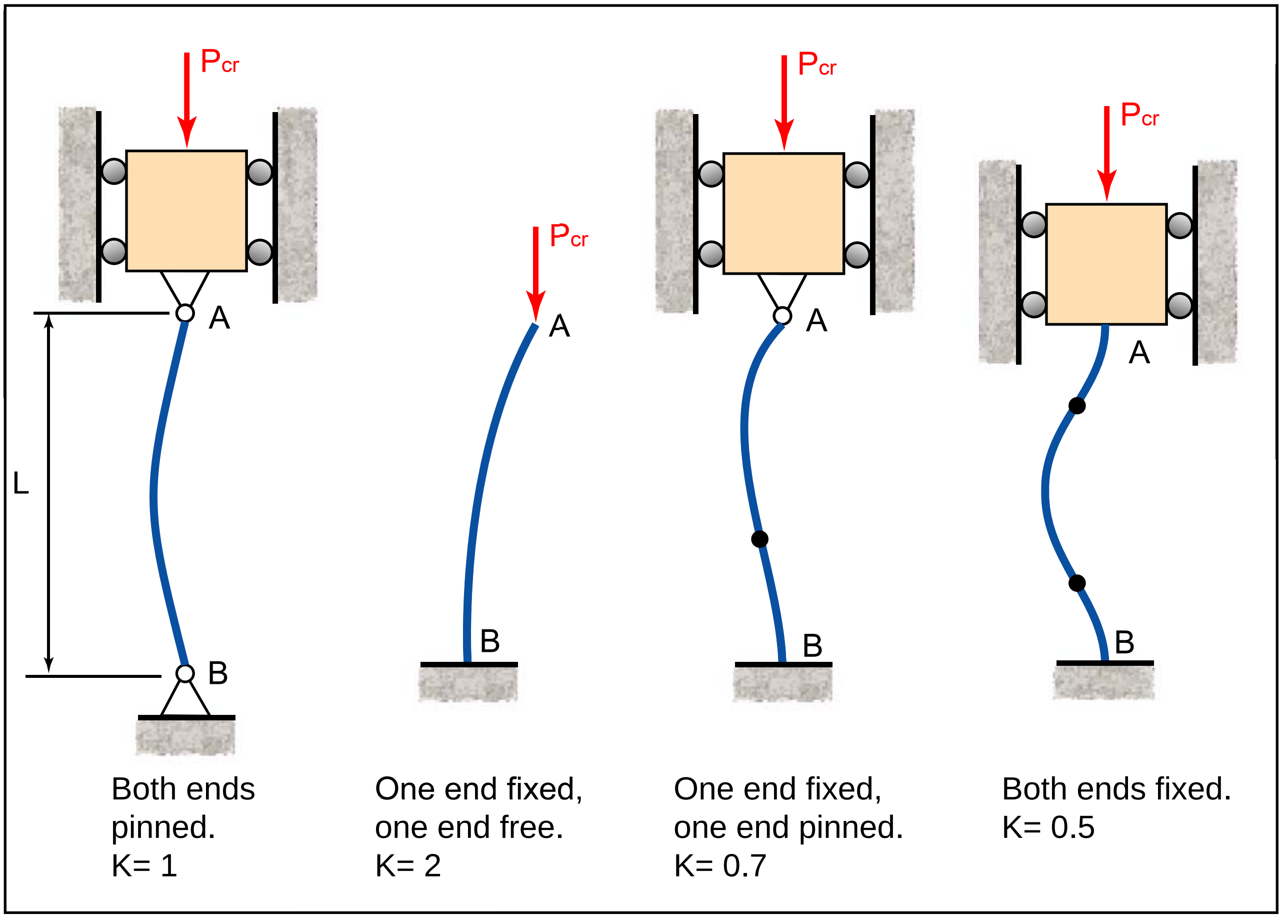
$I_x = 17.2x10^{6mm^4}$

$I_y = 5.56x10^{6mm^4}$

$S_x = 219x10^{3mm^3}$

$S_y = 72.7x10^{3mm^3}$

SUMMARY



Column K values

$$P_{cr} = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2 EI}{(L_e)^2}$$