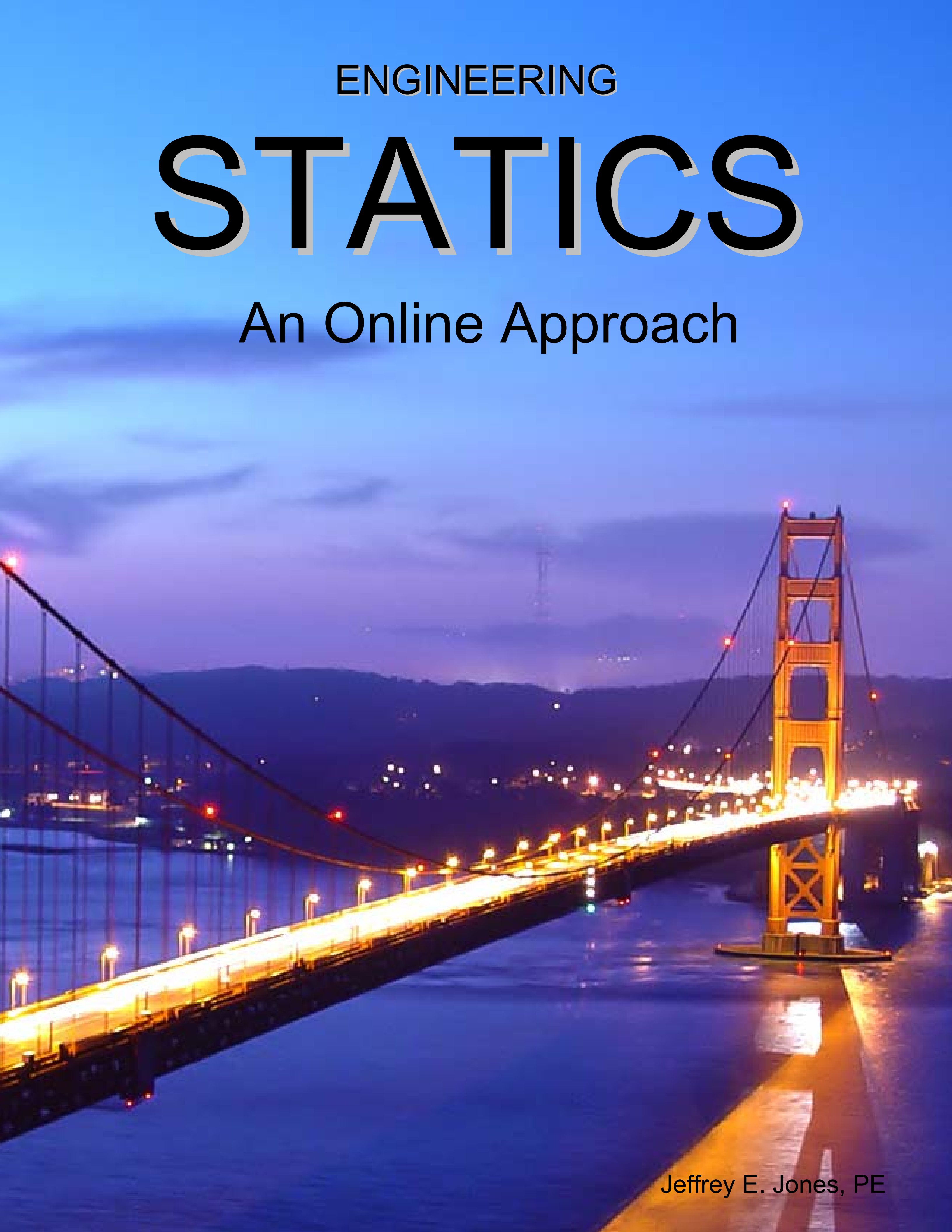
ENGINEERING STATICS

An Online Approach



Jeffrey E. Jones, PE

Prefix	Symbol	Multiplication Factor
tera	T	10^{12} =1 000 000 000 000
giga	G	10^{9} = 1 000 000 000
mega	M	10^{6} = 1 000 000
kilo	k	10^{3} = 1 000
hecto	h	10^{2} = 100
deka	da	10^{1} = 10
deci	d	10^{-1} = .1
centi	c	10^{-2} = .01
milli	m	10^{-3} = .001
micro	μ	10^{-6} = .000 001
nano	n	10^{-9} = .000 000 001
pico	p	10^{-12} = .000 000 000 001

Greek Alphabet					
Α	α	Alpha	Ν	ν	Nu
Β	β	Beta	Ξ	ξ	Xi
Γ	γ	Gamma	Ο	ο	Omicron
Δ	δ	Delta	Π	π	Pi
Ε	ε	Epsilon	Ρ	ρ	Rho
Ζ	ζ	Zeta	Σ	σ	Sigma
Η	η	Eta	Τ	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
Ι	ι	Iota	Φ	φ	Phi
Κ	κ	Kappa	Χ	χ	Chi
Λ	λ	Lambda	Ψ	ψ	Psi
Μ	μ	Mu	Ω	ω	Omega

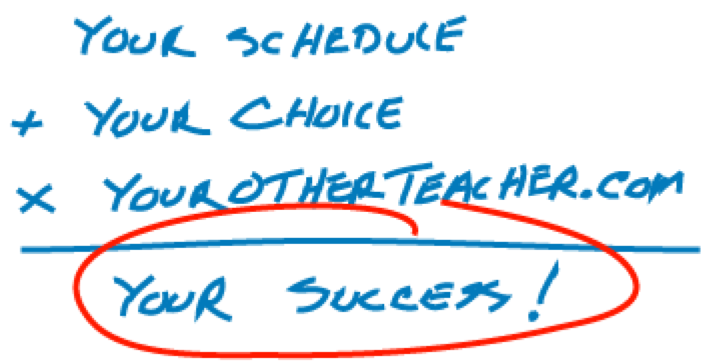
STATICS- AN ONLINE APPROACH

STUDENTS…

This is your YourOtherTeacher.com Statics Companion. Inside you'll find over 300 Statics example problems for use in class or for extra practice outside of class with YourOtherTeacher.com's corresponding online Canvas Statics course. You'll find problems solved by your online instructor, in a detailed, step-by-step, whiteboard video format. Watch only the lessons you want, when you want, and use this companion to follow along. It's easy, and it will help you pass your class.

INSTRUCTORS…

This is your static's companion, too. Use it in class as a source of example problems and save your students the time and energy of copying problem statements and figures off the board. Encourage your students to watch any of these problems solved…anytime they want. Think of YourOtherTeacher.com as your online TA, with unlimited office hours. Contact your YOT representative for free access at admin@yourotherteacher.com.



Cover Photograph:
The Golden Gate Bridge in San Francisco, California.

How to use this book: This video companion contains the screen shots of the problems that are solved on a students Learning Management System such as Canvas or www.YourOtherTeacher.com. The solutions are not included here since it is the authors' belief that more can be learned from the words and gestures in a video than can ever be written. It is suggested that the students write the solutions down as presented in the videos since memory is greatly increased by the writing process.

Avoid looking for problems that are similar to your homework. This would be very short sighted. It is better to understand the concepts than to get the solution for one problem. If you understand the concepts then you can solve any problem that may appear on a test or a problem you may encounter in industry. The saying "Give a man a fish, he eats for a day, teach him how to fish, he eats for a lifetime" is the motto for YourOtherTeacher.com.

Jeff Jones holds a bachelor's and master's degree in civil engineering from San Jose State University in San Jose, CA. His concentration was in structural engineering and applied mechanics. He is also the author of Strength of Materials- An Online Approach, Engineering Drawing- An Online Approach, as well as many others. He has personally recorded over 300 hours of video on the website www.YourOtherTeacher.com which helps 1000's of students every year towards their goal of becoming an engineer. After graduation, Jones was a senior structural engineer at Bechtel Corporation for 10 years where he became a registered professional engineer in California. Since then, he has served in various roles as a Professor, Department Chair, lead instructor, author, chairman and executive with 40+ years of proven experience. Jeff Jones is the recipient of the prestigious "Community College Teacher of the Year" award, Awarded by the American Society of Engineering Educators (ASEE-PSW). He was also awarded Cuesta College's highest honor "Teaching Excellence Award".

Contents

1- INTRODUCTION

2- STATICS OF PARTICLES

3- RIGID BODIES: EQUIVALENT SYSTEMS OF FORCES

4- EQUILIBRIUM OF RIGID BODIES

5- DISTRIBUTED FORCES: CENTROIDS AND CENTERS OF GRAVITY

6- ANALYSIS OF STRUCTURES

7- FORCES IN BEAMS

8- FRICTION

9- DISTRIBUTED FORCES: MOMENTS OF INERTIA

Chapter 1
Introduction

What is Mechanics?

That science which describes and predicts the conditions of rest or motion of bodies under the action of forces:

-Mechanics of Rigid Bodies

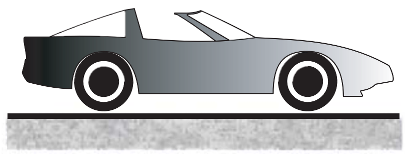
-Mechanics of Deformable Bodies

-Mechanics of Fluids

Fundamental Concepts and Principles

-The Parallelogram Law for the addition of forces:

-The principle of Transmissibility:



-Newton's First Law:

-Newton's Second Law:

-Newton's Third Law:

-Newton's Law of Gravitation:

Numerical Accuracy

Numerical accuracy depends on:
- -accuracy of the given data

- -the accuracy of the computations

Example:

I want to measure the area of my house and I'm so cheap I can't afford a tape measure. But my foot is approximately 1 foot (no pun intended) long. So I measure the length and width of the house accordingly (47.5 by 26.5 foot lengths). Find the area.

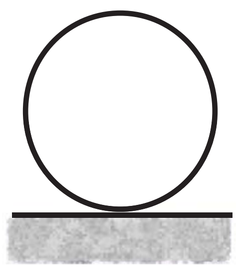
Trial and Error Solutions

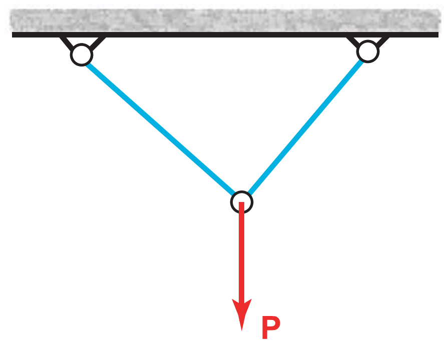
Find x given: 0=73.6 - 100sin(x) - 45cos(x)

Chapter 2
Statics of Particles

Introduction

Particles- when all of the forces converge at a common point:





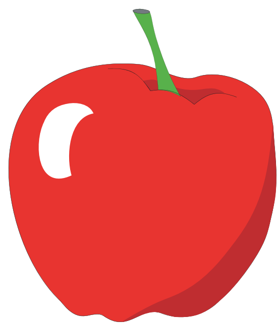
Bodies- when all of the forces *do not* converge on a common point:

Goals

-Replace two or more forces acting on a given particle by a single force having the same effect as the original (2D and 3D).

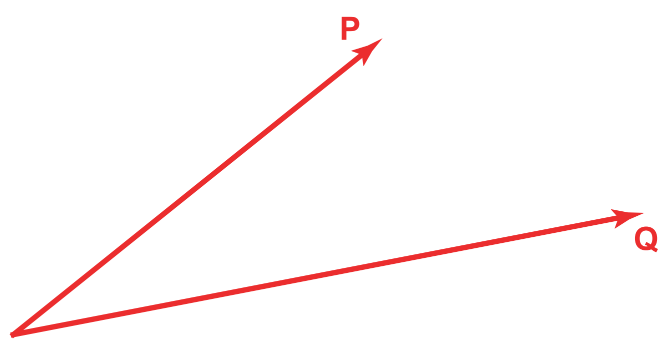
-Equilibrium (Newton's First Law) for 2D and 3D.

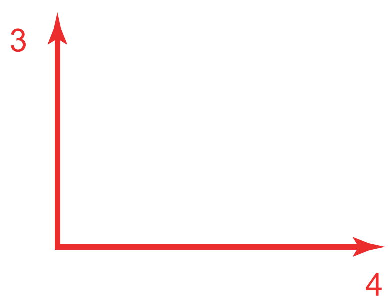
Vectors



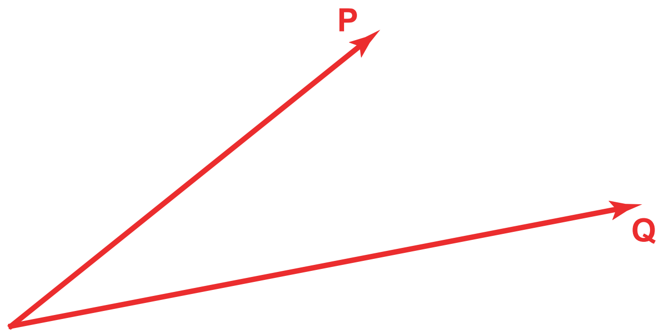
Addition of Vectors

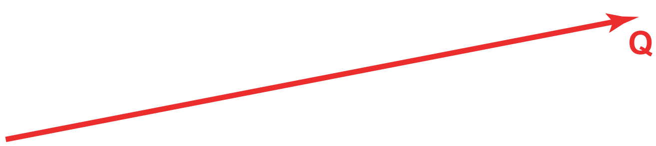
Parallelogram Law

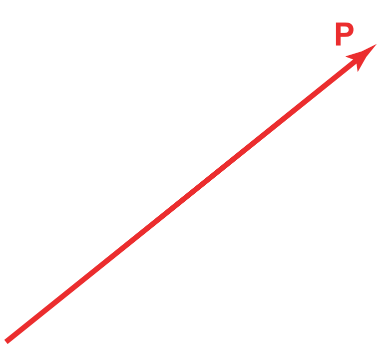




Triangle Rule

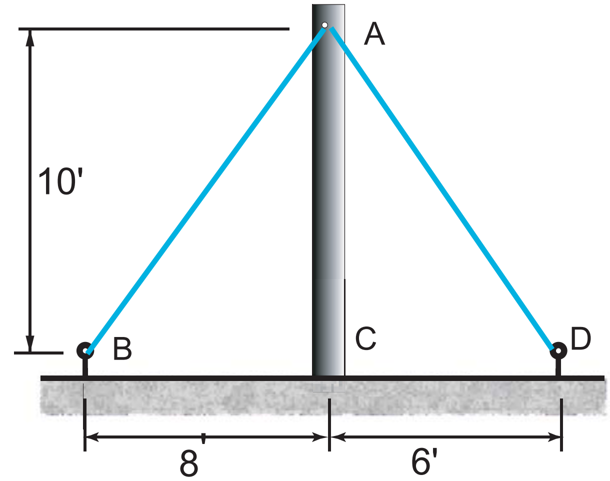




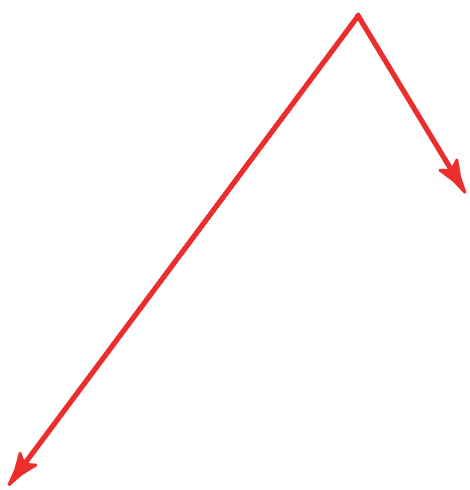


Example

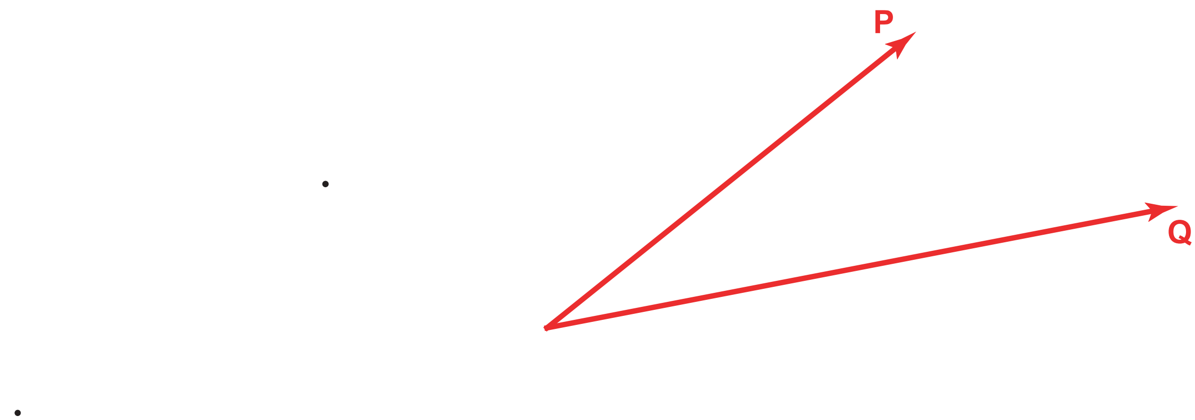
The cable stays AB and AD help support pole AC. Knowing that the tension is 120 lb in AB and 40 lb in AD, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at A using (a) the parallelogram law, (b) the triangle rule. Units: Lb.



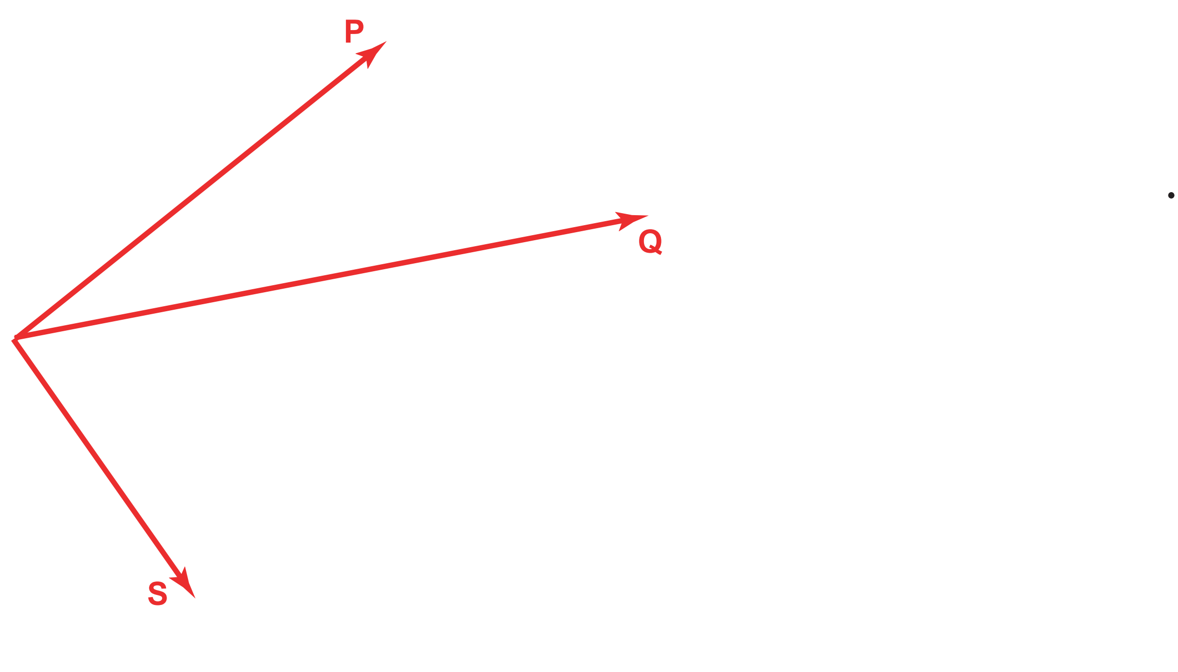
SCALE 1"= 40 lb



Subtraction



Sum of three or more vectors (Polygon Rule)

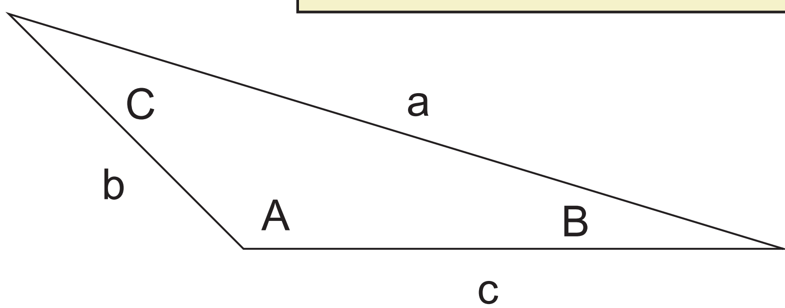


Resultant of Several Concurrent Forces

Law of Sines

In any triangle, the sides are proportional to the sines of the opposite angles, i.e.,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

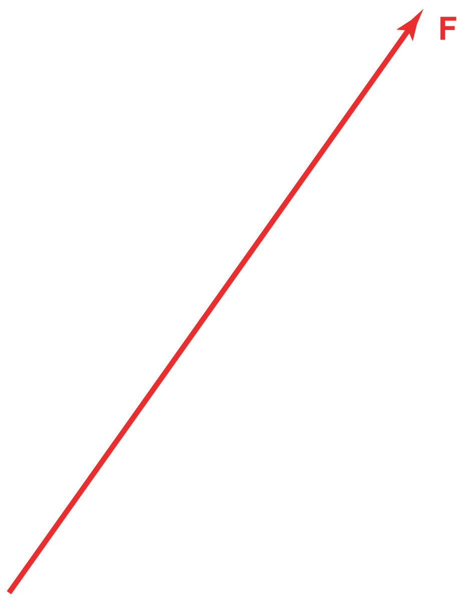


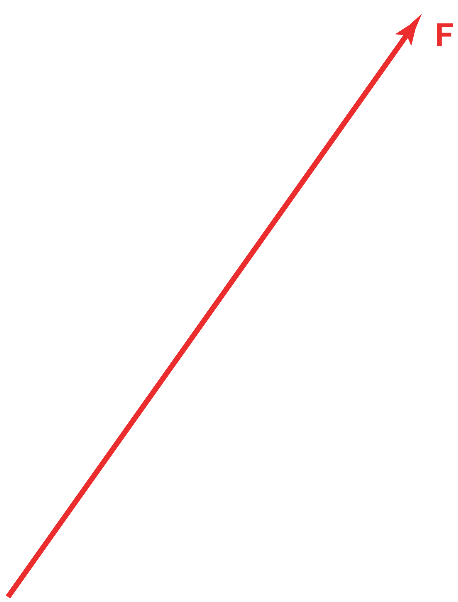
Law of Cosines

In any triangle ABC, the square of any side is equal to the sum of the squares of the other two sides diminished by twice the product of these sides and the cosine of their included angle, i.e.,

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

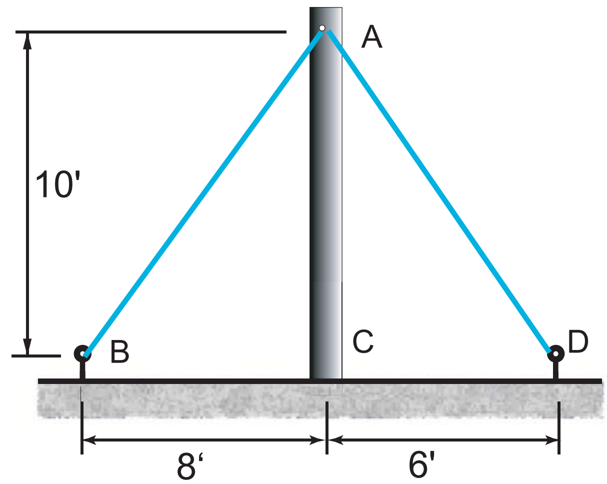
Resolution of a Force into Components

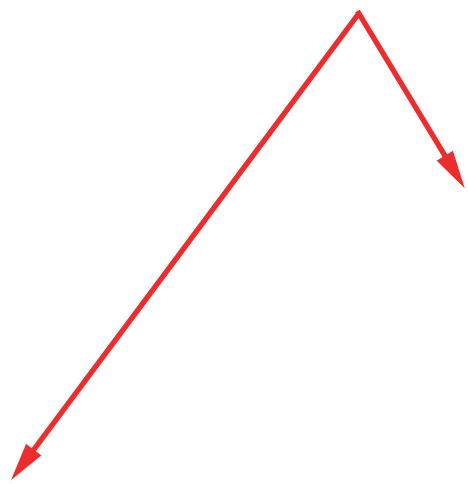




Example

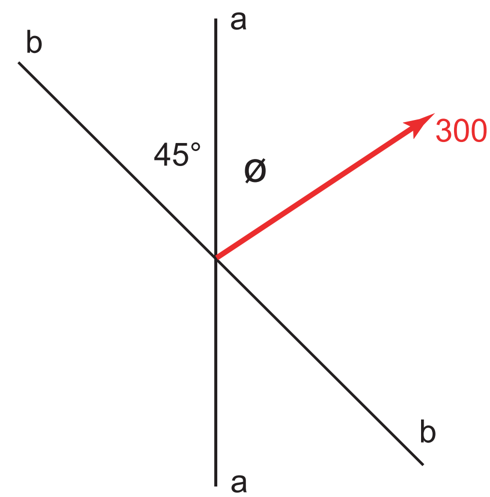
The cable stays AB and AD help support pole AC. Knowing that the tension is 120 lb in AB and 40 lb in AD, determine using trigonometry the magnitude and direction of the resultant of the forces exerted by the stays at A. Units: Lb, ft.





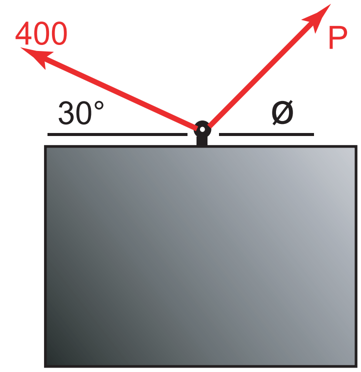
Example

The 300 N force is to be resolved into two components along a-a and b-b. (a) Determine by trigonometry the angle ø, knowing that the component along line a-a is to be 150-N. (b) What is the corresponding value of the component along b-b? Units: N.

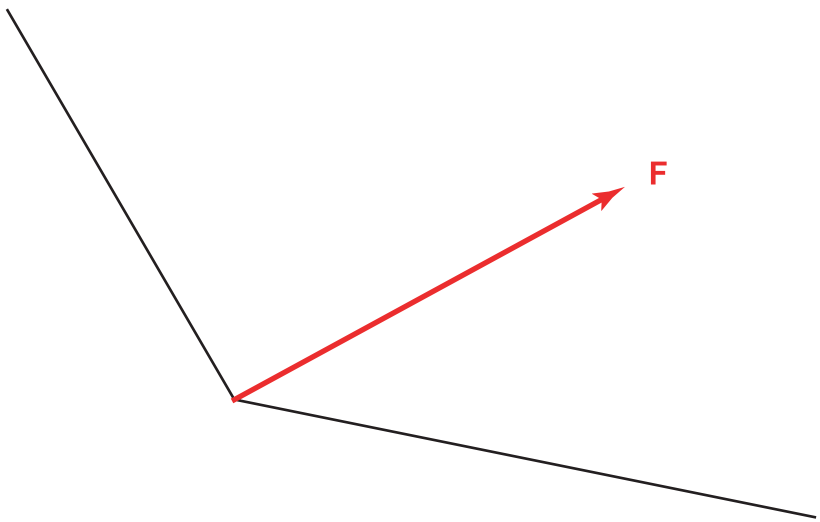


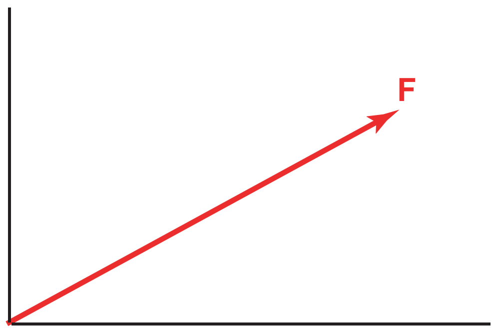
Example

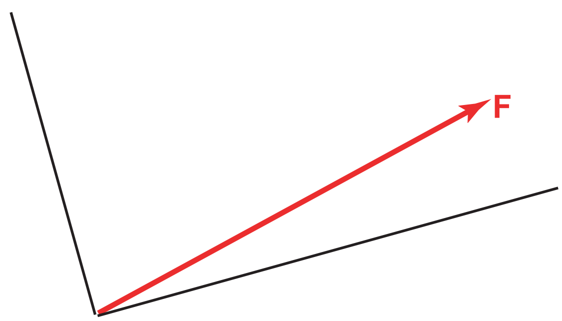
A steel plate is to be lifted straight up. Determine by trigonometry (a) the magnitude and direction of the smallest force P for which the resultant R of the two forces applied at the eye hook is vertical, (b) the corresponding magnitude of R. Units: Lb.



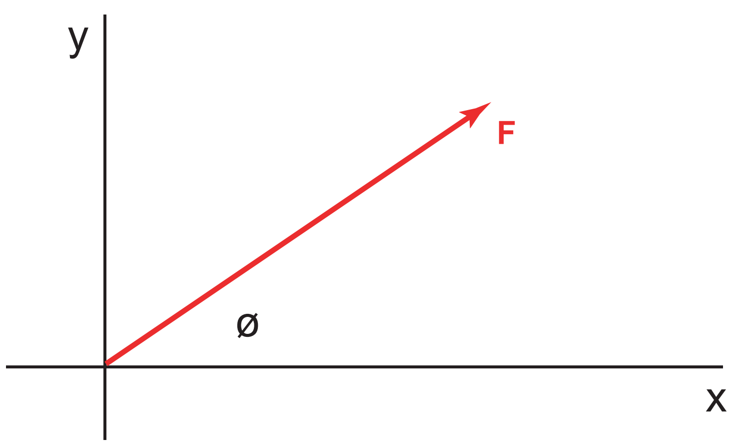
Rectangular Components of a Force. Unit Vectors



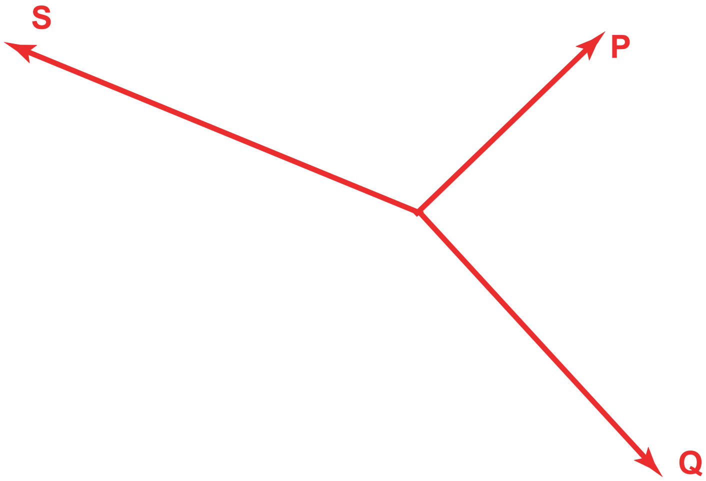


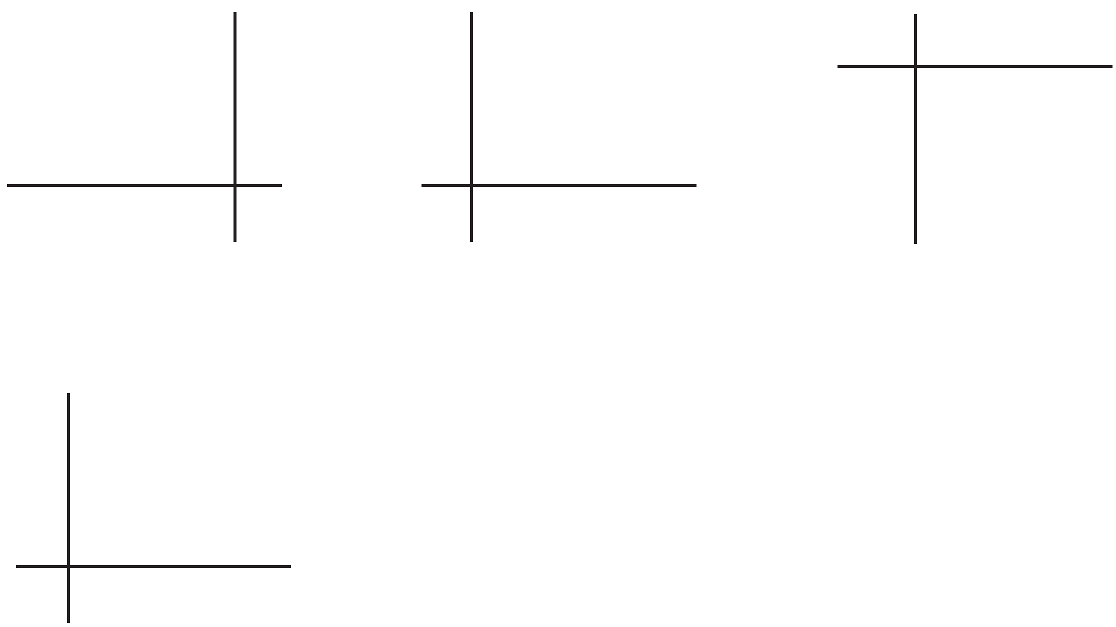


Unit Vectors



Addition of Forces by Summing X and Y Components



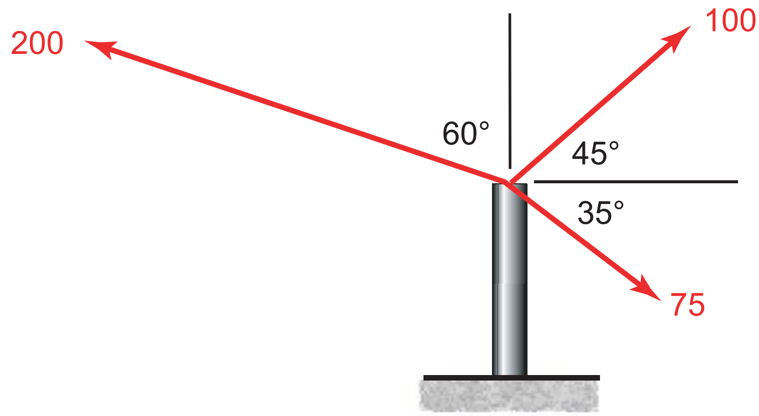


In Summary

The scalar components of Rx and Ry of the resultant R of several forces acting on a particle are obtained by adding algebraically the corresponding scalar components of the given forces.

Example

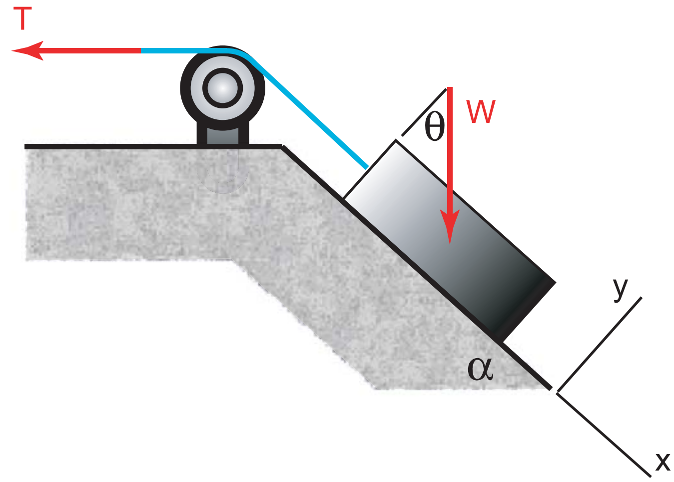
(a)Determine the x and y components of each of the forces shown on the stake. (b) Find the magnitude and direction of the resultant. Units: Lb.



Magnitude	x component	y component
100 lb		
75 lb		
200 lb		

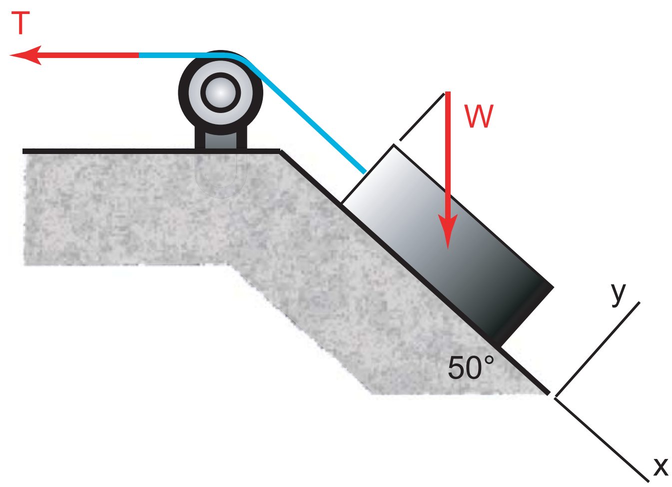
Example

Prove that $\alpha = \theta$.



Example

Find the x and y components of T and W. Units: N.



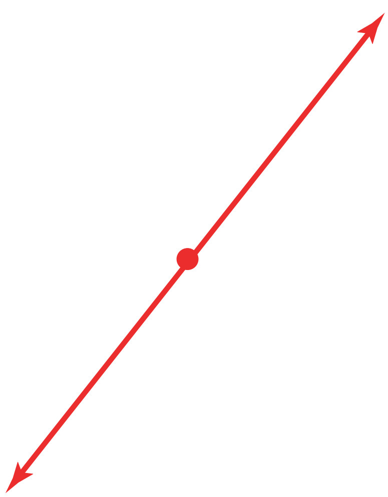
Magnitude	x component	y component

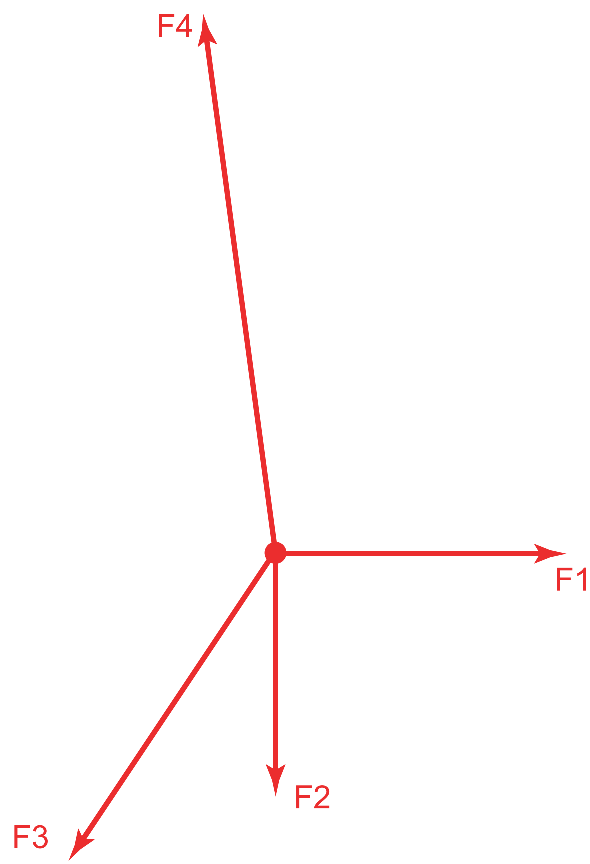
Equilibrium of a Particle

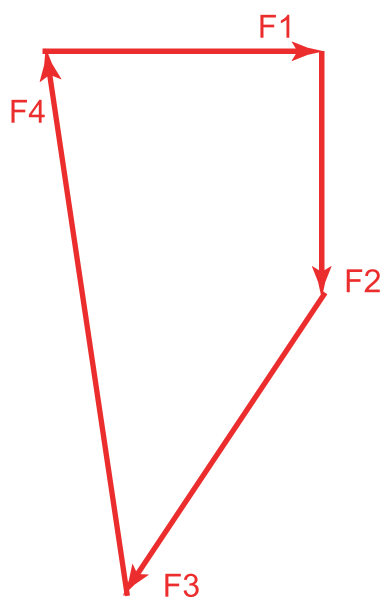
Newton's First Law

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

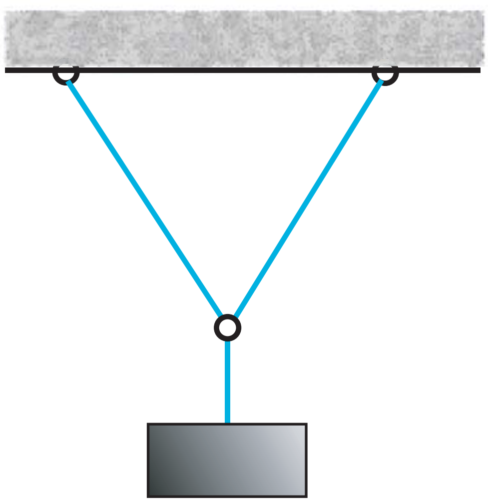
$$\sum F_x = 0$$
$$\sum F_y = 0$$





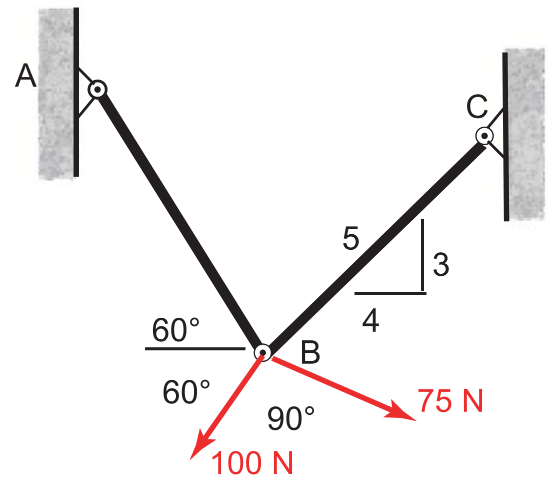


Free-Body Diagram (FBD)



Example

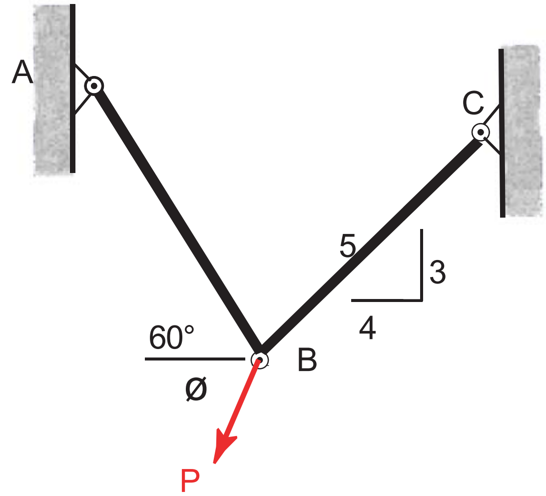
The loads are supported by two rods AB and BC as shown. Find the tension in each rod. Units: N.



Magnitude	x component	y component
100 N		
75 N		

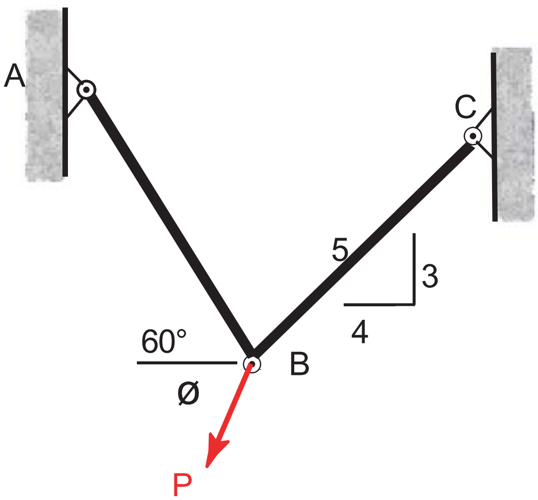
Example

It is known that the maximum allowable tension is 1200 N in rod AB and 600 N in BC. Determine (a) the maximum force P that may be applied at B, (b) the corresponding value of ø. Use the closed polygon method to solve. Units: N.



Example

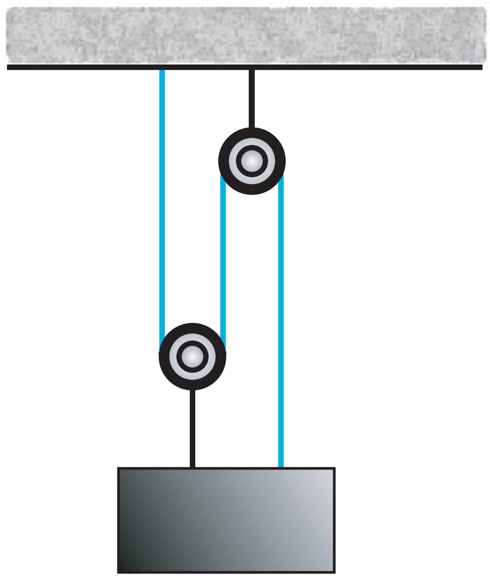
It is known that the maximum allowable tension is 1200 N in rod AB and 600 N in BC. Determine (a) the maximum force P that may be applied at B, (b) the corresponding value of ø. Use the component method to solve. Units: N.

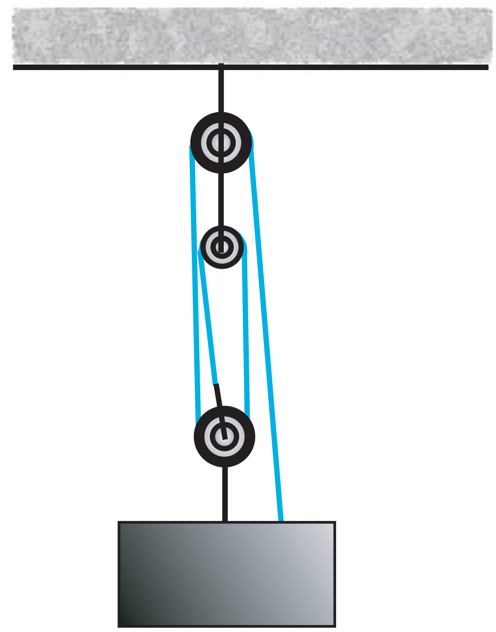


Magnitude	x component	y component

Example

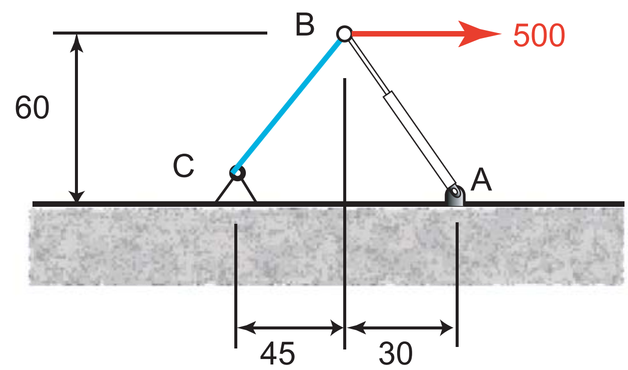
Find the tension in the rope. Assume that all cables are vertical. Note: The tension is the same through-out a continuous cable. This will be proven in another chapter.





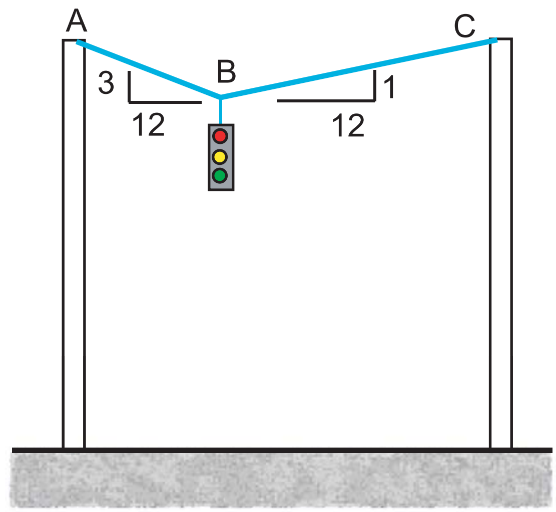
Example

Determine the forces in AB and BC.
Units: Lb, in.



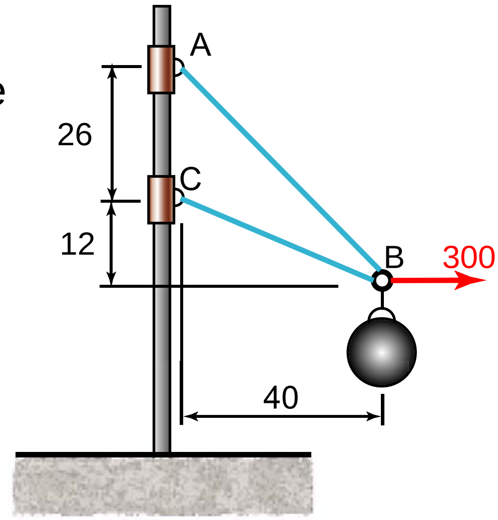
Example

Determine the forces in cables AB and BC due to the 25 lb traffic light. Units: Lb.



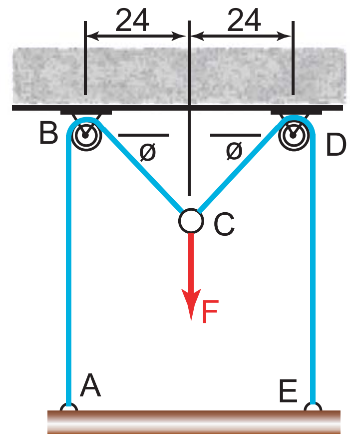
Example

Determine the forces in wires AB and BC. The sphere weighs 100 lbs. Units: Lb, in.



Example

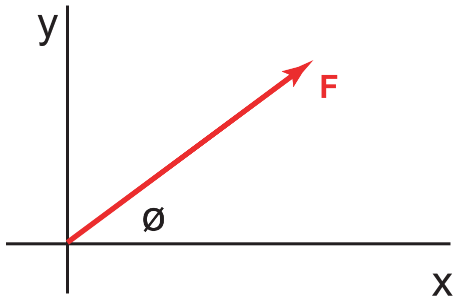
Determine the force F required to lift the 200 lb log when ø= 30°. Units: Lb, in.

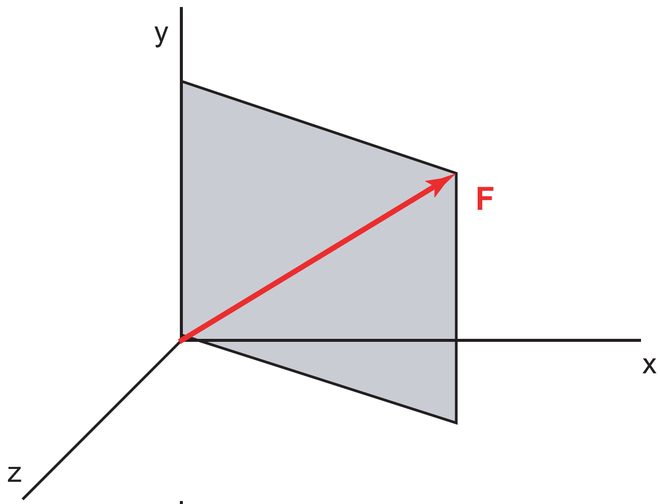


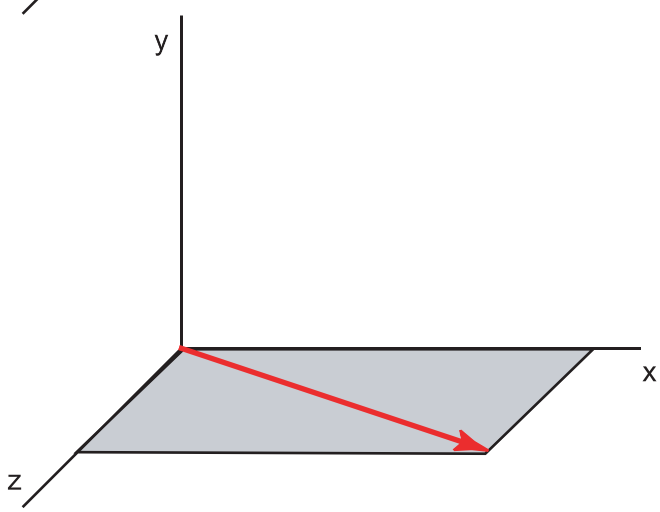
FORCES IN SPACE

Rectangular Components of a Force in Space

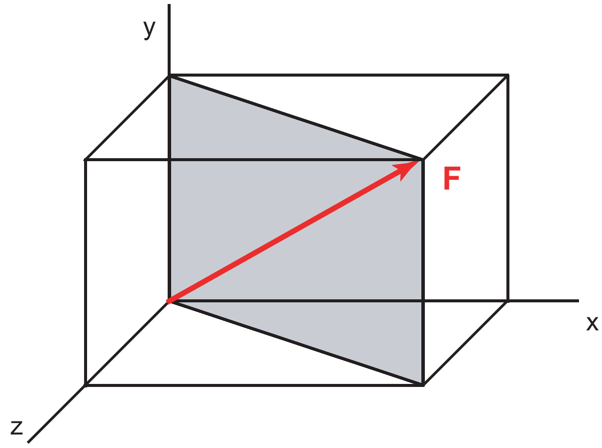
Review



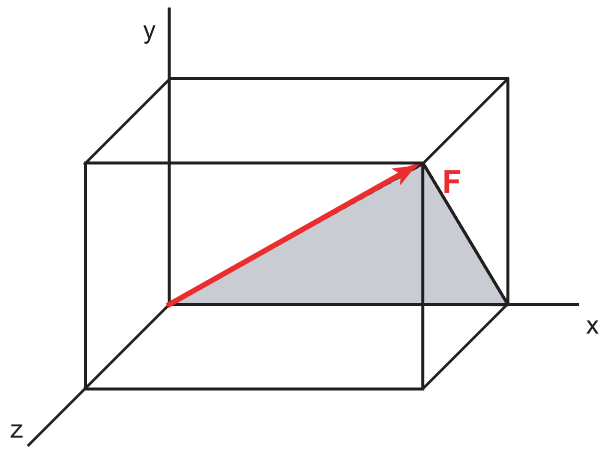




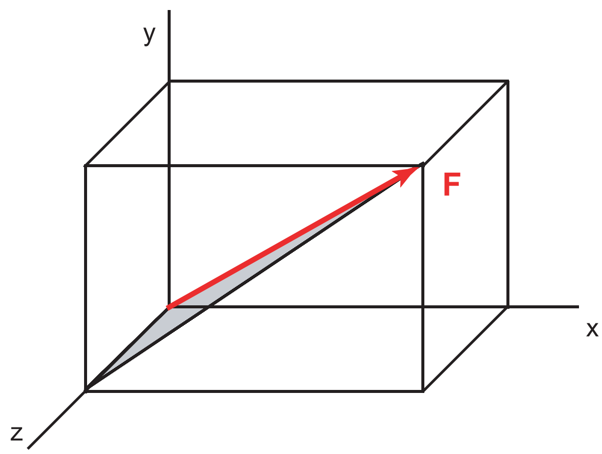
3D Vector Format



$$F_y = F\cos\theta_y$$



$$F_x = F\cos\theta_x$$



$$F_z = F\cos\theta_z$$

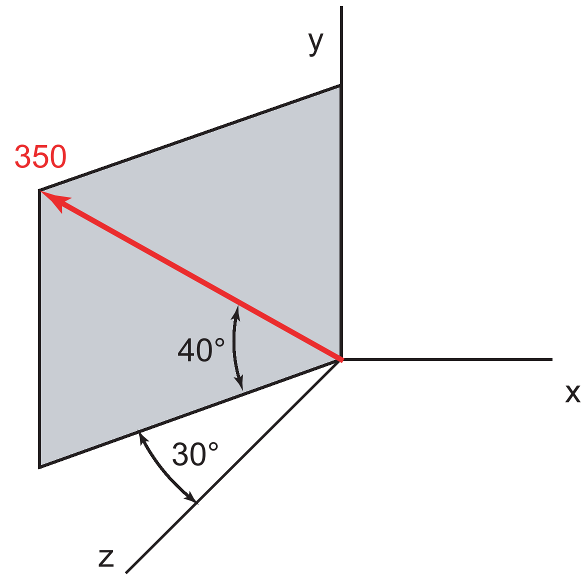
$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

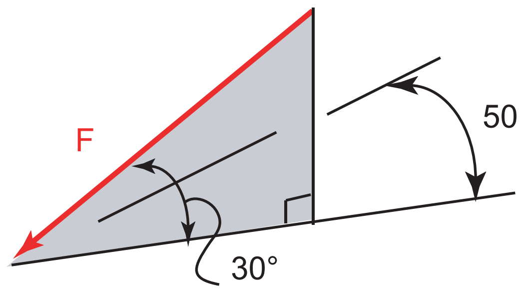
Example

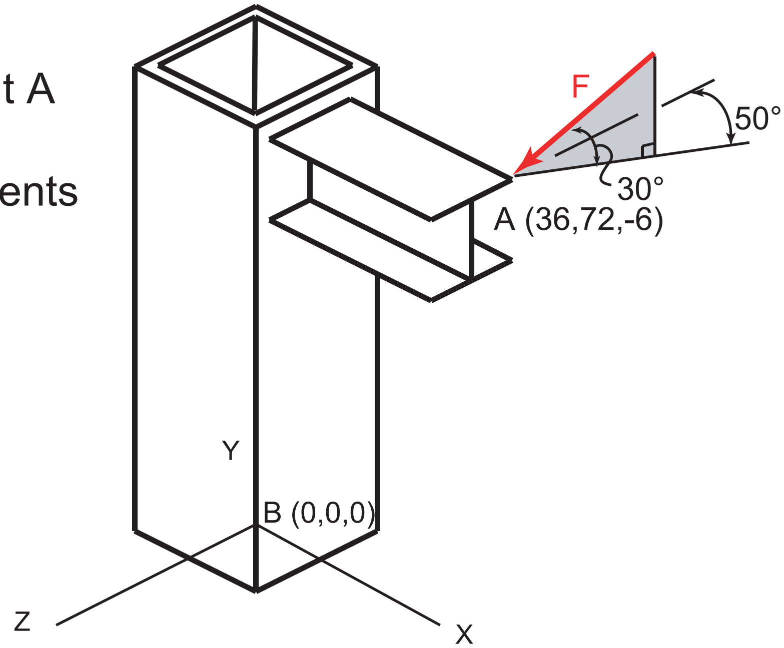
Determine (a) the x, y, and z components of the 350-N force, (b) the angles theta x, y, and z that the force forms with the coordinate axes.



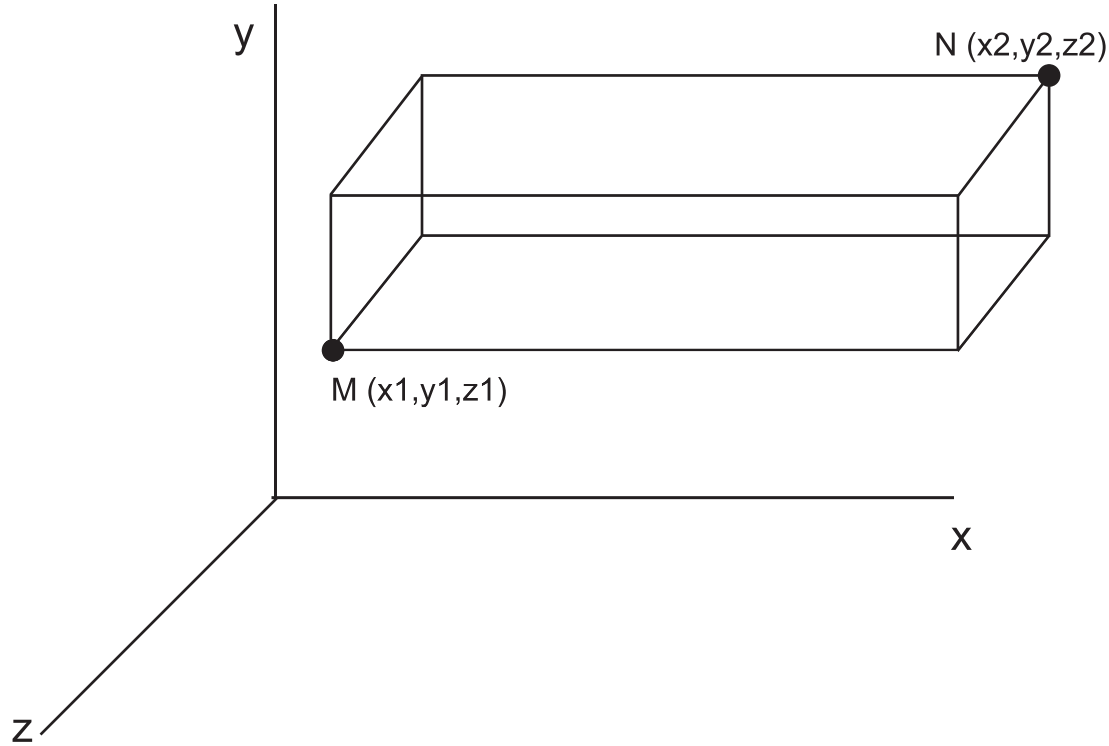
Example

A 300 lb force is applied to point A on the edge of the wide flange beam. Calculate the 3 components of this force. Units: Lb, in.





Force Defined by its Magnitude and Two Points on its Line of Action

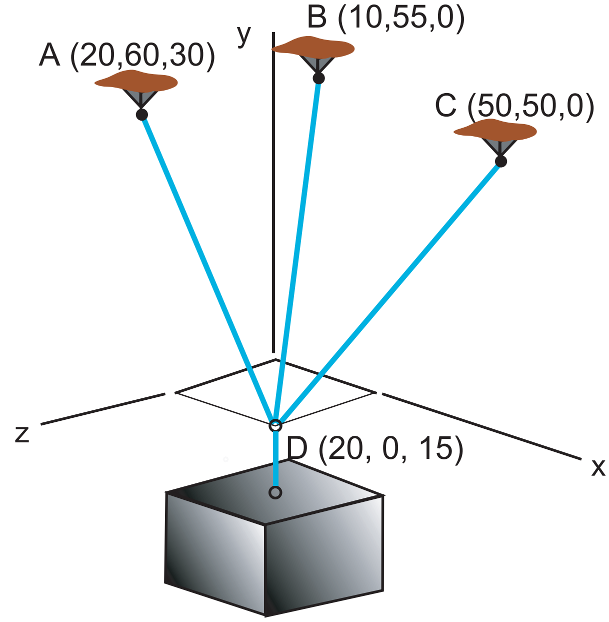


$$\vec{F} = \frac{F}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$F_x = F\frac{d_x}{d} \qquad F_y = F\frac{d_y}{d} \qquad F_z = F\frac{d_z}{d}$$

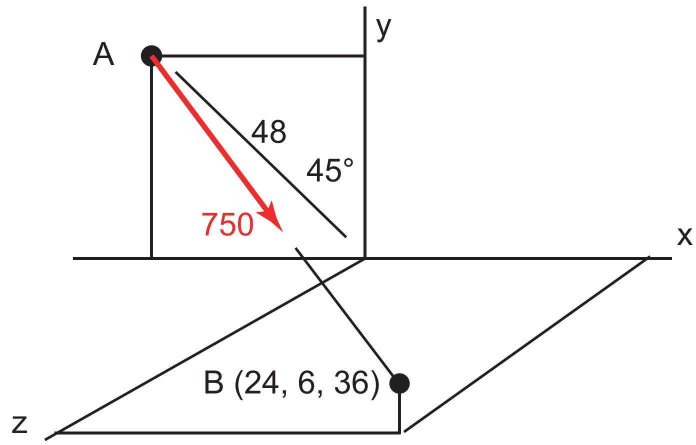
Example

The crate is supported by 3 cables tied to the ring at D. Find the components of each on the ring in terms of their magnitude. Units: in.



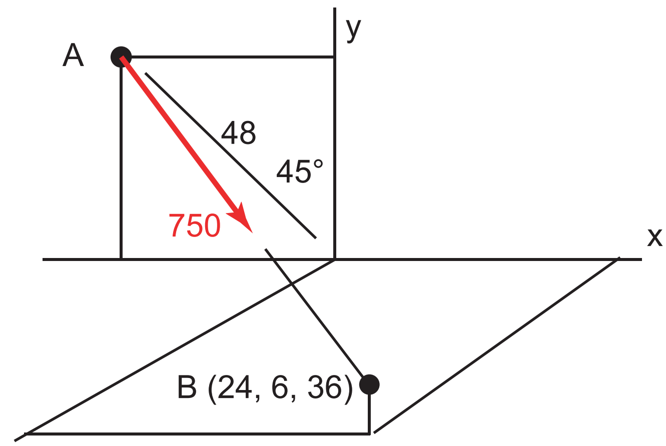
Example

Find the 750 lb force in vector format, then determine the directional angles. Point A is in the xy-plane. Units: Lb, in.



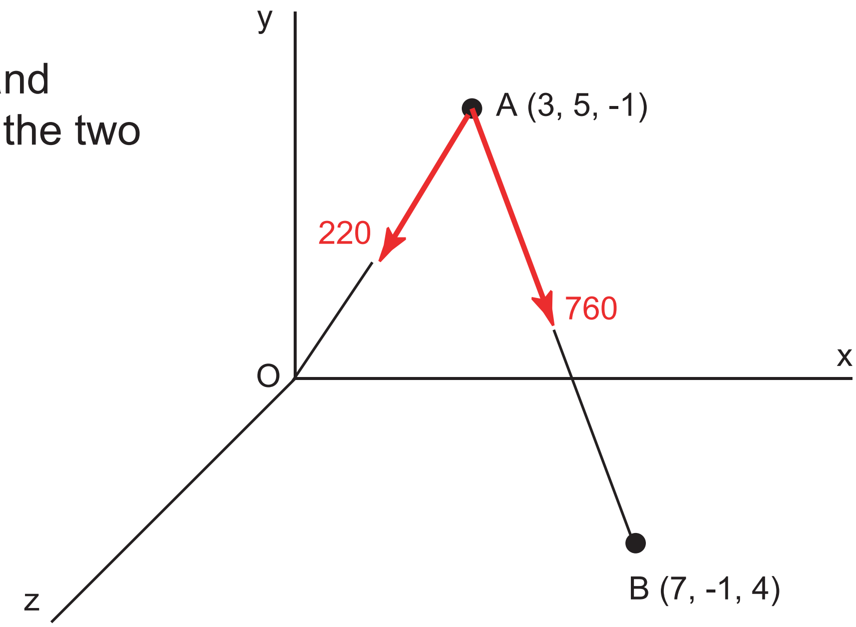
Example

Part 2: Find the 750 lb force in vector format, then determine the direction angles. Point A is in the xy-plane. Units: Lb, in.



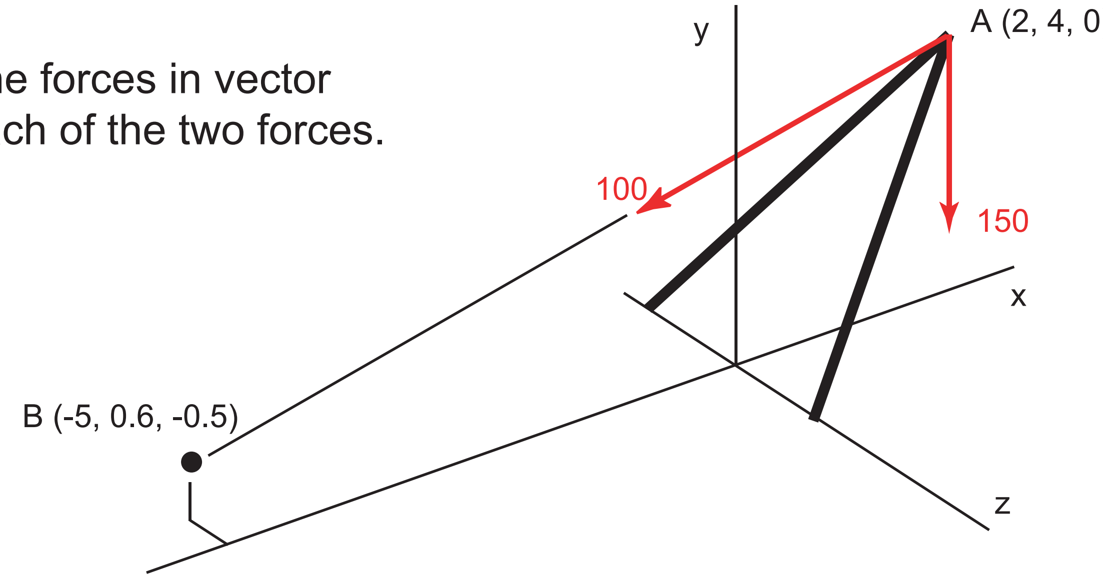
Example

Determine the magnitude and direction of the resultant of the two forces. Units: N, m.



Example

Determine the forces in vector format for each of the two forces.
Units: N, m.

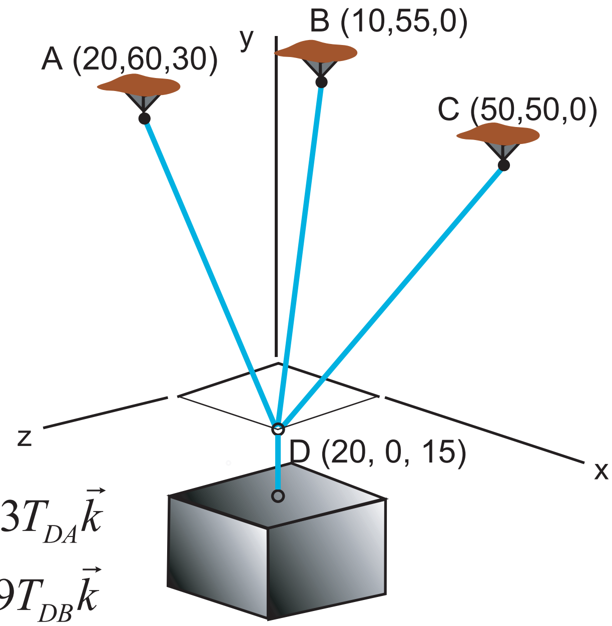


Equilibrium of a Particle in Space

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum F_z = 0$$

Example

The crate is supported by 3 cables tied to the ring at D. Find the tension in each cable and the weight of the crate knowing that the tension in cable DC is 200 lb. Units: Lb, in.



From a previous solution,

$$\vec{T}_{DA} = 0\vec{i} + 0.971T_{DA}\vec{j} + 0.243T_{DA}\vec{k}$$

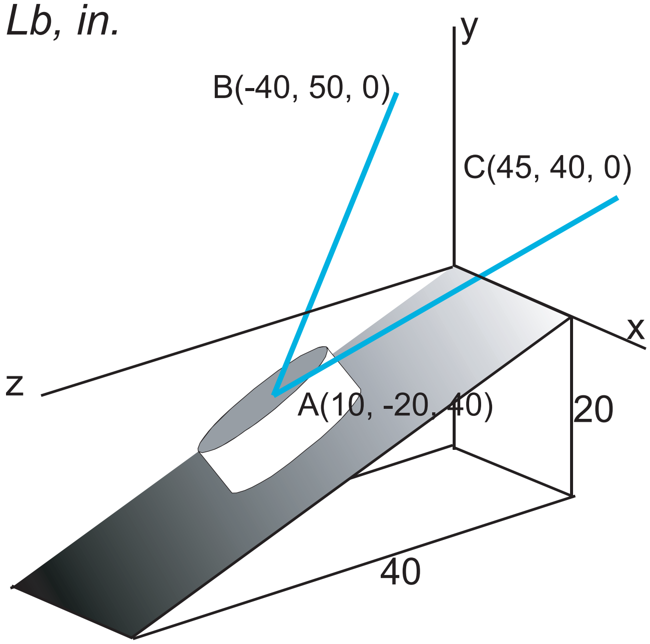
$$\vec{T}_{DB} = -0.173T_{DB}\vec{i} + 0.950T_{DB}\vec{j} - 0.259T_{DB}\vec{k}$$

$$\vec{T}_{DC} = 0.498T_{DC}\vec{i} + 0.830T_{DC}\vec{j} - 0.249T_{DC}\vec{k}$$

$$\vec{W} = 0\vec{i} - W\vec{j} + 0\vec{k}$$

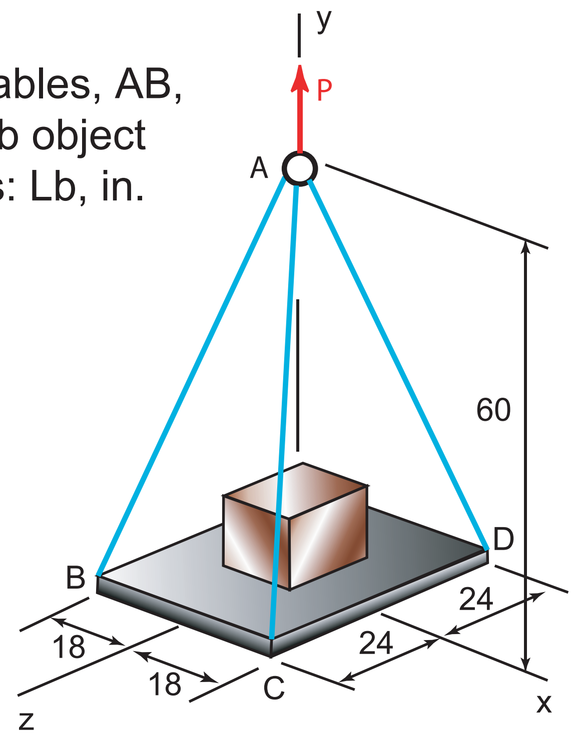
Example

Using two ropes and a roller chute, two workers are unloading a 200 lb cylinder from a truck. Assuming that no friction exists between the cylinder and the chute, determine the tension in each rope. *(Hint: Since there is no friction the force exerted by the chute on the cylinder must be perpendicular to the chute). Units: Lb, in.*



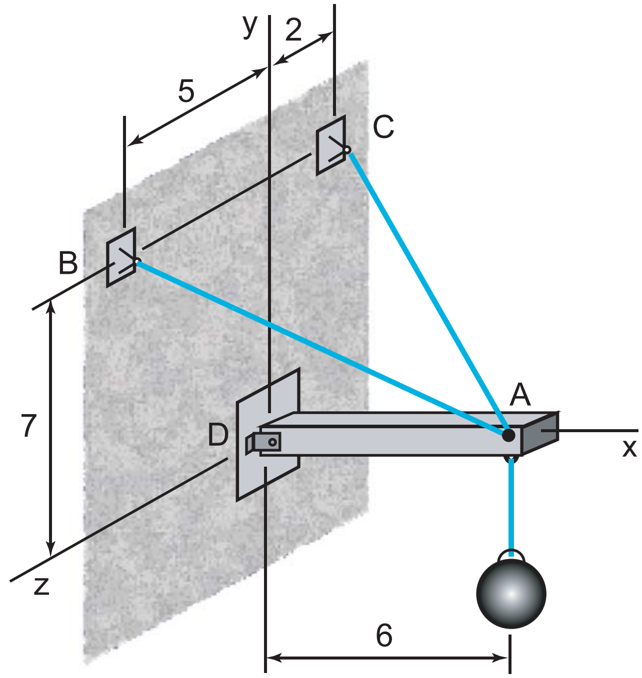
Example

Determine the force in each of the three cables, AB, AC, and AD needed to support the 2200 lb object located in the center of the platform. Units: Lb, in.



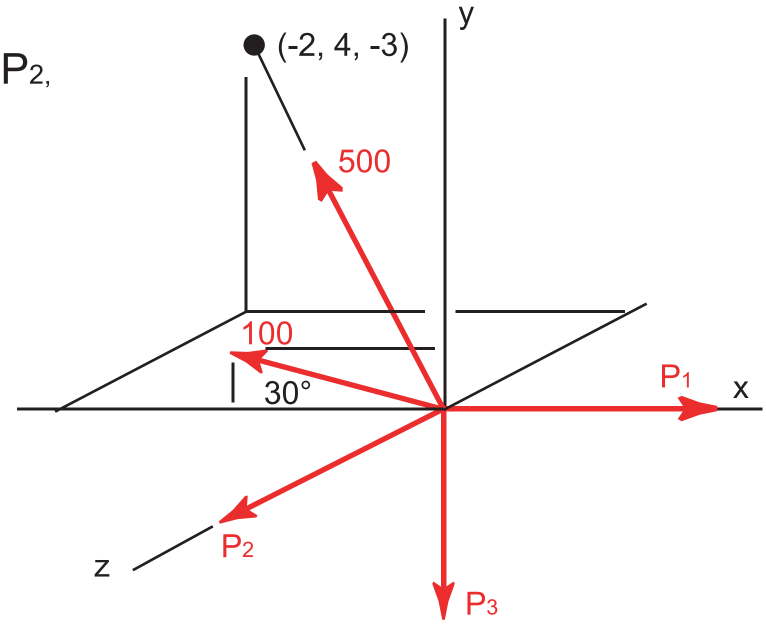
Example

If the resultant force of AB, AC, and the weight is directed along AD, determine the forces in AB and AC due to the 150 lb sphere.
Units: Lb, ft.



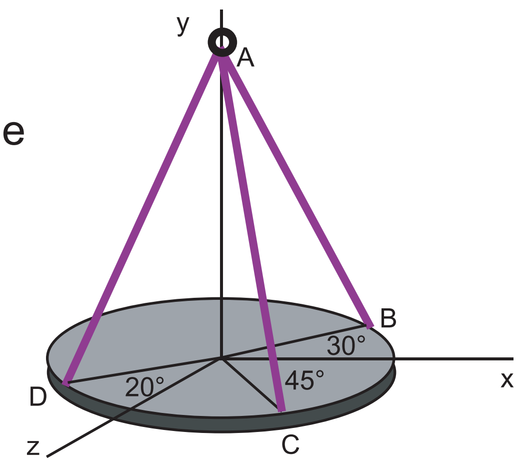
Example

Determine the magnitude of P_1, P_2, and P_3 to maintain equilibrium.
Units: Lb, ft.



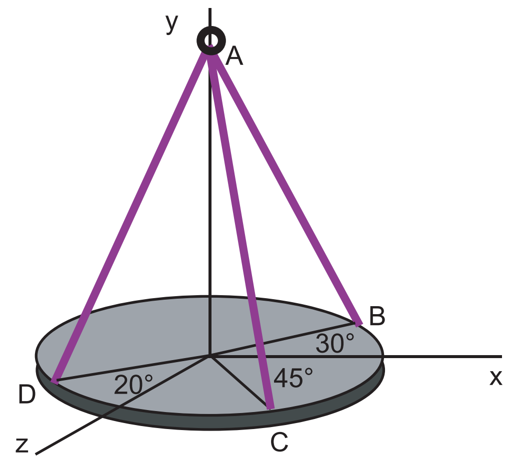
Example

The plate is supported by three wires. Each wire forms a 25° angle with the vertical. If the force in AD is 600 lb, determine the 3 components acting at point D. Also find the directional cosines. Units: Lb.



Example

The plate is supported by three wires. Each wire forms a 25° angle with the vertical. If the x component of AB is 150 lbs, determine the force in AB. Also find the directional cosines. Units: Lb.



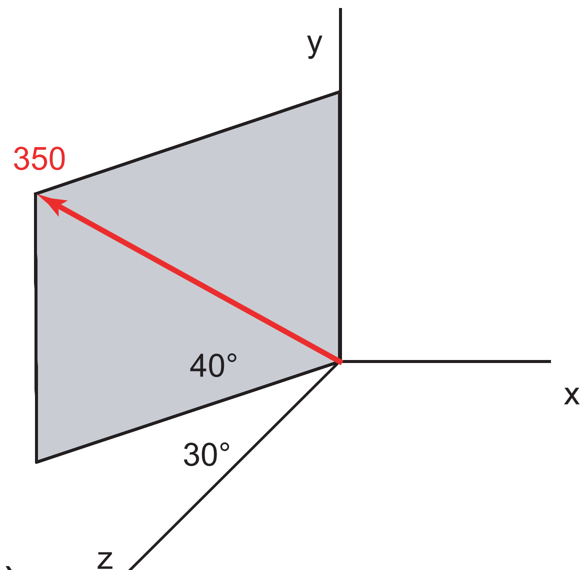
SUMMARY

-Drawing FBDs

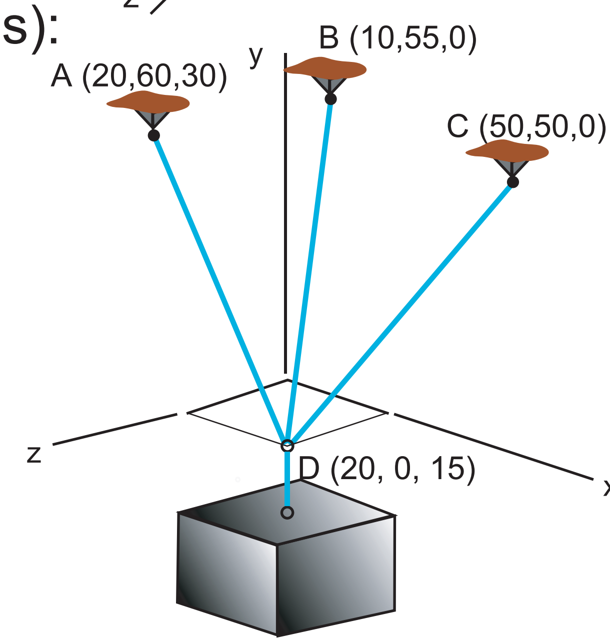
-Equilibrium

-Calculating Forces in 3D

If Given Angles:

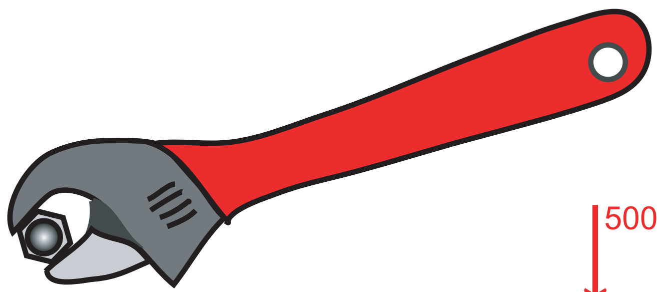


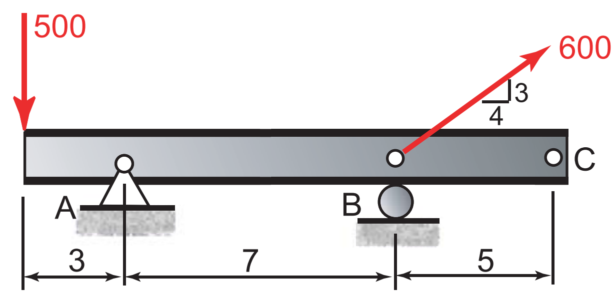
If Given Points (Distances):



Chapter 3
Rigid Bodies: Equivalent Systems of Forces

Introduction





Goals:

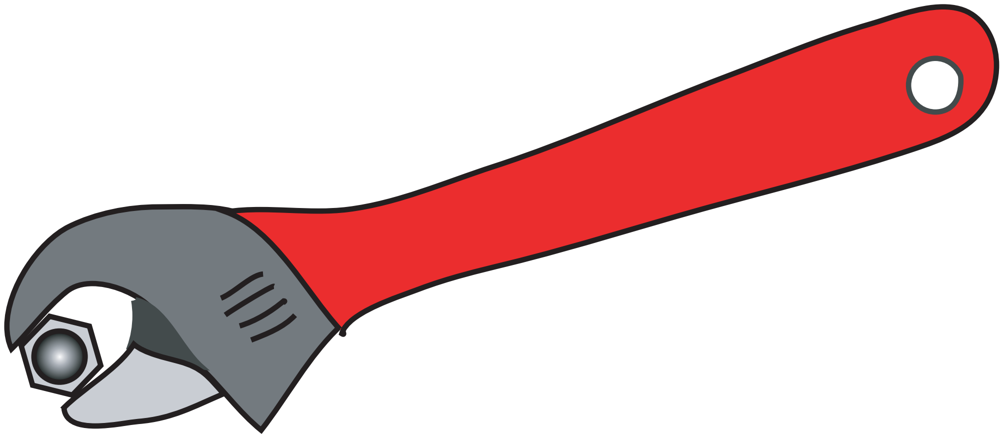
-moments and couples

-priniciple of transmissibility

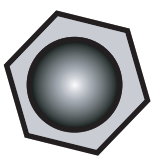
-replace a given system of forces by an equivalent system

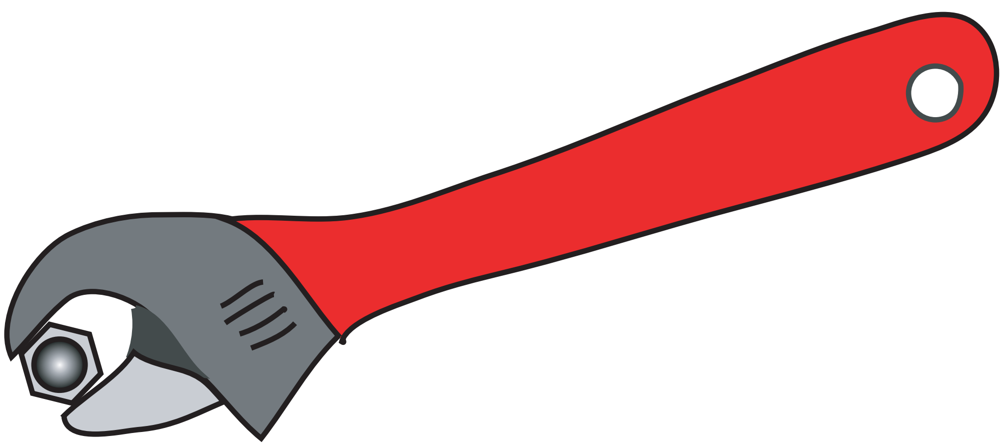
-vector products and scalar products

Moment of a Force about a Point in 2D



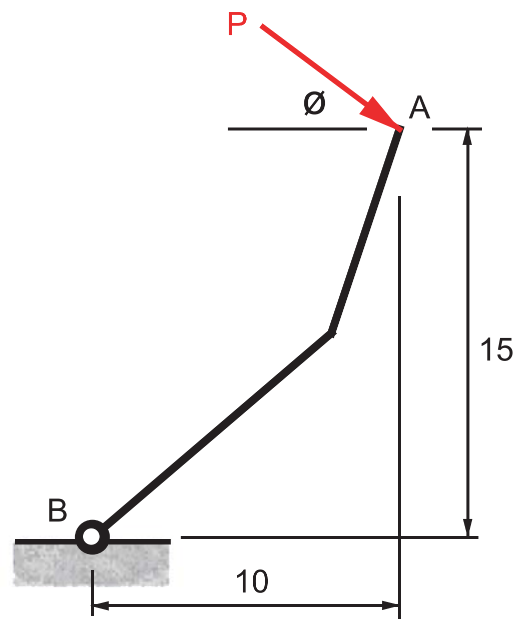
Sign Convention





Example

Determine the smallest force P that will create a 200 lb-in clockwise moment about B. Units: Lb, in.

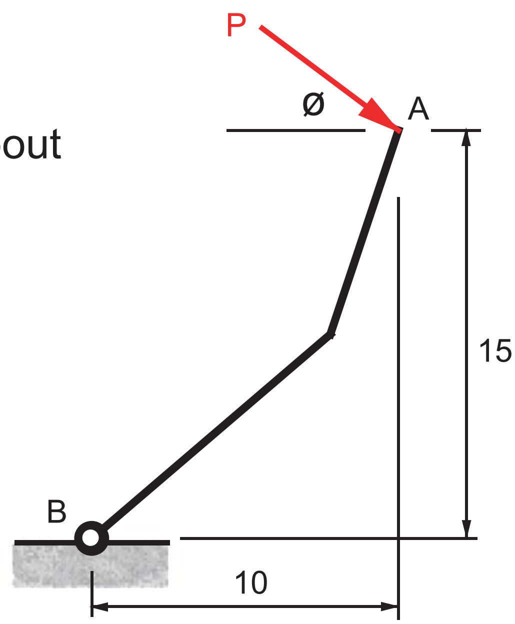


Solution #1:

Example

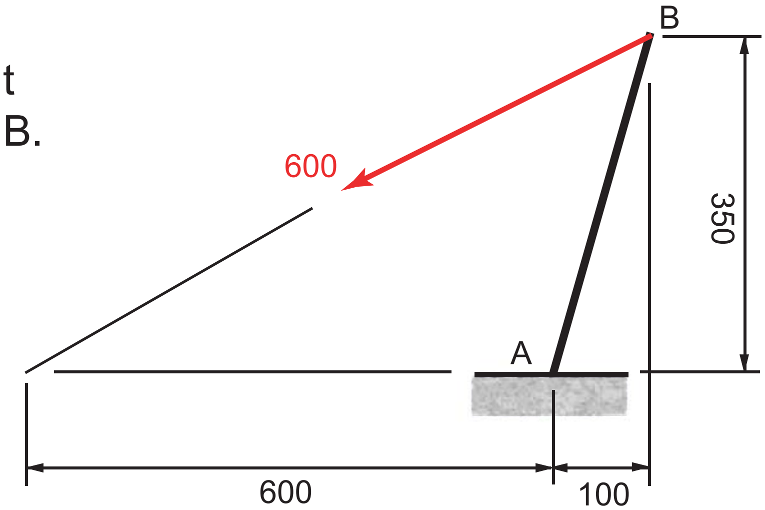
Determine the smallest force P that will create a 200 lb-in clockwise moment about B. Units: Lb, in.

Solution #2:



Example

Calculate the moment about point A due to the 600 N force at point B. Units: N, mm.



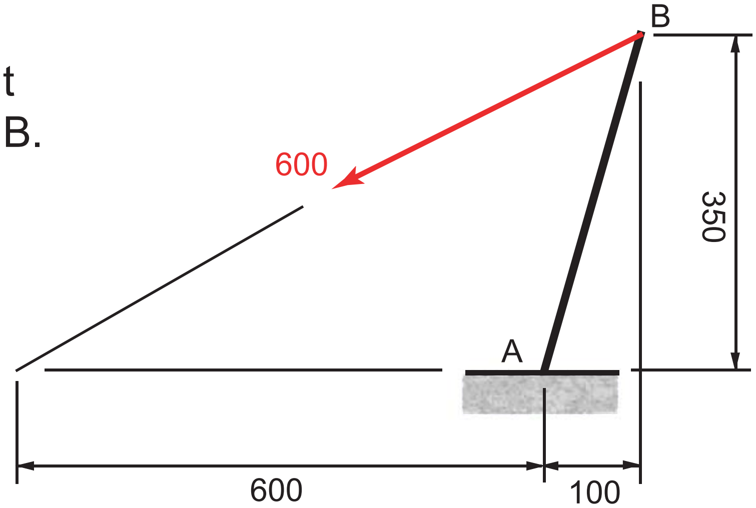
Solution #1:

Solution #2:

Example

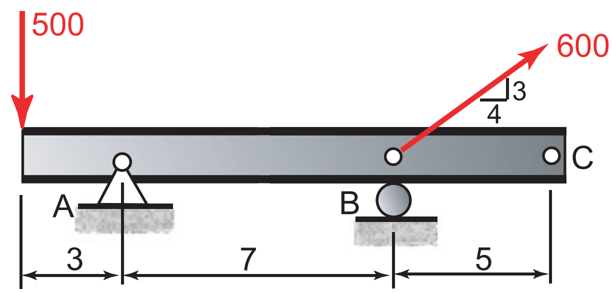
Calculate the moment about point A due to the 600 N force at point B. Units: N, mm.

Solution #3:



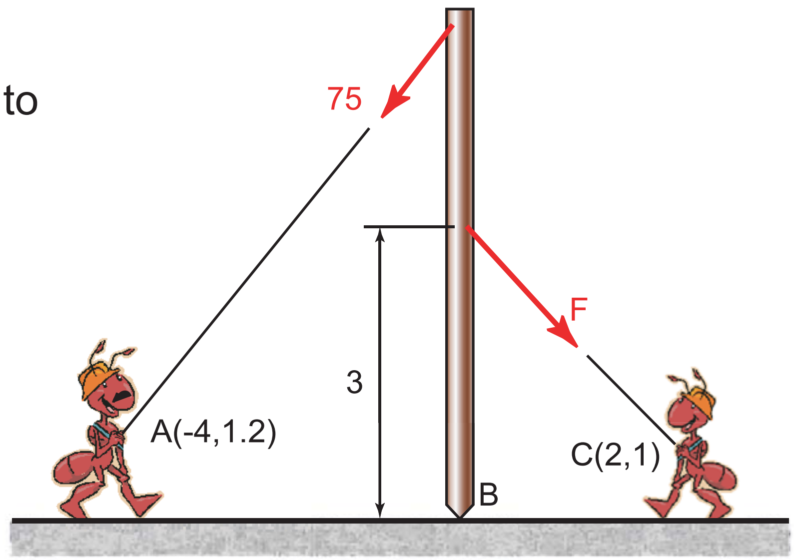
Example

Find the moment of the two forces about point C on the beam.
Units: N, m.



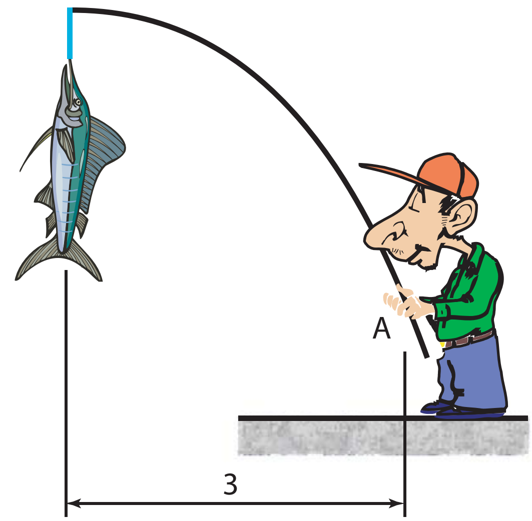
Example

Determine the force F required to prevent the 5 meter pole from tipping. Units: N, m.



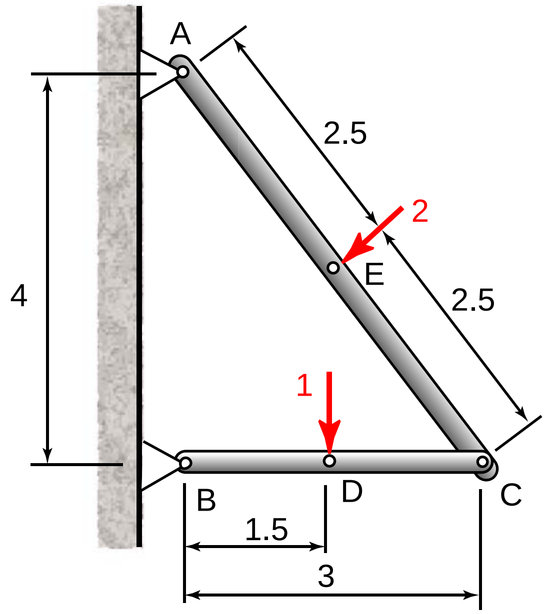
Example

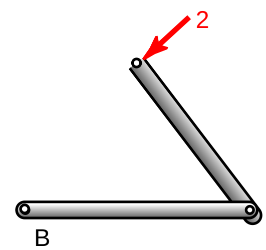
Determine the moment about point A created by the 100 kg fish.
Units: N, m.

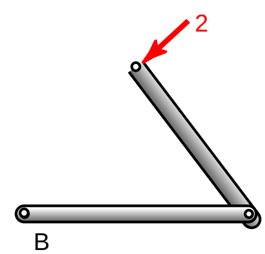


Example

Determine the moment of the load at E about a) A, b) B.
Units: Kips, ft.

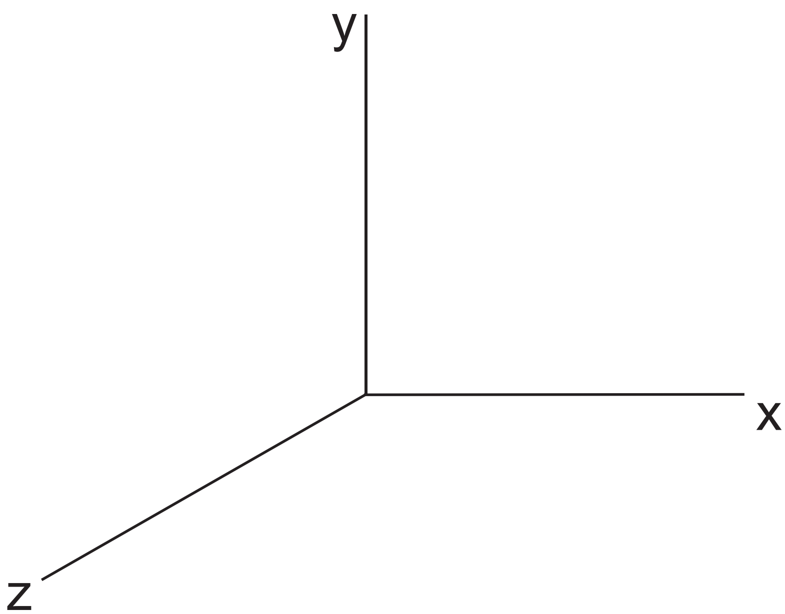






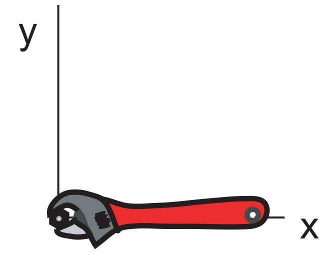
Rectangular Components of the Moment of a Force

Sign Convention for Moments in 3D

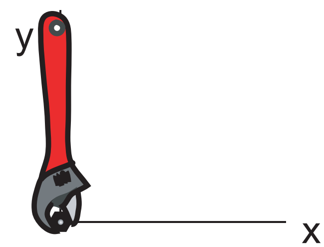


Review

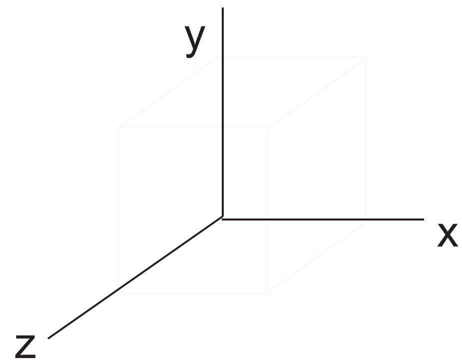
Fy force



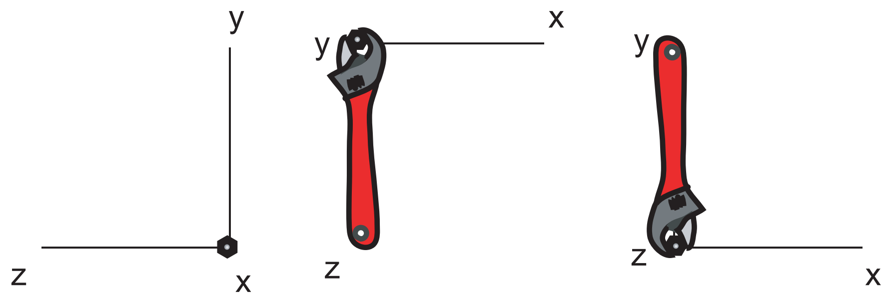
Fx force



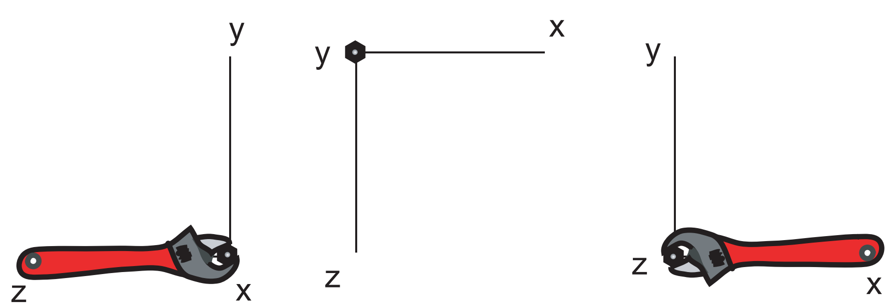
General



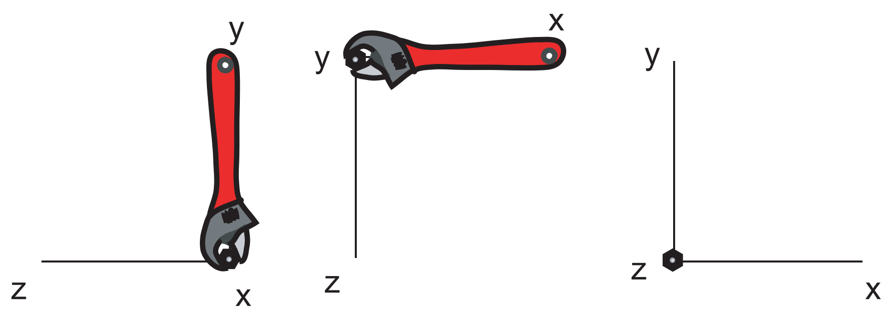
Fx force



Fy force



Fz force

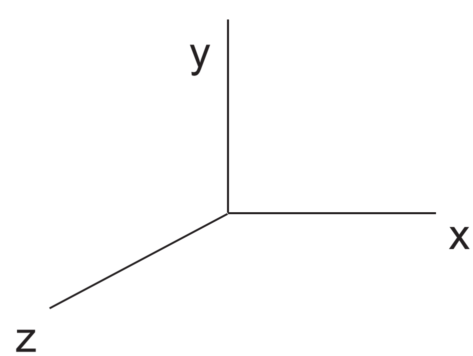


Summary

$$M_x = F_z d_y - F_y d_z$$

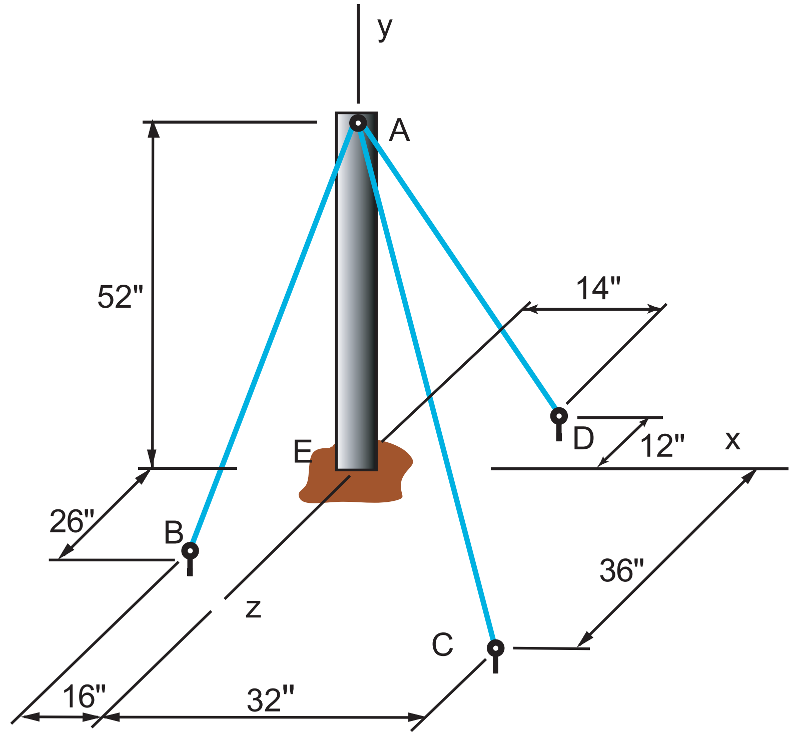
$$M_y = F_x d_z - F_z d_x$$

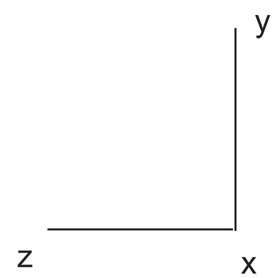
$$M_z = F_y d_x - F_x d_y$$

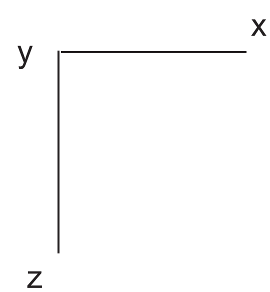


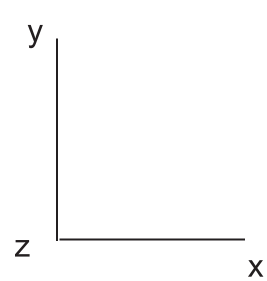
Example

The tension force in wire AB is 600 lb. Calculate the moment at E due to this force.
Units: Lb, in.



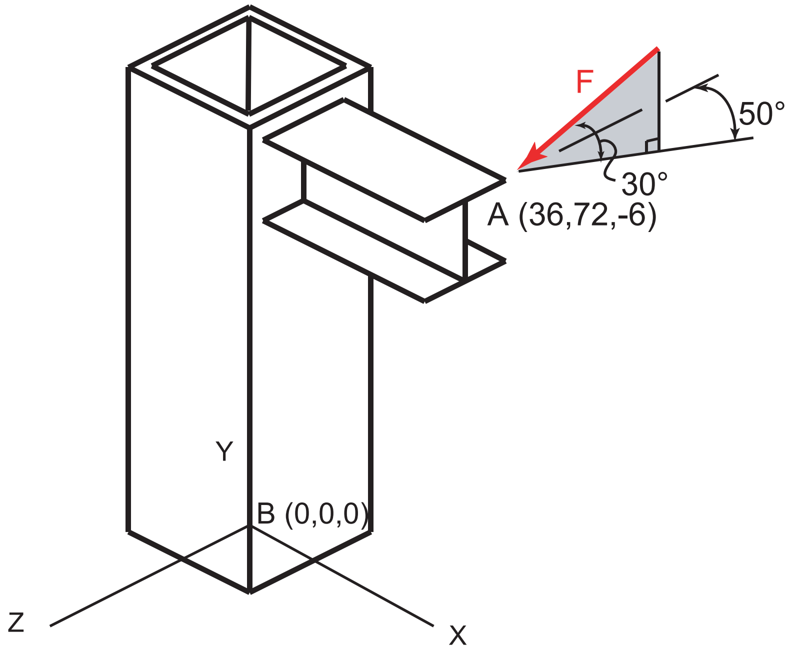


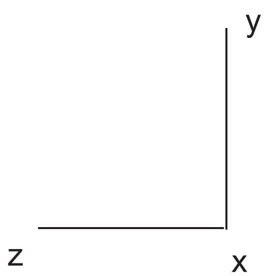


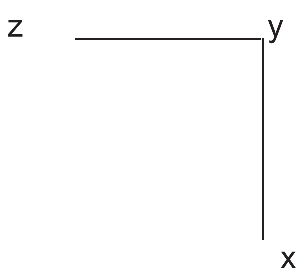


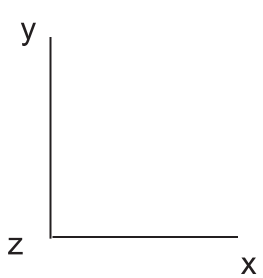
Example

A 300 lb force is applied to point A on the edge of the wide flange beam. Calculate the moment at B due to this force. Units: Lb, in.



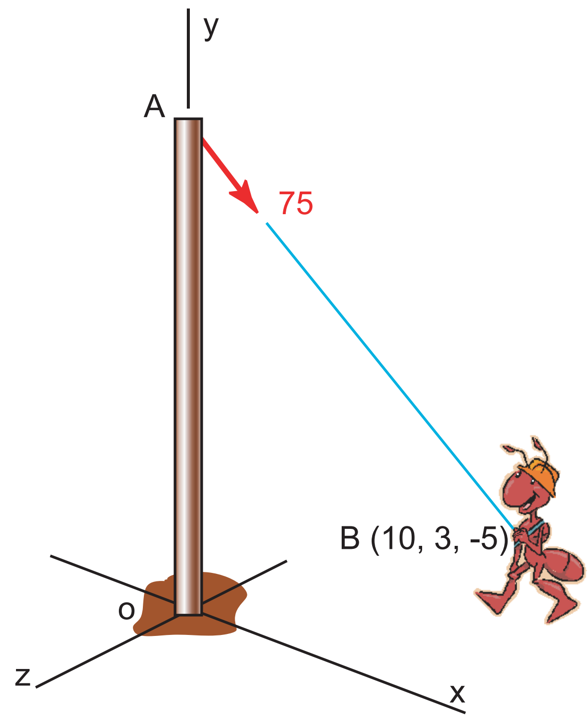


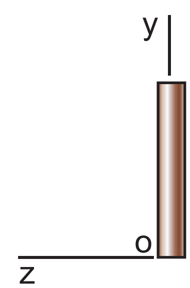


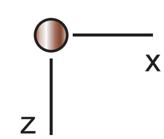


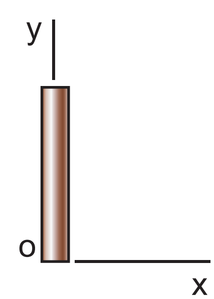
Example

Find the moment about point O due to the 75 lb force applied at the top of the 25 ft pole. Units: Lb, ft.



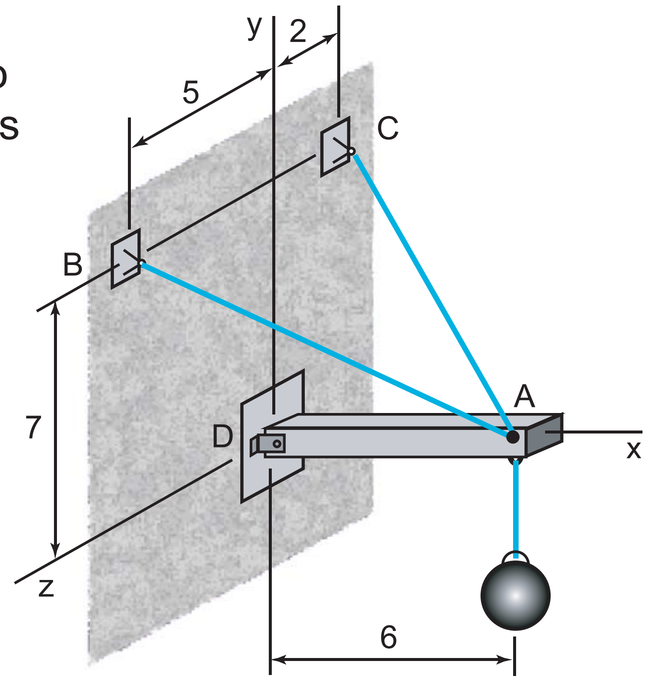


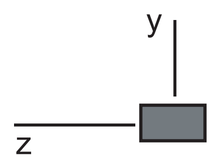


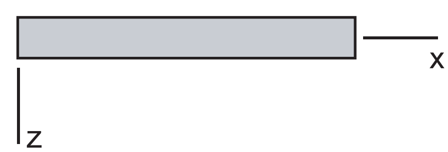


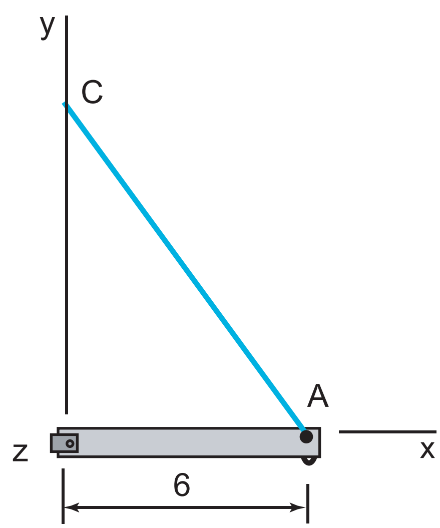
Example

Determine the moment about D due to the force in wire AB if the force in AB is 64.2 lb. Units: Lb, ft.

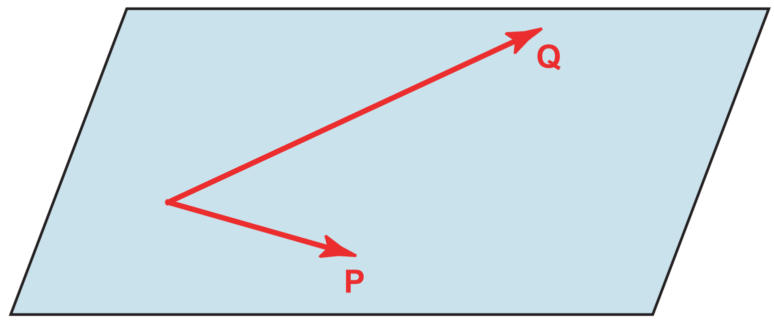




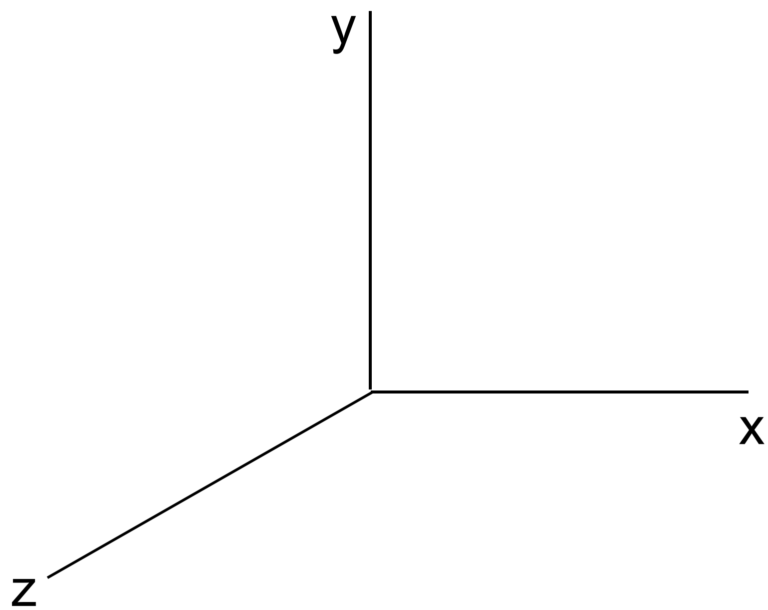




Vector Product of Two Vectors



Vector Product of Two Vectors- continued



$\vec{i} \times \vec{i} =$ $\vec{j} \times \vec{i} =$ $\vec{k} \times \vec{i} =$

$\vec{i} \times \vec{j} =$ $\vec{j} \times \vec{j} =$ $\vec{k} \times \vec{j} =$

$\vec{i} \times \vec{k} =$ $\vec{j} \times \vec{k} =$ $\vec{k} \times \vec{k} =$

$$\begin{aligned}\vec{V} = \vec{P} \times \vec{Q} &= (P_x\vec{i} + P_y\vec{j} + P_z\vec{k}) \times (Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}) \\ &= (P_x\vec{i}) \times (Q_x\vec{i}) + (P_x\vec{i}) \times (Q_y\vec{j}) + (P_x\vec{i}) \times (Q_z\vec{k}) \\ &+ (P_y\vec{j}) \times (Q_x\vec{i}) + (P_y\vec{j}) \times (Q_y\vec{j}) + (P_y\vec{j}) \times (Q_z\vec{k}) \\ &+ (P_z\vec{k}) \times (Q_x\vec{i}) + (P_z\vec{k}) \times (Q_y\vec{j}) + (P_z\vec{k}) \times (Q_z\vec{k})\end{aligned}$$

$$\vec{V} = (P_yQ_z - P_zQ_y)\vec{i} + (P_zQ_x - P_xQ_z)\vec{j} + (P_xQ_y - P_yQ_x)\vec{k}$$

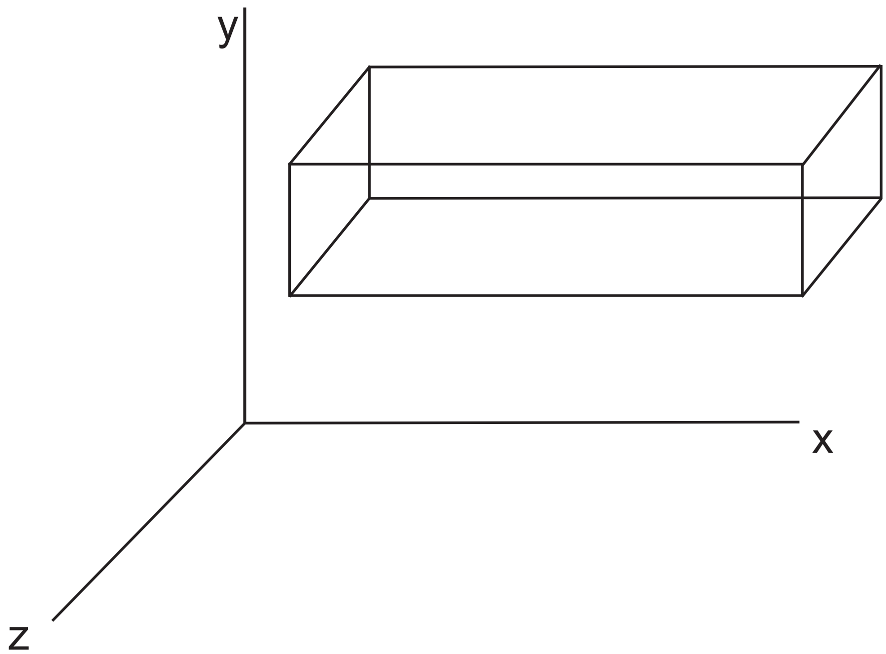
Vectors- continued

$$\vec{V} = \vec{P} \times \vec{Q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

$$\vec{V} = (P_y Q_z - P_z Q_y)\vec{i} + (P_z Q_x - P_x Q_z)\vec{j} + (P_x Q_y - P_y Q_x)\vec{k}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \qquad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Moment of a Force about a Point



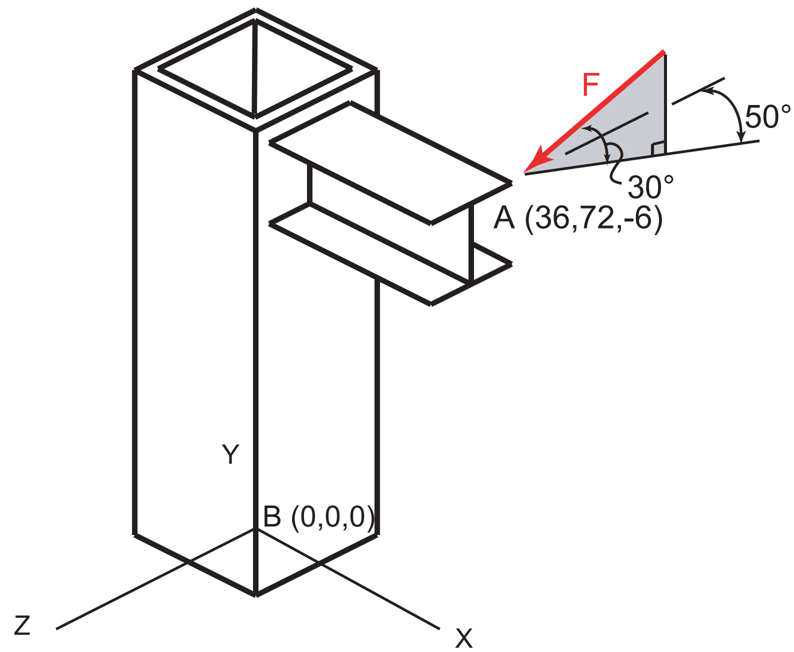
Definition of a Position Vector:

The position vector always starts at the point you want to find the moment about and ends anywhere along the axis of the force.

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

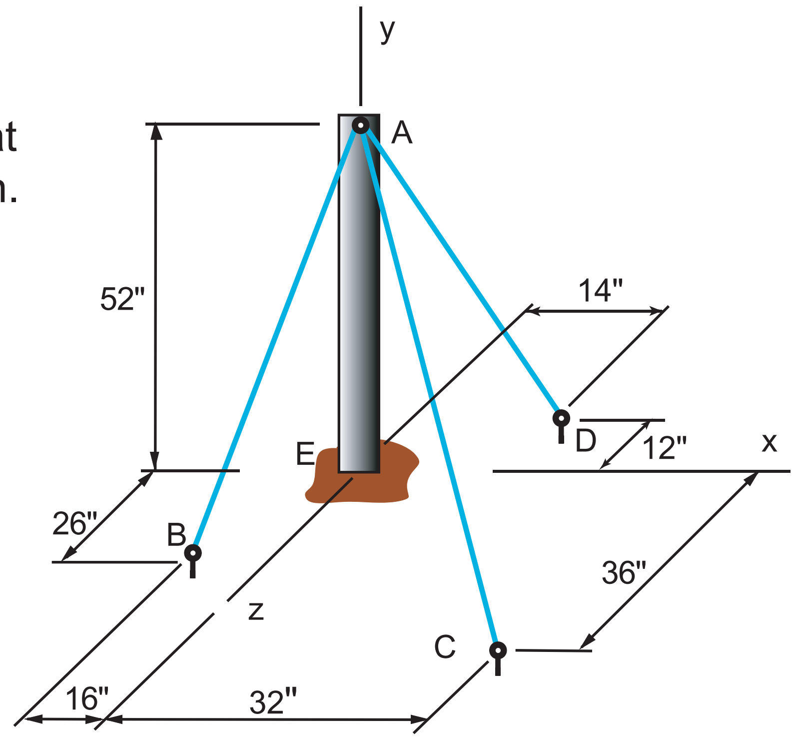
Example

A 300 lb force is applied to point A on the edge of the wide flange beam. Calculate the moment at B due to this force. Use vector products to solve. Units: Lb, in.



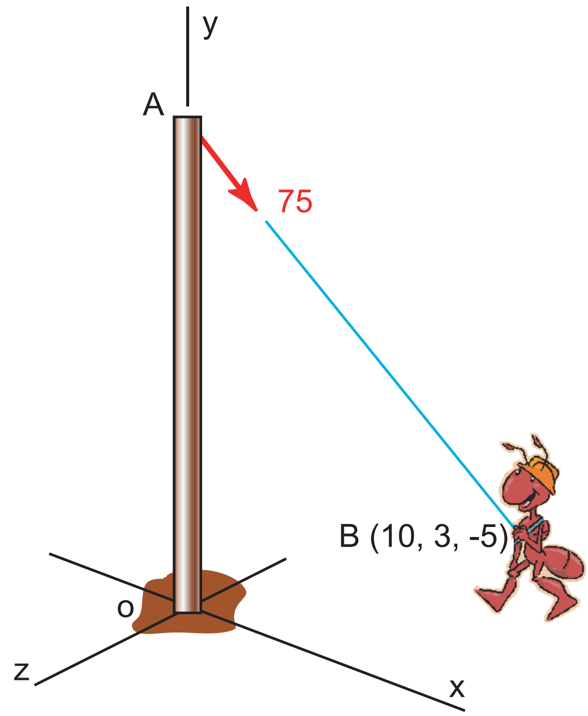
Example

The tension force in wire AB is 600 lb. Calculate the moment at E due to this force. Units: Lb, in.



Example

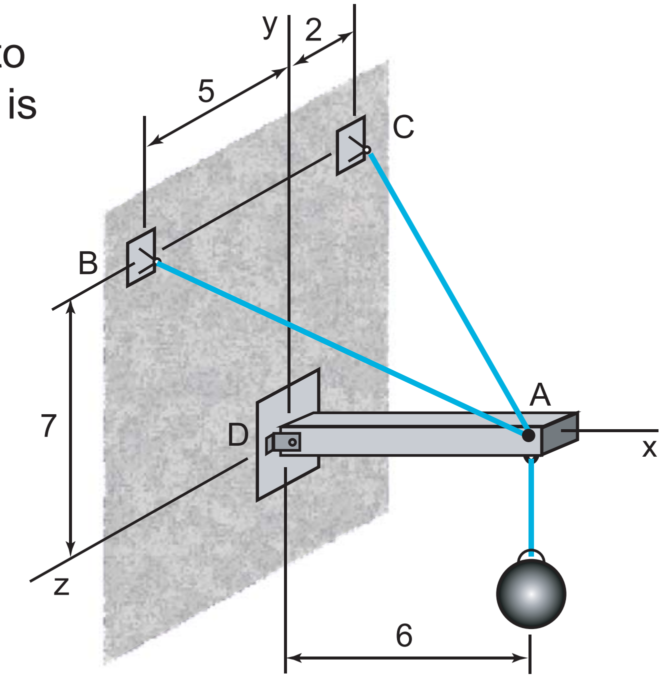
Find the moment about point O due to the 75 lb force applied at the top of the 25 ft pole. Units: Lb, ft.



Example

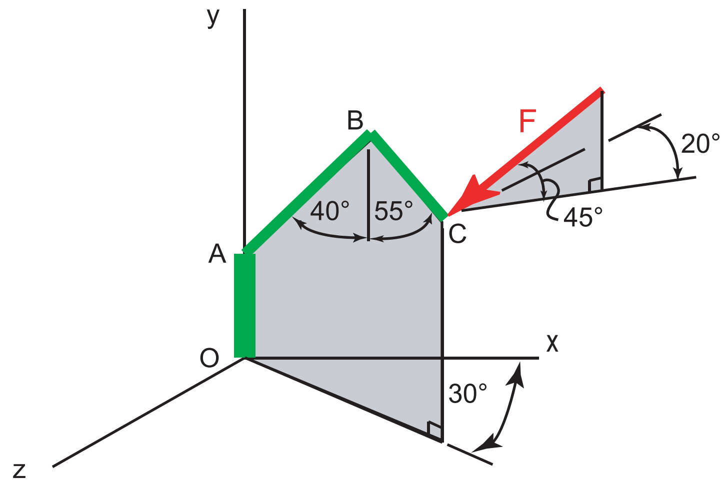
Determine the moment about D due to the force in wire AB if the force in AB is 64.2 lb.

Units: Lb, ft.

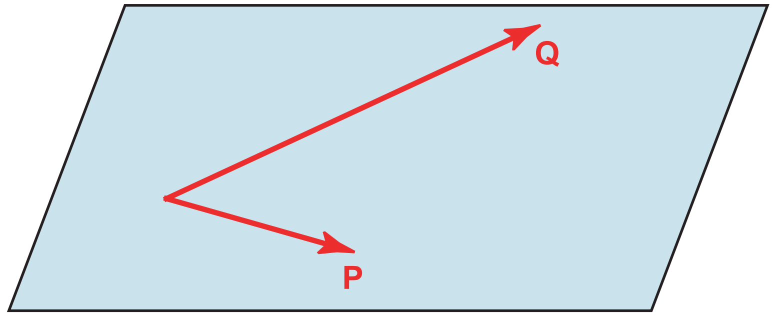


Example

Determine the moment about point O if AO= 150, AB= 400, BC= 300, and F= 5.5 N. Points OABC are in one plane. Units: N, mm.



Scalar Product of Two Vectors



$\vec{i} \cdot \vec{i} =$ $\qquad$ $\vec{j} \cdot \vec{i} =$ $\qquad$ $\vec{k} \cdot \vec{i} =$

$\vec{i} \cdot \vec{j} =$ $\qquad$ $\vec{j} \cdot \vec{j} =$ $\qquad$ $\vec{k} \cdot \vec{j} =$

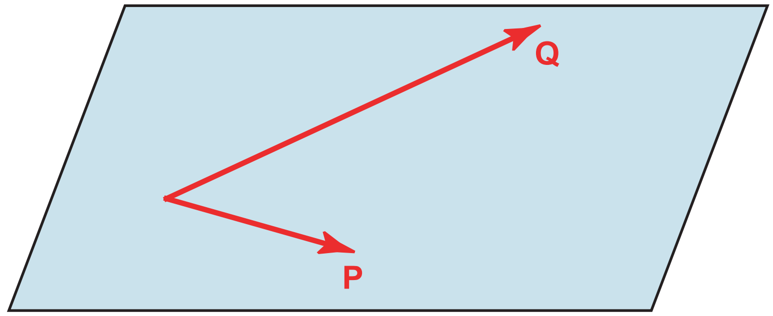
$\vec{i} \cdot \vec{k} =$ $\qquad$ $\vec{j} \cdot \vec{k} =$ $\qquad$ $\vec{k} \cdot \vec{k} =$

$$\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z$$

Angle Formed by Two Given Vectors

$$\vec{P} = P_x\vec{i} + P_y\vec{j} + P_z\vec{k}$$

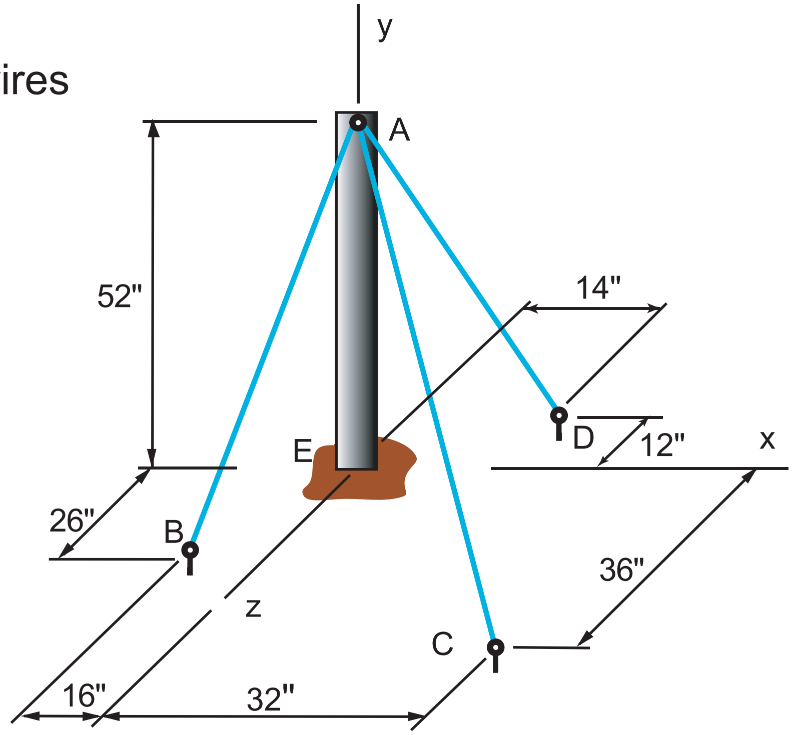
$$\vec{Q} = Q_x\vec{i} + Q_y\vec{j} + Q_z\vec{k}$$



$$\cos\theta = \frac{P_xQ_x + P_yQ_y + P_zQ_z}{PQ}$$

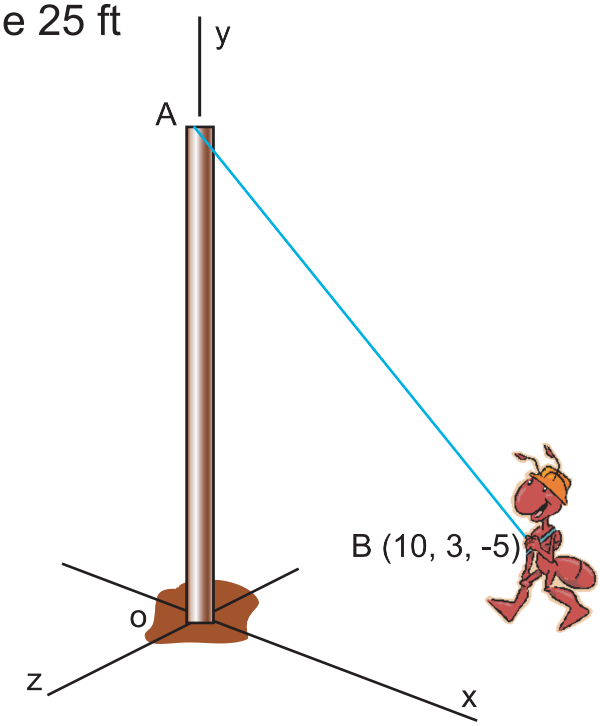
Example

Determine the angle between wires AB and AC. Units: In.



Example

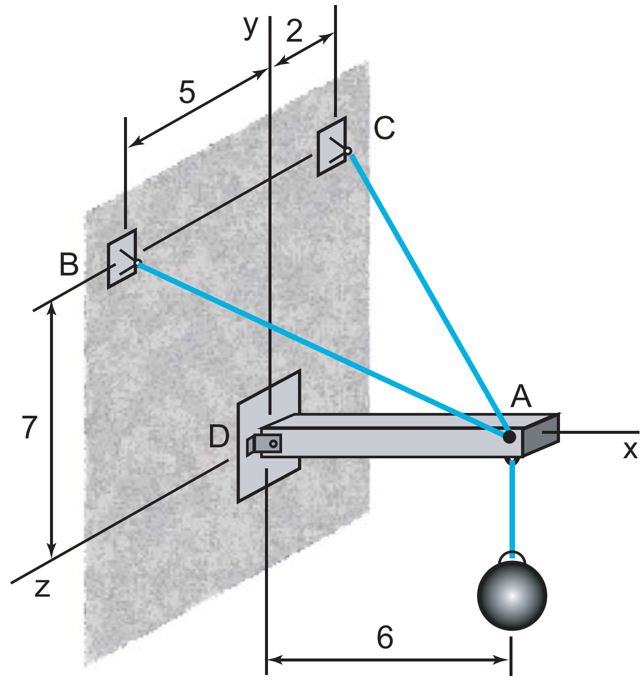
Determine the angle between AB and the 25 ft pole. Units: ft.



Example

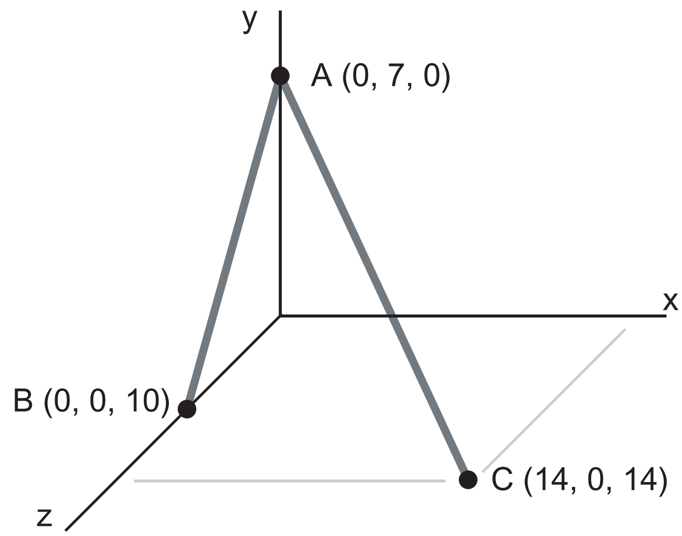
Determine the angle between AB and AC.

Units: Lb, ft.



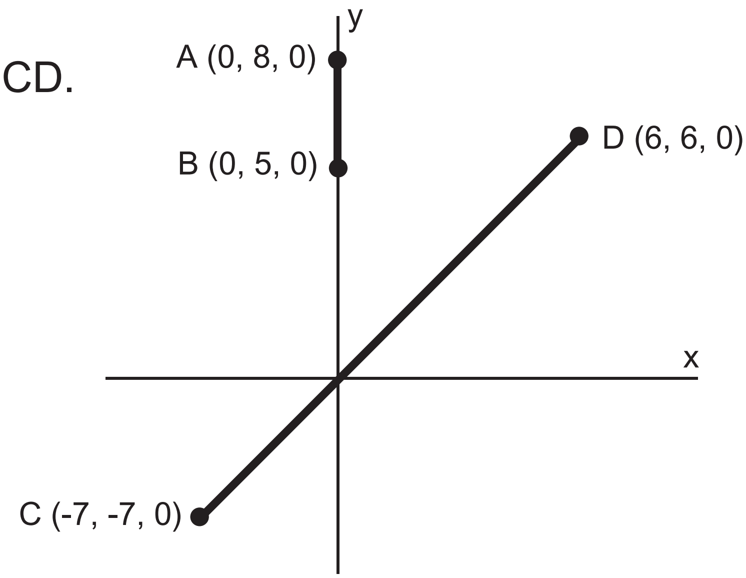
Example

Find the angle between AB and AC.
Units: Ft.



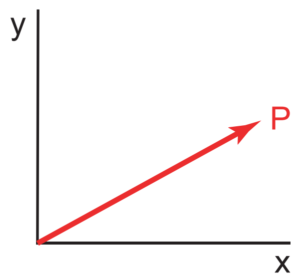
Example

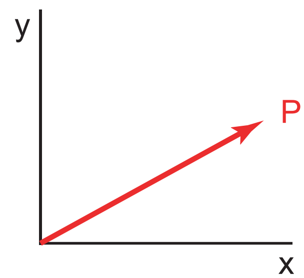
Find the angle between AB and CD.
Units: Ft.

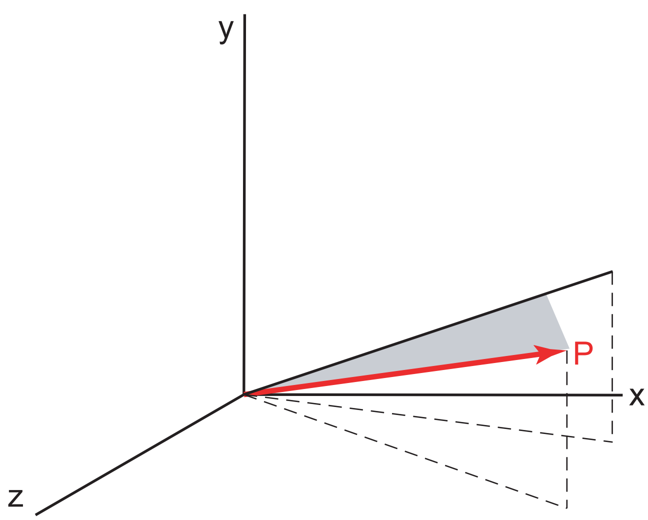


Mixed Triple Product of Three Vectors
Projection of a Vector on a Given Axis

Review



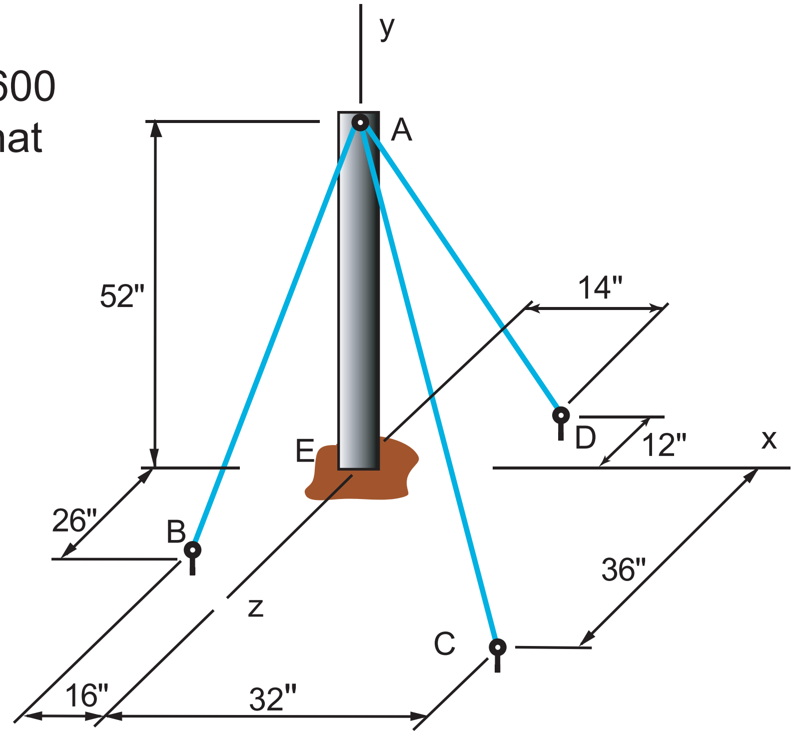




$$P_{OL} = \vec{P} \bullet \vec{\lambda}_{OL}$$

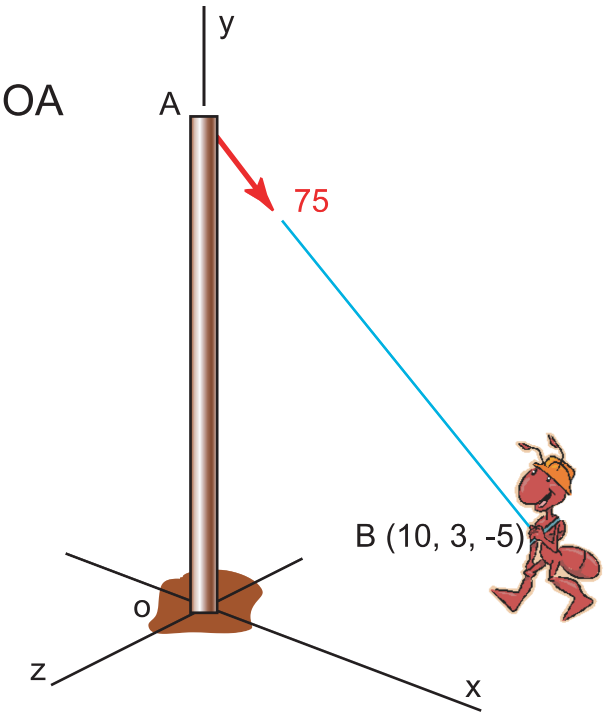
Example

The tension force in wire AB is 600 lb. Calculate the projection of that force on AC. Units: Lb, in.



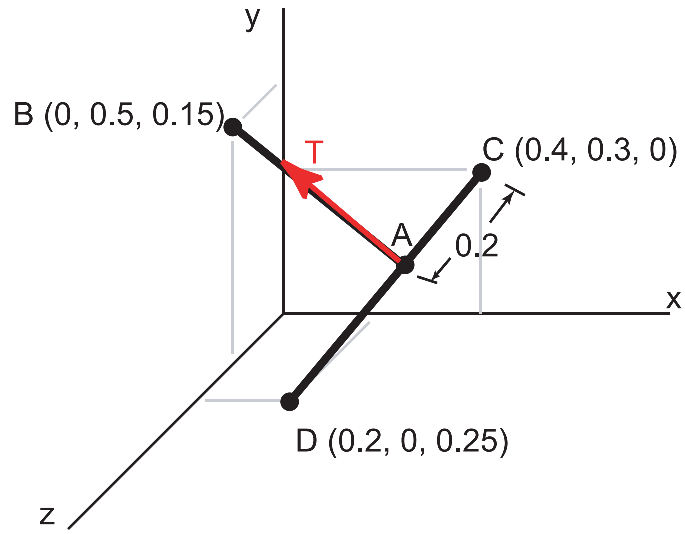
Example

Determine the component of F onto the OA axis of the 25 m pole. Units: N, m.



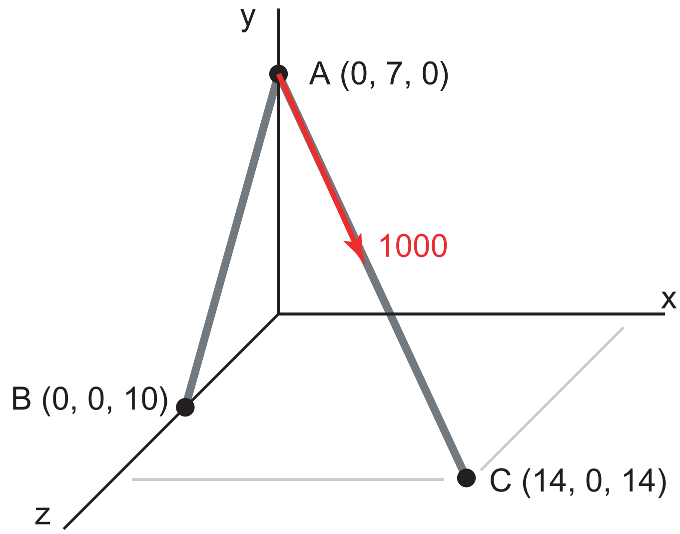
Example

Find the component (projection) of the 50 N force along AB onto CD.
Units: N, m.

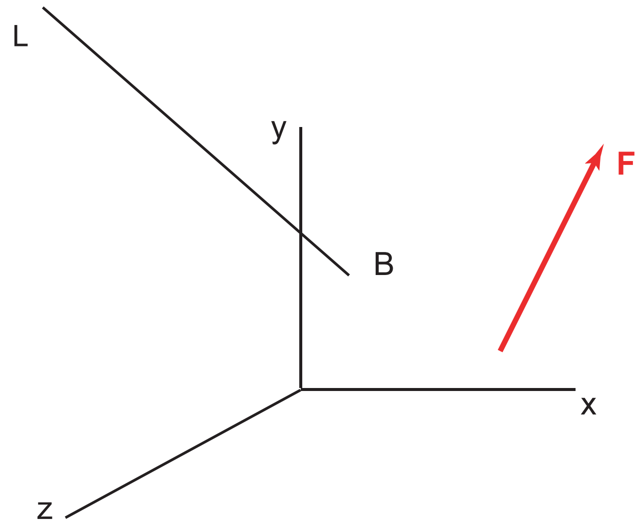


Example

Find the component (projection) of the 1000 lb force onto AB.
Units: Lb, in.



Moment of a Force about a Given Axis



Recall,

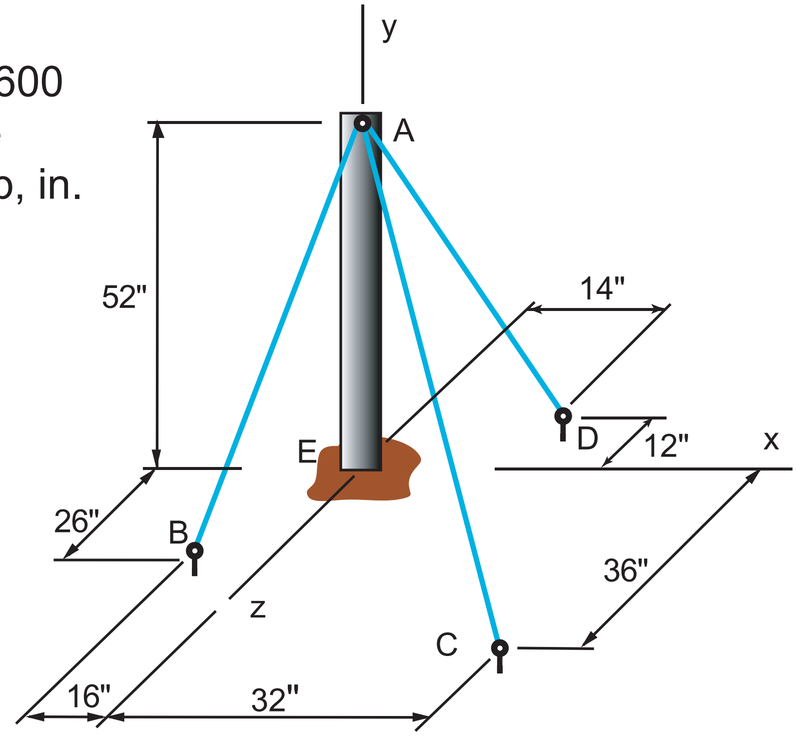
$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_{BL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Note: This is the easiest way to solve moments about a line. It is normally far more difficult to look into the axis as we did earlier in the chapter.

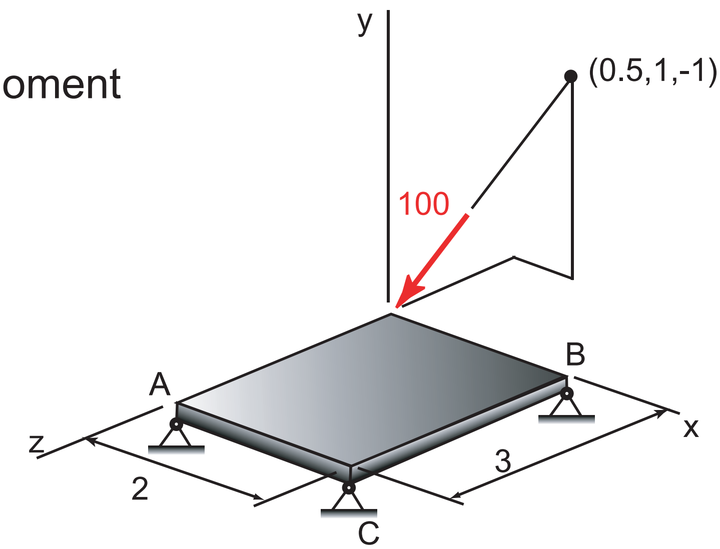
Example

The tension force in wire AB is 600 lb. Compute the moment of the tension about line EC. Units: Lb, in.



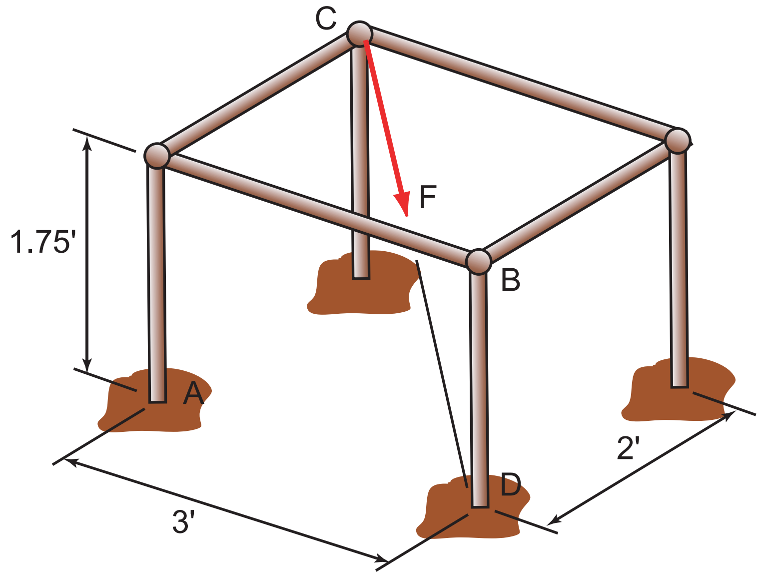
Example

Determine the magnitude of the moment about AB due to the 100 lb force.
Units: Lb, ft.



Example

Determine the magnitude of the moment about AB. The force F= 100 lb. Units: Lb, ft.



Observation

-When finding moments about a ***point:***

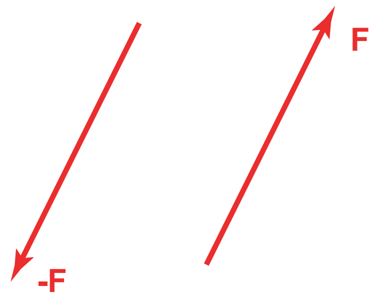
> The position vector must always start at the point and ends anywhere along the force.

-When finding moments about an ***axis:***

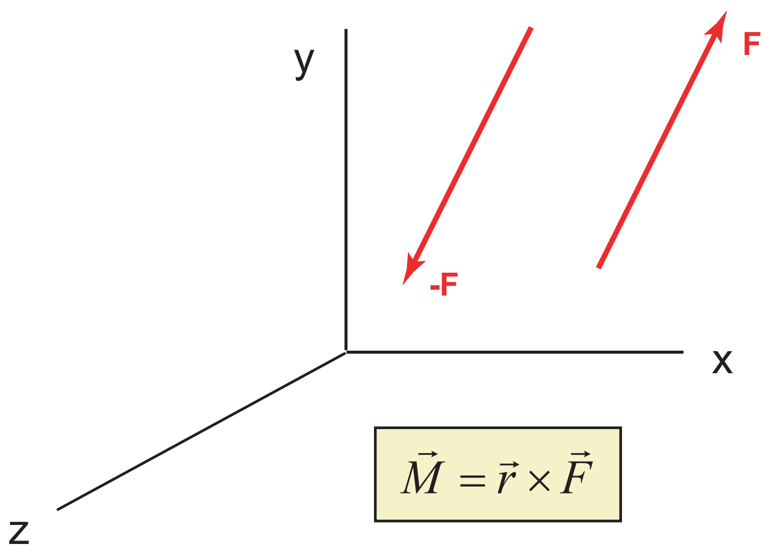
> The position vector can start anywhere along the axis and ends anywhere along the force.

Moment of a Couple

Two forces **F** and -**F** having the same magnitude, parallel lines of action, and opposite sense are said to form a couple.

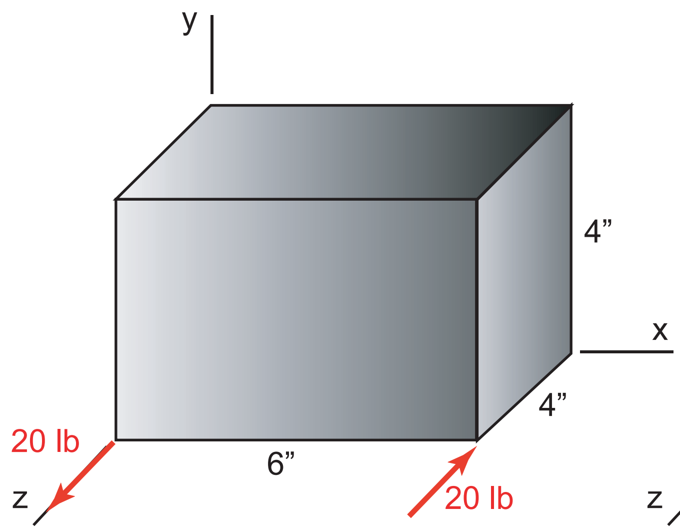


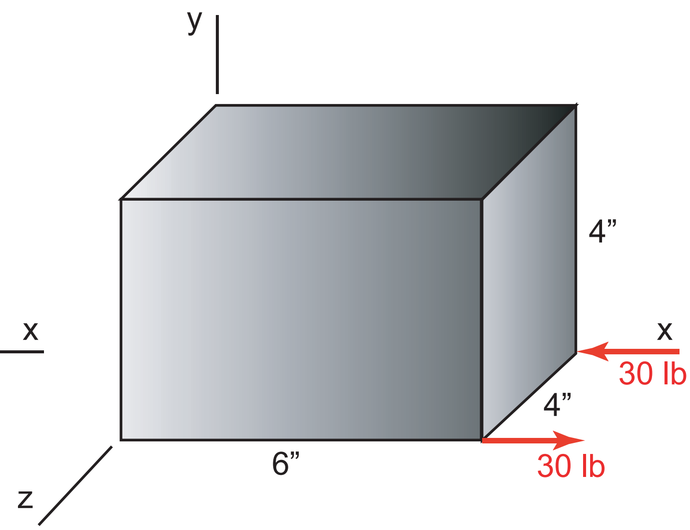
$$M = Fd_{\perp}$$

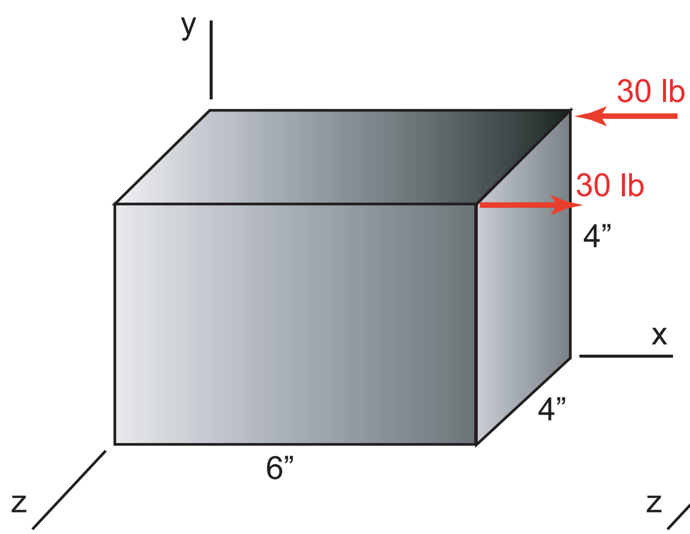


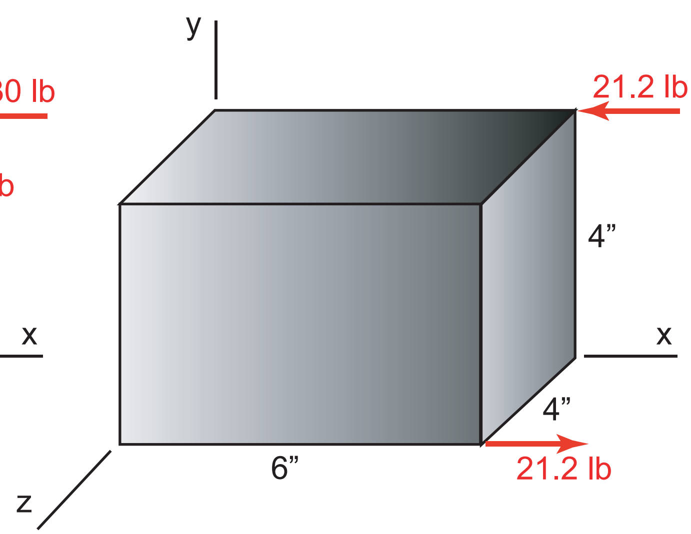
$$\vec{M} = \vec{r} \times \vec{F}$$

Equivalent Couples



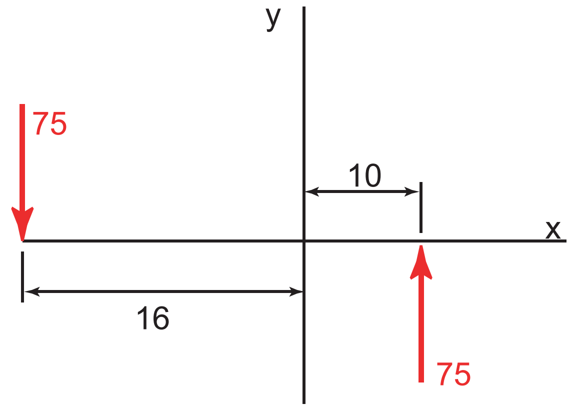






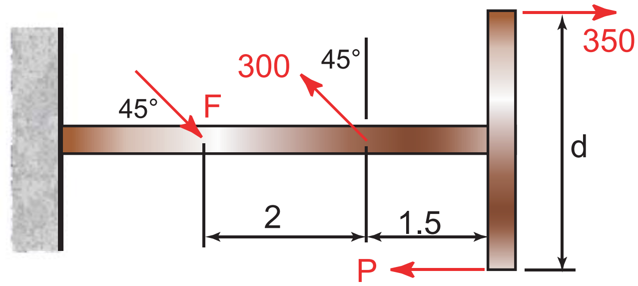
Example

Determine the magnitude and direction of the couple.
Units: Lb, in.



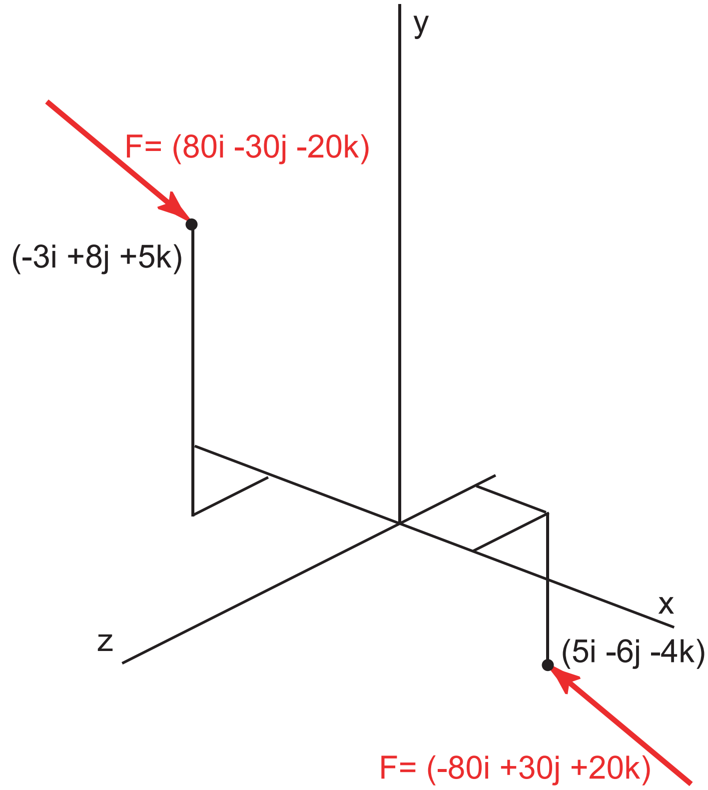
Example

If the resultant of two couples is to be zero, determine the magnitudes of P and F, and the distance d. Units: N, m.

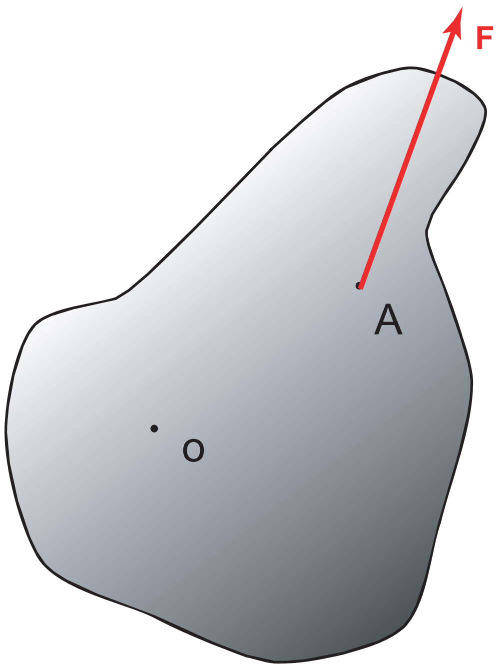


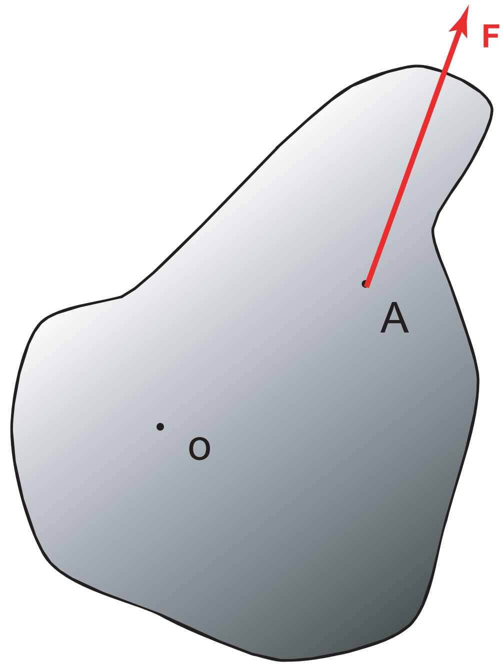
Example

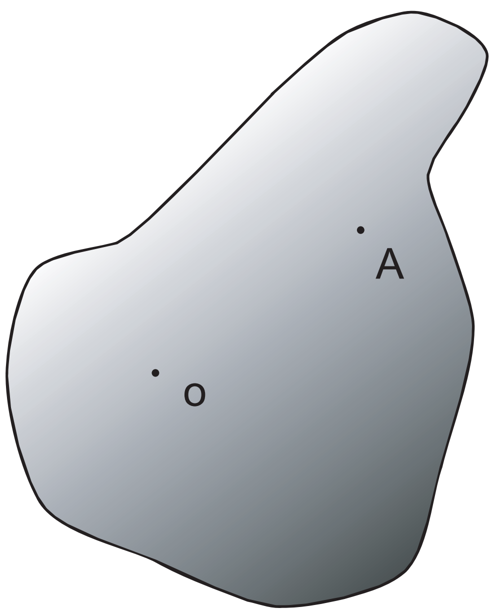
Determine the magnitude of the couple. Units: Lb, in.



Resolution of a Given Force into a Force at O and a Couple





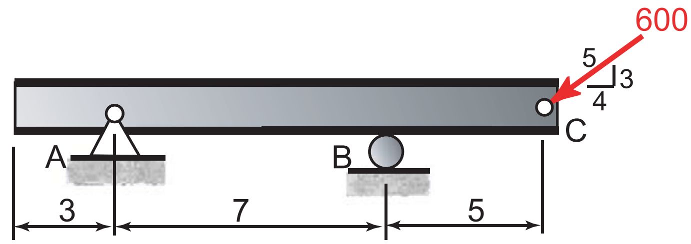


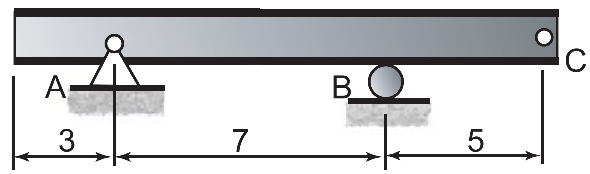
Conclusion:
You can move a force to a new location provided you add in the appropiate moment.

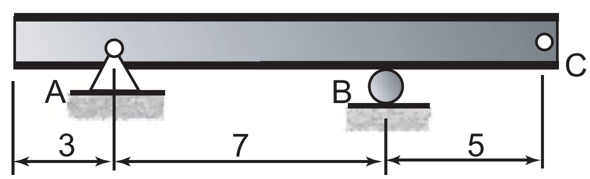
Example

Find the equivalent force-couple system at point A and B on the beam.

Units: Lb, ft.

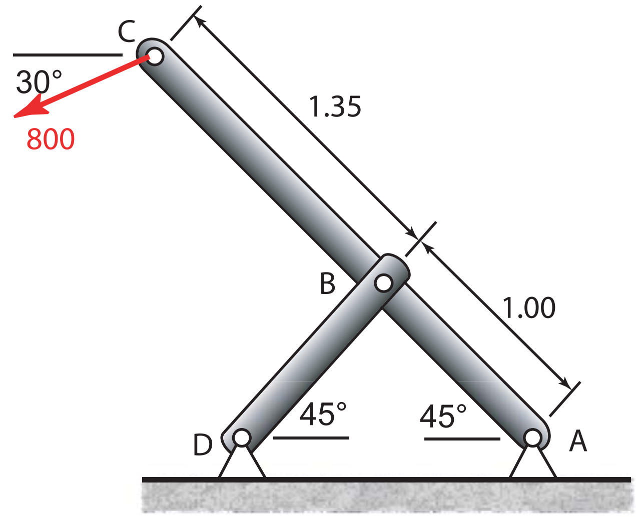


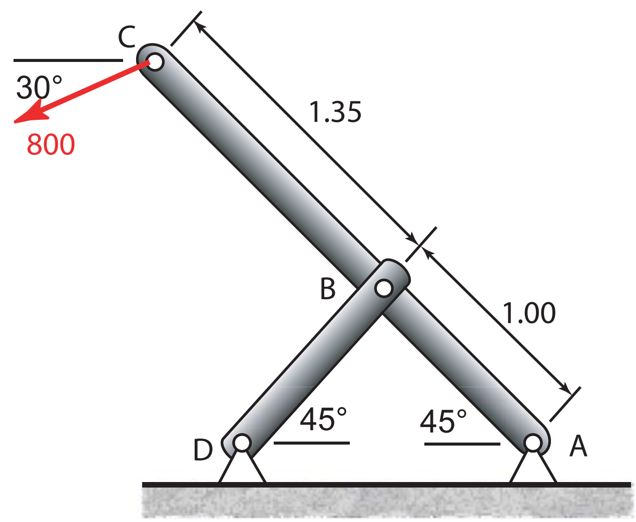




Example

Find the equivalent force-couple system at point A and B on the beam. Units: N, m.



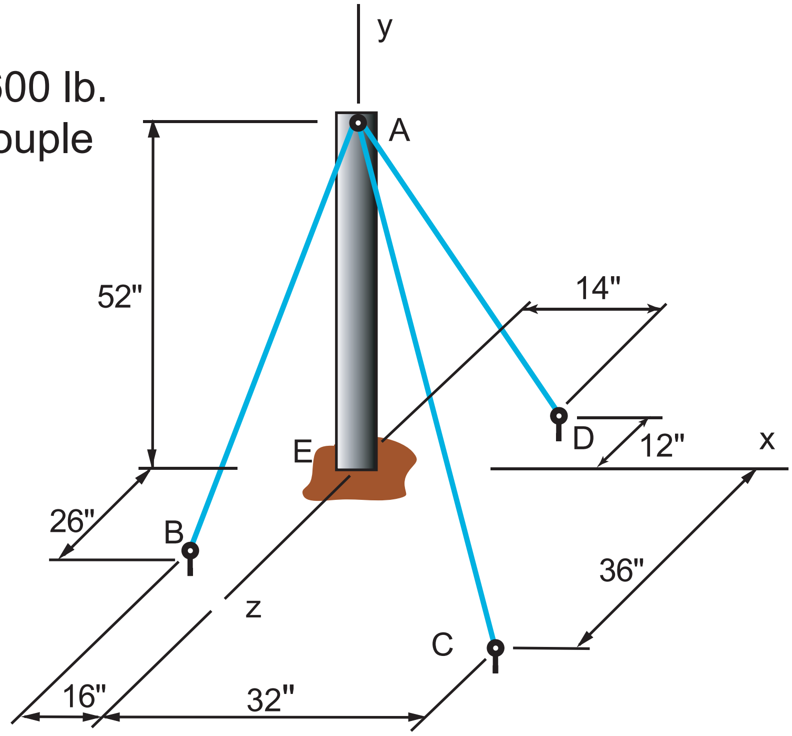


Example

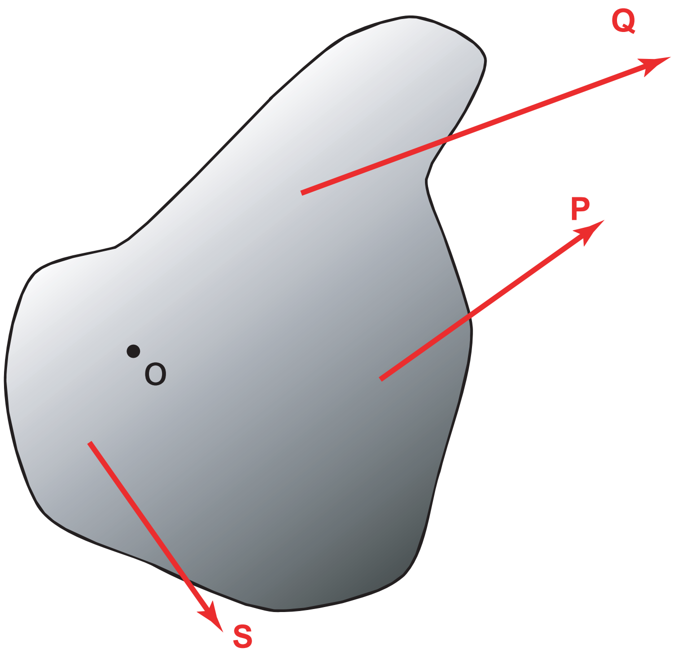
The tension force in wire AB is 600 lb. Calculate the equivalent force-couple system of this force at point E.
Units: Lb, in.

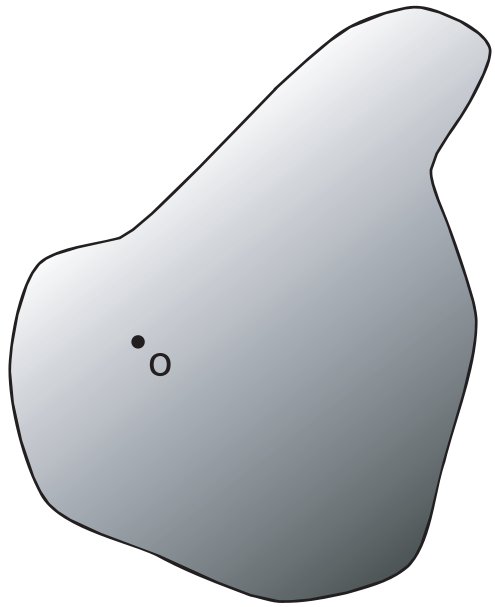
From a previous solution,

$$\vec{T}_{AB} = -159\vec{i} - 517\vec{j} + 259\vec{k}$$



Reduction of a System of Forces to One Force and a Couple

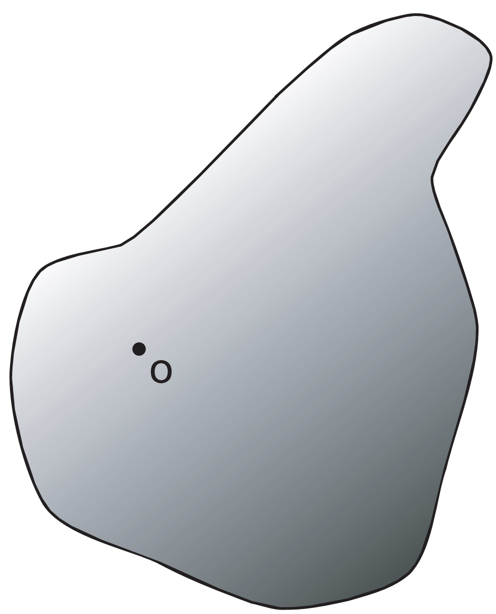




Conclusion:

$$\vec{M}_O = \sum\left(\vec{r} \times \vec{F}\right)$$

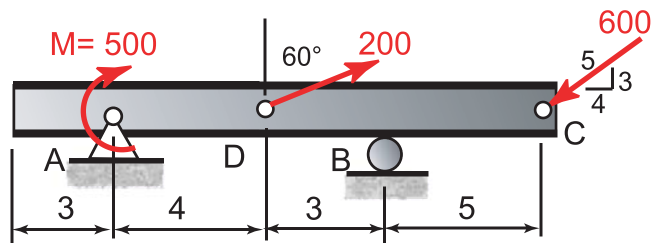
Moving a Moment to a New Location

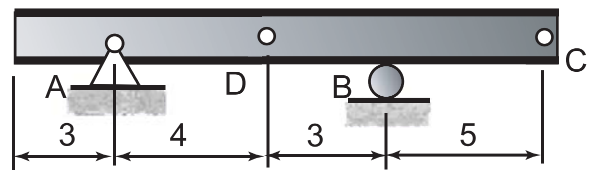


Conclusion:
To move a moment to a new location... just do it!

Example

Replace the forces and moment by an equivalent force-couple system at point D on the beam. Units: Lb, ft.





Example

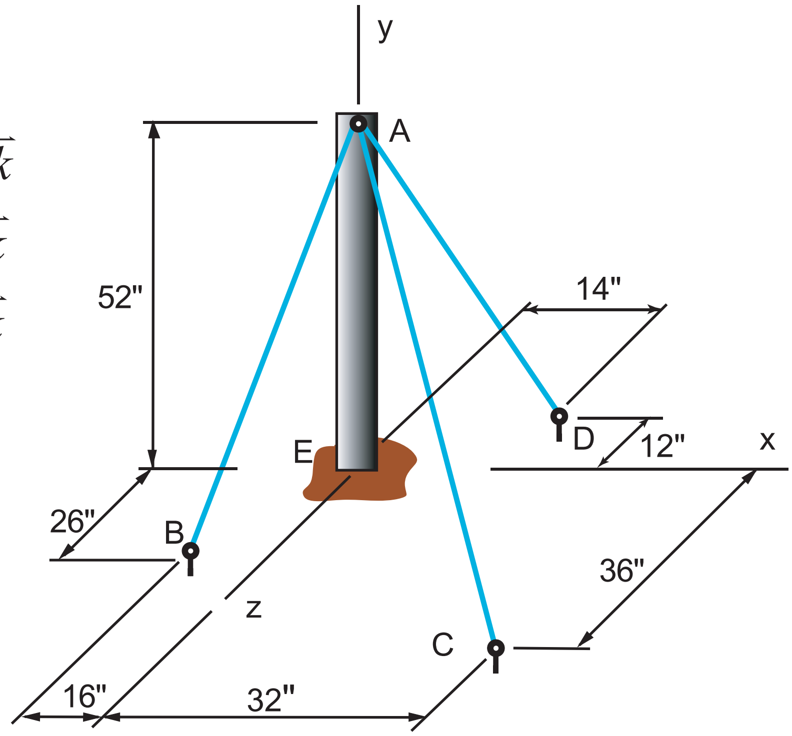
Replace the forces in the three wires at A by an equivalent force-couple system at point E. Units: Lb, ft.

From a previous solution,

$$\vec{T}_{AB} = -159\vec{i} - 517\vec{j} + 259\vec{k}$$

$$\vec{T}_{AC} = 135\vec{i} - 220\vec{j} + 152\vec{k}$$

$$\vec{T}_{AD} = 127\vec{i} - 471\vec{j} - 109\vec{k}$$



Example

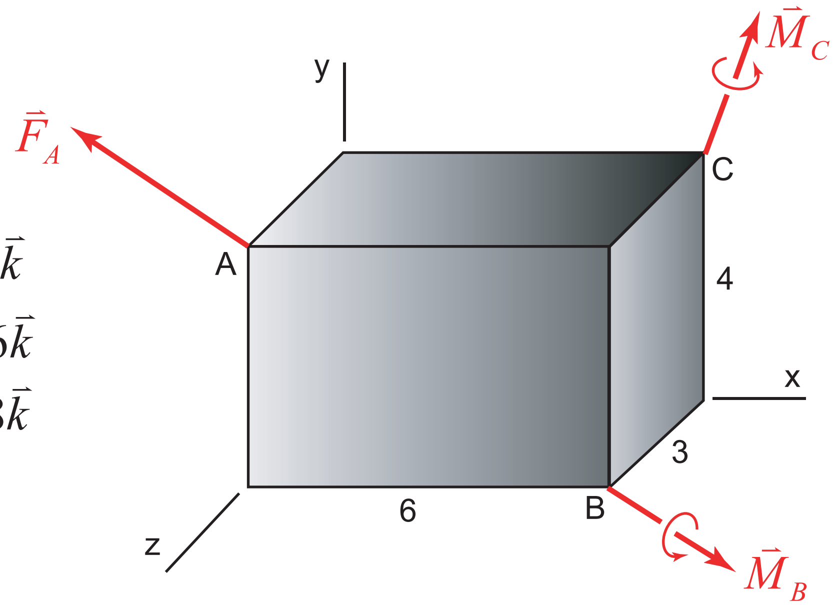
Replace the force and moments by an equivalent force-couple system at point B. Units: Lb, in.

Given:

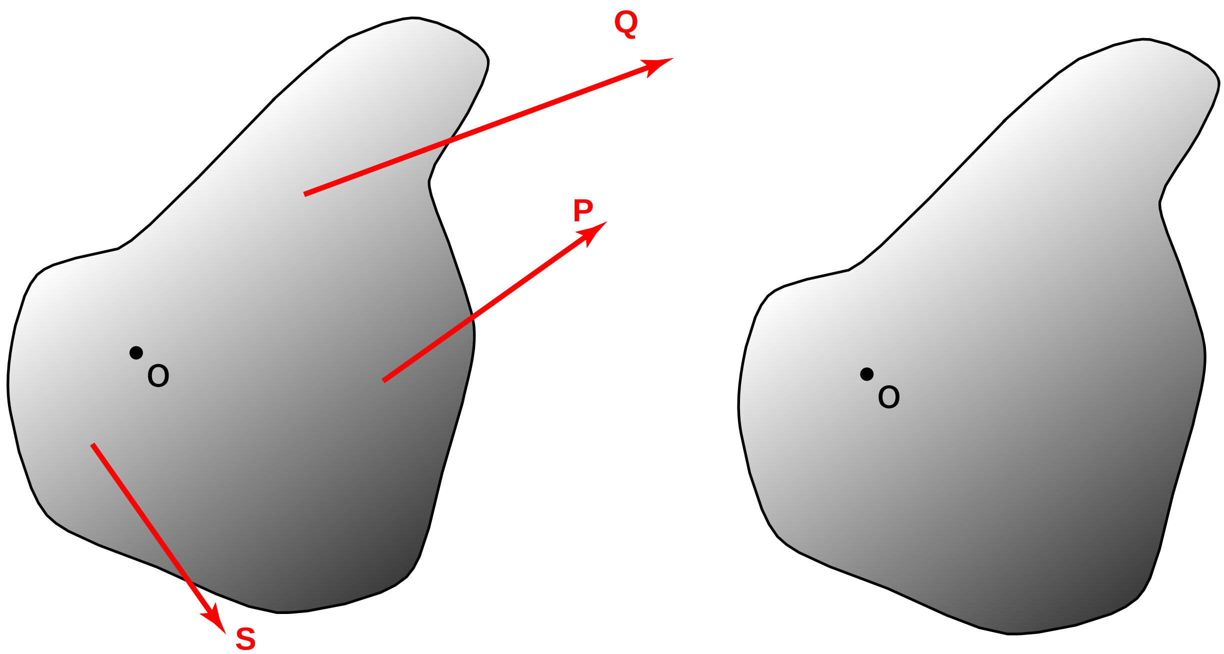
$$\vec{F}_A = -117\vec{i} + 95.6\vec{j} + 210\vec{k}$$

$$\vec{M}_C = 48.4\vec{i} + 15.4\vec{j} - 33.6\vec{k}$$

$$\vec{M}_B = 32.6\vec{i} - 56.2\vec{j} + 14.8\vec{k}$$

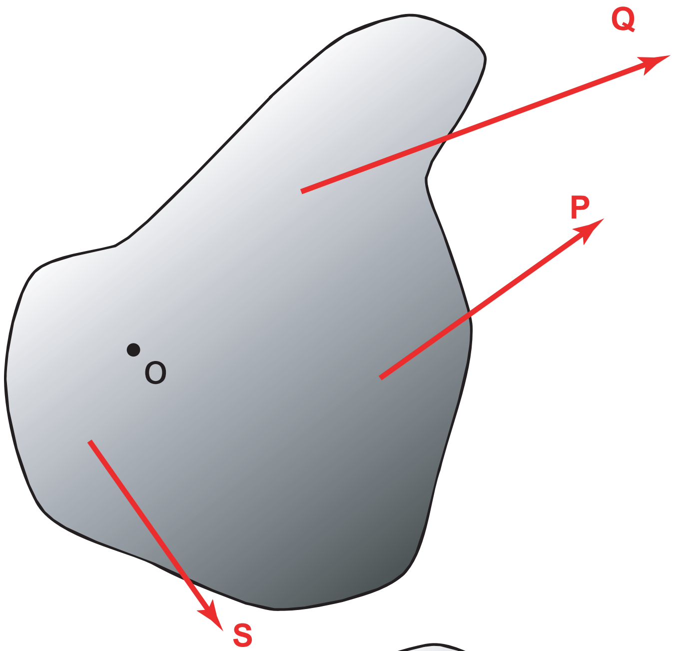


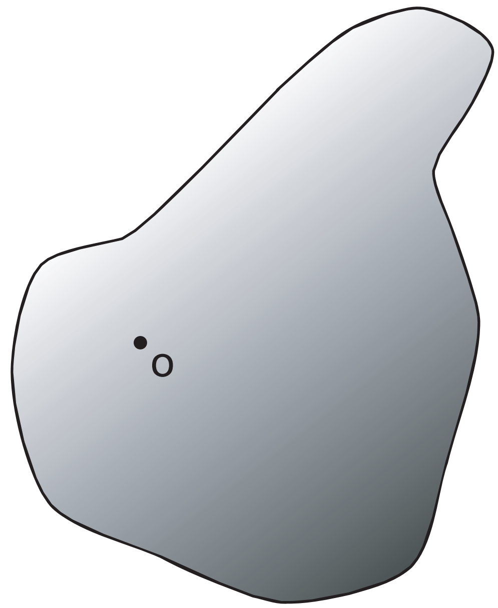
Equivalent System of Forces

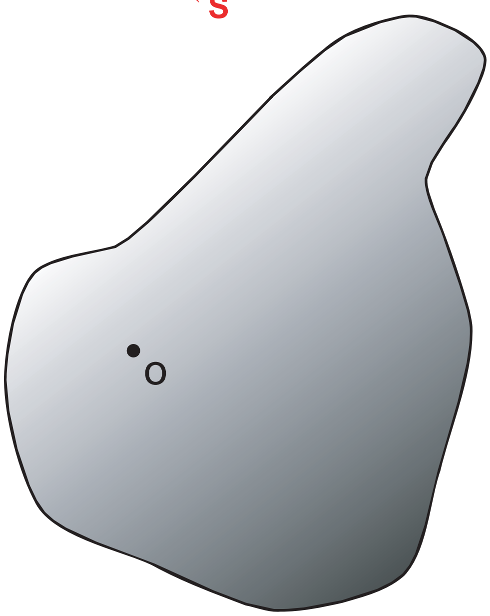


Two systems are equivalent if they have the same:
-Translational effect
-Rotational effect

Further Reduction of a System of Forces



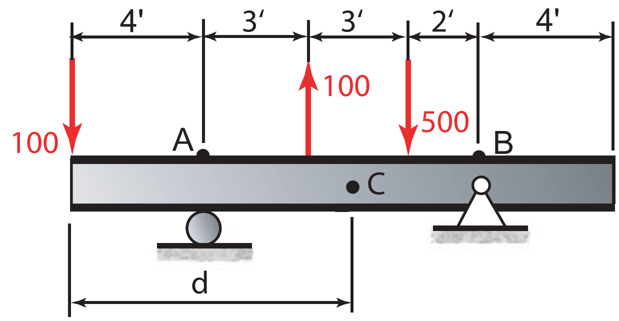


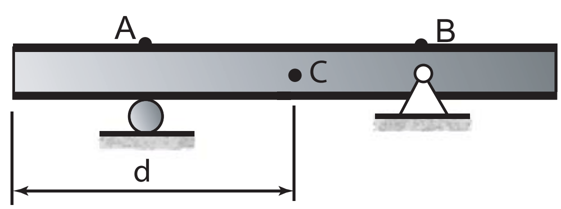


A system of forces can be replaced by a single force placed at a certain location.

Example

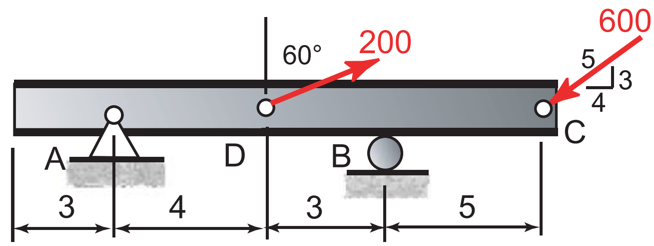
Replace this system with a single force at C and determine the distance d. Units: Lb, ft.

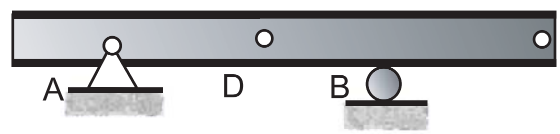




Example

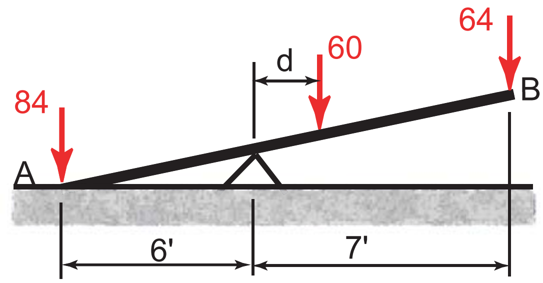
Replace the two forces with an equivalent force. Specify the location along the beam's centerline that this force must pass through. Units: N, m.





Example

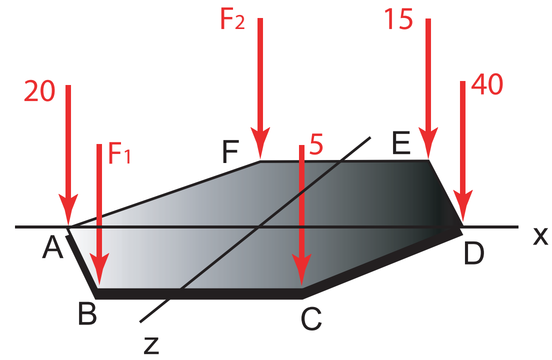
Two children are sitting at the ends of a seesaw. Where should a third child sit so that the seesaw is perfectly balanced if the third child weighs 60 lb? Units: Lb, ft.

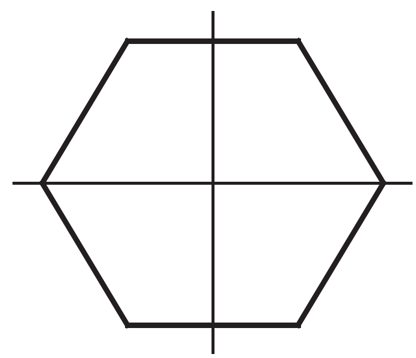


Example

A concrete foundation mat in the shape of a regular hexagon of side 14 ft support four column loads as shown. Determine the magnitudes of the additional loads which must be applied at B and F if the resultant of all six loads is to pass through the center of the mat.

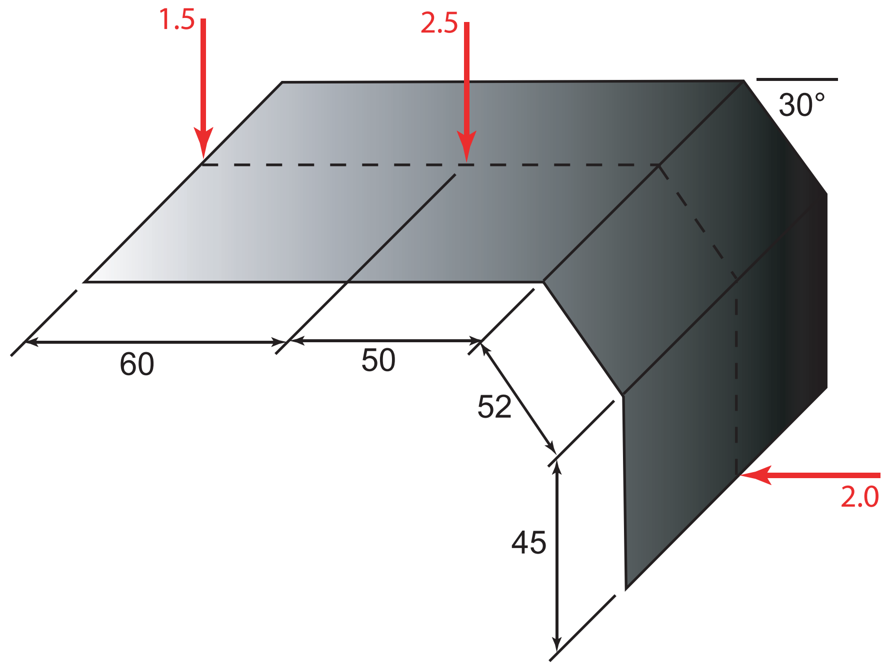
Units: Ft, kip (1 kip= 1000 pounds)



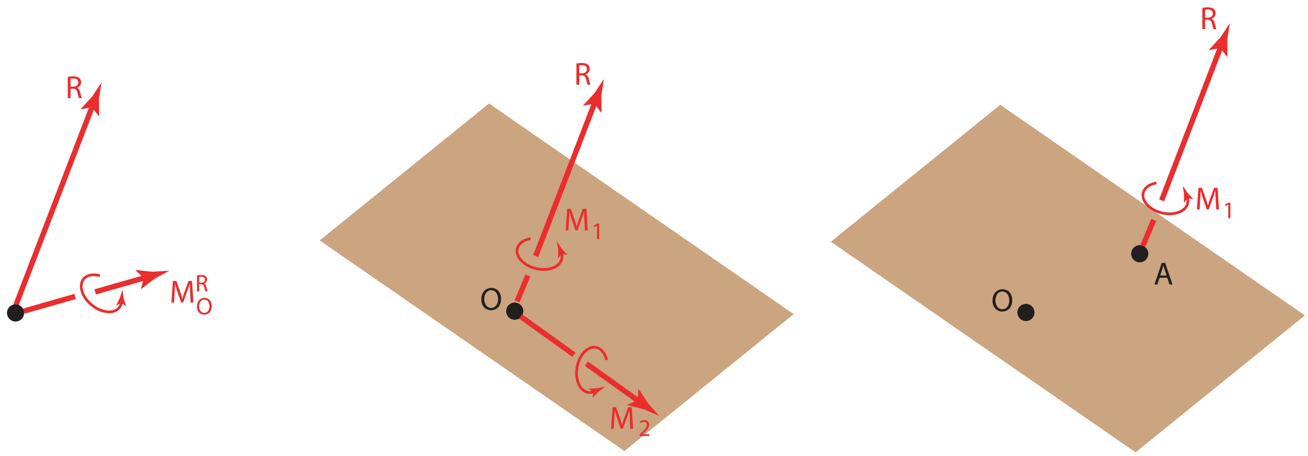


Example

Find the resultant of the three loads and its location. Units: kN, mm.



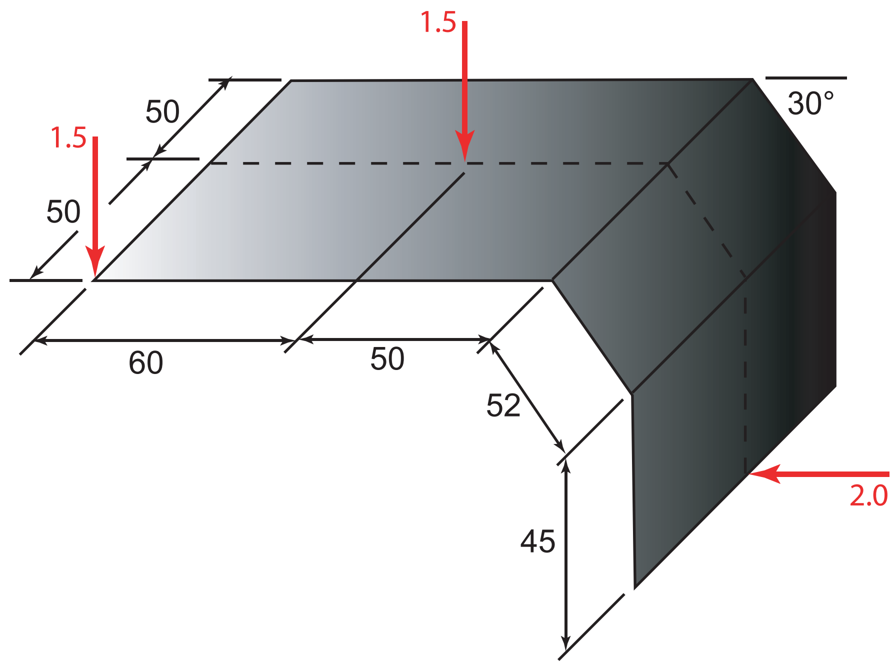
Reduction of a System of Forces to a Wrench



$$\rho = \frac{\vec{R} \bullet \vec{M}_O^R}{R^2}$$

Example

Replace the three loads with an equivalent wrench and determine (a) the magnitude and direction of R, (b) the pitch of the wrench, (c) the point where the wrench intersects the yz plane. Units: kN, mm.



Summary

-Moments about a point in 2D

-Moments about a point in 3D

-Moments about a line

-Equivalent force couple systems

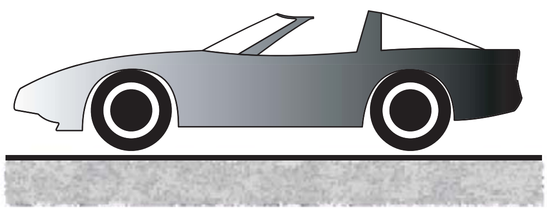
-Equivalent systems

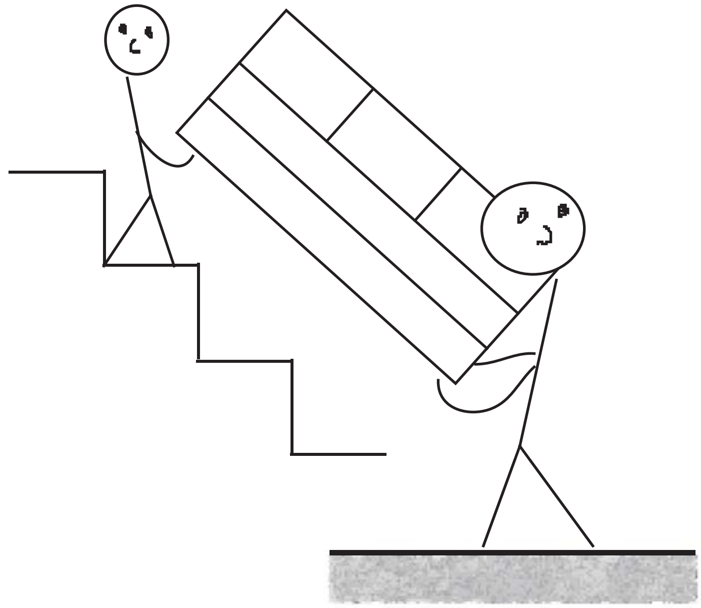
Chapter 4
Equilibrium of Rigid Bodies

Introduction

A particle remains at rest or continues to move in a straight line with uniform velocity if the resultant forces acting on it are zero, in other words:

$$\sum \vec{F} = 0$$
$$\sum \vec{M} = 0$$





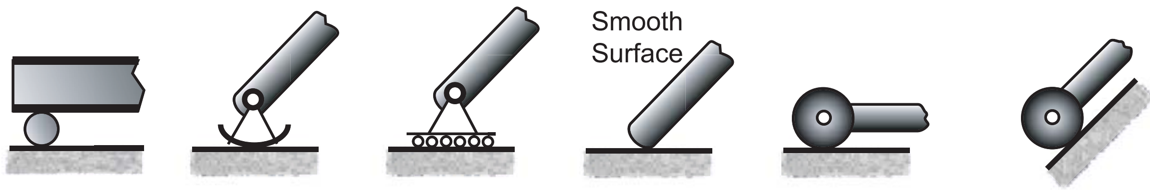
Goals:

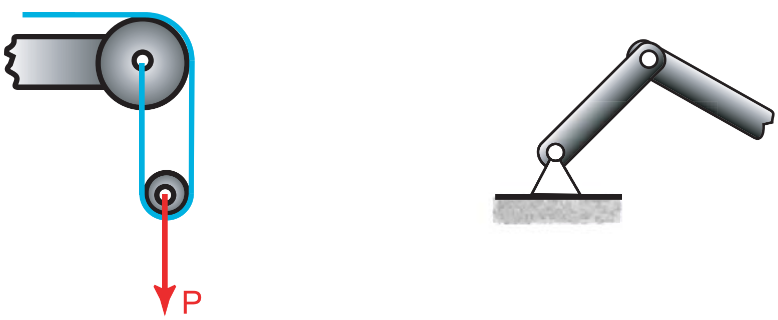
-Master drawing FBDs
-Map (planning what you are going to do)
-Equilibrium in 2D
-Mastering moments about a point
-Two force bodies
-Three force bodies
-Equilibrium in 3D
-Mastering moments about an axis or a line

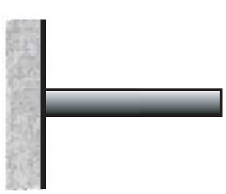
Reactions at Supports and Connections for a Two-Dimensional Structure

General Rule:

-If it can move then there
-If it can't move then there
-If it can rotate then there
-If it can't rotate then there





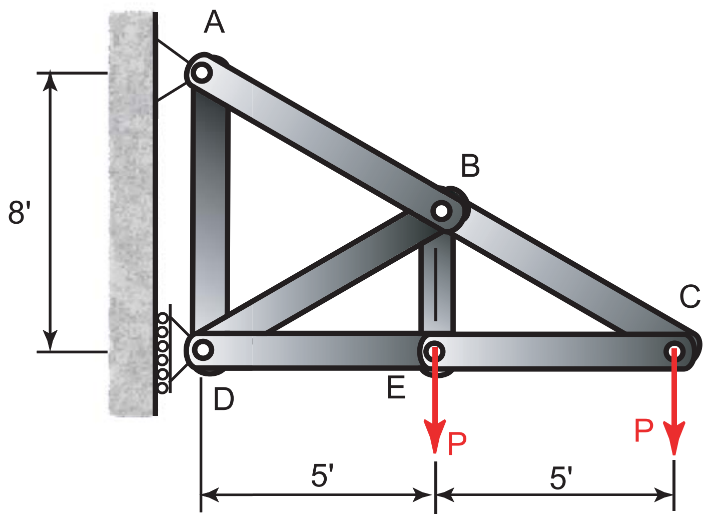


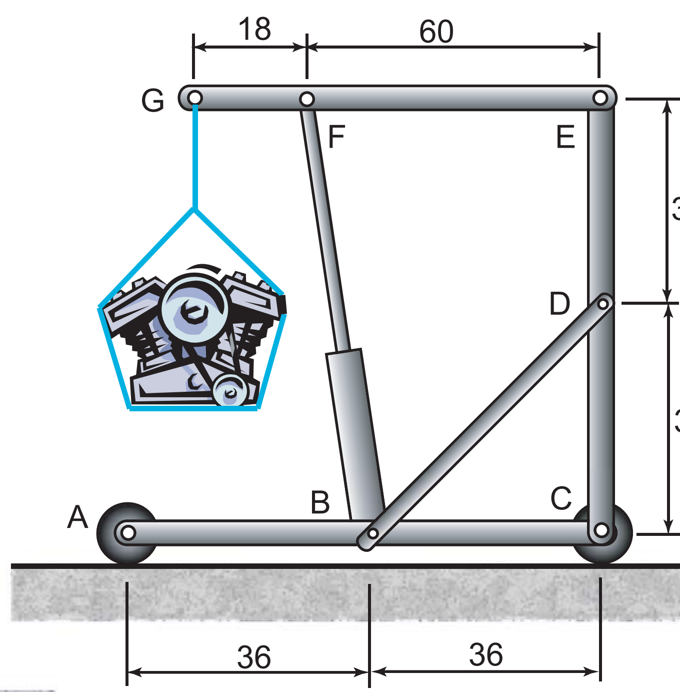
Sample Free-Body Diagrams

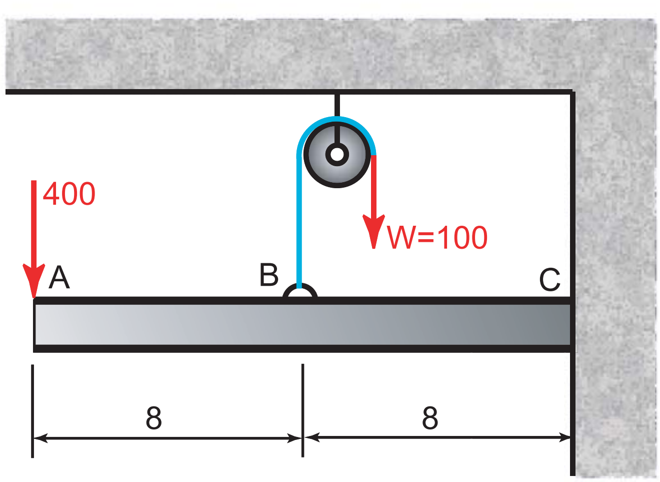
General Approach:

Map:

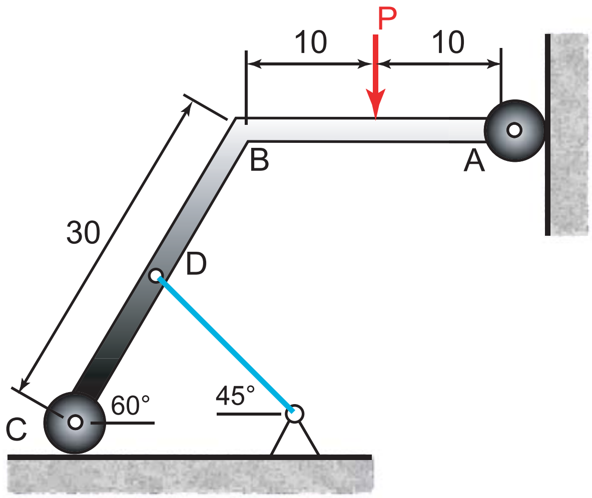
Sample Free-Body Diagrams

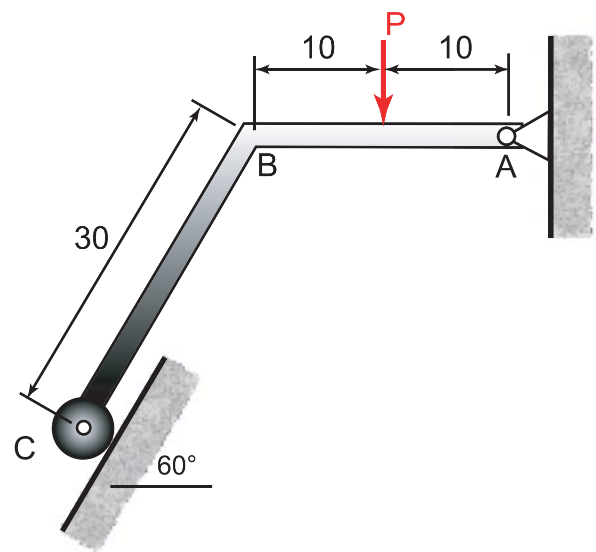


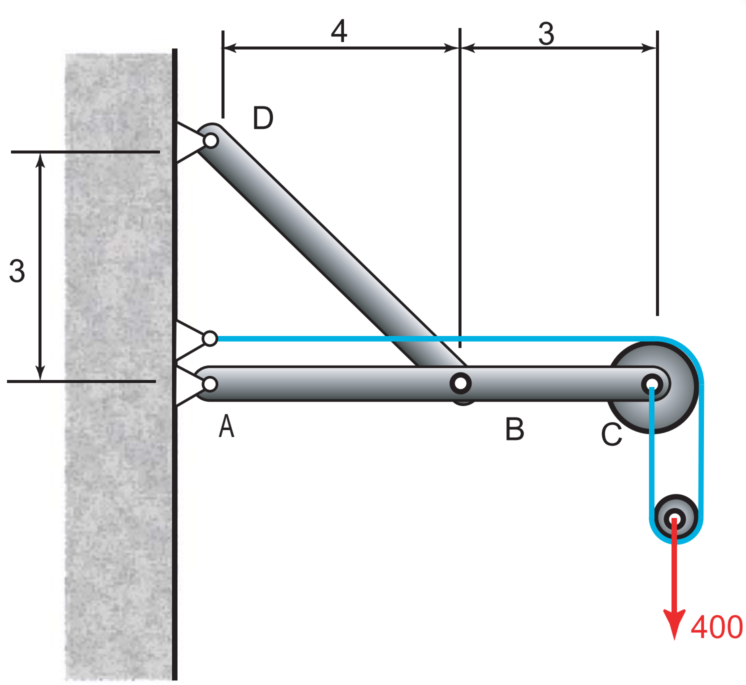




Sample Free-Body Diagrams



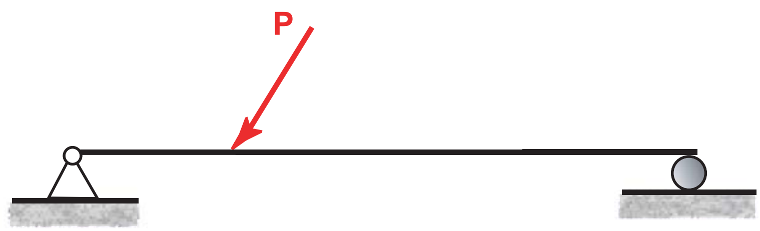




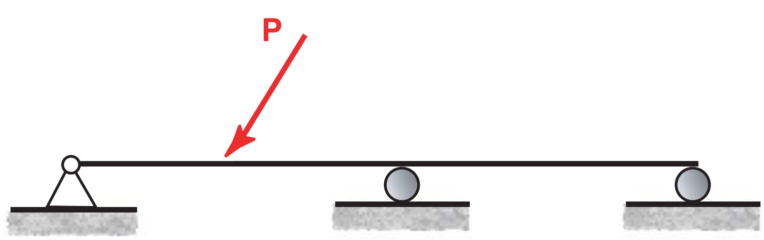
Constraints and Statical Determinacy

Sum of the forces and sum of the moments are always valid, but may be insufficient to solve the problem.

Statically Determinate

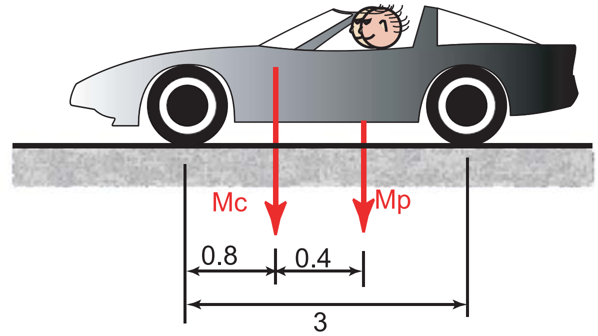


Statically Indeterminate



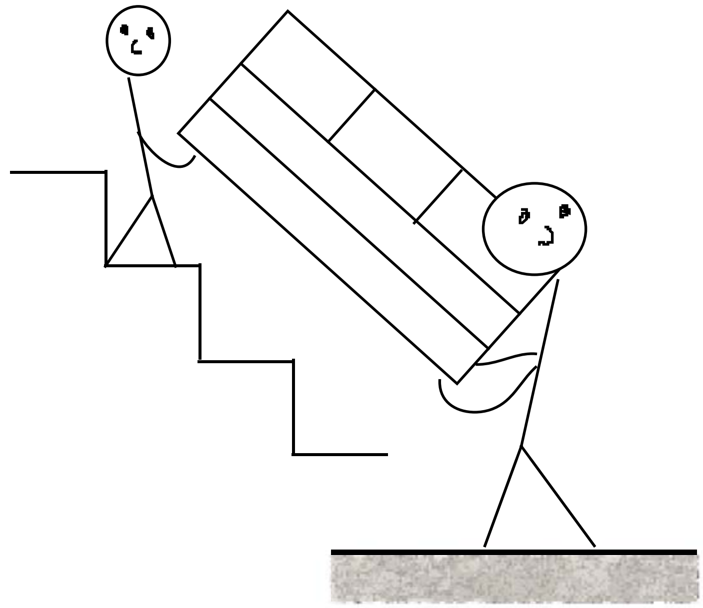
Example

The car weighs 1400 kg (Mc) and the two passengers weigh 100 kg each (Mp). Determine the reactions at each of the two (a) rear wheels A, (b) front wheels B. Units: N, m.



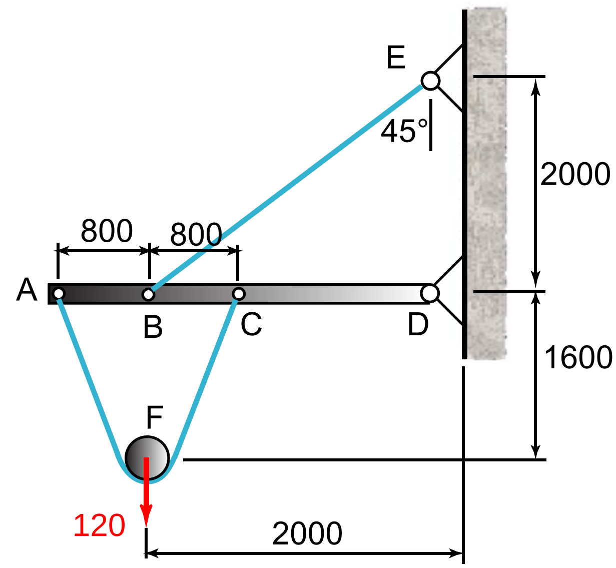
Example

Determine the reactions that each person needs to support. The 6 ft. couch weighs 100 lb and is tilted 45°. Assume that the upper person is only able to support a reaction perpendicular to the couch (no friction) whereas the lower person can support parallel and perpendicular to the couch. Units: Lb, ft.



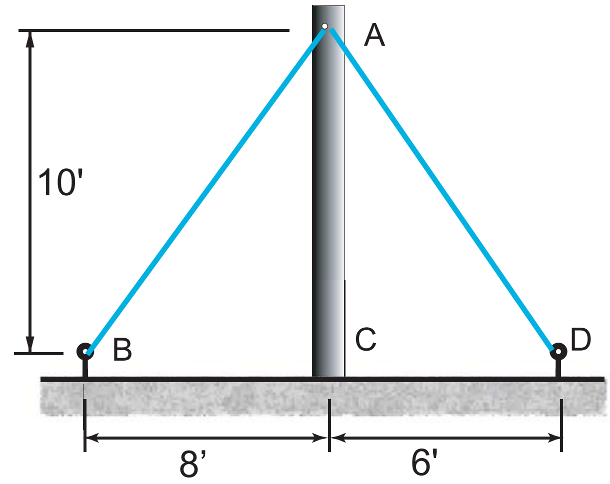
Example

Determine the reactions at D and the tension in BE. The wire connected at A and C is continuous.
Units: N, mm.



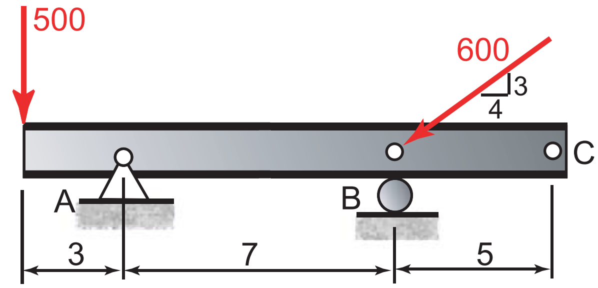
Example

The cable stays AB and AD help support pole AC. Knowing that the tension is 140 lb in AB and 40 lb in AD, determine the reactions at C. Units: Lb, ft.



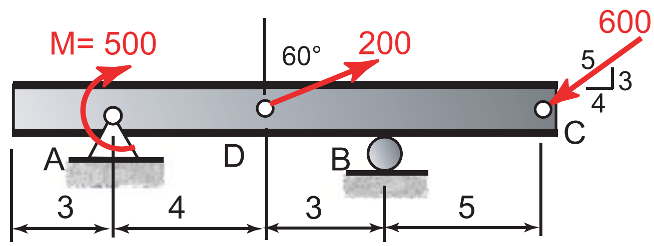
Example

Determine the reactions at supports A and B.
Units: Lb, ft.



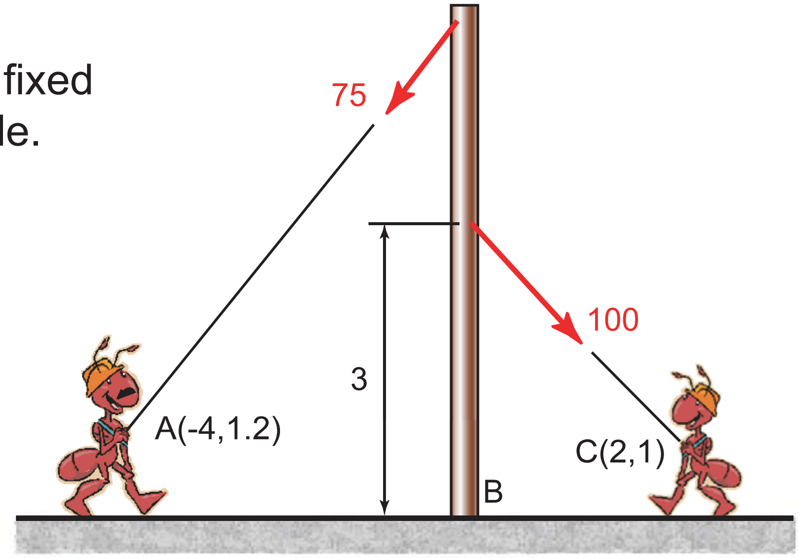
Example

Determine the reactions at A and B. Units: Lb, ft.



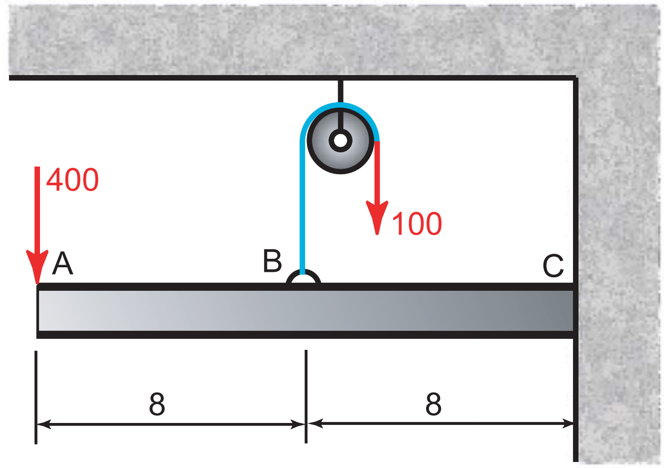
Example

Determine the reactions at the fixed support (point B) of the 5 m pole. Units: N, m.



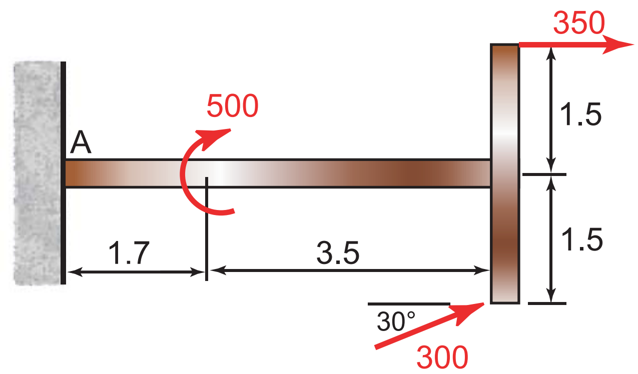
Example

Find the reactions at the fixed support C. Units: Lb, ft.



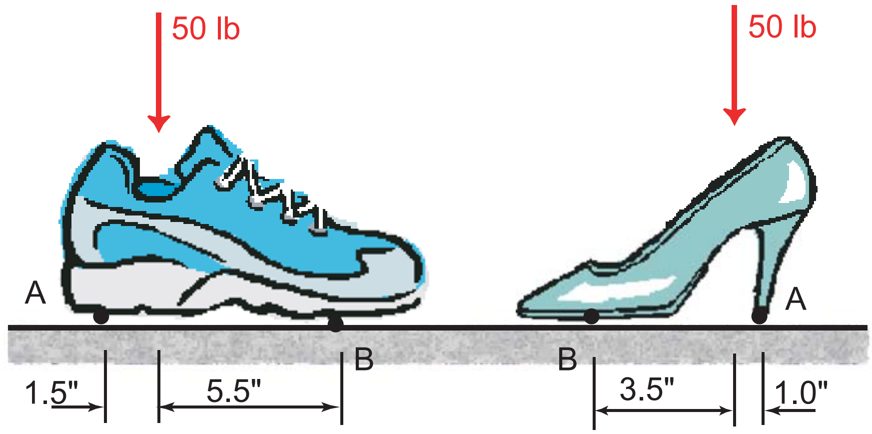
Example

Determine the reactions at fixed support A. Units: N, m.



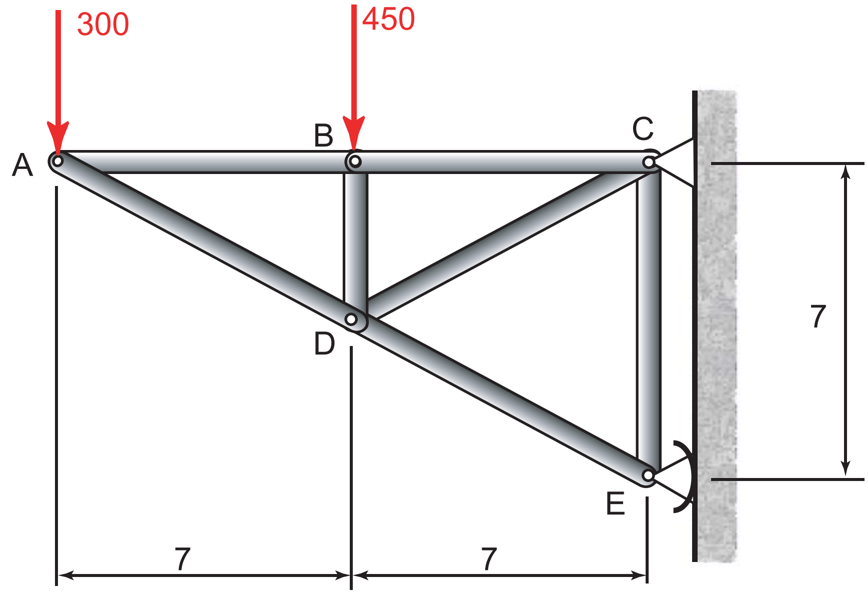
Example

Determine the reactions at A and B for each type of shoe. Units: Lb, in.



Example

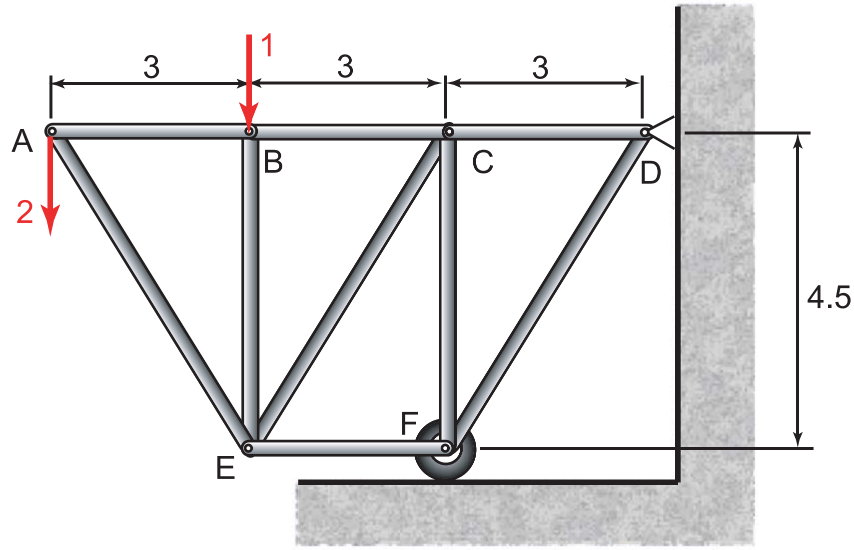
Determine the reactions at C and E.
Units: Lb, ft.



Example

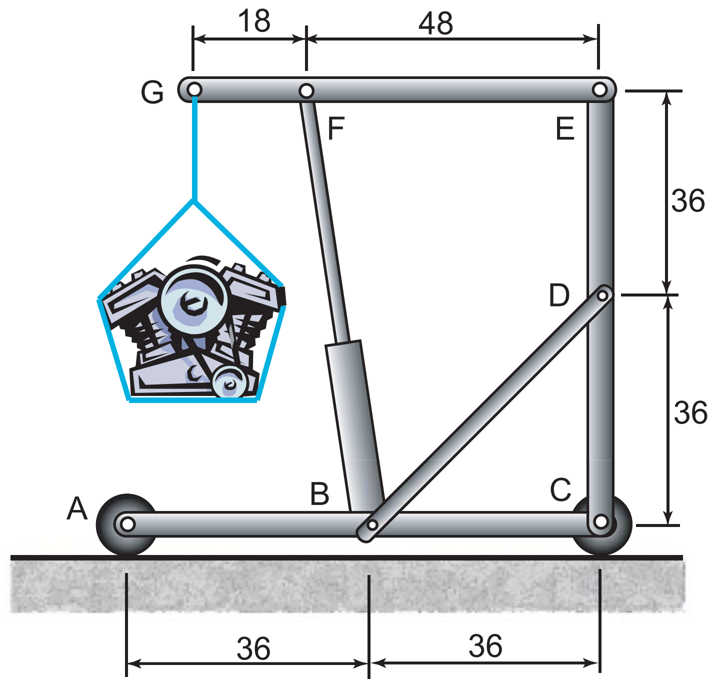
Determine the reactions at D and F.

Units: kN, m.



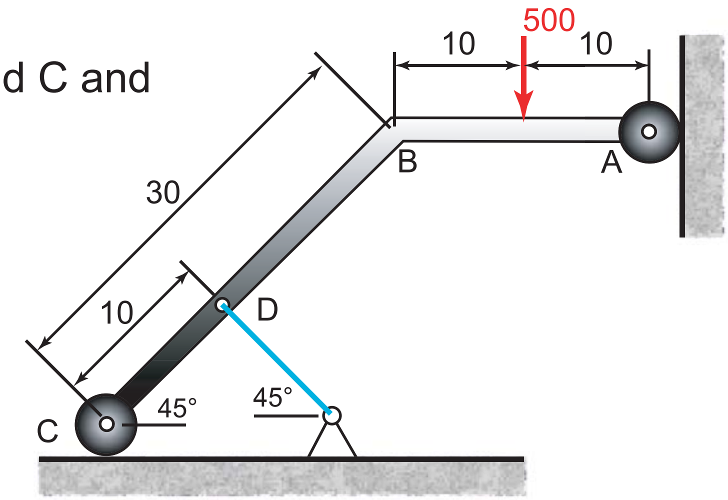
Example

Find the reactions at A and C due to the 650 lb engine.
Units: Lb, in.

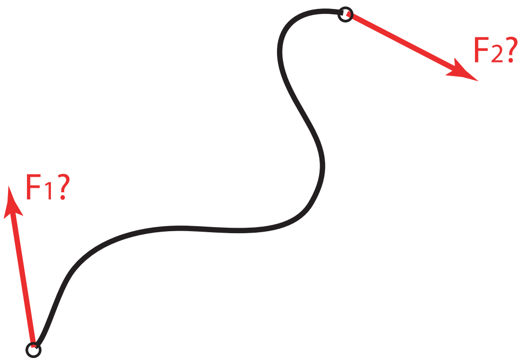


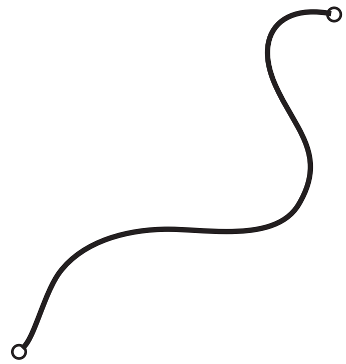
Example

Find the reactions at rollers A and C and the tension in the rope.
Units: Lb, ft.



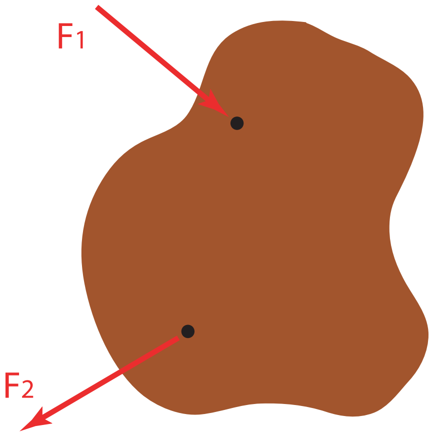
Equilibrium of a Two-Force Body

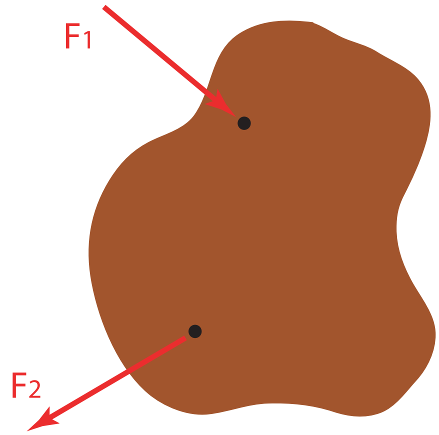




Conclusion:
For any member that is pinned at both ends and no loads between the ends, the resultant forces must pass through each other.

Equilibrium of a Three-Force Body

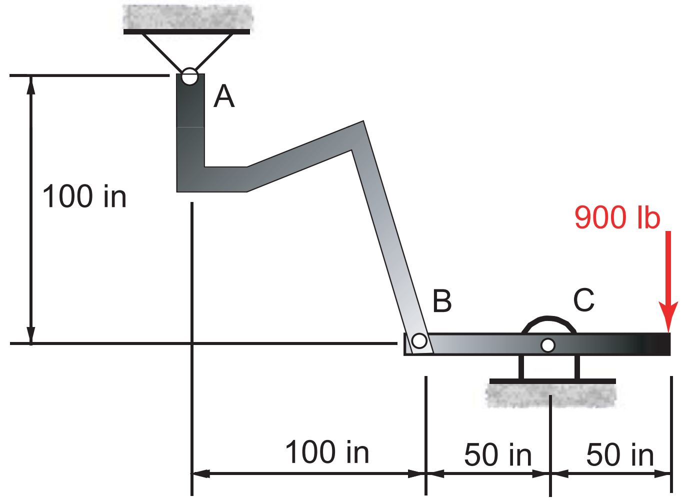




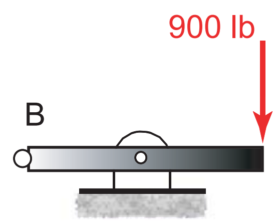
Conclusion: When the direction of two of the three forces are known, the third force must pass through the intersection of the other two.

Example

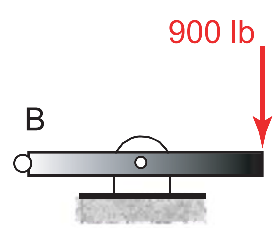
Find the reactions at A and C.
Units: Lb, in.



Solution 1

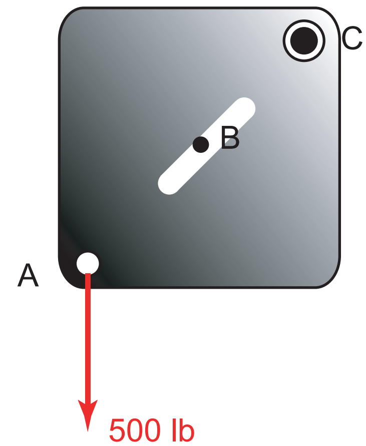


Solution 2



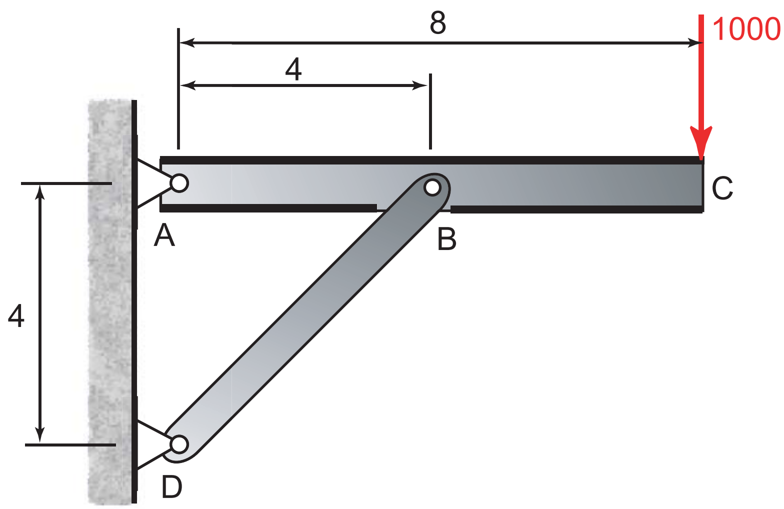
Example

The square plate has two holes at A and C. At A a 500 lb load is applied. At C a nail is inserted and is attached to the wall so as to prevent movement in 2 directions. Another nail at the 45° slot B prevents movement perpendicular to the slot. Find the reactions at B and C knowing that B is in the center of the plate. Units: Lb, in.



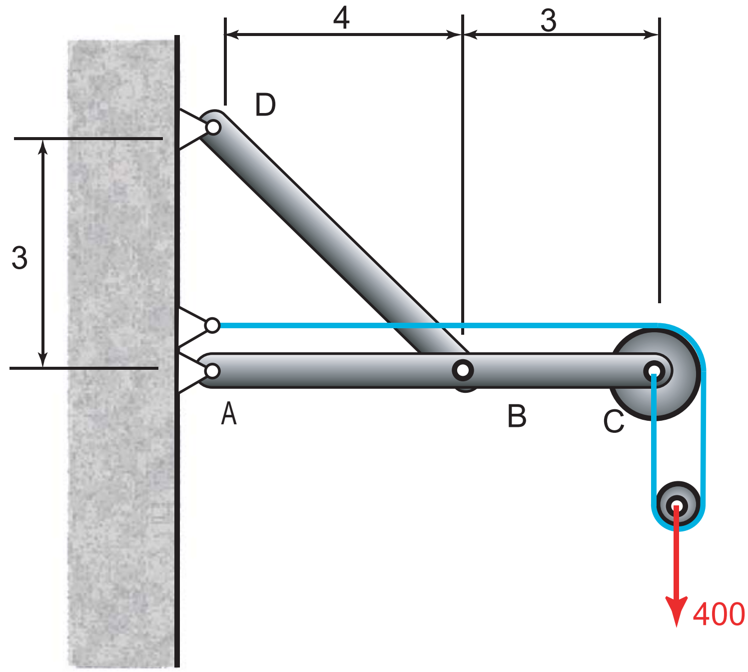
Example

Determine the reactions at A and D. Units: Lb, ft.

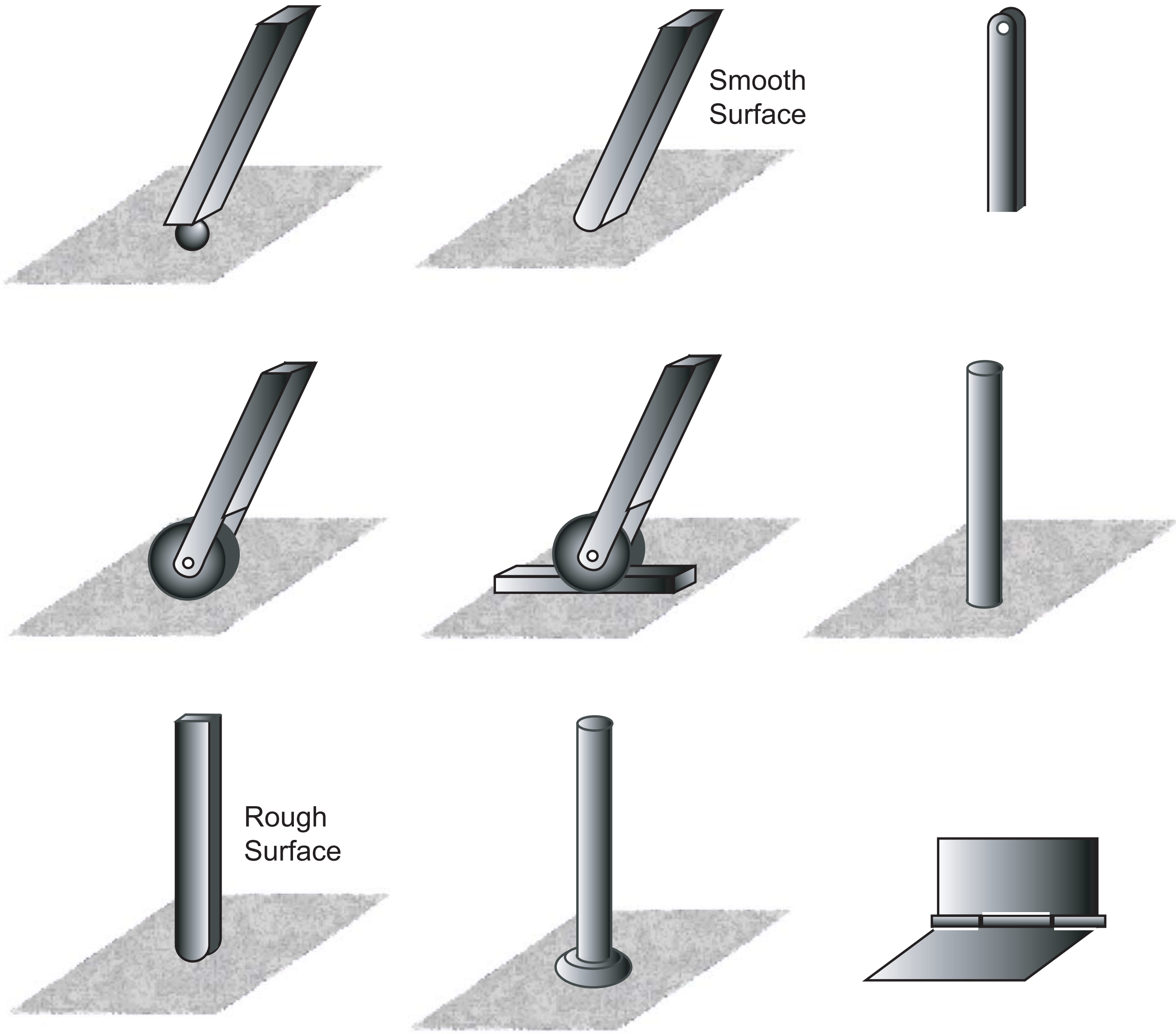


Example

Determine the reactions at A, D, and the tension in the rope. The radius of the smaller pulley is 2.5" and the larger is 5”. Units: Lb, ft.



Reactions at Supports and Connections for a Three-Dimensional Structure



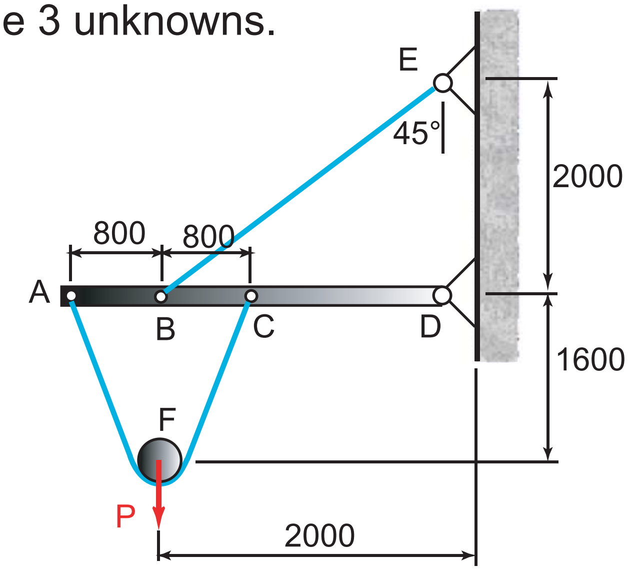
Why do we normally ignore the moments for hinges and bearings?

Equilibrium of a Rigid Body in 3D

$$\sum \vec{F} = 0$$
$$\sum \vec{M} = 0$$

General Approach for Solving Three Dimensional Problems

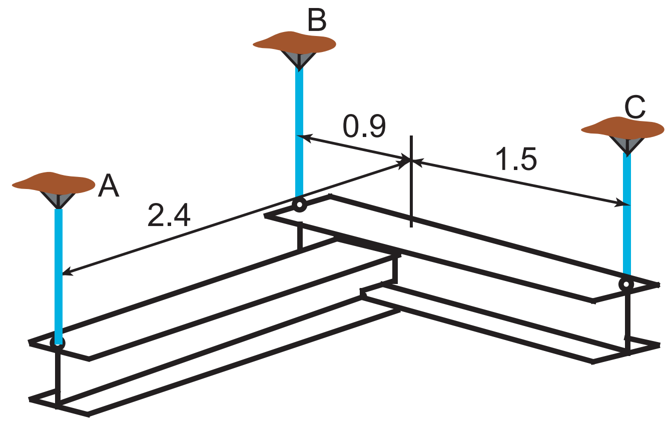
Recall for 2D problems we normally start by taking moments about a point that has 2 out of the 3 unknowns.



For 3D problems

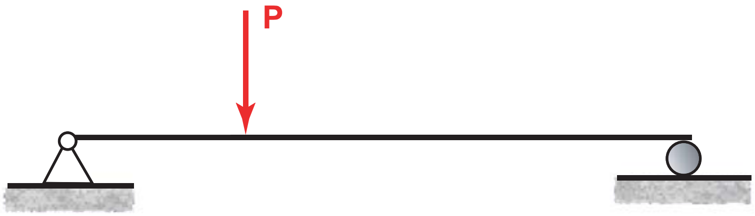
Example

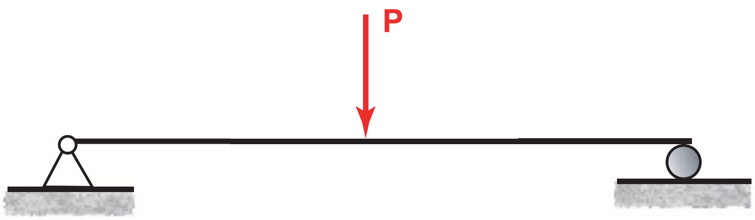
The two steel I-beams, each with a mass of 100 kg are welded together at right angles and lifted by the vertical cables so that the beams remain in a horizontal plane. Compute the tension in each of the cables A, B, C. Units: N, m.

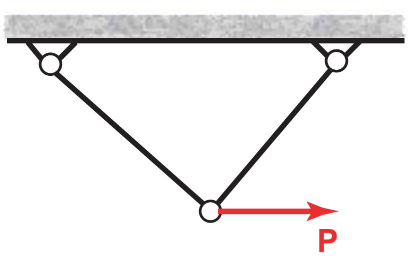


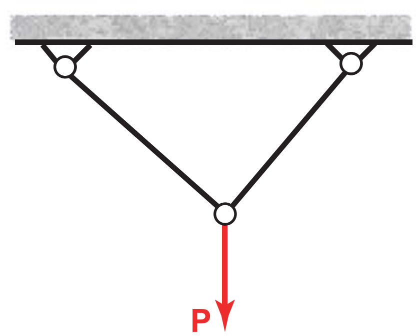
Symmetry

A structure whose geometry and loads are symmetric, will behave symmetrically.



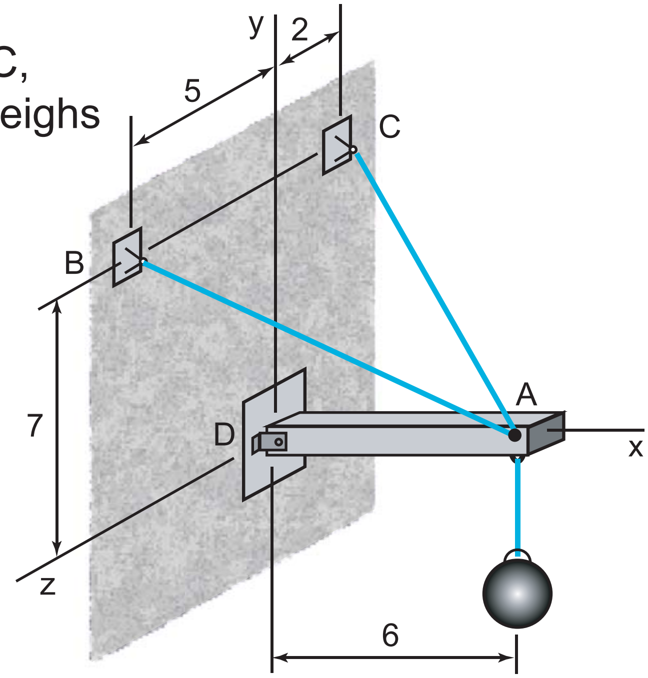






Example

Calculate the tension in cables AB, AC, and the reactions at D. The sphere weighs 150 lb.
Units: Lb, ft.



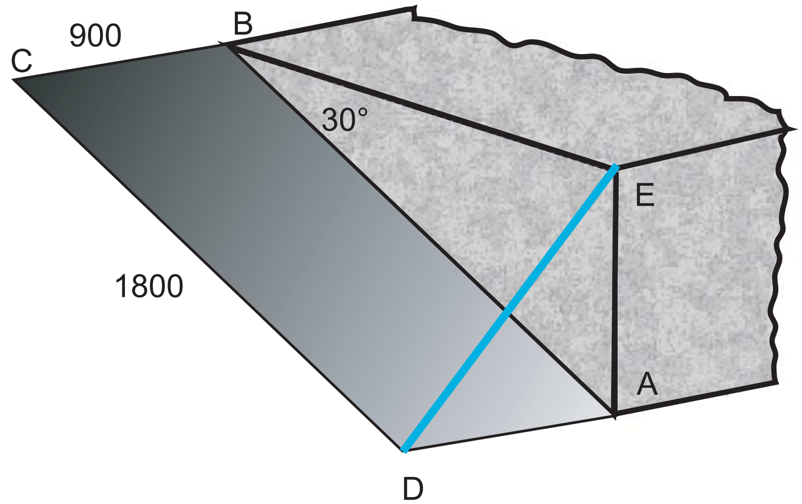
From a previous solution,

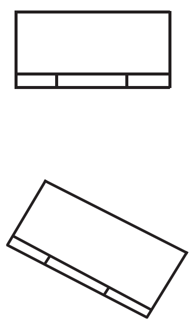
$$\vec{T}_{AB} = T_{AB}(-0.571\vec{i} + 0.667\vec{j} + 0.476\vec{k})$$

$$\vec{T}_{AC} = T_{AC}(-0.636\vec{i} + 0.742\vec{j} - 0.212\vec{k})$$

Example

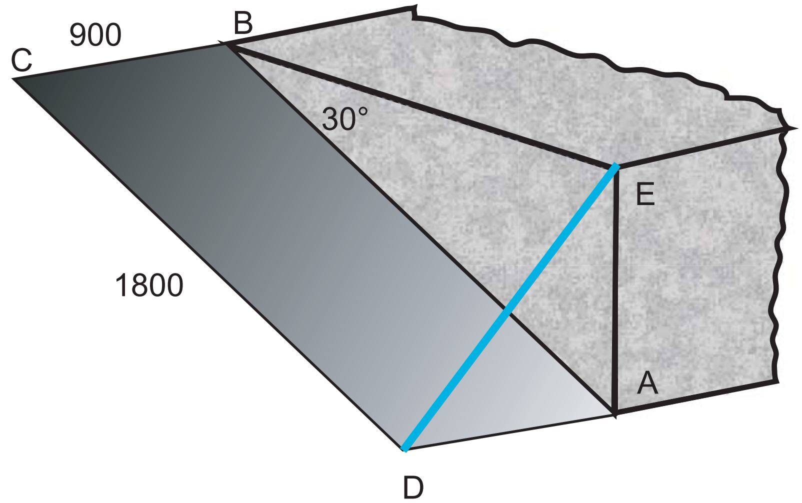
The cellar door weighs 50 kg. Hinge A can support thrust along the hinge axis AB, whereas hinge B supports force normal to the hinge axis only. Find the tension T in the wire ED. Units: N, mm.





Example

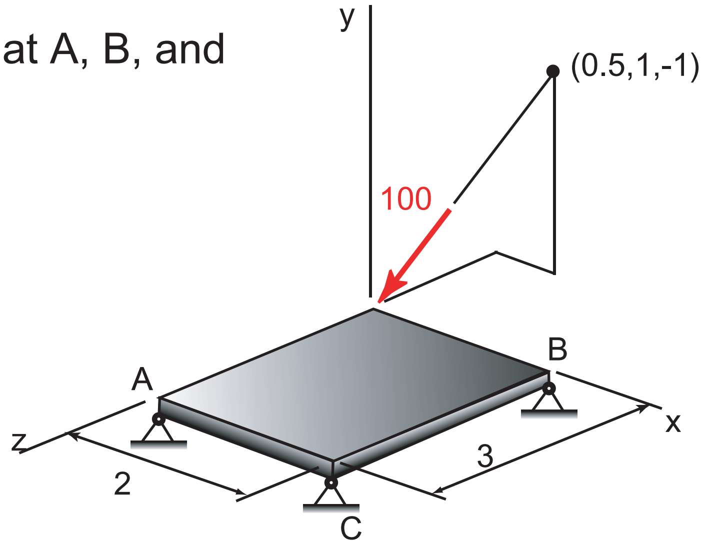
The cellar door weighs 50 kg. Hinge A can support thrust along the hinge axis AB, whereas hinge B supports force normal to the hinge axis only. Find the reactions at A and B.



Example

Determine the vertical reactions at A, B, and C. Units: N, m.

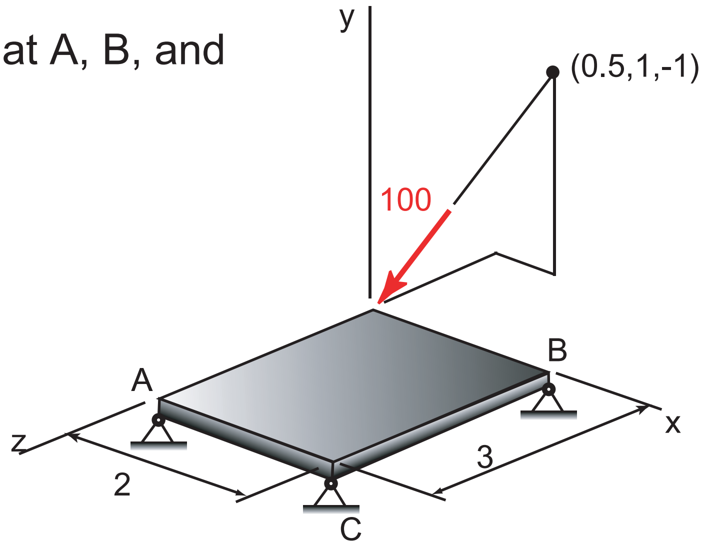
Solution #1:



Example

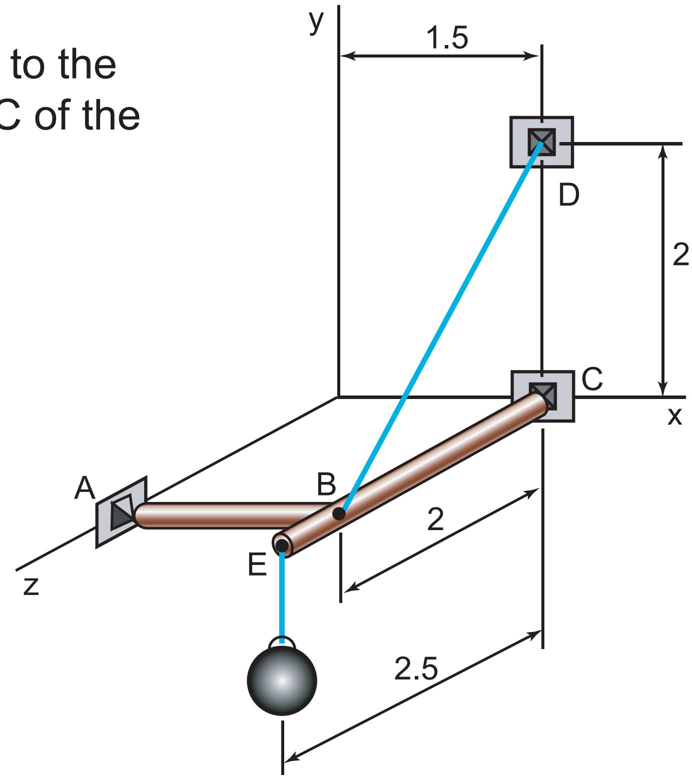
Determine the vertical reactions at A, B, and C. Units: N, m.

Solution #2:



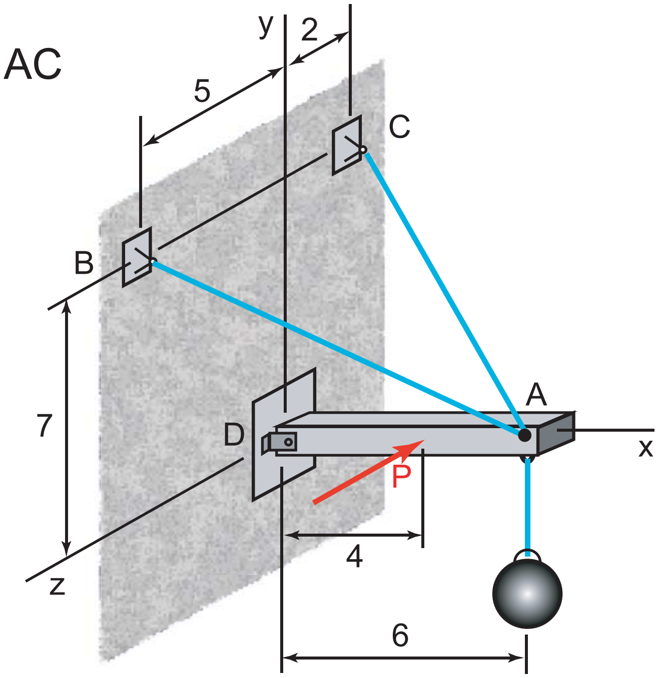
Example

Determine the tension in BD due to the 100 kg sphere. Supports A and C of the right angle pipes are pinned.
Units: N, m.



Example

Find the force P so that the tension in AC is 0. The sphere weighs 150 lb.
Units: Lb, ft.



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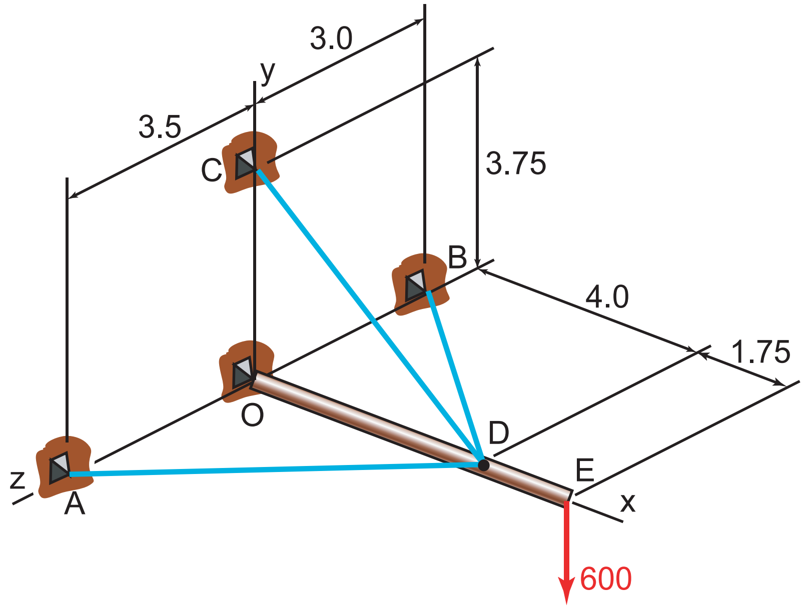
$$\vec{T}_{AC} = T_{AC}(-0.636\vec{i} + 0.742\vec{j} - 0.212\vec{k})$$

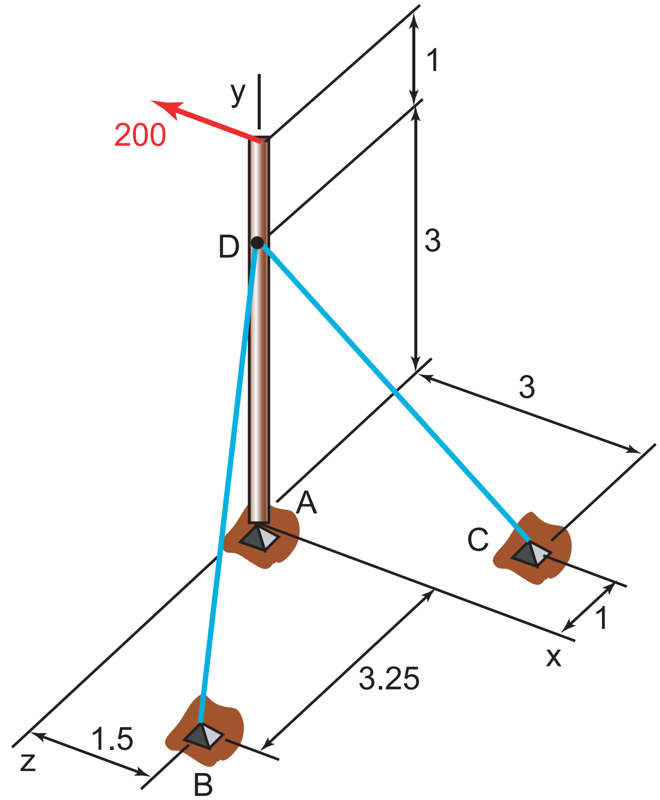
Example

Determine the tension in CD.
Units: N, m.



Example

Determine the force in each wire.
Units: N, m.

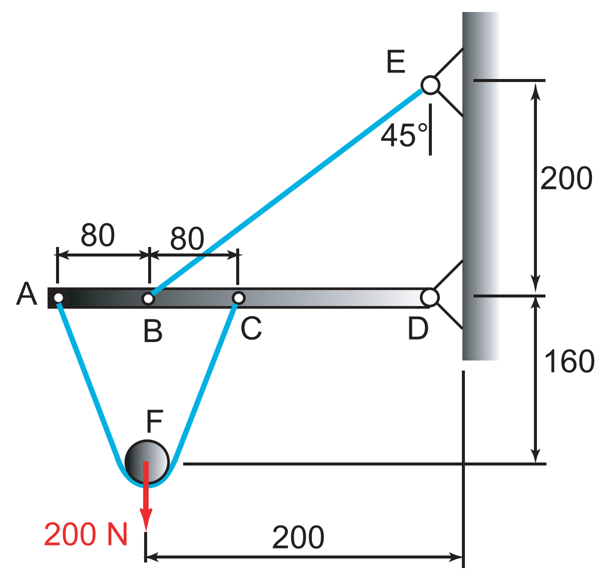


Summary

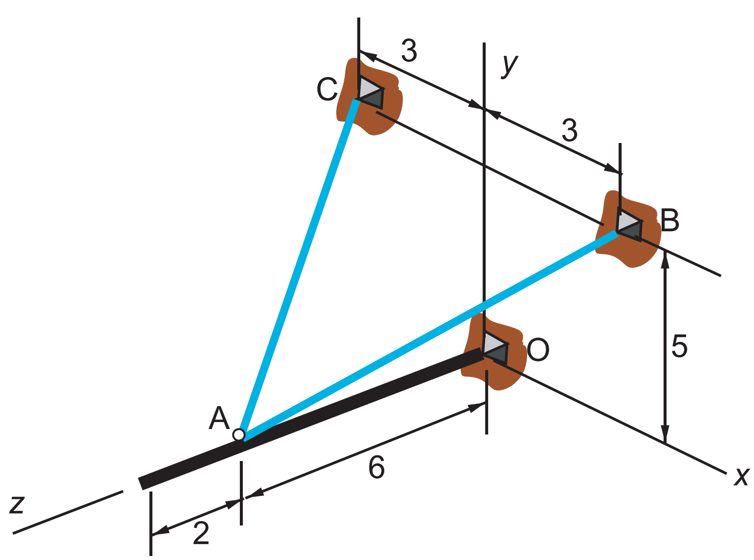
FBD

2D Equilibrium

Mapping

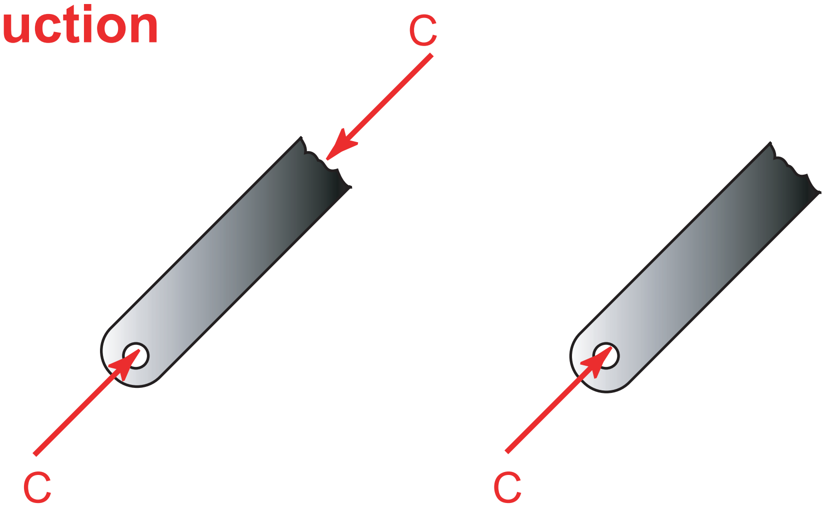


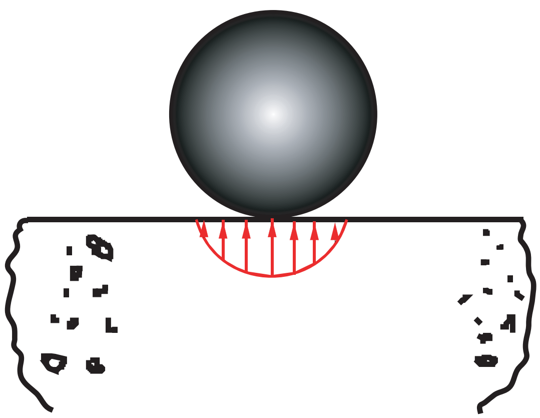
3D Equilibrium



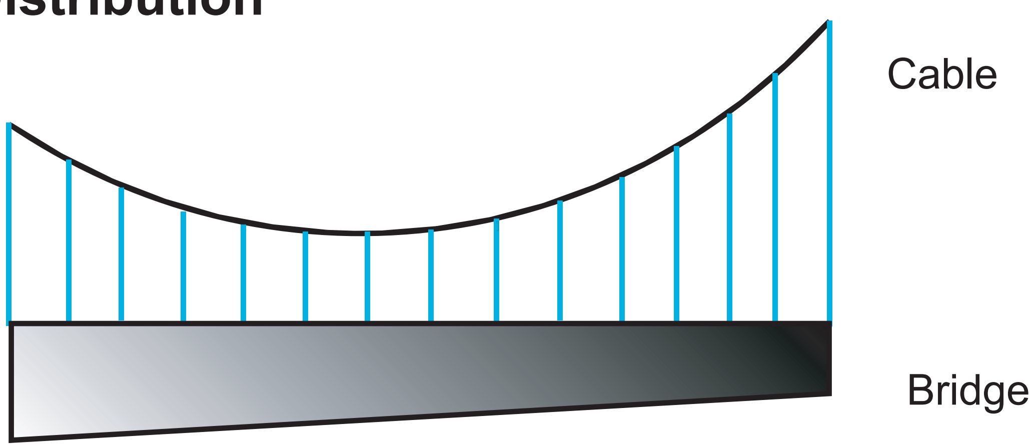
Chapter 5
Distributed Forces

Introduction

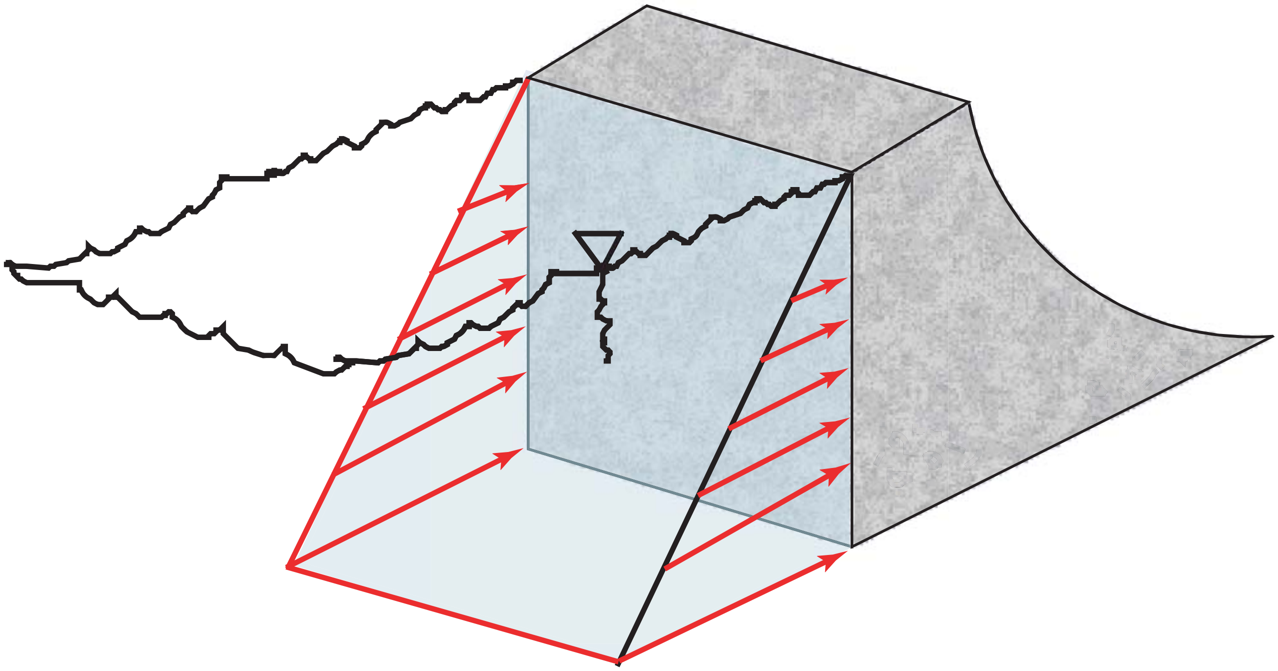




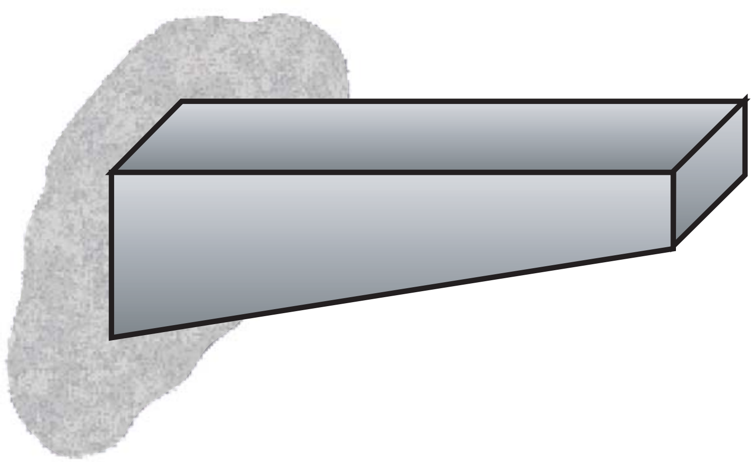
Line Distribution



Area Distribution



Volume Distribution

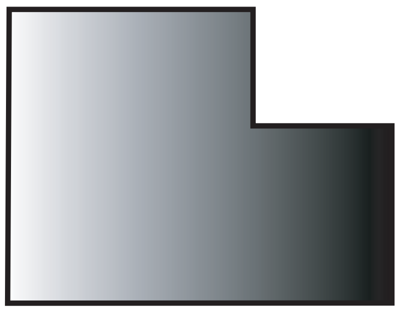


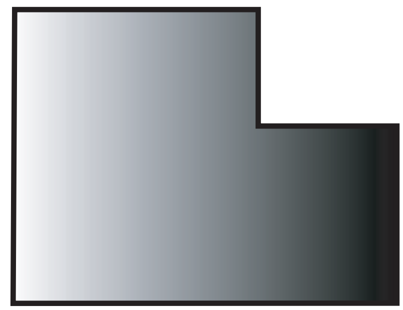
Center of Gravity of a Two-Dimensional Body

Composite Plates and Wires

Example

Find a general equation to locate the center of mass. Assume uniform thickness and homogeneous (same material).





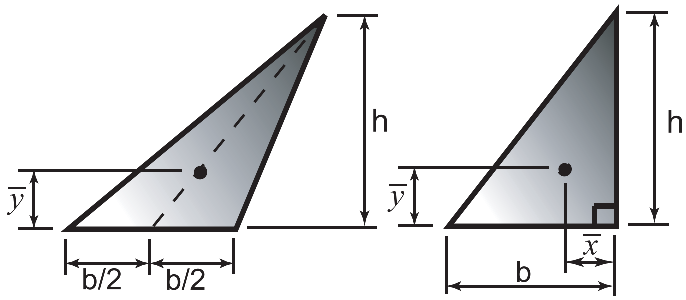
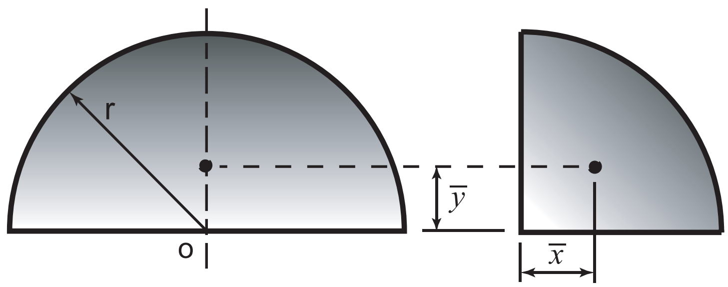
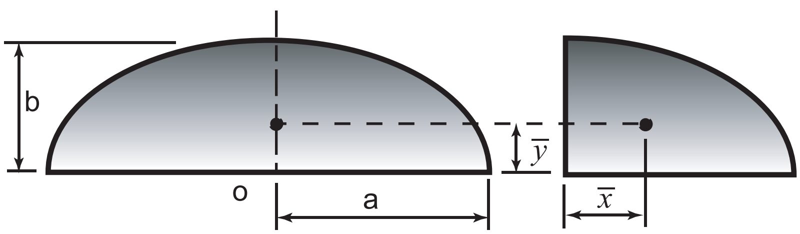
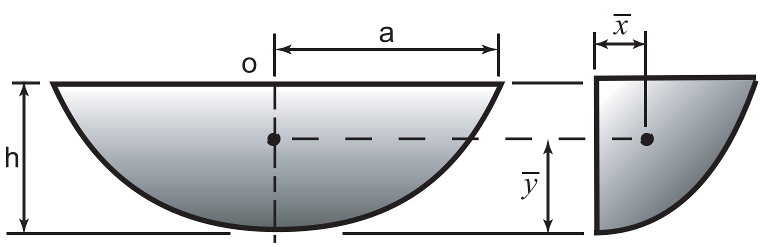
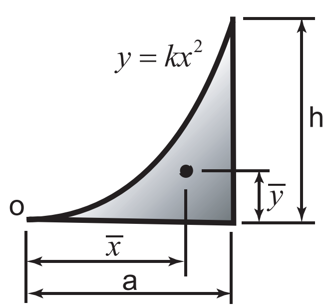
$$\overline{X} = \frac{\sum \overline{x} A}{\sum A} \qquad \overline{Y} = \frac{\sum \overline{y} A}{\sum A} \qquad \overline{Z} = \frac{\sum \overline{z} A}{\sum A}$$

First Moment of Areas and Lines

$$\overline{X} = \frac{\sum \overline{x} A}{\sum A} \qquad \overline{Y} = \frac{\sum \overline{y} A}{\sum A} \qquad \overline{Z} = \frac{\sum \overline{z} A}{\sum A}$$

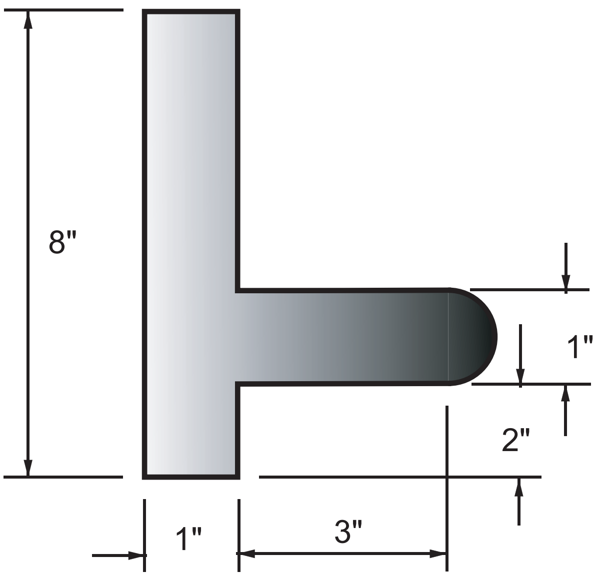
$$Q_x = \sum \overline{x} A \qquad Q_y = \sum \overline{y} A \qquad Q_z = \sum \overline{z} A$$

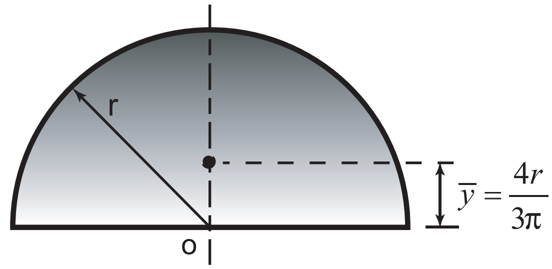
Centroids of Areas

Shape		$\bar{x}$	$\bar{y}$	Area
Triangle		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Semi-circle		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-circle		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semi-ellipse		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Quarter-ellipse		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Parabola		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Semi-parabola		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$

Example

Determine the centroid of the plane area. Units: In.





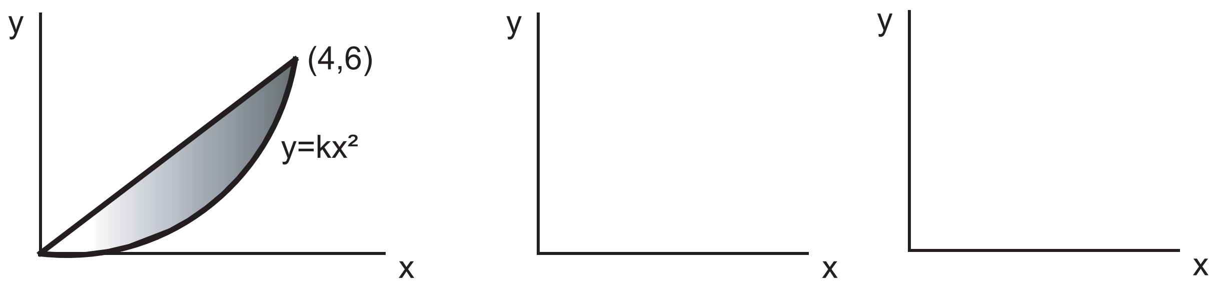
Part	Area	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$

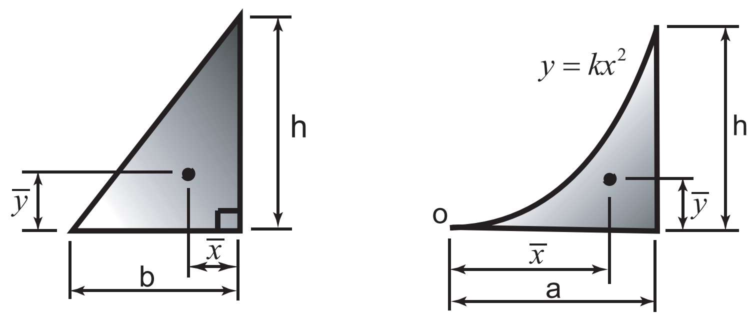
$$\bar{X} = \frac{\sum \bar{x}A}{\sum A}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A}$$

Example

Determine the centroid of the plane area. Units: In.





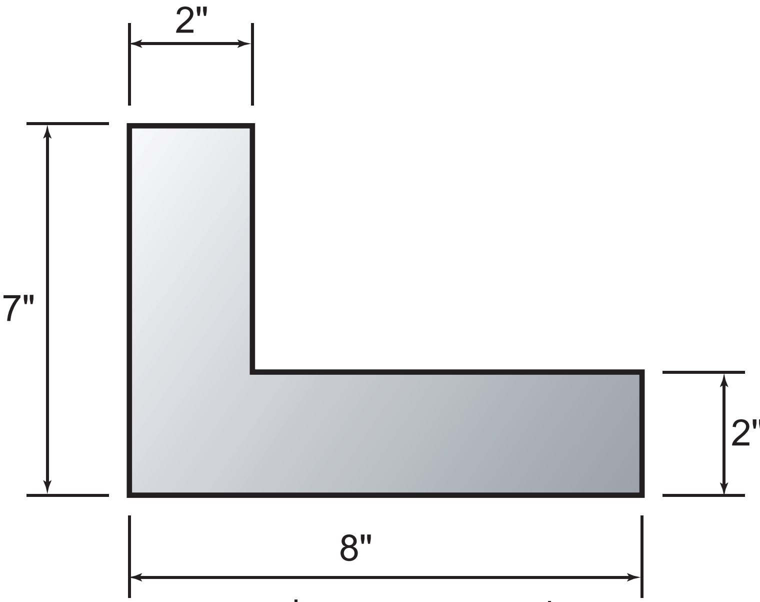
Part	Area	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A}$$

Example

Determine the centroid of the plane area. Units: In.



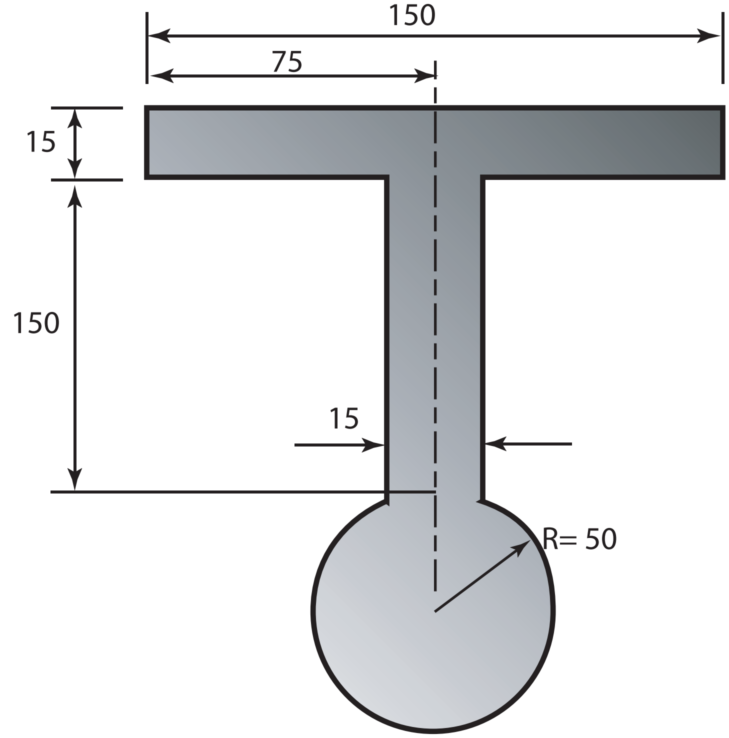
Part	Area	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
	______			______	______

$$\overline{X} = \frac{\sum \bar{x} A}{\sum A}$$

$$\overline{Y} = \frac{\sum \bar{y} A}{\sum A}$$

Example

Determine the centroid of the plane area. Units: mm.

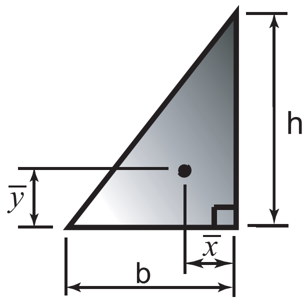


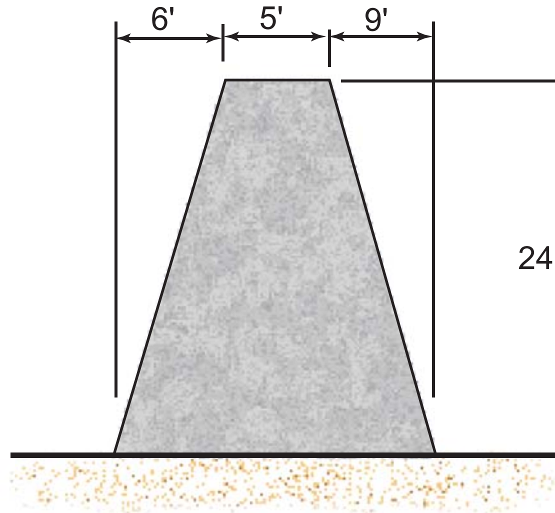
Part	Area	$\bar{y}$	$\bar{y}A$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A}$$

Example

Determine the X centroid of the dam.
Units: Ft.



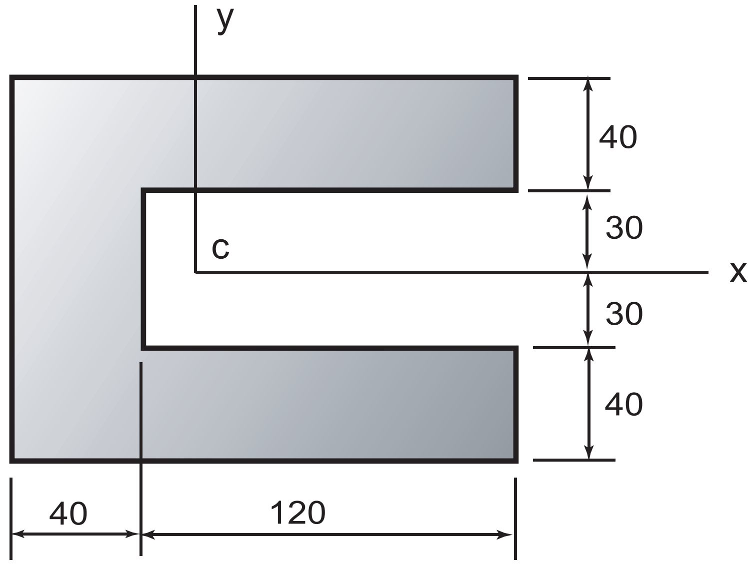


Part	Area	$\bar{x}$	$\bar{x}A$

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A}$$

Example

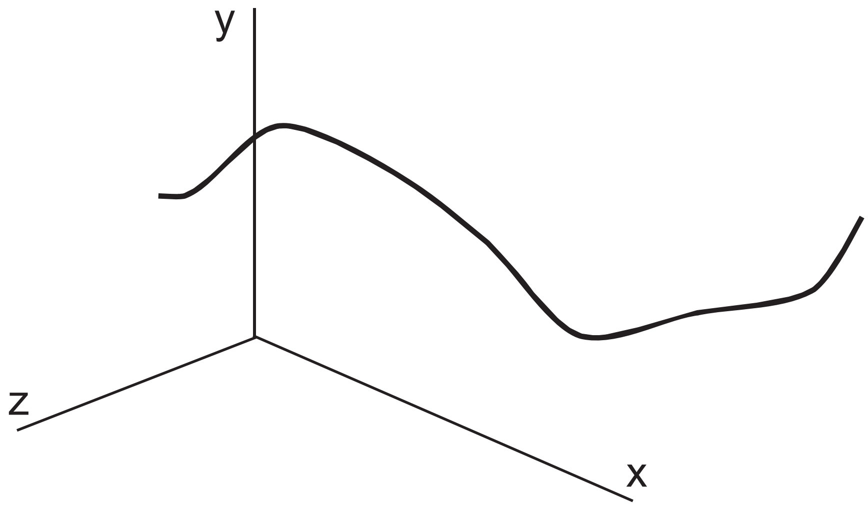
Determine the centroid of the plane area. Units: mm.



Part	Area	$\bar{x}$	$\bar{x}A$

$$\overline{X} = \frac{\sum \bar{x}A}{\sum A}$$

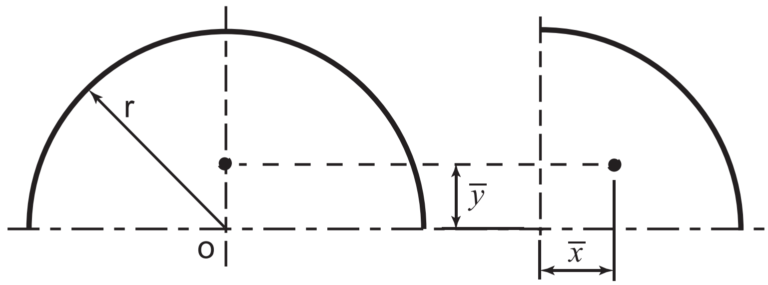
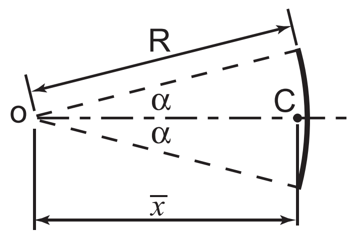
Centroids of Lines



$$\overline{X} = \frac{\sum \overline{x} L}{\sum L} \qquad \overline{Y} = \frac{\sum \overline{y} L}{\sum L} \qquad \overline{Z} = \frac{\sum \overline{z} L}{\sum L}$$

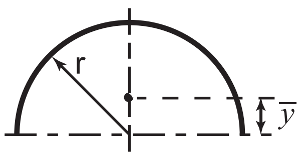
$$\overline{X} = \frac{\int_0^L x\,dL}{L} \qquad \overline{Y} = \frac{\int_0^L y\,dL}{L} \qquad \overline{Z} = \frac{\int_0^L z\,dL}{L}$$

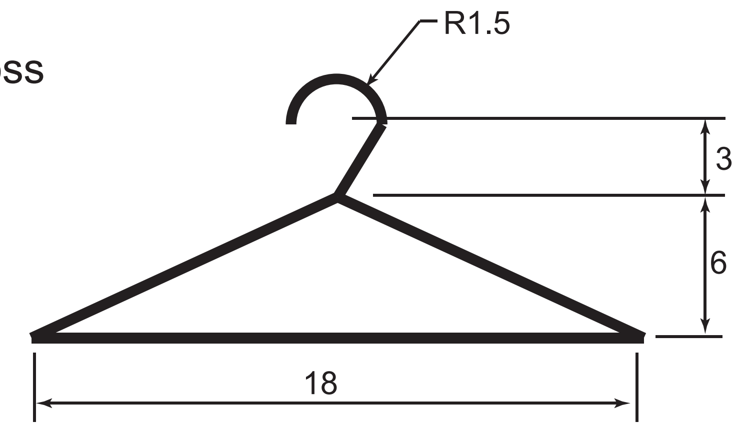
Centroids of Lines

<table>
<tr><th>Shape</th><th></th><th>$\bar{x}$</th><th>$\bar{y}$</th><th>L</th></tr>
<tr><td>Half-Circle</td><td rowspan="2">
</td><td>0</td><td>$\frac{2r}{\pi}$</td><td>πr</td></tr>
<tr><td>Quarter-Circle</td><td>$\frac{2r}{\pi}$</td><td>$\frac{2r}{\pi}$</td><td>$\frac{\pi r}{2}$</td></tr>
<tr><td>Arc</td><td>
</td><td>$\frac{r \sin\alpha}{\alpha}$</td><td>0</td><td>$2\alpha r$</td></tr>
</table>

Example

A thin steel wire of uniform cross section is bent into the shape shown. Locate the center of gravity. Units: In.





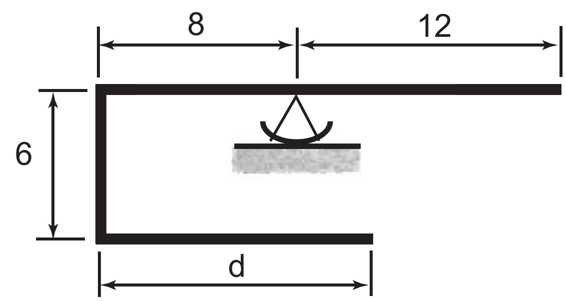
Part	Length	$\bar{x}$	$\bar{y}$	$\bar{x}L$	$\bar{y}L$

$$\bar{X} = \frac{\sum \bar{x}L}{\sum L}$$

$$\bar{Y} = \frac{\sum \bar{y}L}{\sum L}$$

Example

A thin steel wire of uniform cross section is bent into the shape shown. Determine the distance d to keep it aligned as shown. Units: In.

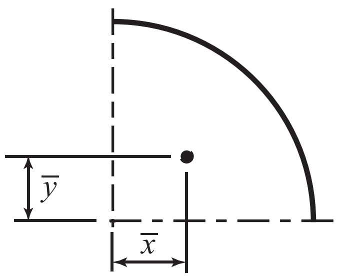


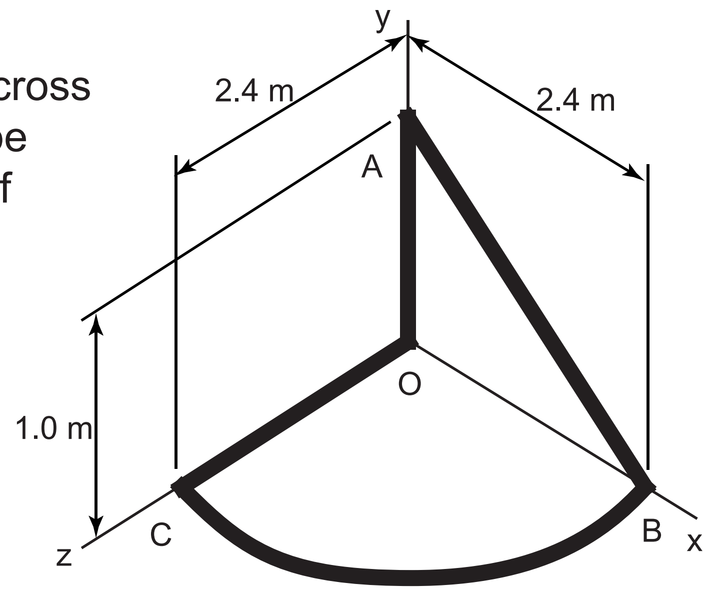
Part	Length	$\bar{x}$	$\bar{y}$	$\bar{x}L$	$\bar{y}L$
	____			____	____

$$\bar{X} = \frac{\sum \bar{x}L}{\sum L}$$

Example

A thin steel wire of uniform cross section is bent into the shape shown. Locate the center of gravity.





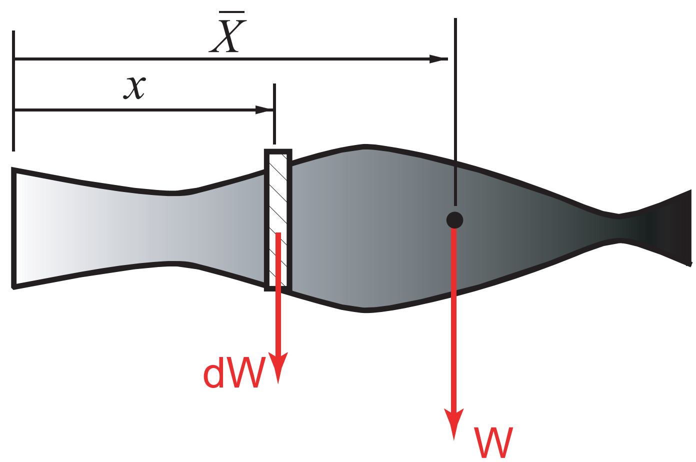
Part	Length	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\bar{x}L$	$\bar{y}L$	$\bar{z}L$

$$\bar{X} = \frac{\sum \bar{x}L}{\sum L}$$

$$\bar{Y} = \frac{\sum \bar{y}L}{\sum L}$$

$$\bar{Z} = \frac{\sum \bar{z}L}{\sum L}$$

Determination of Centroids by Integration



$$W = \int_0^L dW \qquad dm = xdW$$

$$\bar{X}W = \int_0^L xdW$$

Since the moments in each system must be equal,

$$\bar{X} = \frac{\int_0^L xdW}{W} \qquad \bar{Y} = \frac{\int_0^L ydW}{W} \qquad \bar{Z} = \frac{\int_0^L zdW}{W}$$

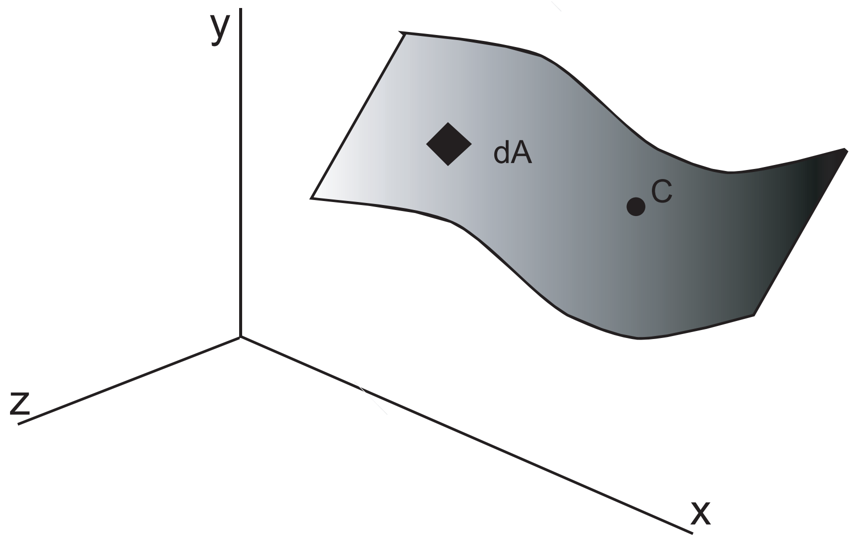
or with W= Mg and dW= gdM,

$$\bar{X} = \frac{\int_0^L xdM}{M} \qquad \bar{Y} = \frac{\int_0^L ydM}{M} \qquad \bar{Z} = \frac{\int_0^L zdM}{M}$$

or with M= ρV and dM= ρdV,

$$\bar{X} = \frac{\int_0^L x\rho dV}{\rho V} \qquad \bar{Y} = \frac{\int_0^L y\rho dV}{\rho V} \qquad \bar{Z} = \frac{\int_0^L z\rho dV}{\rho V}$$

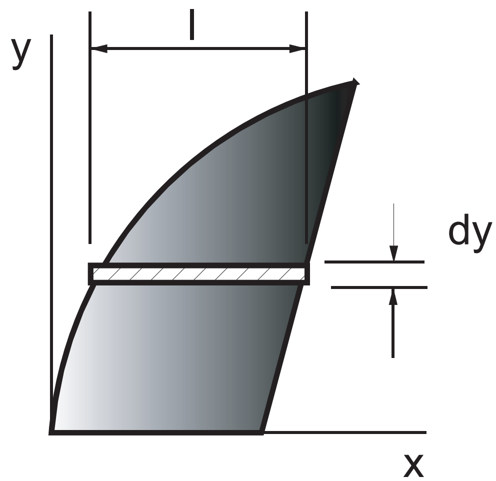
Areas

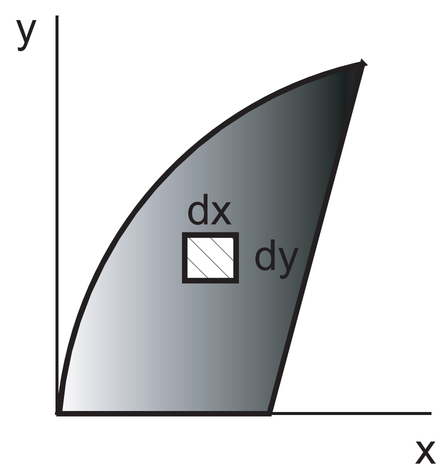


$$\overline{X} = \frac{\int_0^L x dA}{A} \qquad \overline{Y} = \frac{\int_0^L y dA}{A} \qquad \overline{Z} = \frac{\int_0^L z dA}{A}$$

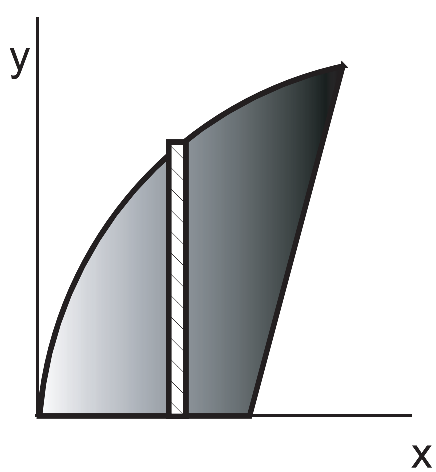
Choice of Element Integration

Order of Element

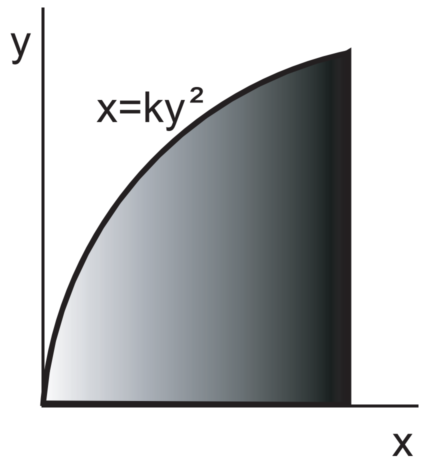


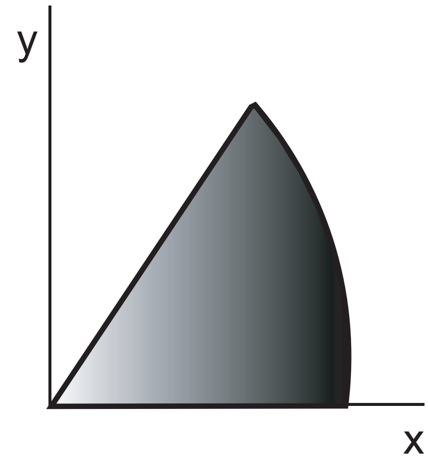


Continuity

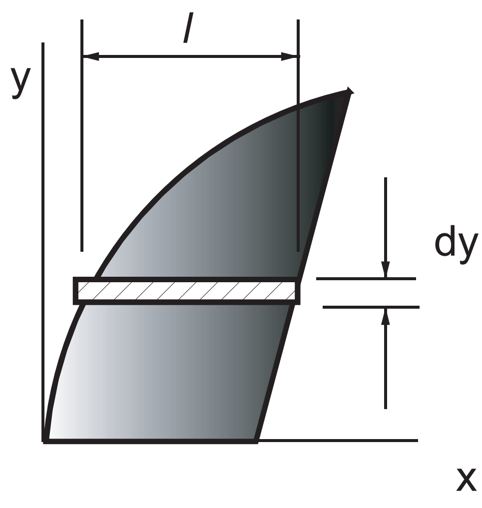


Choice of Coordinates



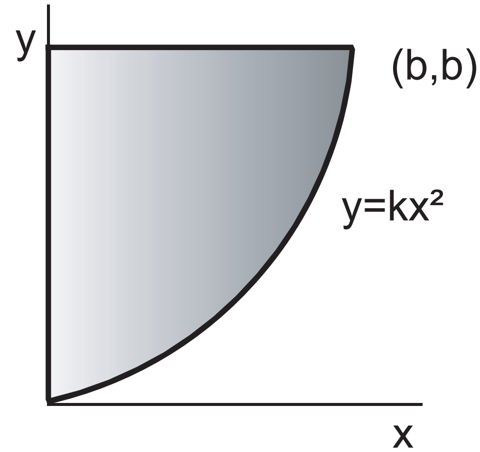


Centroidal Coordinates of the Element



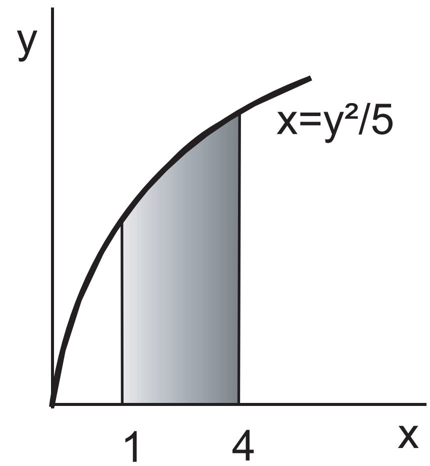
Example

Using integration, determine the coordinates of the centroid of the shaded area.



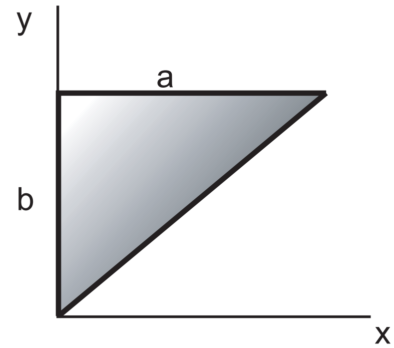
Example

Using integration, determine the coordinates of the centroid of the shaded area.



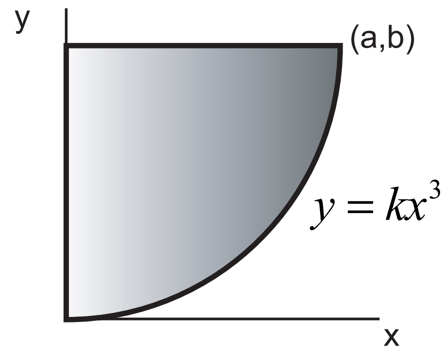
Example

Determine the centroid of the area.
Use integration.



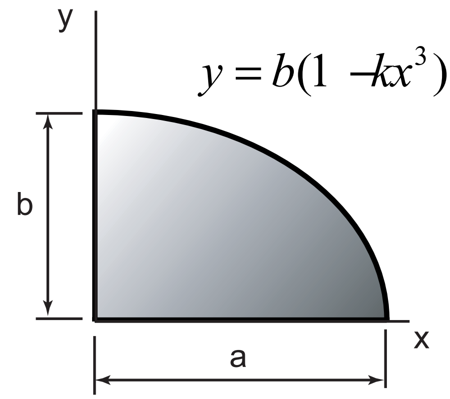
Example

Determine the centroid of the area. Use integration.



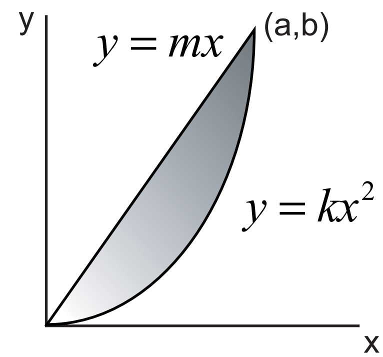
Example

Determine the x centroid of the area.
Use integration.



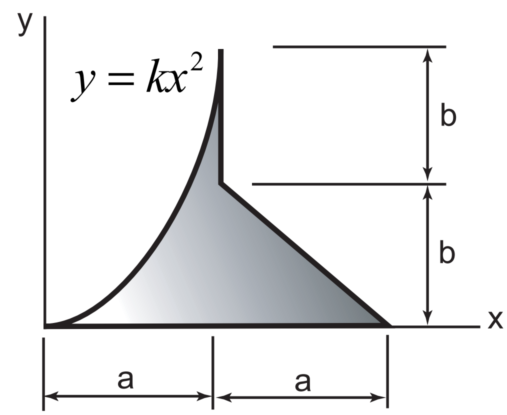
Example

Determine the centroid of the area.
Use integration.



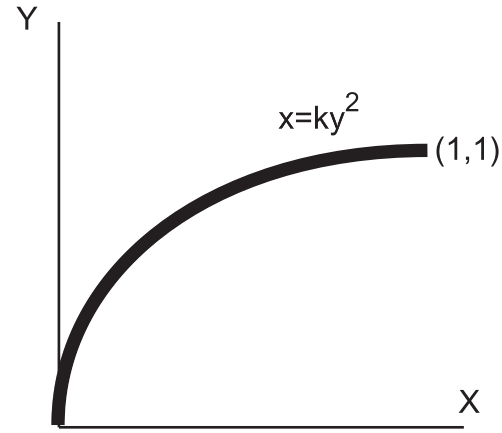
Example

Determine the x centroid of the area.
Use integration.



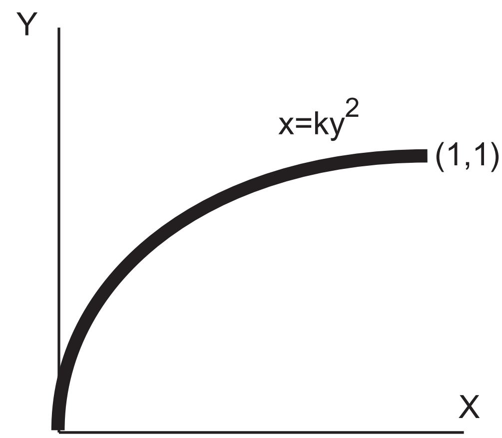
Example

Determine the length of the homogeneous rod. Use integration.



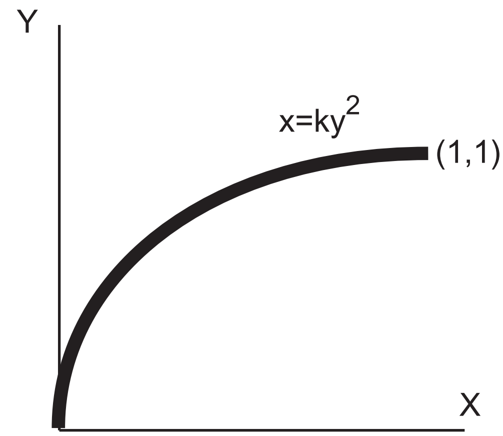
Example

Determine the x centroid of the homogeneous rod. Use integration.



Example

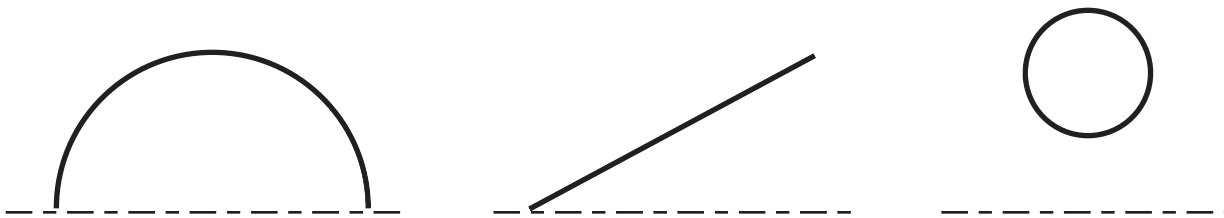
Determine the y centroid of the homogeneous rod. Use integration.



Theorems of Pappus-Guldinus: Theorem I

The area of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid of the curve while the surface is being generated.

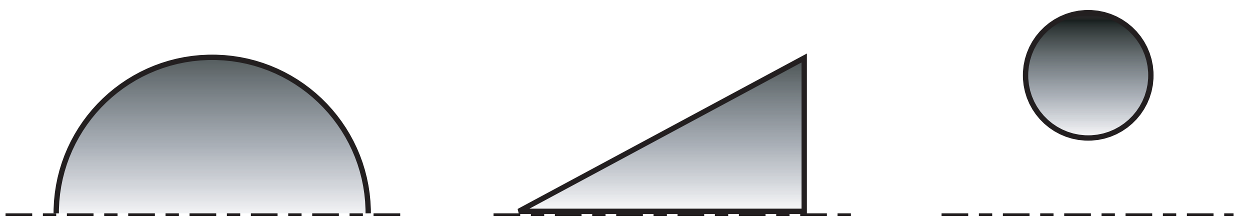
$$A = 2\pi \bar{y} L$$



Theorems of Pappus-Guldinus-Theorem II

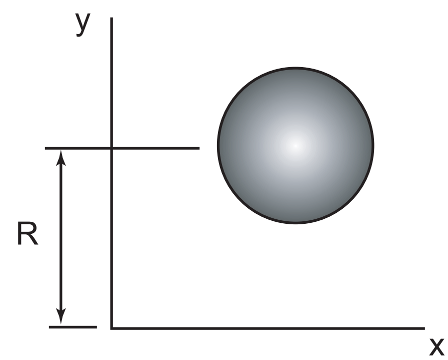
The volume of a body of revolution is equal to the generating area times the distance traveled by the centroid of the area while the body is being generated.

$$V = 2\pi \bar{y} A$$



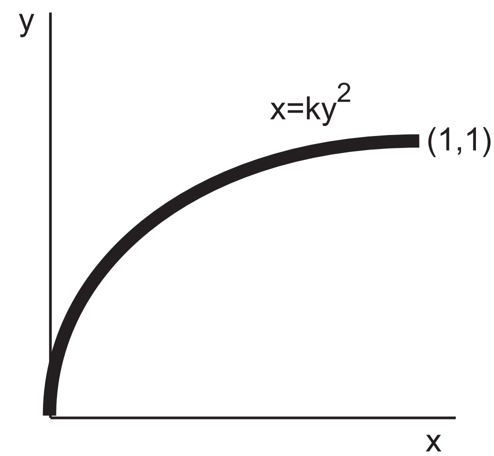
Example

Determine the surface area of a circle with radius r rotated about the x-axis forming a torus.



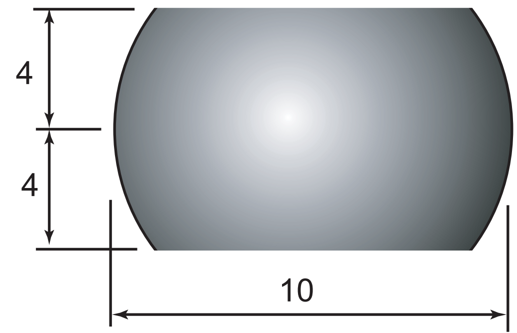
Example

Determine the surface area of the parabolic shape if it is rotated 180˚ about the x-axis. Units: in.



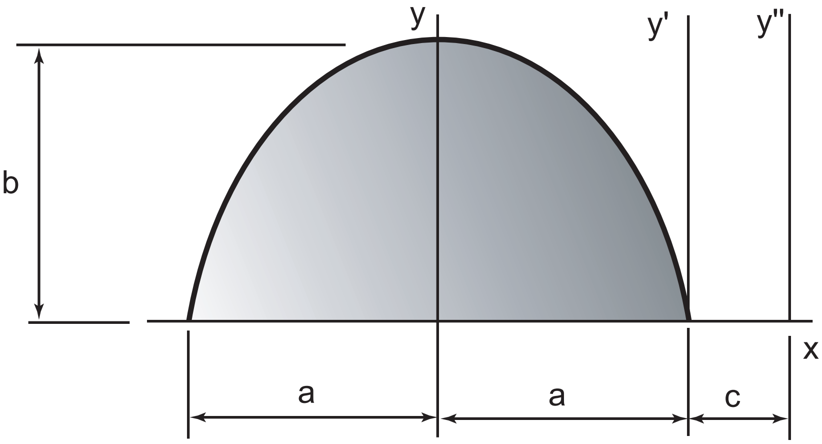
Example

Determine the surface area and volume of a ball that has been cut off on the top and bottom as shown. Units: In.



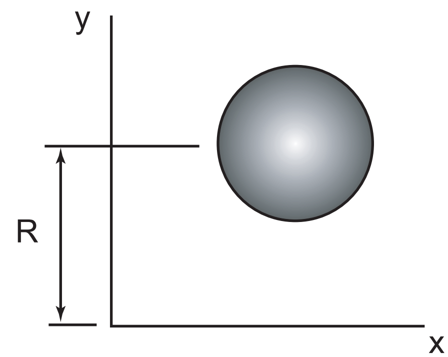
Example

Determine the volume of the semi-elliptical shape rotated about the y, y' and y'' axis.



Example

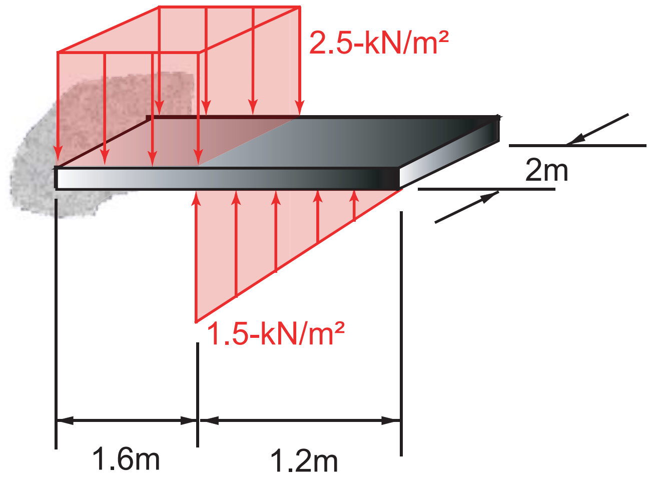
Determine volume of a circle with radius r rotated about the x-axis forming a torus.

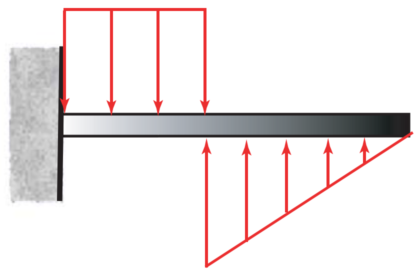


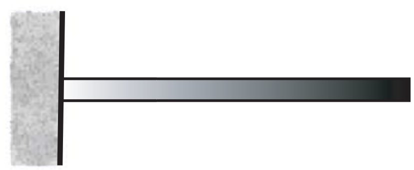
Distributed Loads on Beams

Example

Determine the support reactions of the cantilever beam. Units: kN, m.







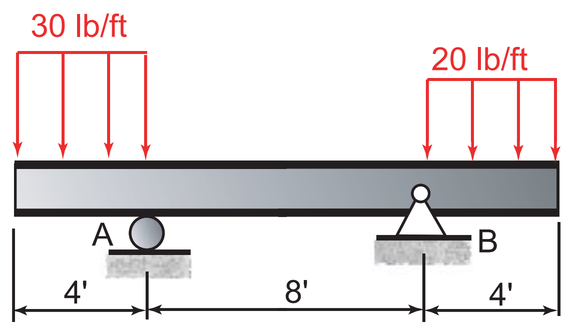
Conclusion:

-When given a distributed load with the units of force per length the resultant is equal to the area under the curve.

-When given a distributed load with the units of force per area the resultant is equal to the volume under the curve.

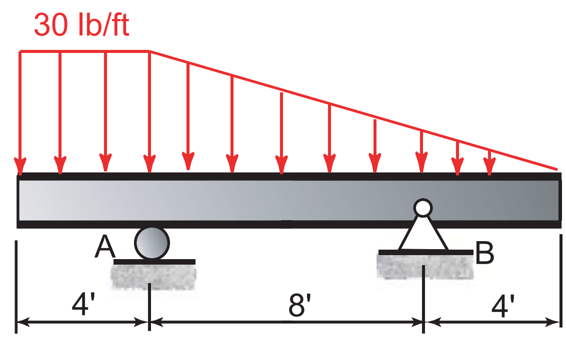
Example

Determine the reactions at A and B.
Units: Lb, ft.



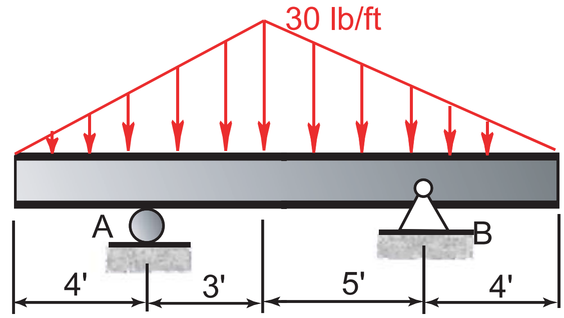
Example

Determine the reactions at A and B.
Units: Lb, ft.



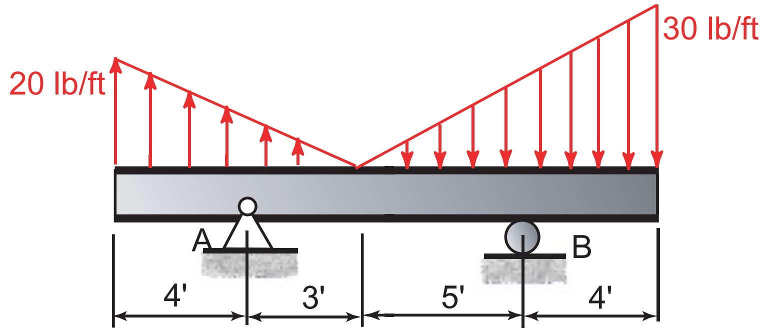
Example

Determine the reactions at A and B.
Units: Lb, ft.

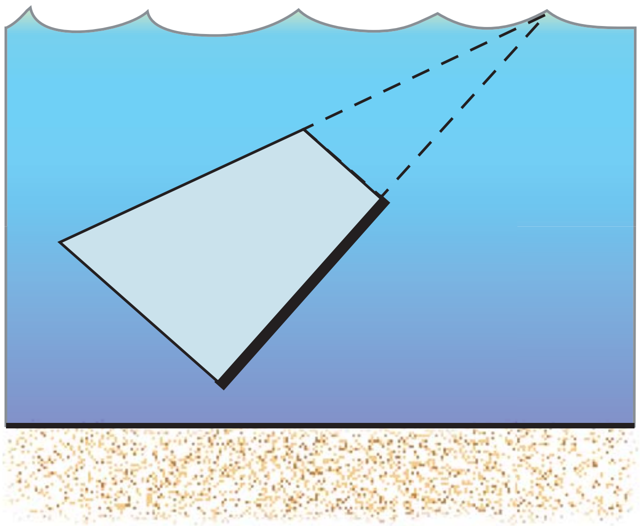


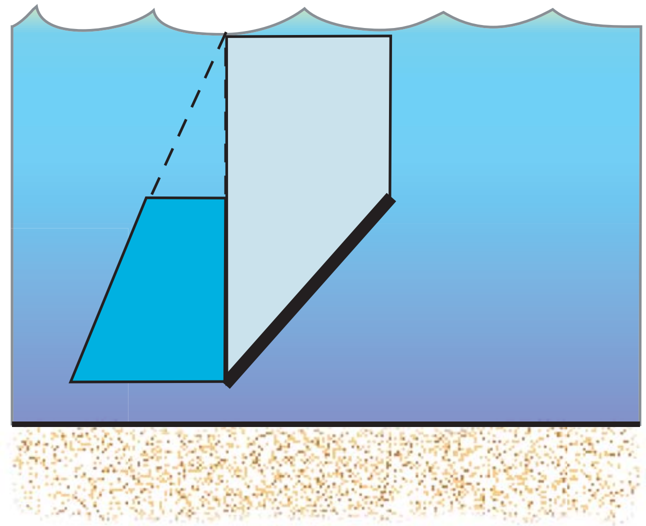
Example

Determine the reactions at A and B. Units: Lb, ft.

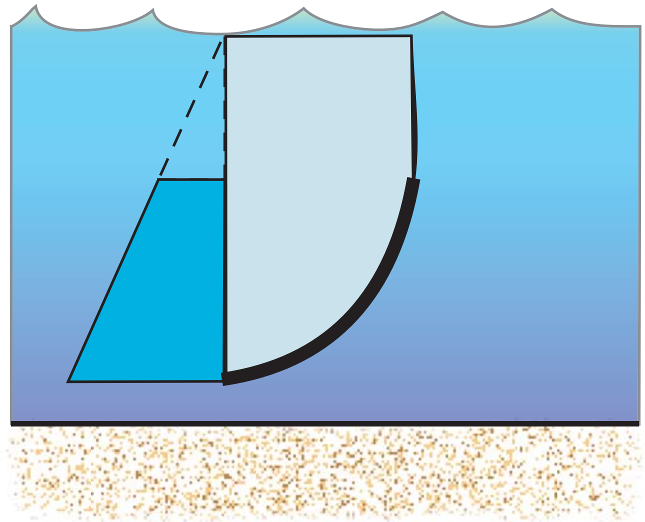


Forces on Submerged Surfaces- Flat Surfaces



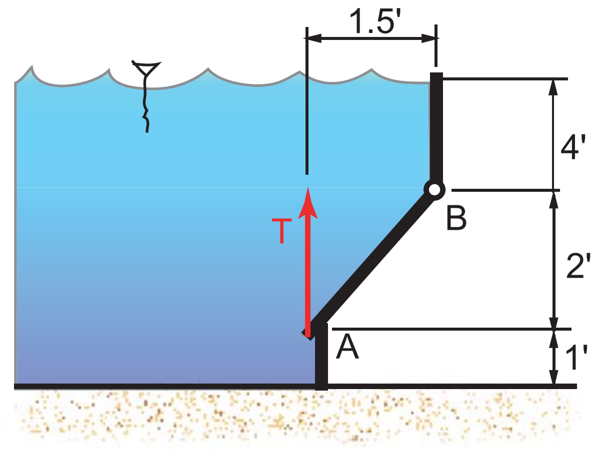


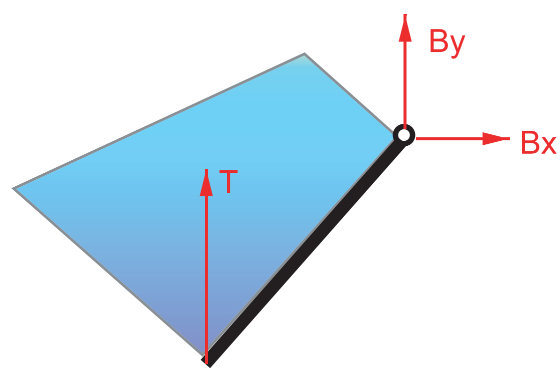
Forces on Submerged Surfaces- Curved Surfaces



Example

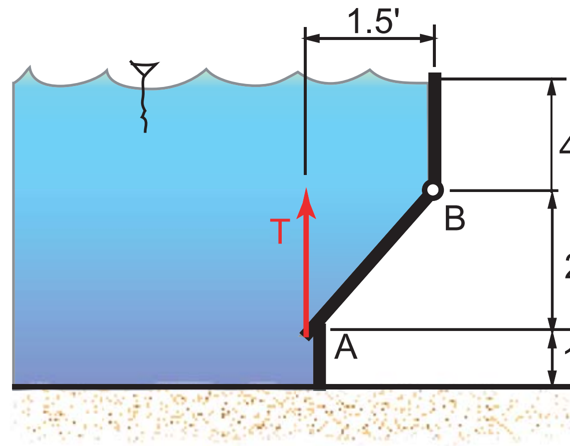
Determine the minimum tension required to open the gate. The gate AB is 2.25 ft wide.
Units: Lb, ft.

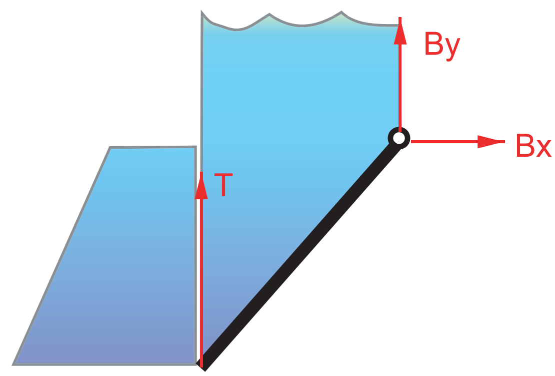




Example

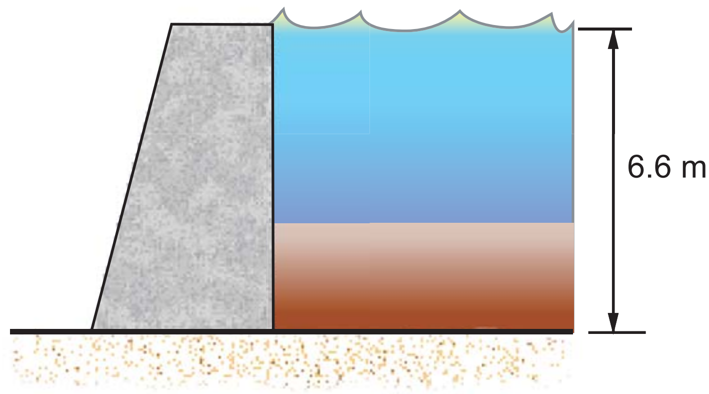
Determine the minimum tension required to open the gate. The gate AB is 2.25 ft wide.
Units: Lb, ft.

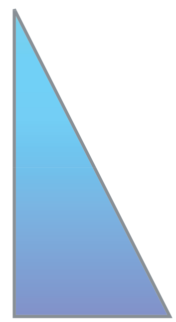


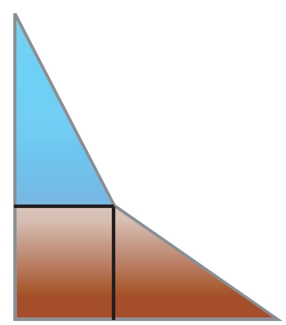


Example

The dam is designed to withstand the additional force caused by silt. Assuming that silt is equivalent to a liquid of density of 1800 kg/m³ and considering a 1 m wide section of dam, determine the force acting on the dam face for a slit accumulation of depth 2 m. Units: N, m.

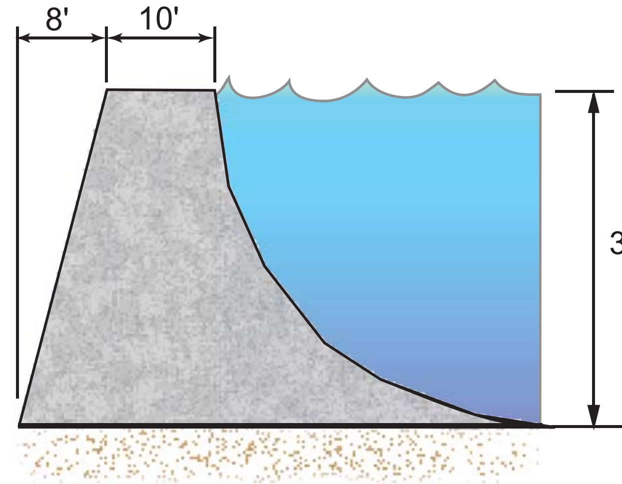






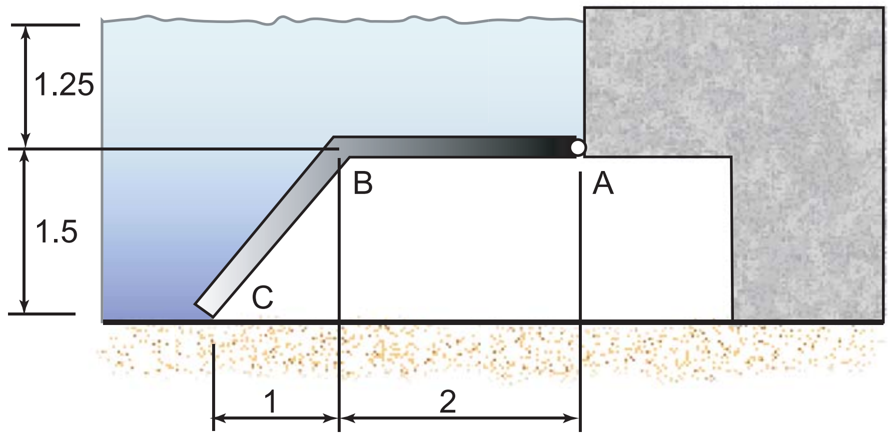
Example

Determine the resultant force due to the water on the face of the dam. Also find the forces under the concrete dam. Units: Lb, ft.



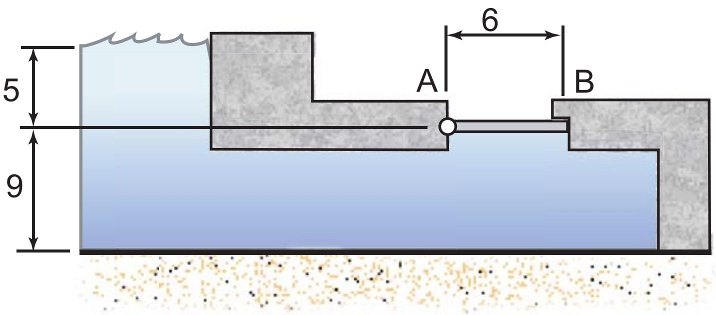
Example

Find the reactions at A and C. Point C is a frictionless surface. The width of the gate is 1.5 m. Units: N, m



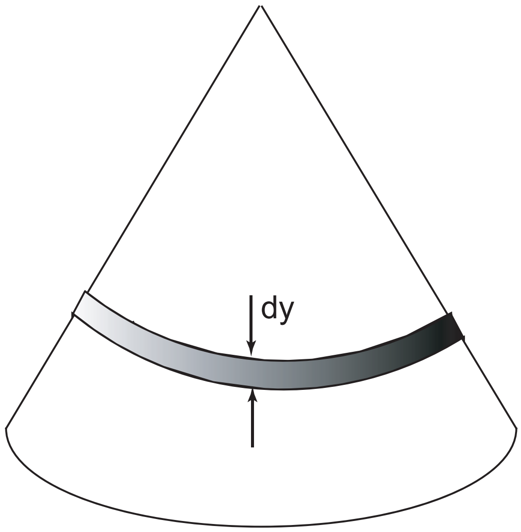
Example

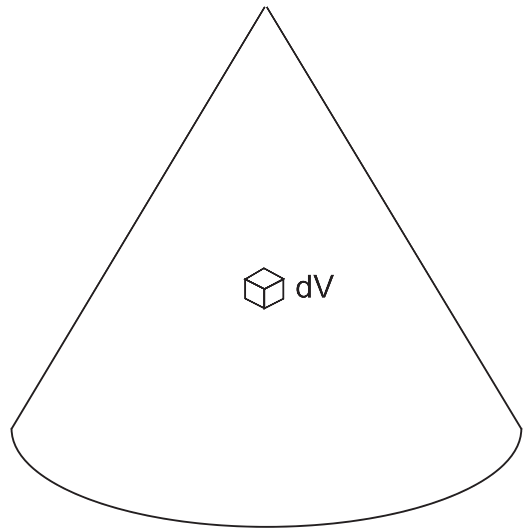
Determine the reactions at A and B. The gate is 8' wide.
Units: Ft



Center of Gravity of a Three-Dimensional Body

Centroid of a Volume

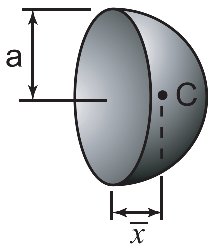
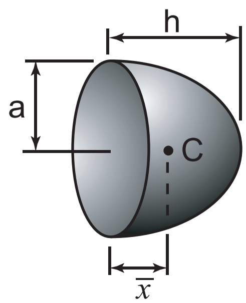
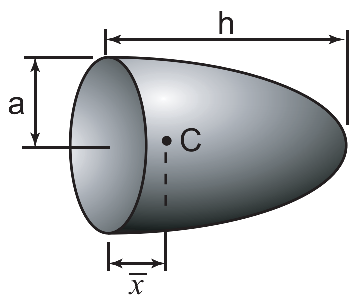
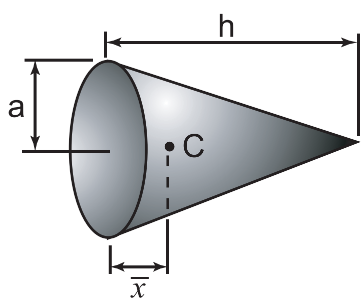
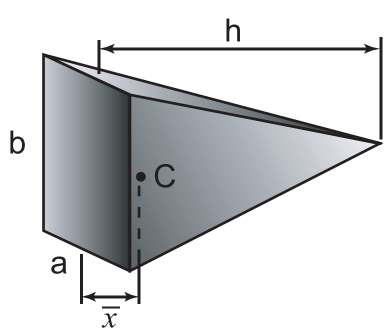




$$\overline{X} = \frac{\sum \overline{x}V}{\sum V} \qquad \overline{Y} = \frac{\sum \overline{y}V}{\sum V} \qquad \overline{Z} = \frac{\sum \overline{z}V}{\sum V}$$

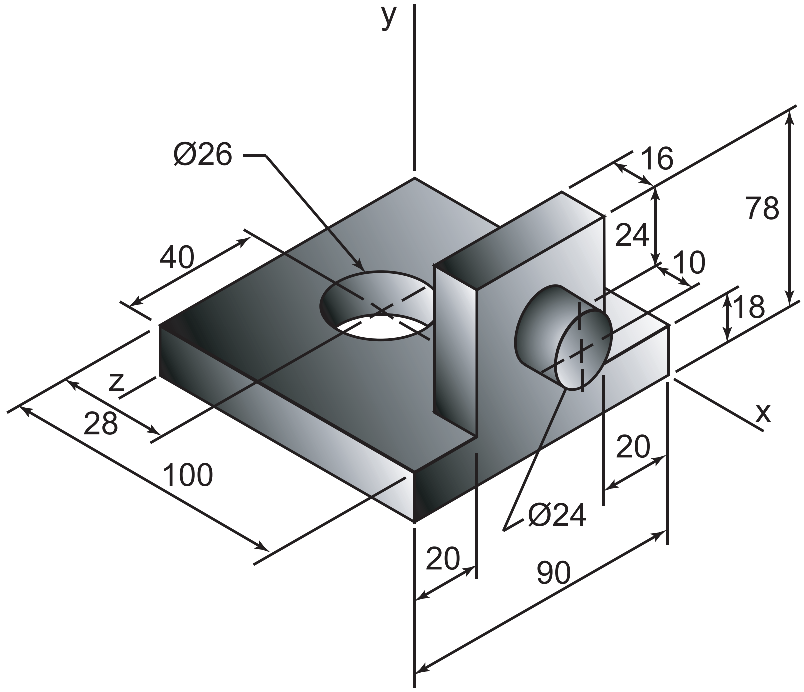
$$\overline{X} = \frac{\int_0^L x\,dV}{\int_0^L dV} \qquad \overline{Y} = \frac{\int_0^L y\,dV}{\int_0^L dV} \qquad \overline{Z} = \frac{\int_0^L z\,dV}{\int_0^L dV}$$

Centroids of Volumes

Shape		$\bar{x}$	Volume
Hemisphere		$\frac{3a}{8}$	$\frac{2}{3}\pi a^3$
Semiellipsoid of revolution		$\frac{3h}{8}$	$\frac{2}{3}\pi a^2 h$
Paraboloid of revolution		$\frac{h}{3}$	$\frac{1}{2}\pi a^2 h$
Cone		$\frac{h}{4}$	$\frac{1}{3}\pi a^2 h$
Pyramid		$\frac{h}{4}$	$\frac{1}{3}abh$

Example

For the machine element shown, locate the centroid. Units: mm.



Part	Volume	X_{el}	Y_{el}	Z_{el}	$X_{el}V$	$Y_{el}V$	$Z_{el}V$
	____				____	____	____

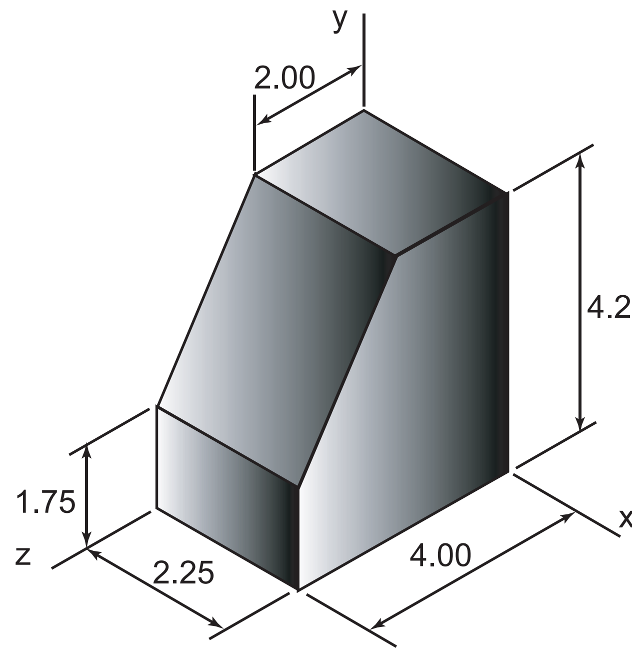
$$\overline{X} = \frac{\sum \overline{x}V}{\sum V}$$

$$\overline{Y} = \frac{\sum \overline{y}V}{\sum V}$$

$$\overline{Z} = \frac{\sum \overline{z}V}{\sum V}$$

Example

For the machine element shown, locate the centroid. Units: in.



Part	Volume	Xel	Yel	Zel	XelV	YelV	ZelV

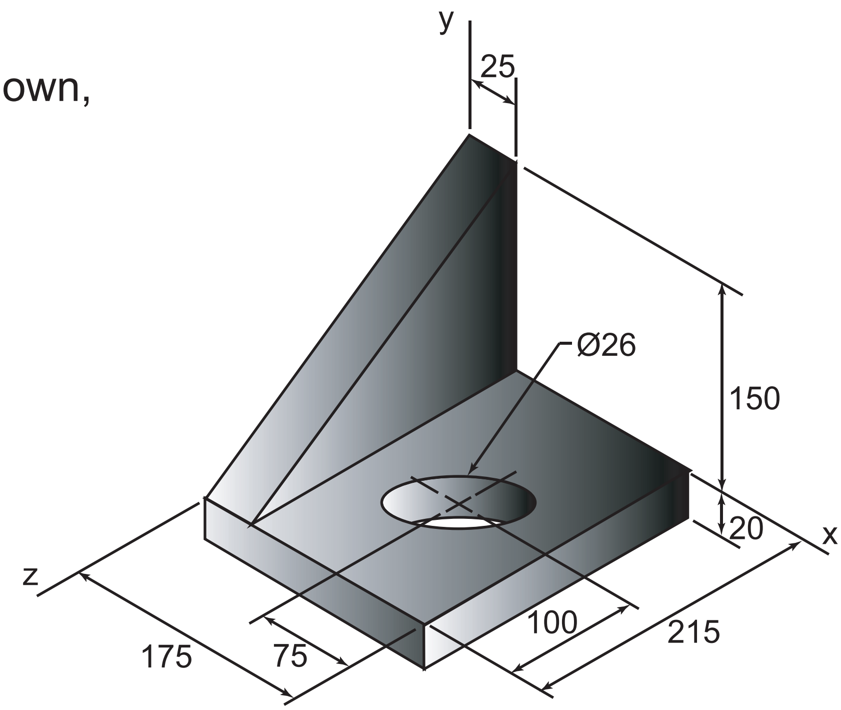
$$\overline{X} = \frac{\sum \overline{x}V}{\sum V}$$

$$\overline{Y} = \frac{\sum \overline{y}V}{\sum V}$$

$$\overline{Z} = \frac{\sum \overline{z}V}{\sum V}$$

Example

For the machine element shown, locate the centroid.
Units: mm.



Part	Volume	X_{el}	Y_{el}	Z_{el}	$X_{el}V$	$Y_{el}V$	$Z_{el}V$
	____				____	____	____

$$\bar{X} = \frac{\sum \bar{x}V}{\sum V}$$

$$\bar{Y} = \frac{\sum \bar{y}V}{\sum V}$$

$$\bar{Z} = \frac{\sum \bar{z}V}{\sum V}$$

Summary

Centroids Vs. Center of Masses

Centroids of Lines

$$\overline{X} = \frac{\sum \overline{x}L}{\sum L} \qquad \overline{Y} = \frac{\sum \overline{y}L}{\sum L} \qquad \overline{Z} = \frac{\sum \overline{z}L}{\sum L}$$

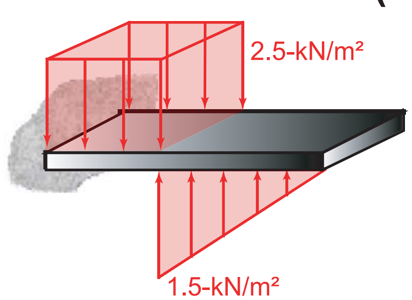
Centroids of Areas

$$\overline{X} = \frac{\sum \overline{x}A}{\sum A} \qquad \overline{Y} = \frac{\sum \overline{y}A}{\sum A} \qquad \overline{Z} = \frac{\sum \overline{z}A}{\sum A}$$

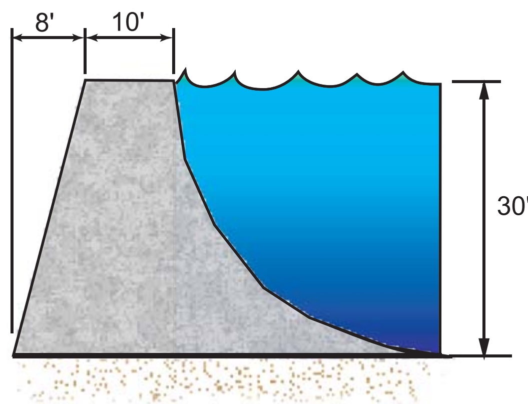
Centroids of Volumes

$$\overline{X} = \frac{\sum \overline{x}V}{\sum V} \qquad \overline{Y} = \frac{\sum \overline{y}V}{\sum V} \qquad \overline{Z} = \frac{\sum \overline{z}V}{\sum V}$$

Distributed Loads (Force/Length)



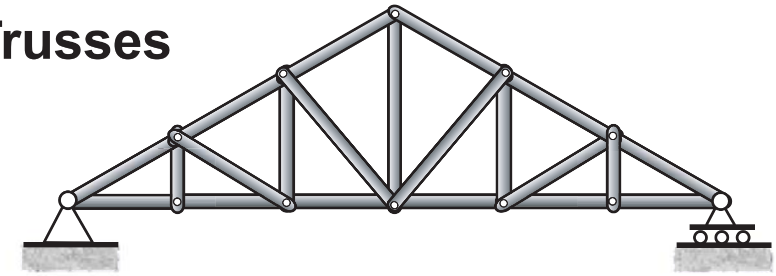
Pressures (Force/Area)



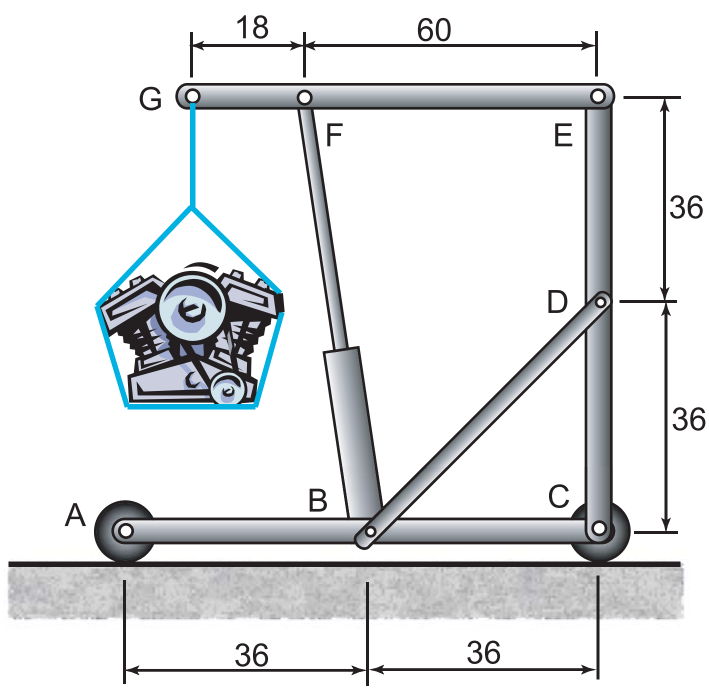
Chapter 6
Analysis of Structures

Introduction

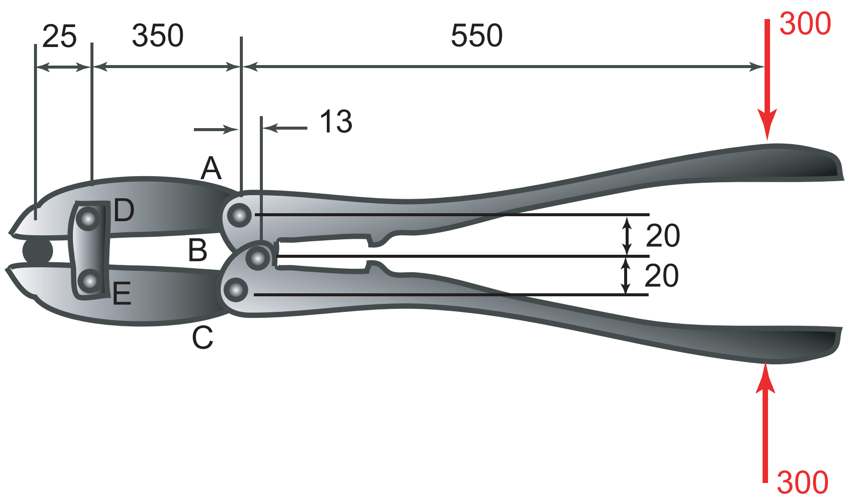
Trusses



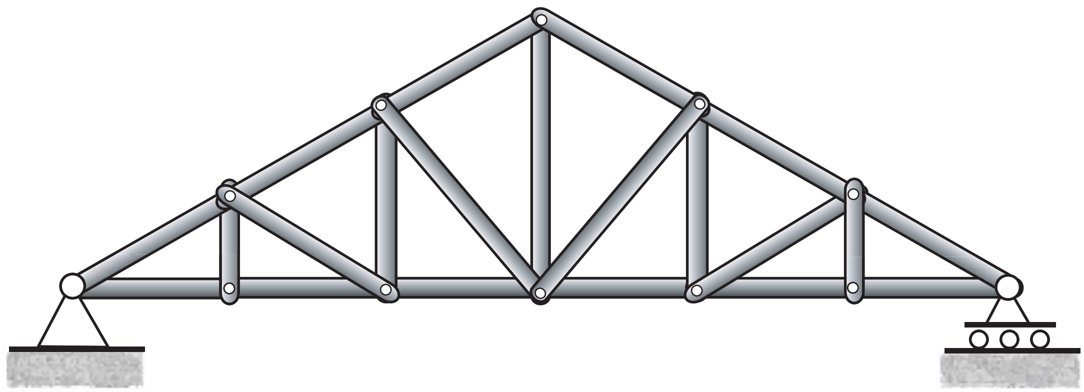
Frames



Machines



Plane Trusses



-Loads act only at the joints

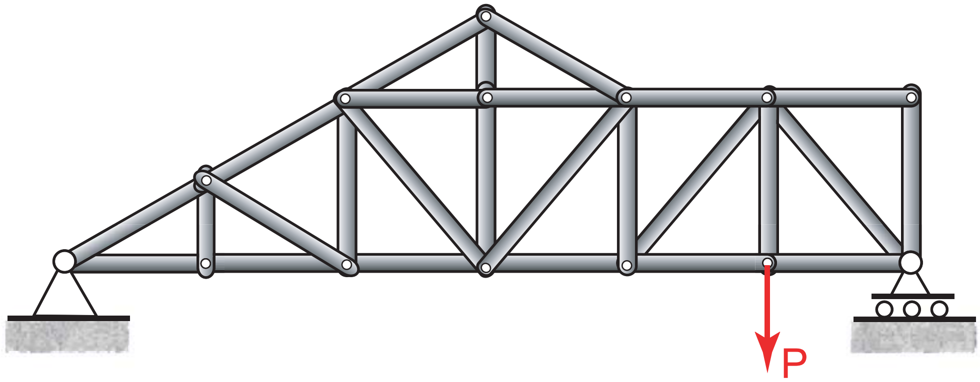
-Pinned connections

-Two force members (tension/compression)

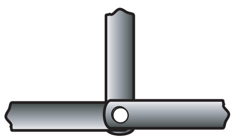
-Basic element is the triangle

Bridge Truss in Santa Margarita

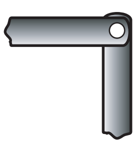
Method of Joints
Joints Under Special Loading Conditions- Tricks



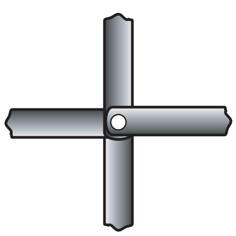
Trick #1

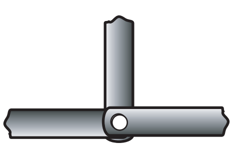


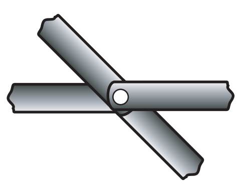
Trick #2



Trick #3



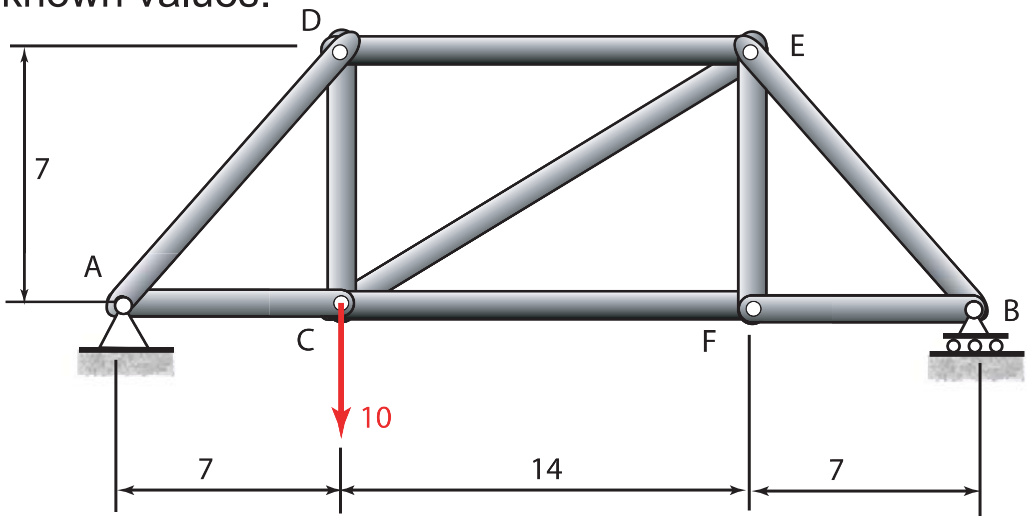


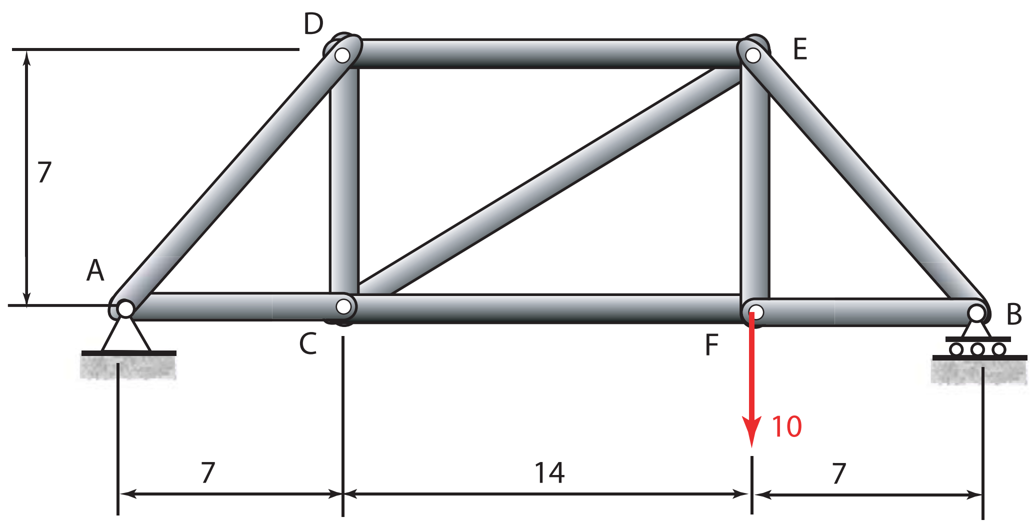


NOTE: No additional loads applied to the joints.

Example

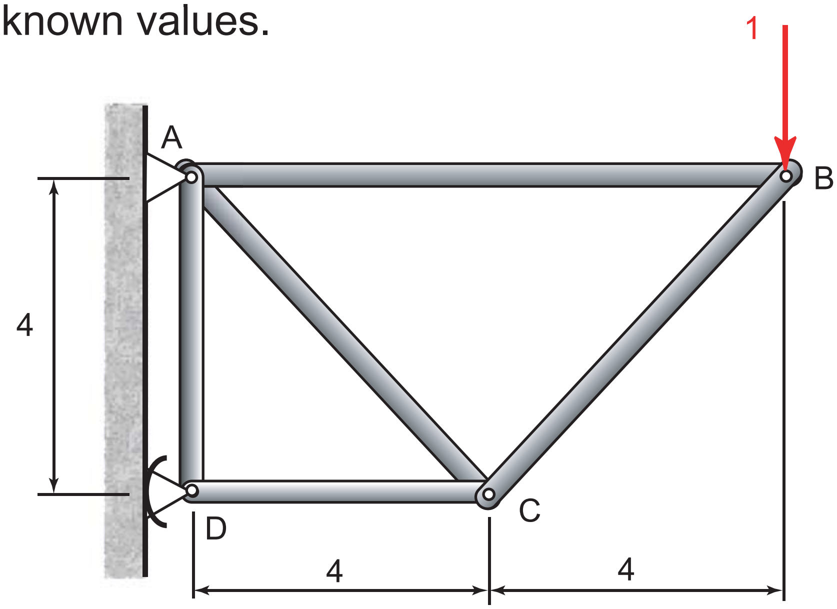
Using the tricks, find the zero force members and also any members of known values.

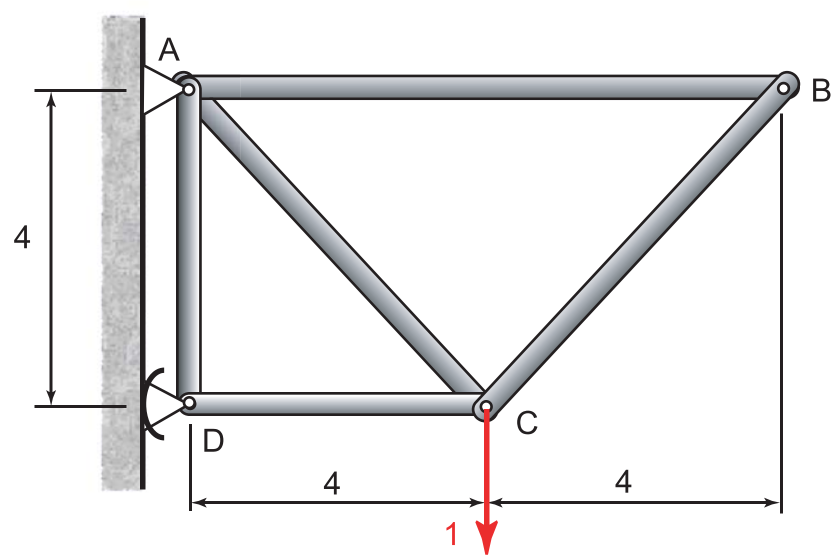




Example

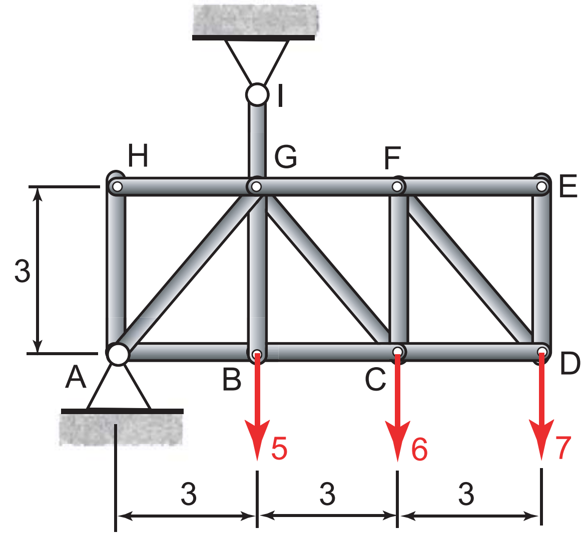
Using the tricks, find the zero force members and also any members of known values.





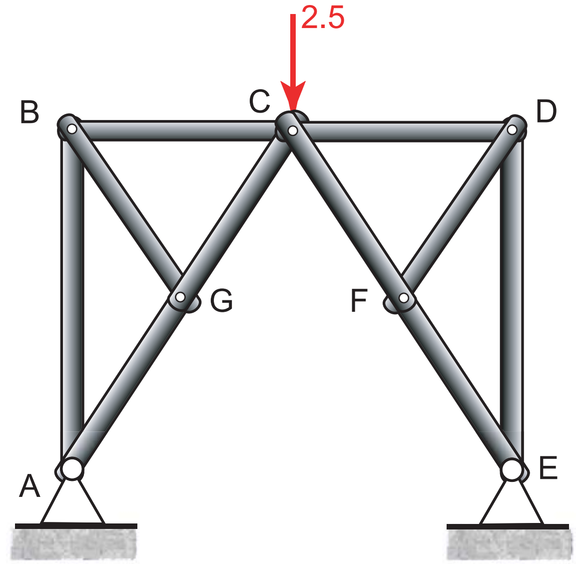
Example

Using the tricks, find the zero force members and also any members of known values.

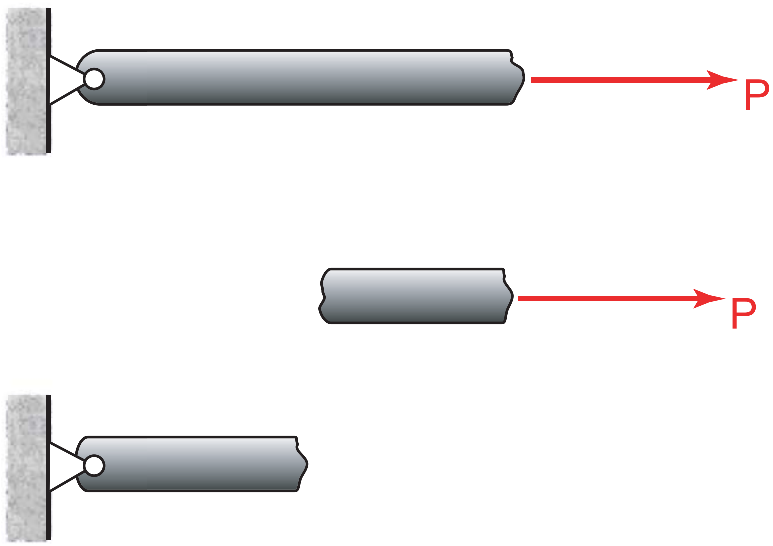


Example

Using the tricks, find the zero force members and also any members of known values.



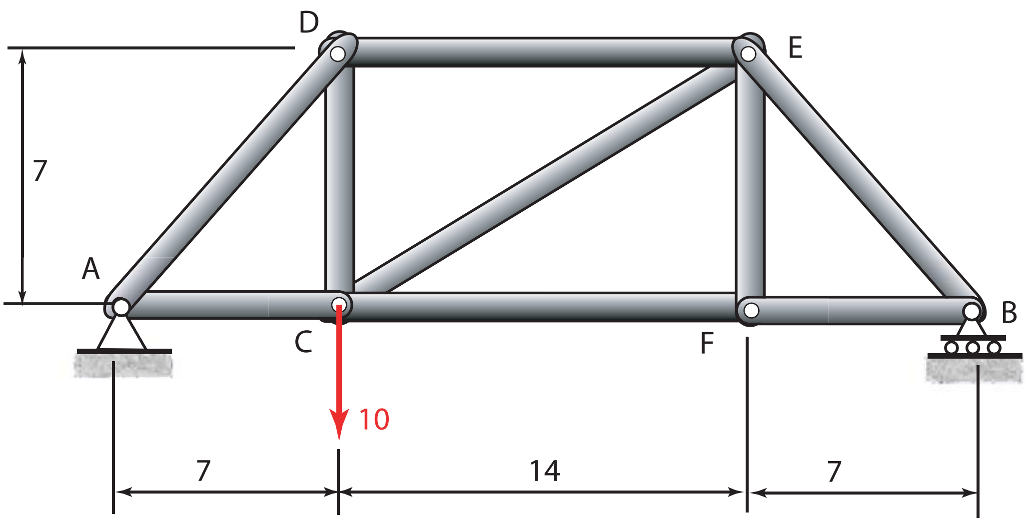
Review of FBDs



NEWTON'S THIRD LAW: The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.

Conclusion:
If a bar is in tension, then no matter what FBD you draw it is still in tension. If a bar is in compression, then no matter what FBD you draw it is still in compression.

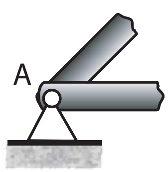
Relationships Between FBDs

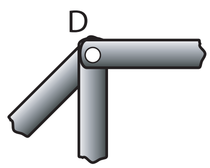


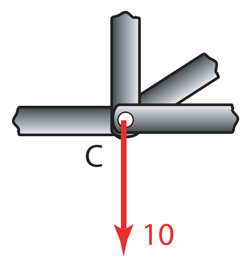
FBD- Joint A

FBD- Joint D

FBD- Joint C

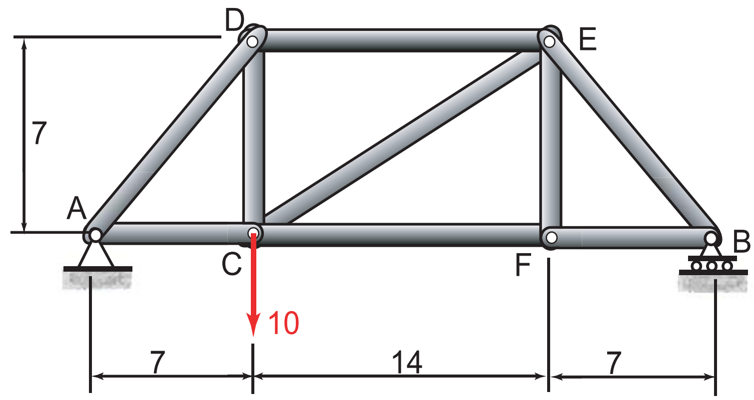




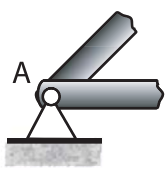


Example

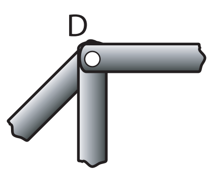
Determine the force in each member of the truss. Note the presence of any zero-force members. Units: kN, m.



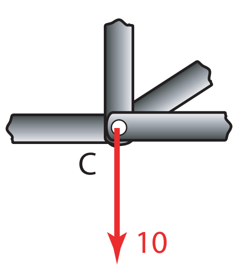
FBD- Joint A



FBD- Joint D

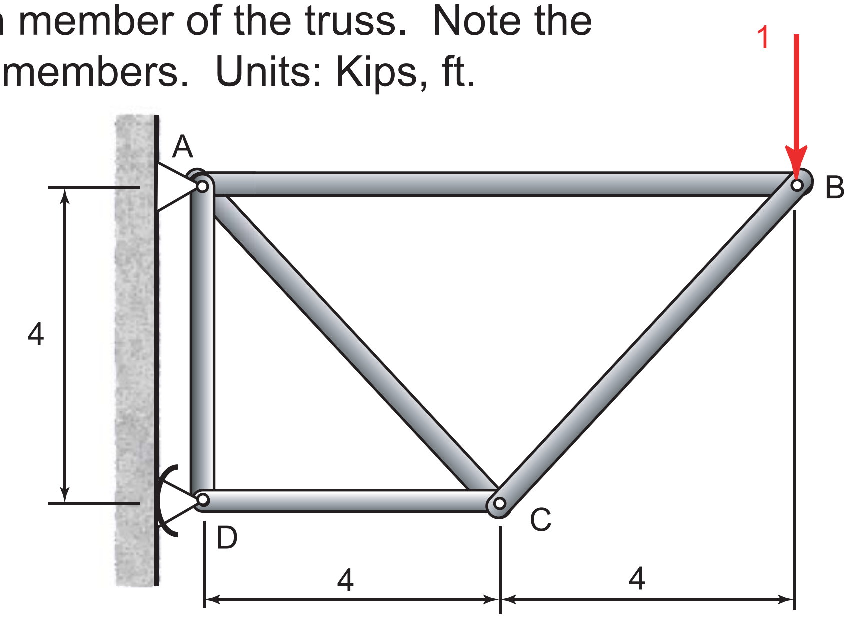


FBD- Joint C



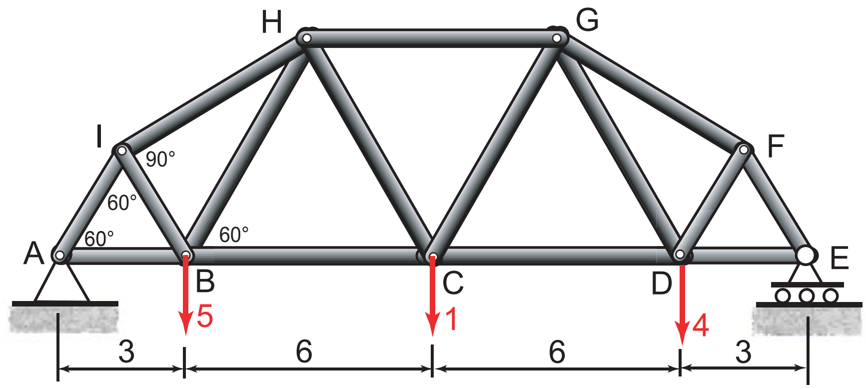
Example

Determine the force in each member of the truss. Note the presence of any zero-force members. Units: Kips, ft.



Example

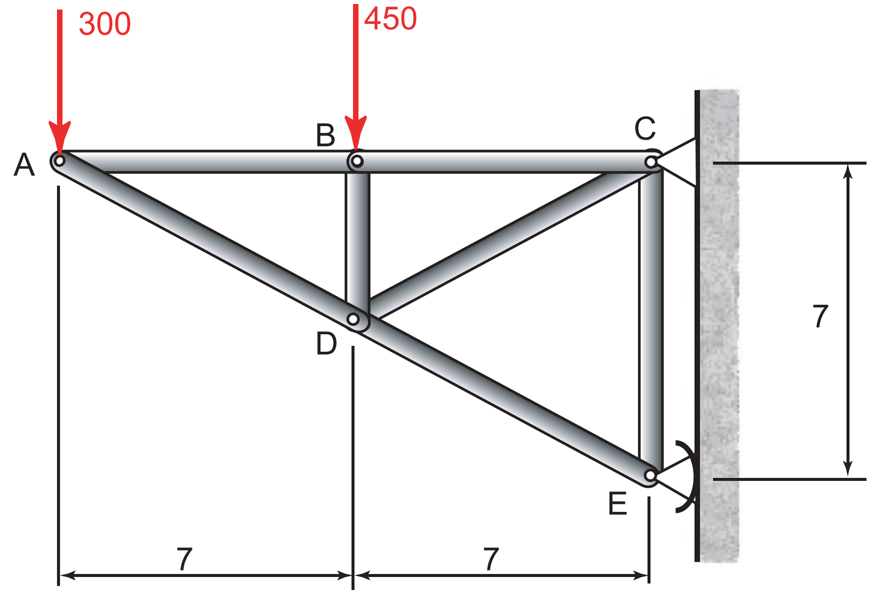
Determine the forces in members BI and BH. Units: kN, m.



Example

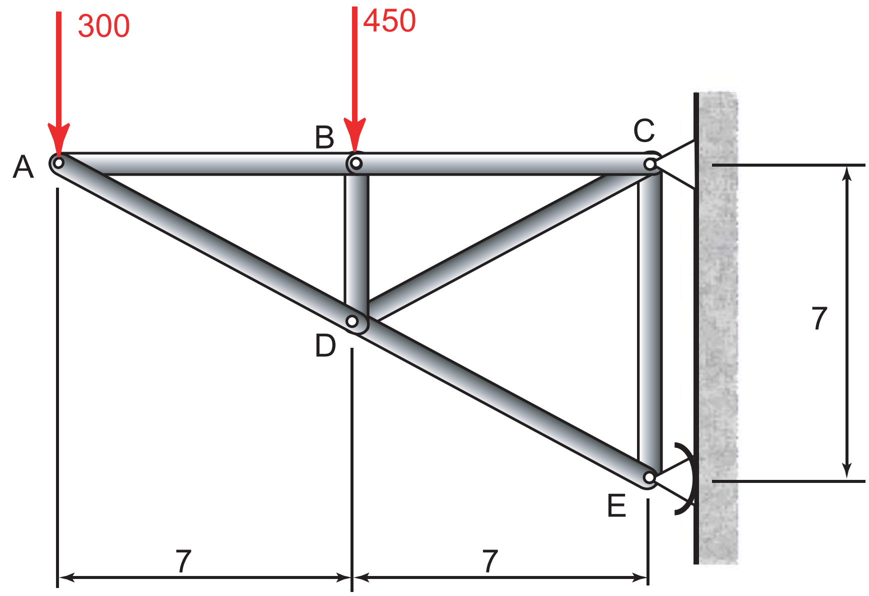
Determine the forces in AB and AD.

Units: kN, m.



Example

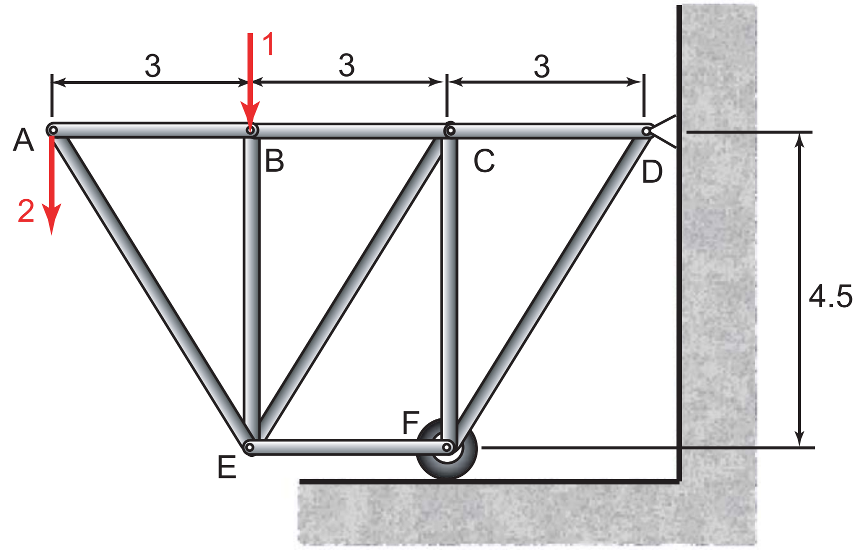
Determine the forces in CD and DE.
Units: kN, m.



Example

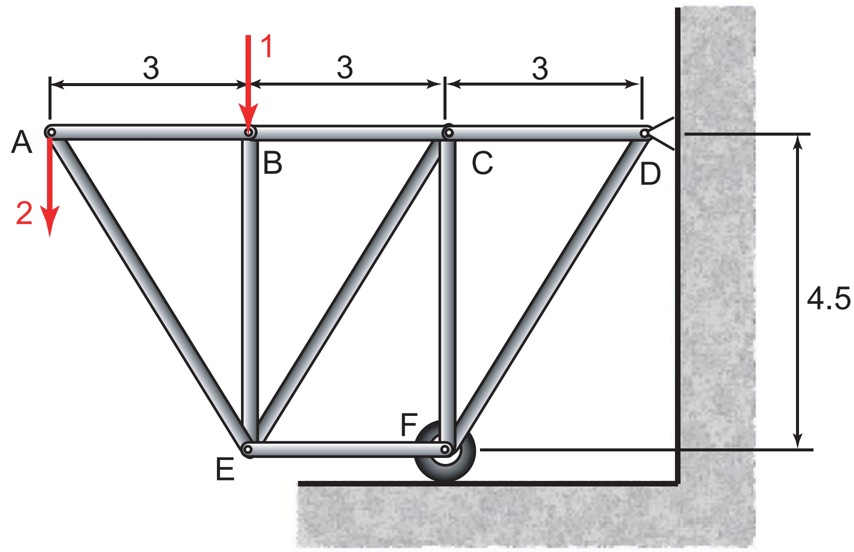
Determine the forces in AB and AE.

Units: kN, m.



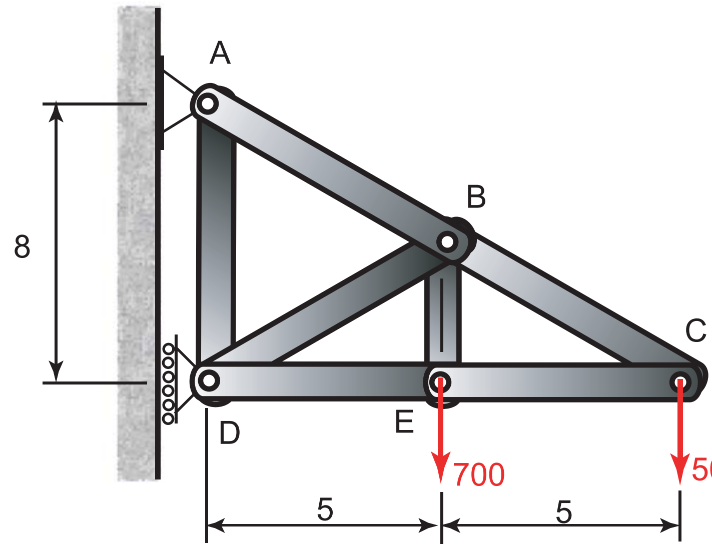
Example

Determine the forces in CD and DF.
Units: kN, m.



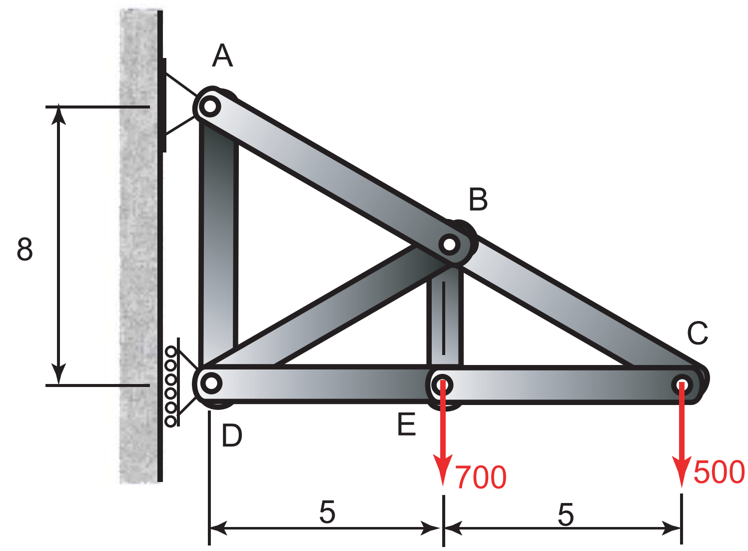
Example

Determine the forces in BC and EC. Units: Lb, ft.



Example

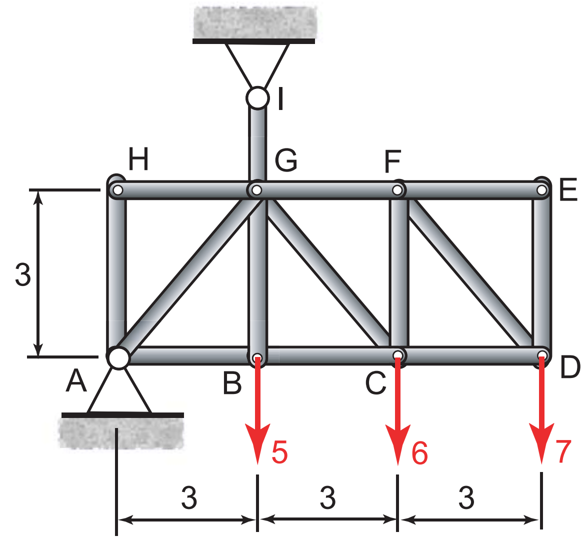
Determine the forces in AB and AD. Units: Lb, ft.



Method of Sections

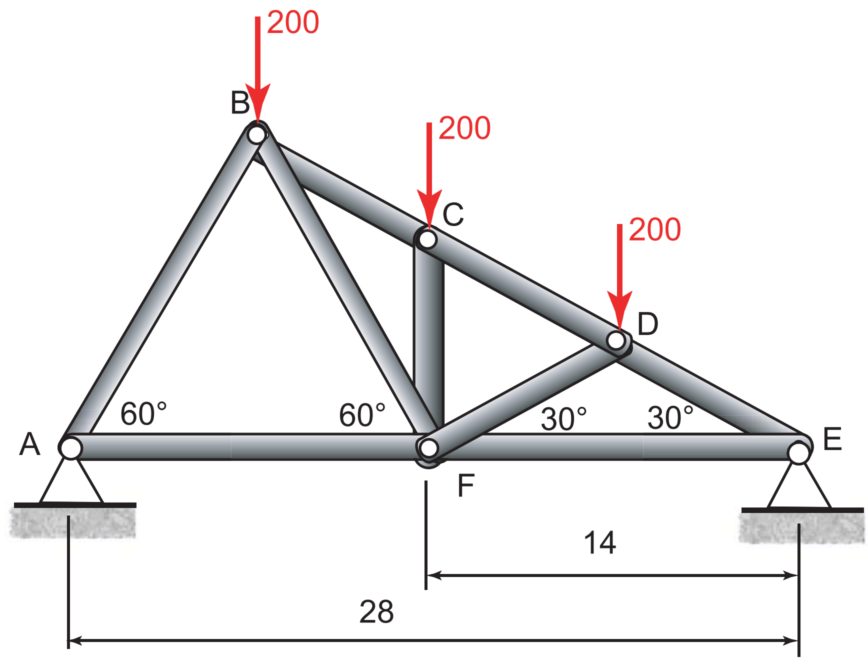
Example

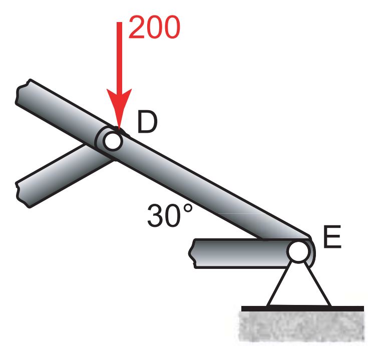
Determine the force in member CG.
Units: kN, m.



Example

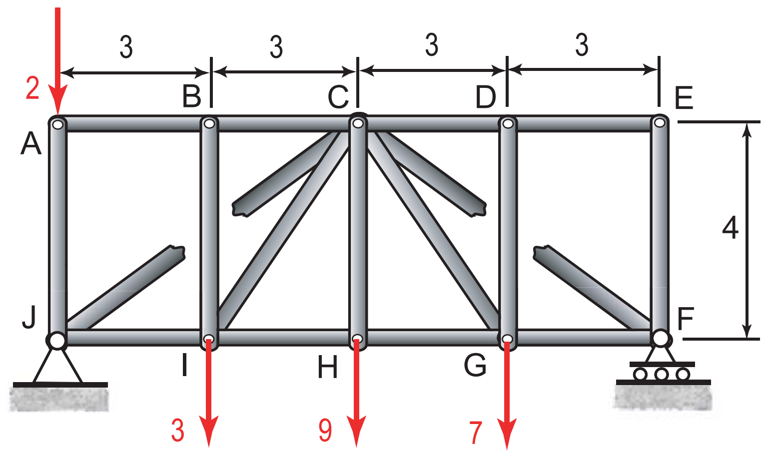
Determine the forces in members CD, CF, and EF. Ignore any horizontal reactions at the supports. Units: Lb, ft.





Example

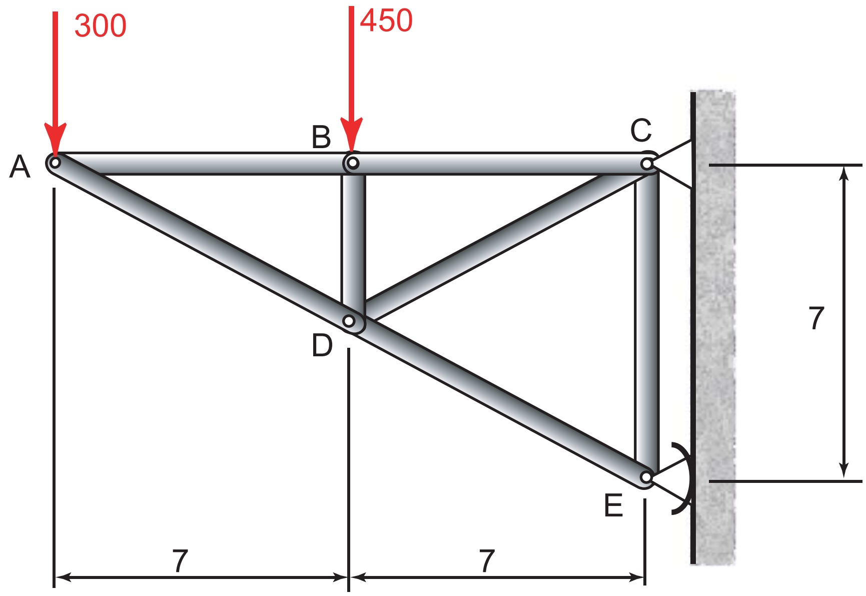
Compute the forces in members BC, CJ, CI, and HI. The members CJ and CF pass behind BI and DG. Units: Kips, ft. 1 Kip= 1000 lb.



Example

Determine the forces in BC, CD, and DE.

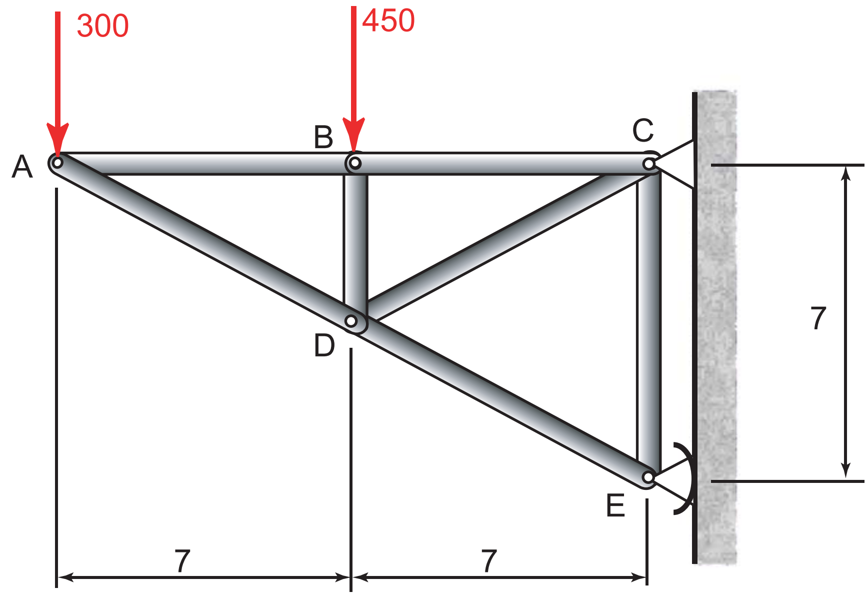
Units: kN, m.



Example

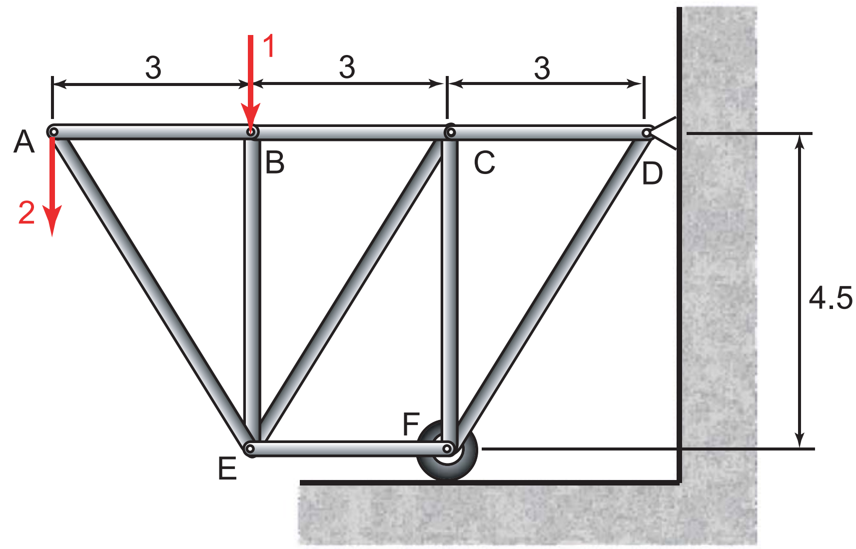
Determine the forces in BC, BD, and AD.

Units: kN, m.



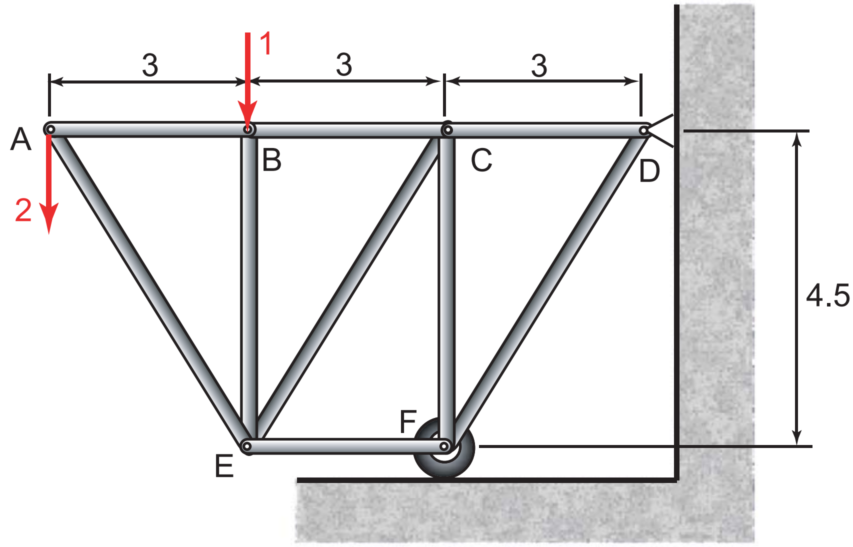
Example

Determine the forces in BC, CE, and EF.
Units: kN, m.



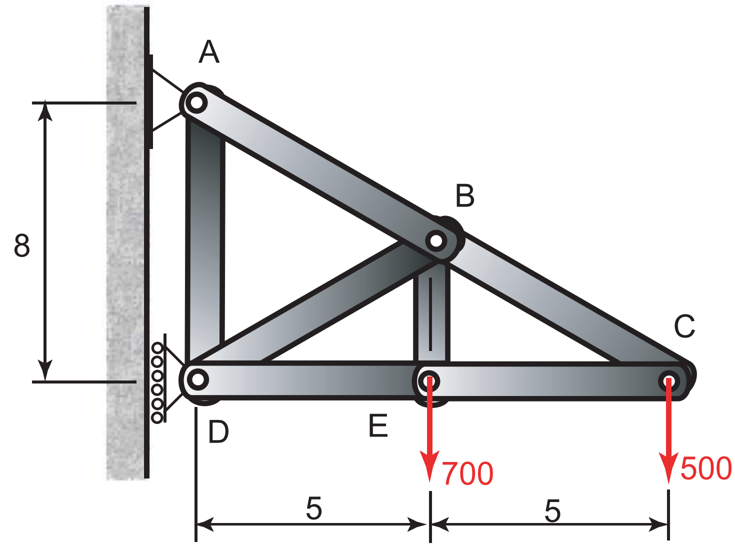
Example

Determine the forces in CD and DF.
Units: kN, m.



Example

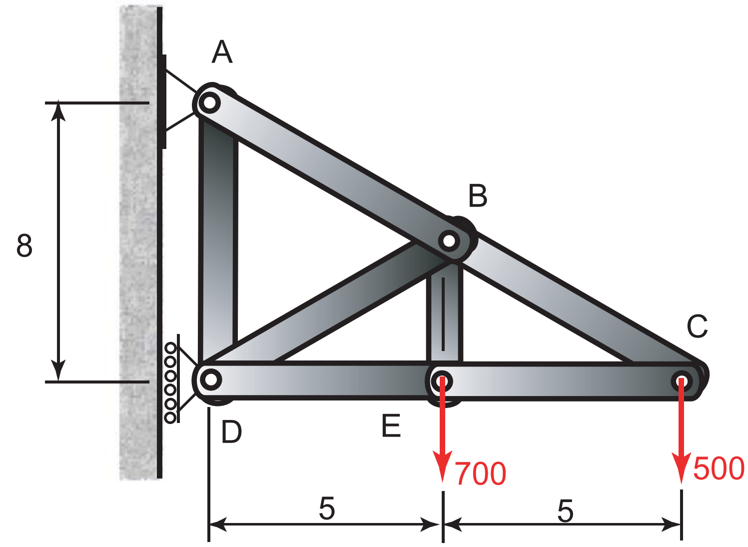
Determine the forces in AB, BD, and DE.
Units: Lb, ft.



Example

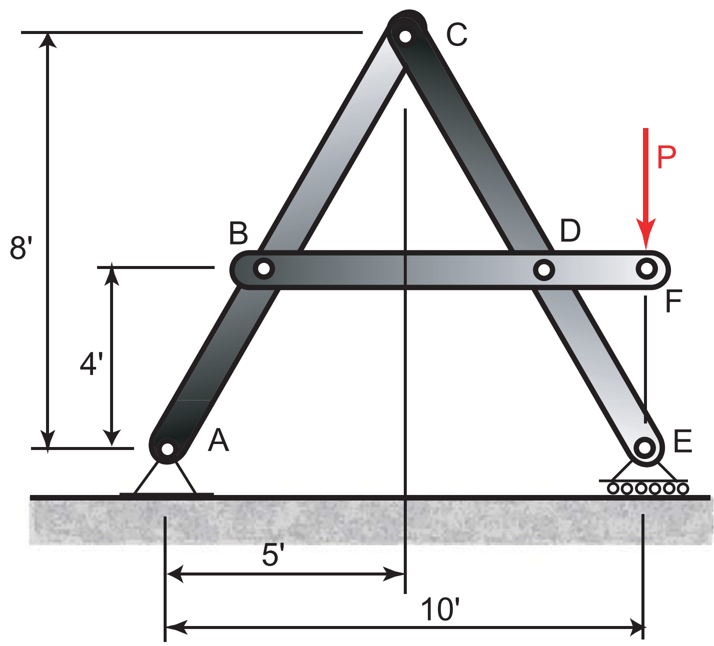
Determine the forces in AB, BD, and EC.

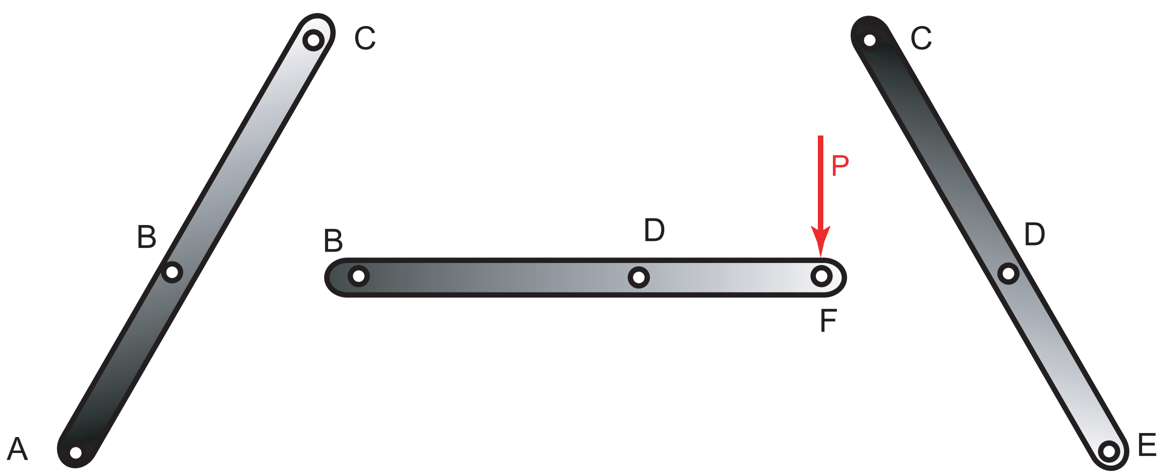
Units: Lb, ft.



FRAMES and MACHINES

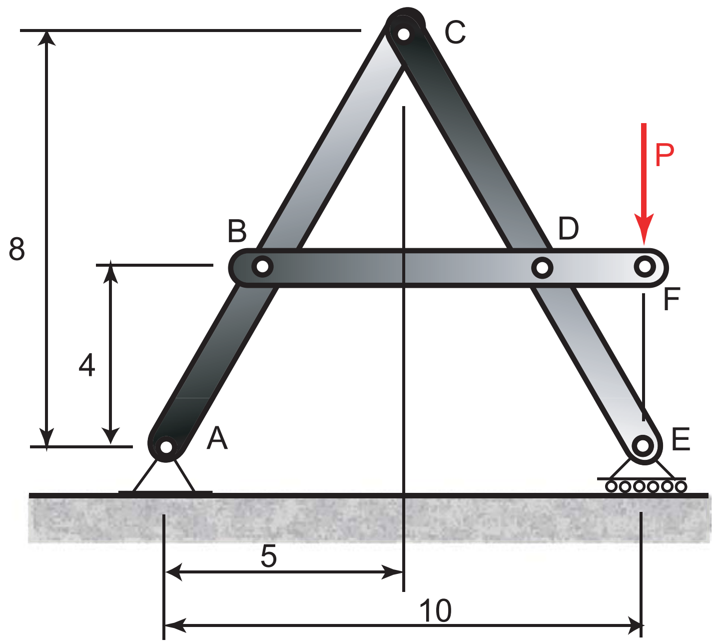
A structure is called a FRAME or MACHINE if at least one of its individual members is a multi-force member. If the structure is intended to move then we call it a MACHINE, if its not intended to move as in a building then its called a frame. No matter what you call them they are both analyzed the same.

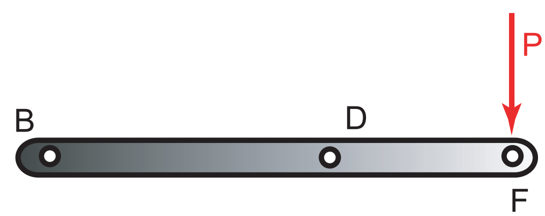


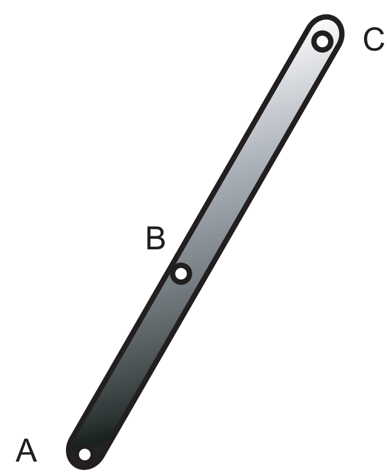


Example

Compute the force supported by the pin at B. P= 5000 lb. Units: Lb, ft.

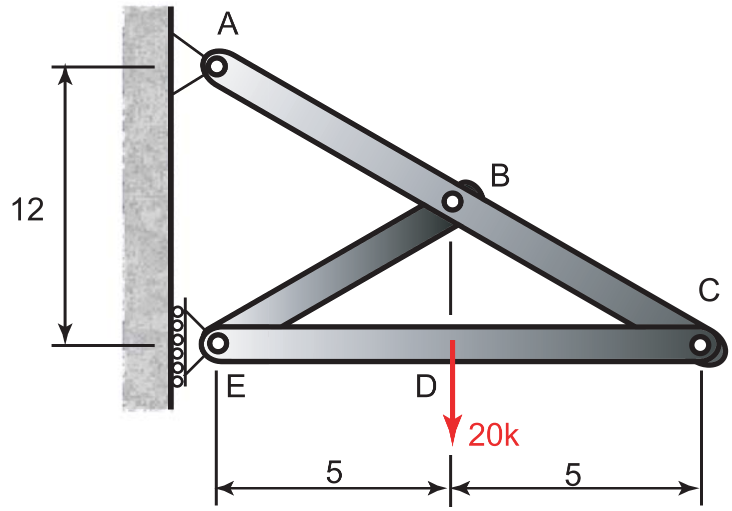


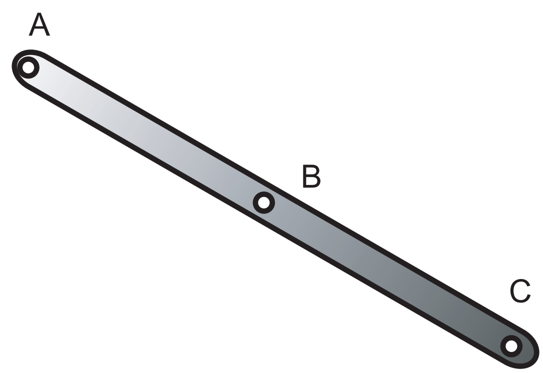




Example

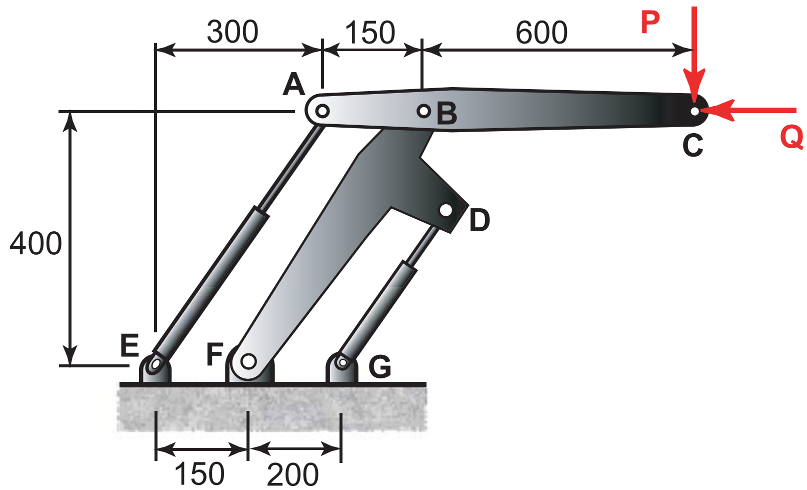
For the frame and loading shown, determine the components of all forces acting on member ABC.
Units: Kips, ft.

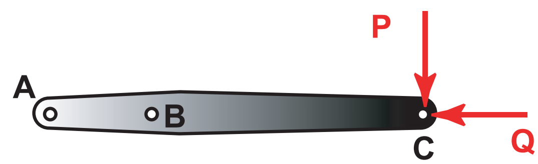


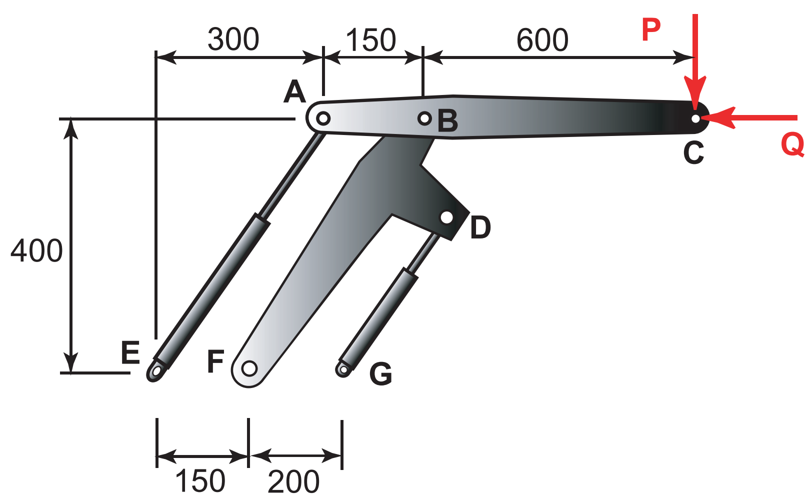


Example

Knowing that in the position shown the cylinders are parallel, determine the force exerted by each cylinder when P= 190 N and Q= 95 N. Dimensions: N, mm.

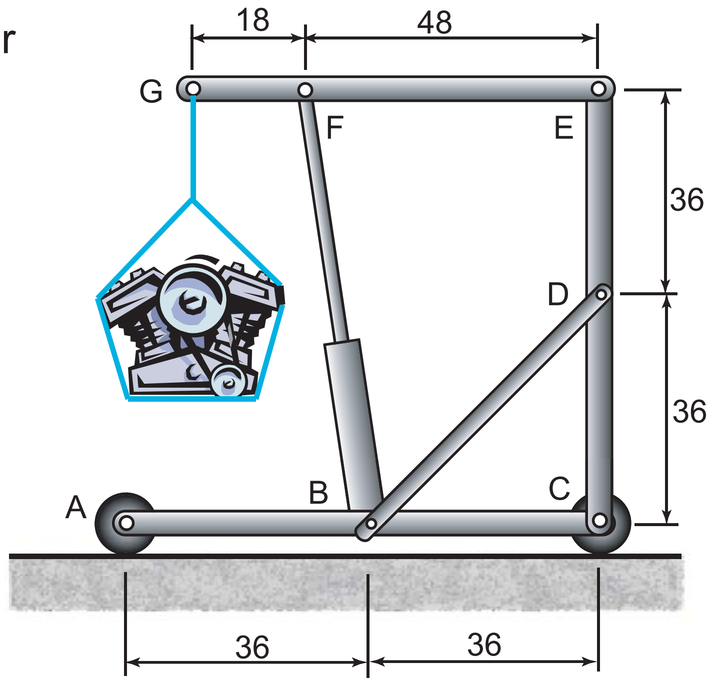






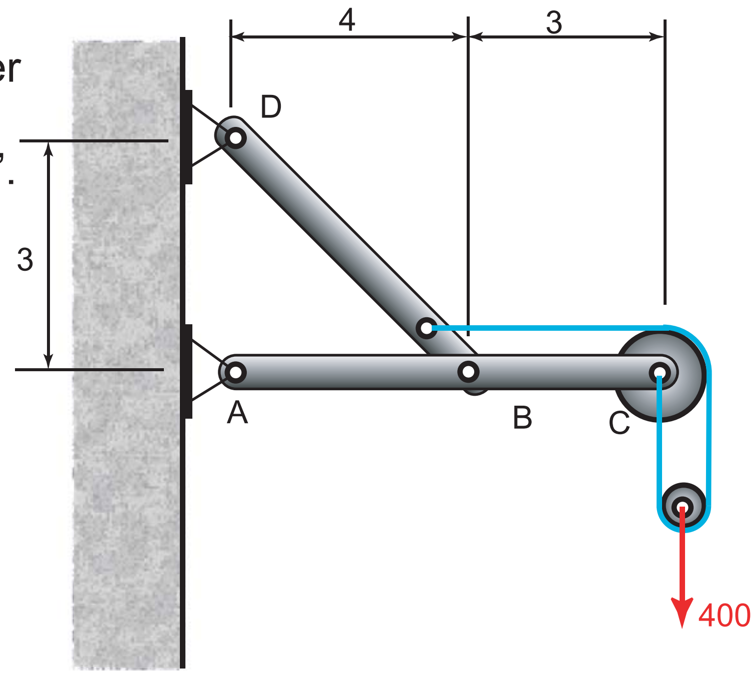
Example

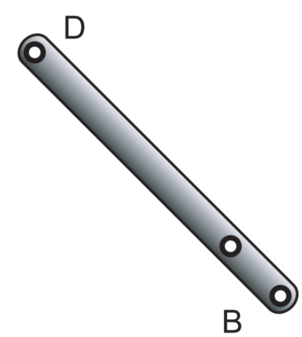
Determine the forces on member EFG due to the 650 lb engine. Units: Lb, in.

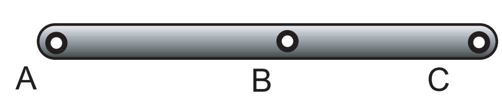


Example

Determine the forces on member ABC. The radius of the small pulley is 2.5" and the larger is 5”. Units: Lb, ft.

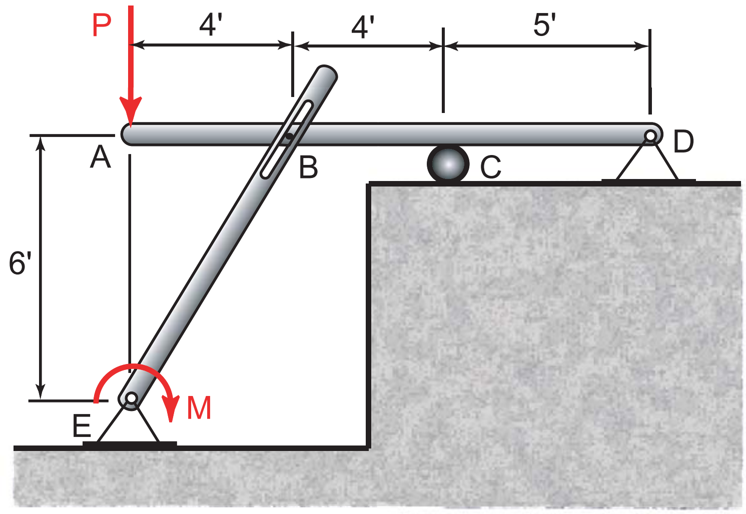


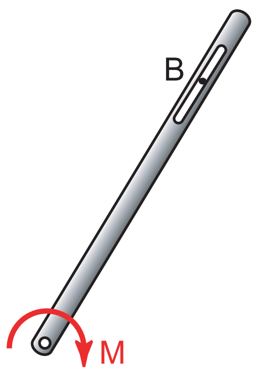


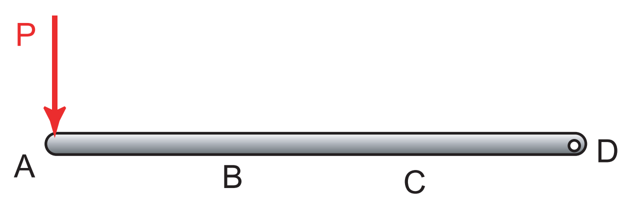


Example

Determine the forces on member ABCD due to P= 500 lb and M= 700 ft-lb. Units: Lb, ft.

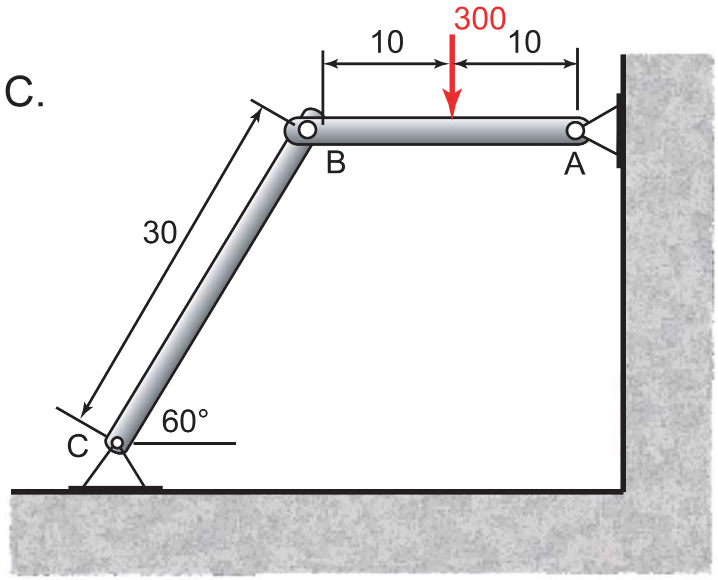






Example

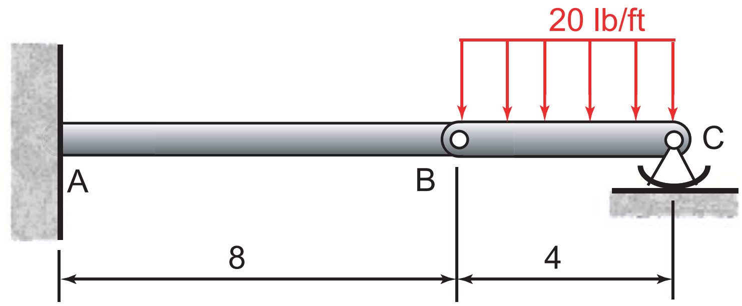
Determine the reactions at A and C.
Units: Lb, in.

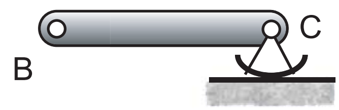


Example

Determine the reactions at A and C.

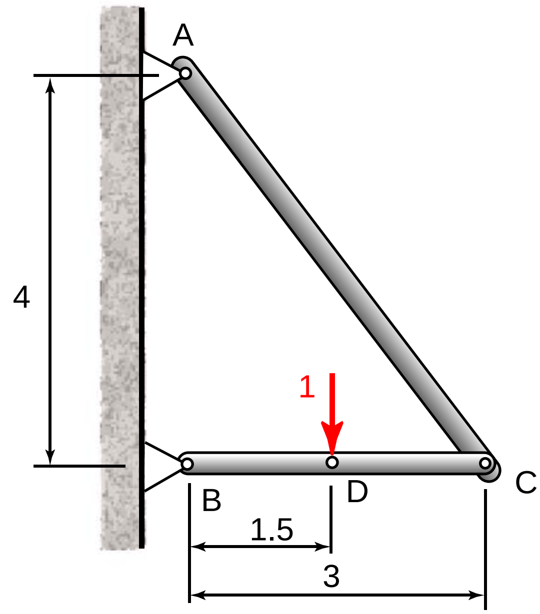
Units: Lb, ft.





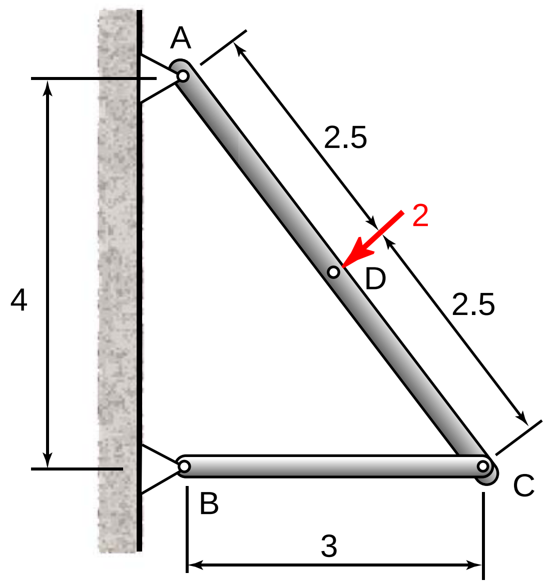
Example

Determine the support reactions   Units: Kips, ft.



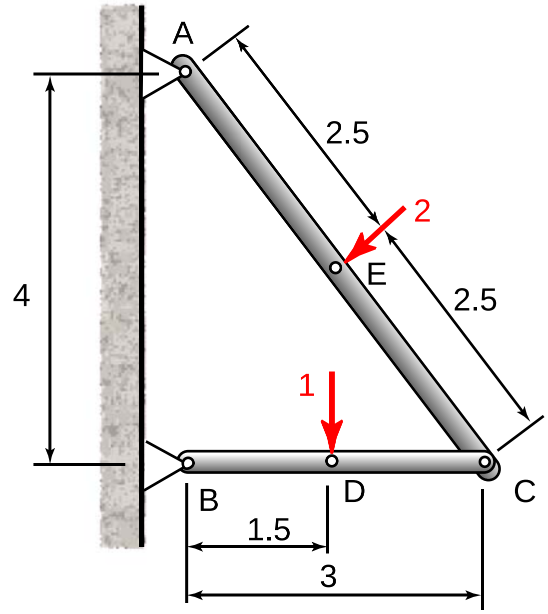
Example

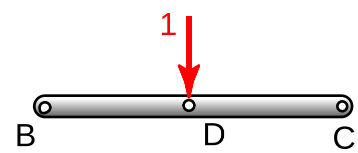
Determine the support reactions   Units: Kips, ft.



Example

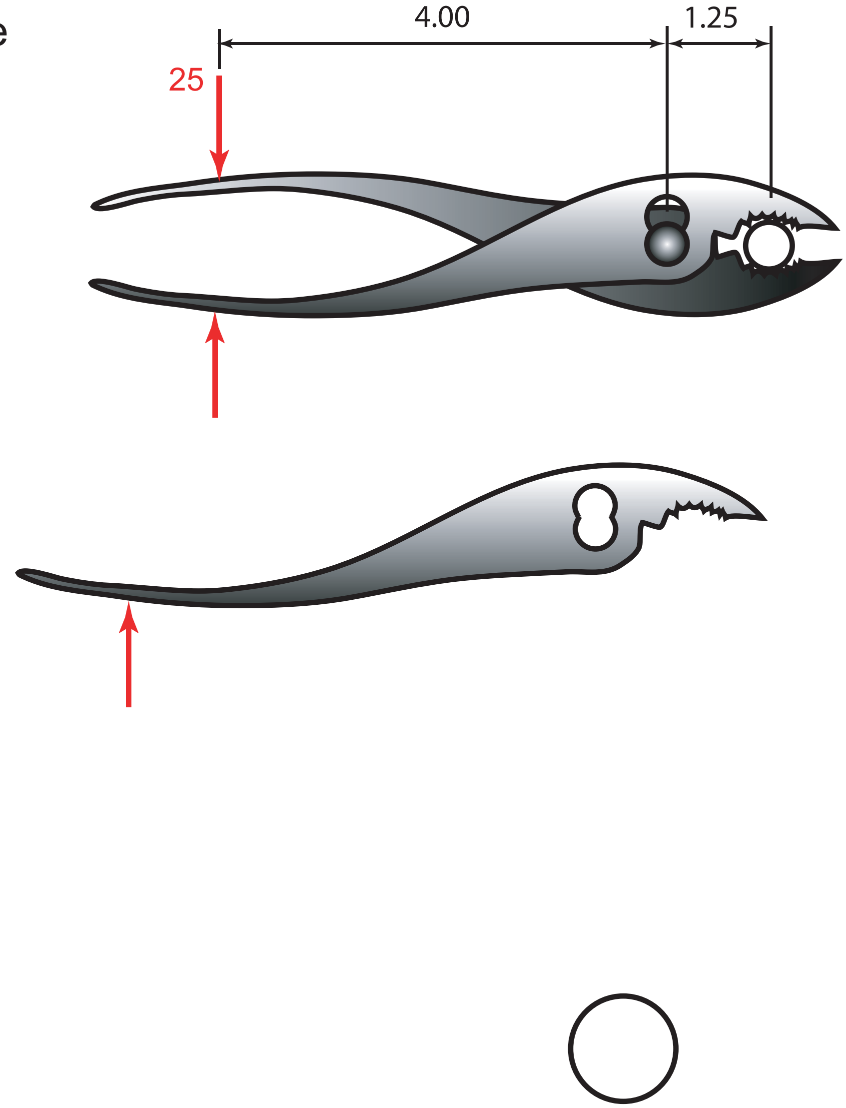
Determine the support reactions   Units: Kips, ft.





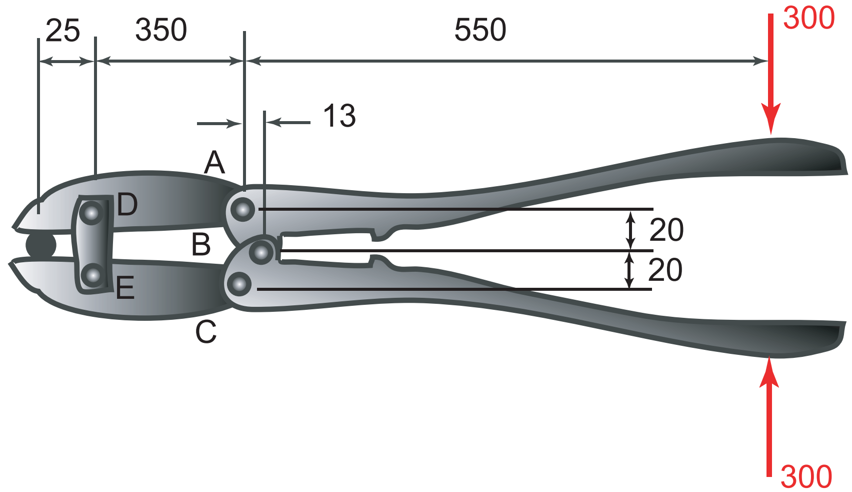
Example

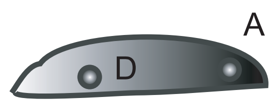
Determine the clamping force exerted on the pipe.
Units: Lb, in.

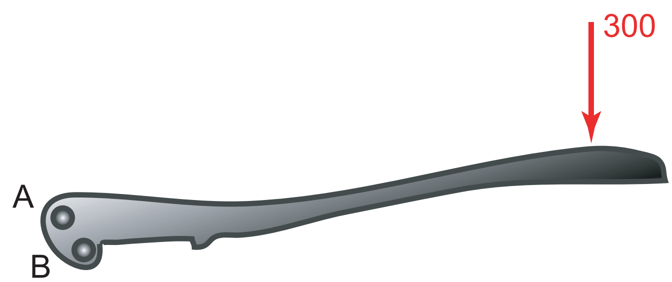


Example

Determine the cutting force exerted on the rod.
Units: N, mm.

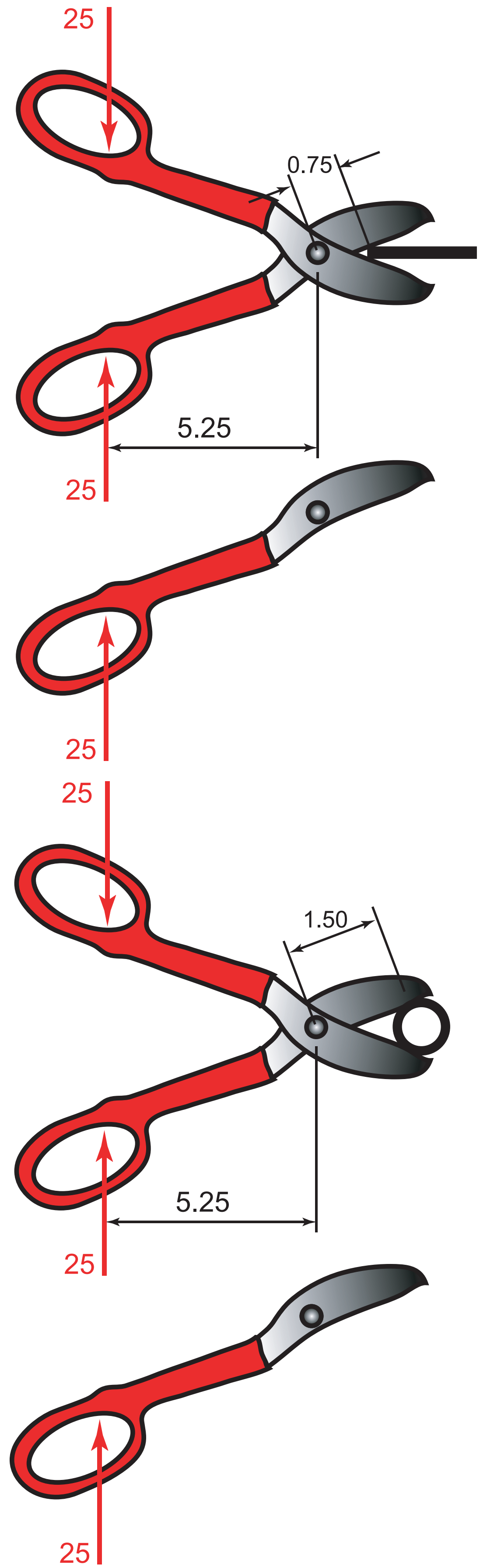






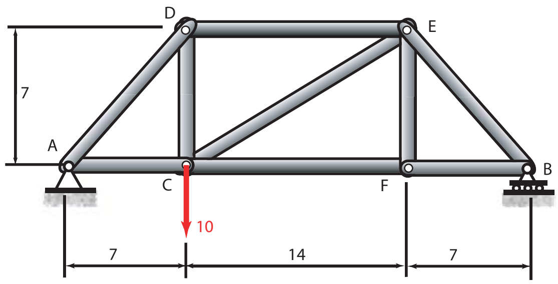
Example

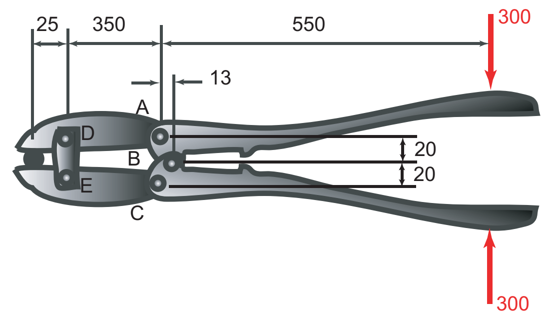
Determine the cutting force exerted on the a) thin sheet metal and b) the pipe. Ignore any friction between the blades and the object. Units: Lb, in.



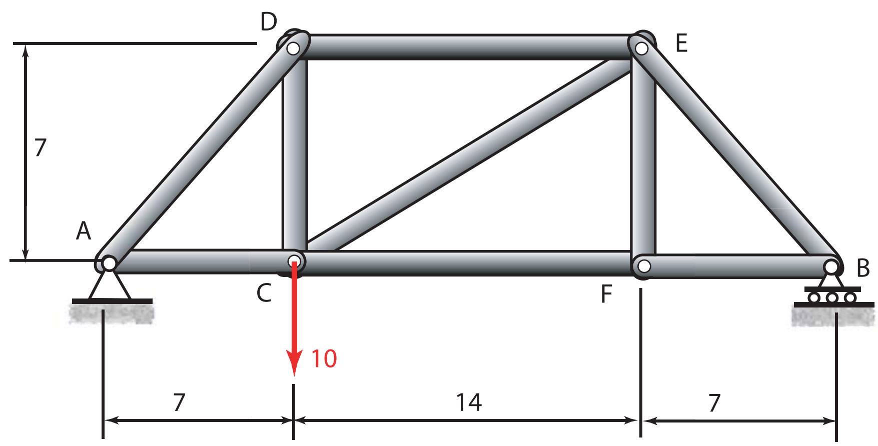
Summary

Two Force Members

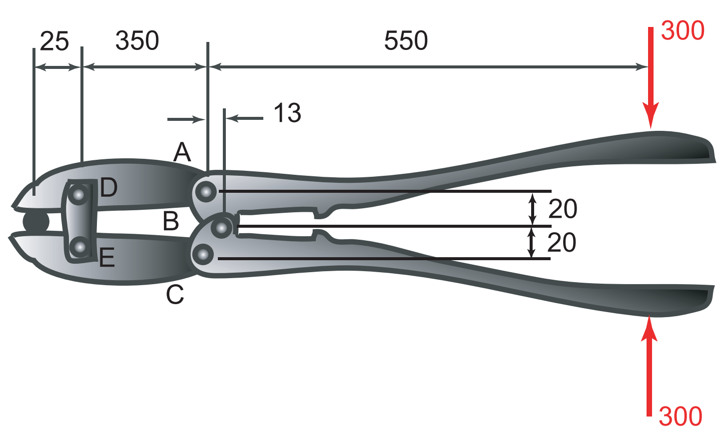




Method of Joints and Method of Sections

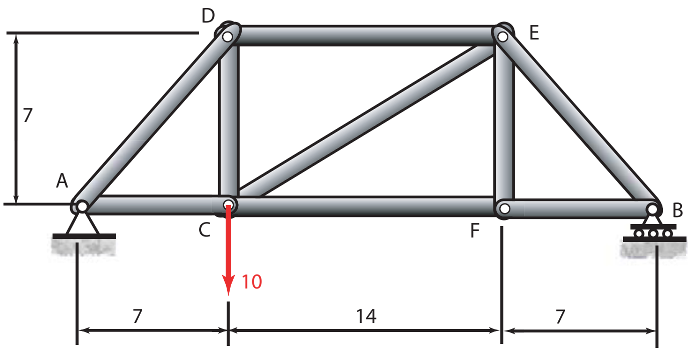


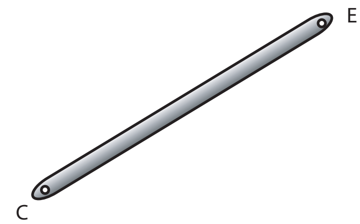
Frames and Machines (Multi-Force Members)

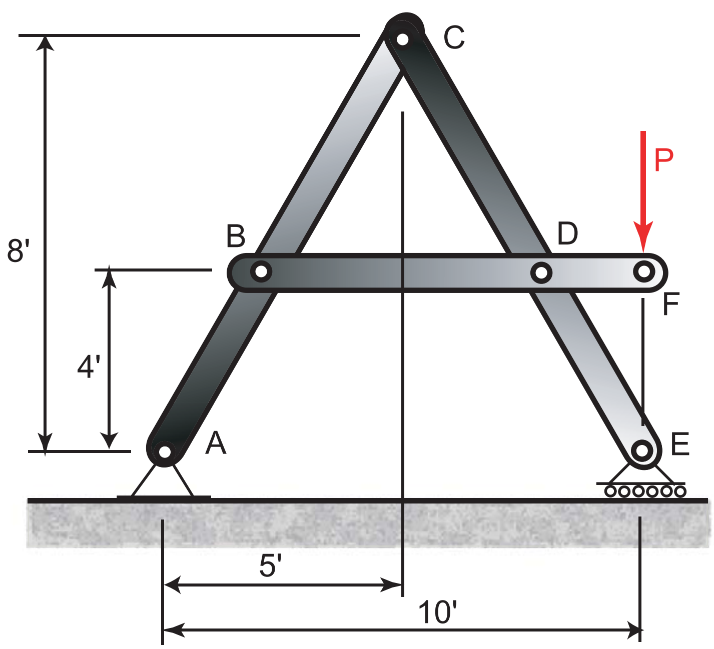


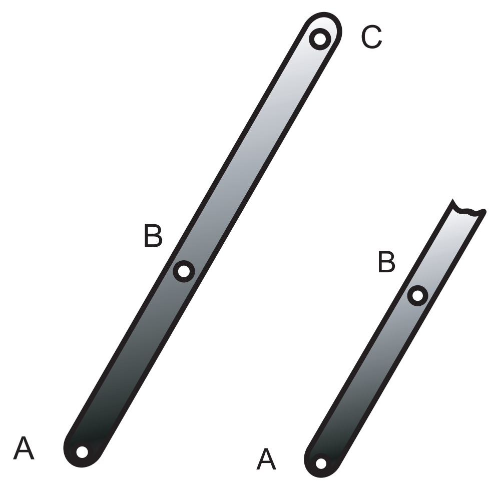
Chapter 7
Forces in Beams

Introduction
Internal Forces in Members



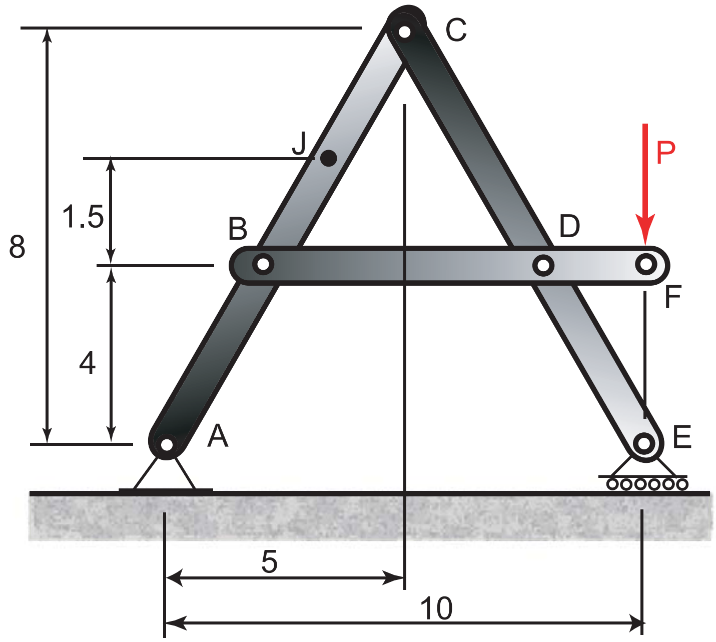


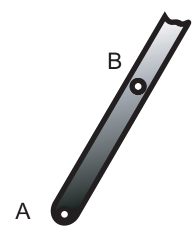




Example

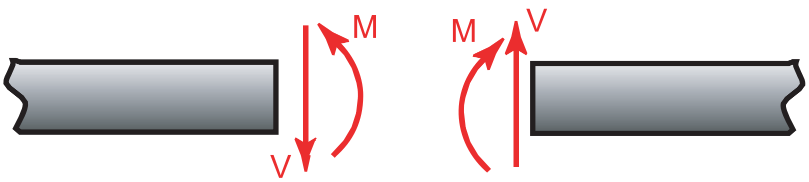
Determine the internal forces at point J. P= 5000 lb.
Units: Lb, ft.



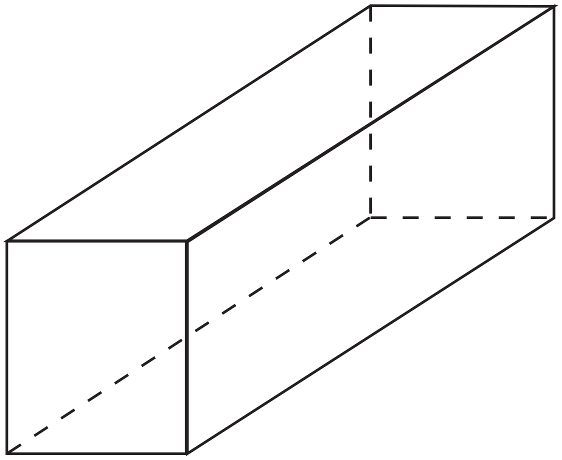


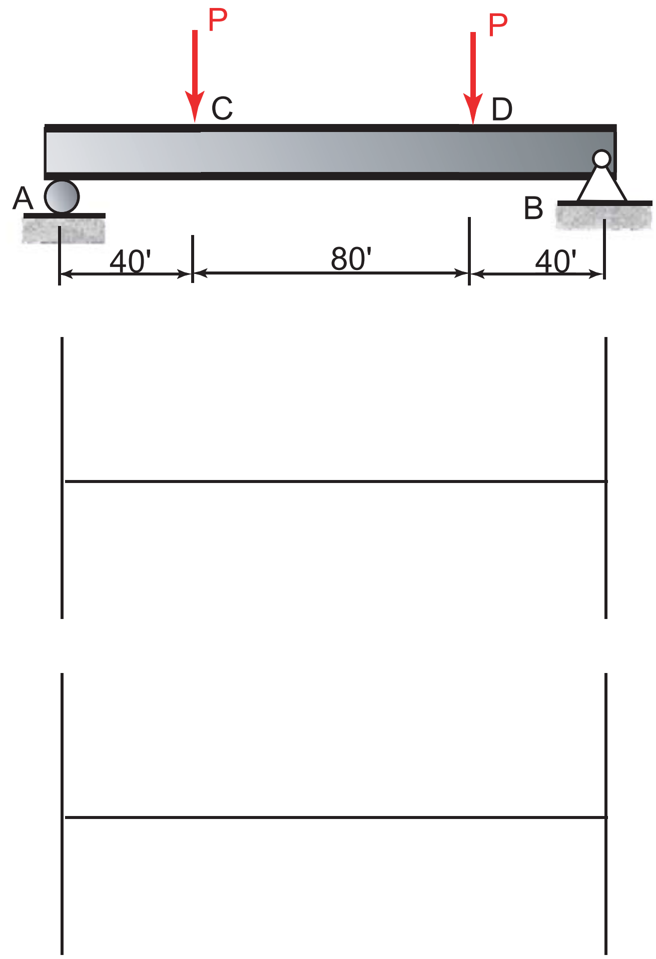
Shear and Bending Moment Diagrams

Sign Convention



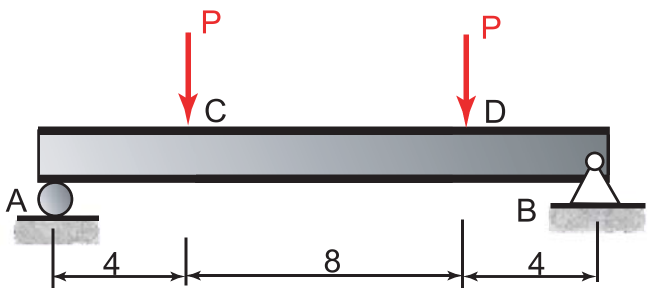
Why do we Need to Draw Shear and Bending Diagrams?



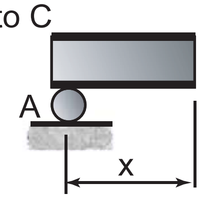
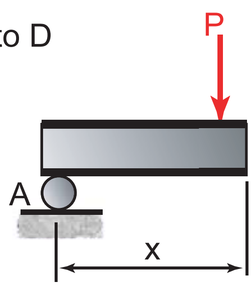
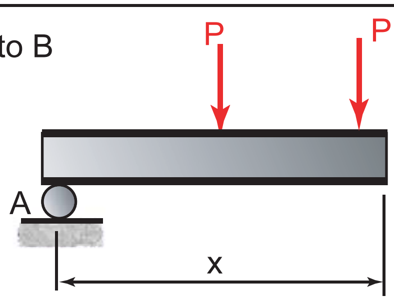


Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.

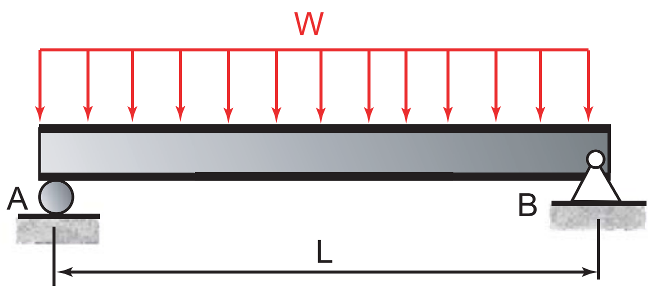


Support Reactions

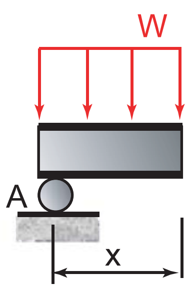
Segment	Shear	Moment
A to C 		
C to D 		
D to B 		

Example

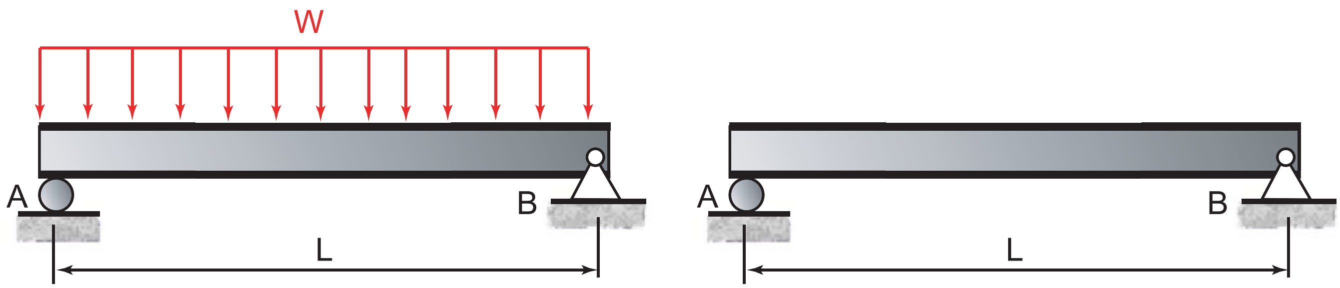
Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes.



Support Reactions

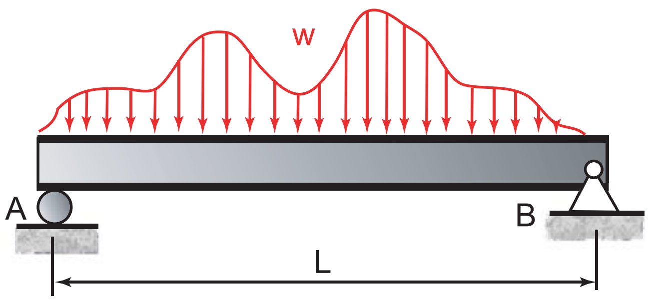
Segment	Shear	Moment
		

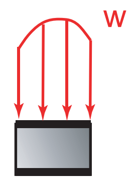
Caution:



When drawing FBDs, always use the original loading and not the equivalent.

Relations Among Load, Shear, and Bending Moment





$$\Delta V = -w\Delta x$$

The change in shear is equal to the area under the load curve.

$$\frac{dV}{dx} = -w$$

The slope of the shear diagram is equal to the value of the w load.

$$\frac{dM}{dx} = V$$

The slope of the moment diagram is equal to the value of the shear.

$$\Delta M = V\Delta x$$

The change in moment is equal to the area under the shear curve.

Observations about the Shape of Shear/ Moment Diagrams

Shear Diagrams:

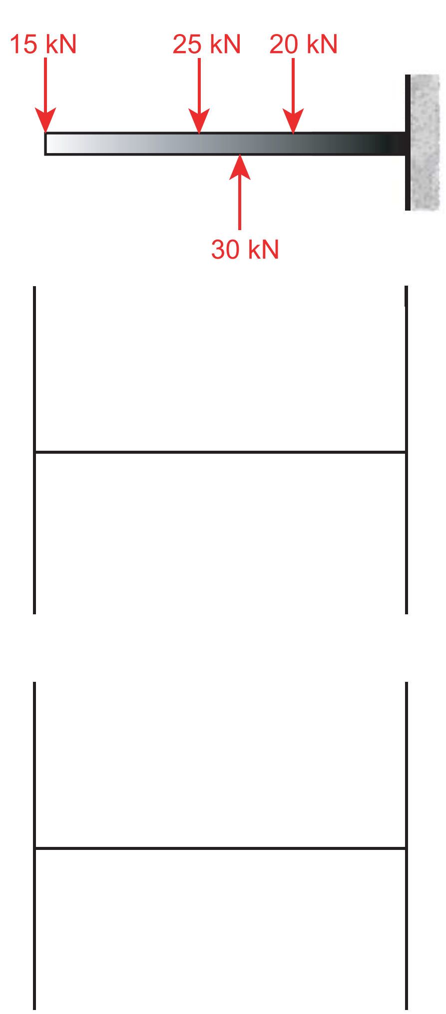
- -Are a plot of forces (note the units).
- -Discontinuities occur at concentrated forces.

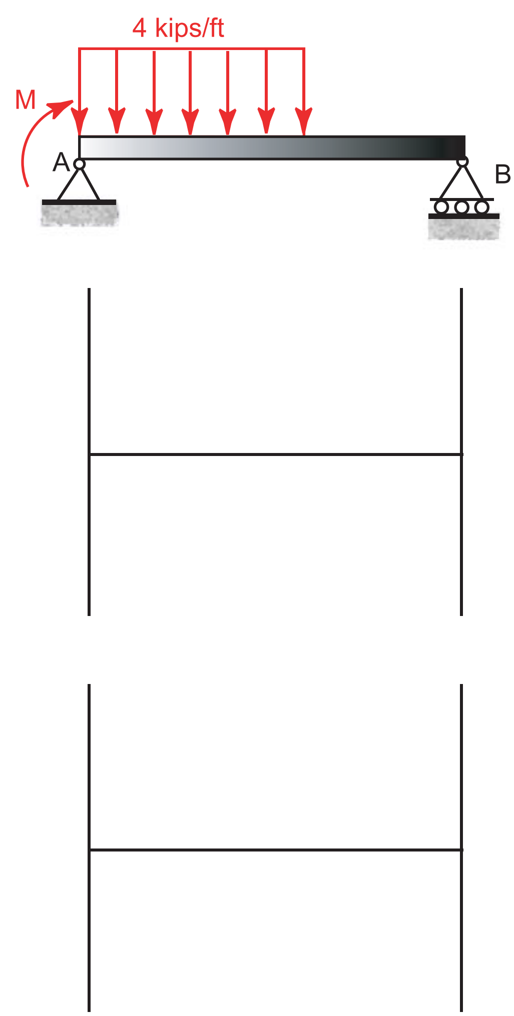
Moment Diagrams:

- -Are a plot of moments (note the units).
- -Discontinuities occur at concentrated moments.

Miscellaneous:

- -Check your work by noting that you always start and end at zero.
- -Always use the original loading and not the equivalent.

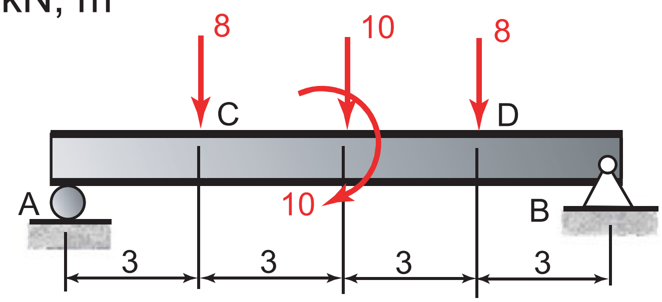




Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: kN, m

Support Reactions



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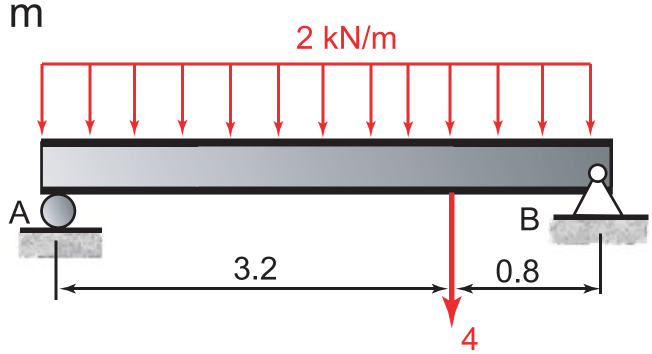
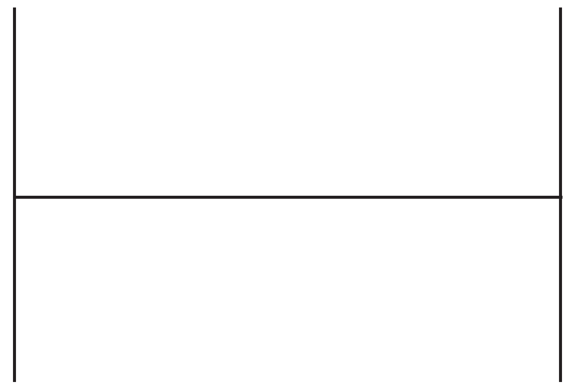
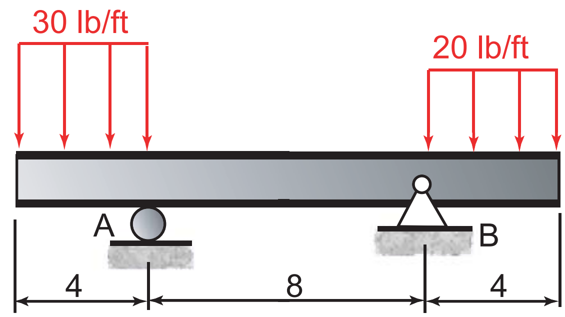


Support Reactions



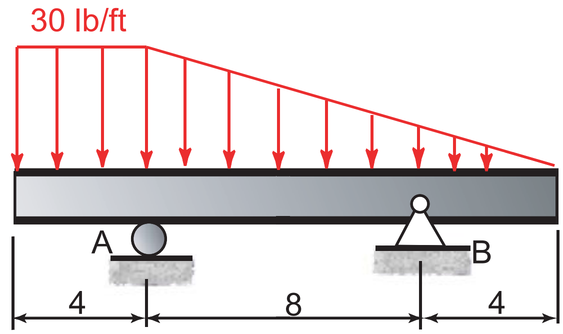
Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.



Example

Sketch the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.



Example

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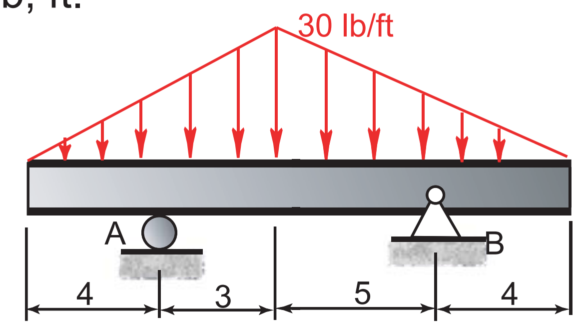
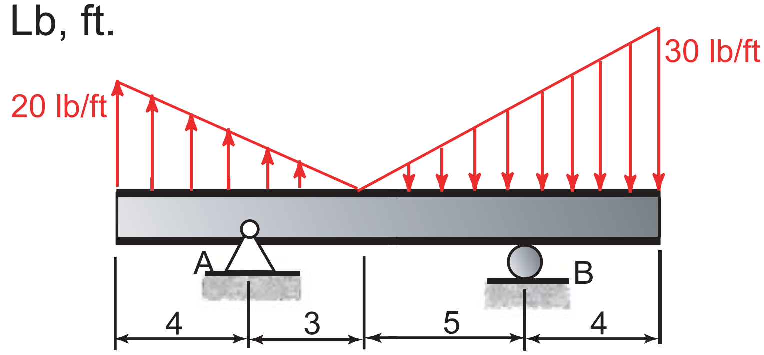


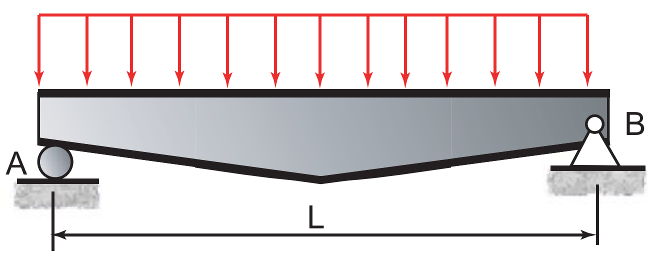
Example

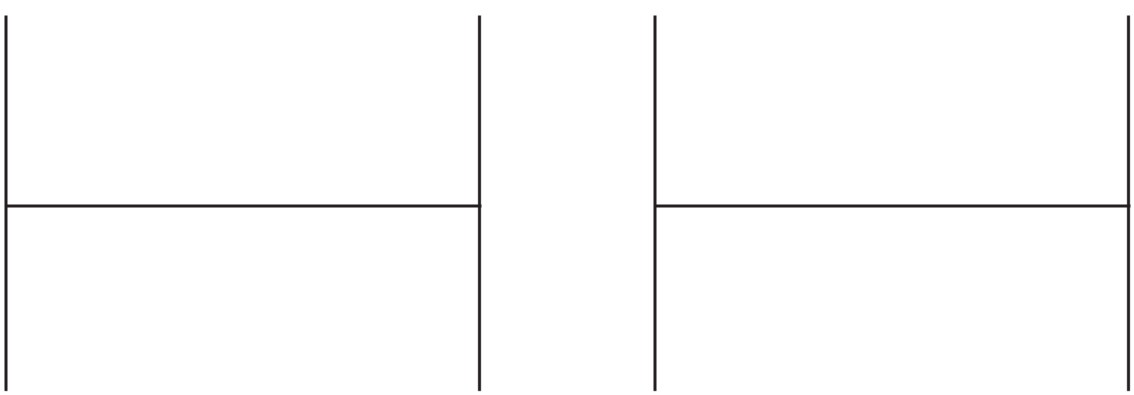
Sketch the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. Units: Lb, ft.



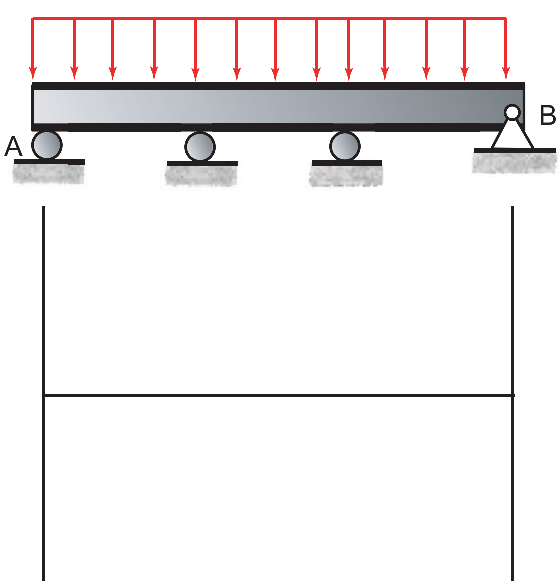
Example

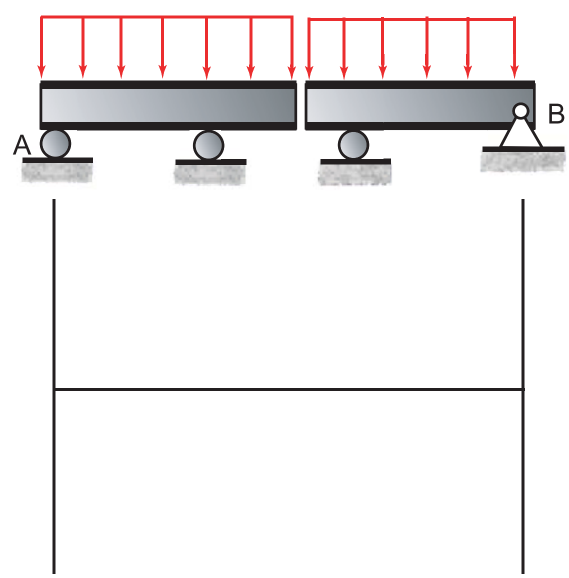
Here is an example of how the shape of the girder reflects the shear and bending diagrams.





So why did they put that gap in the bridge?

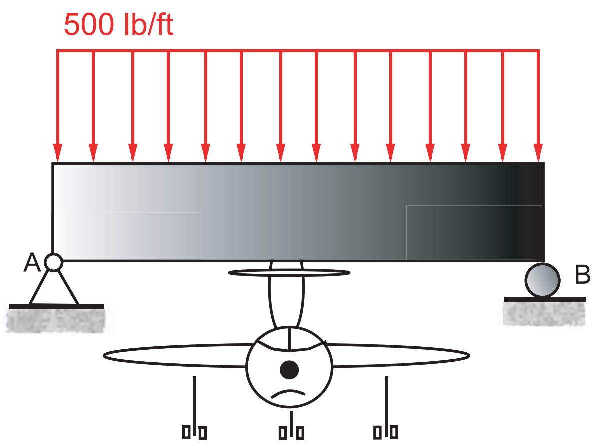


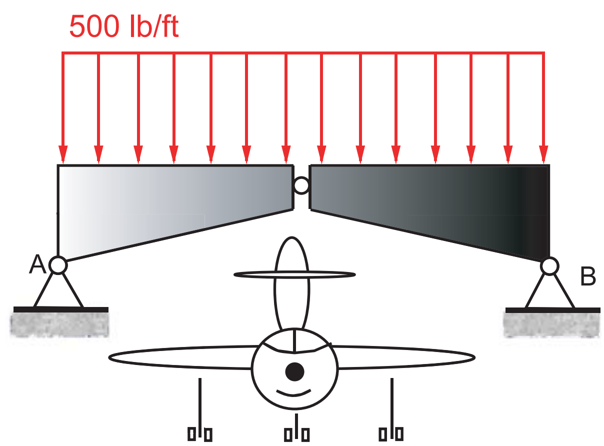


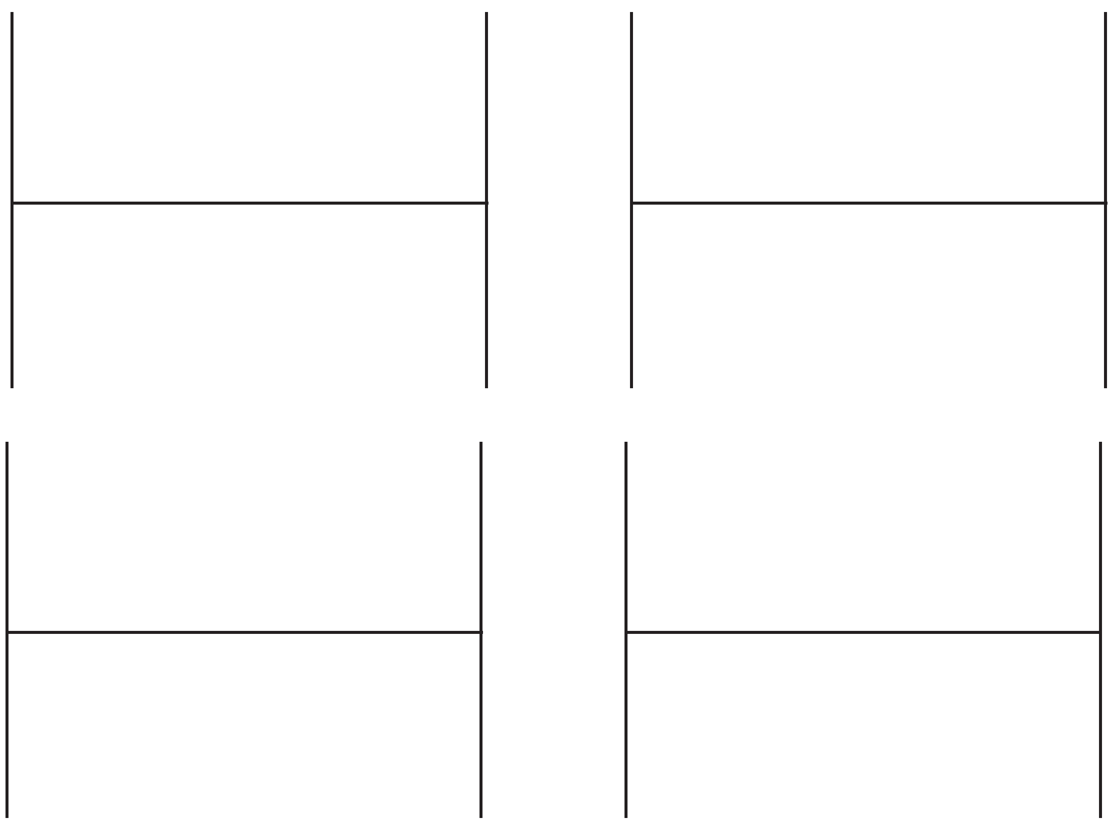
Pins

Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. The addition of the internal pin at the center of the beam allows additional head room because rather than the moment being a maximum in the center it becomes zero. This design is used at Wings Air West in SLO. Total span= 75 ft.



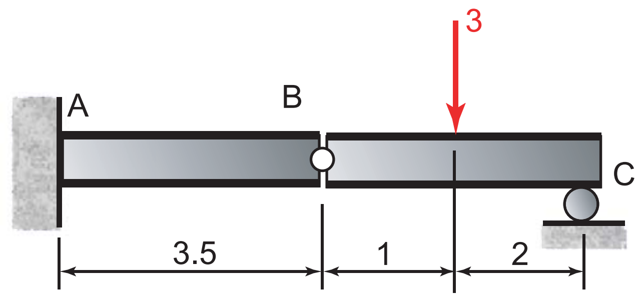


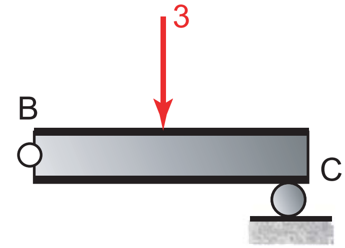


Example

Draw the shear and bending-moment diagrams for the beam and loading shown. Label all points of change, maximums and minimums, and the axes. This example demonstrates that with the addition of an internal pin we get an additional equation, otherwise we would have too many unknowns.

Support Reactions





Summary

Sign Convention

Shear Diagrams

Moment Diagrams

Observations about the Shape of Shear/ Moment Diagrams

Shear Diagrams:

-Are a plot of forces (note the units).

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Moment Diagrams:

-Are a plot of moments (note the units).

-Discontinuities occur at concentrated moments.

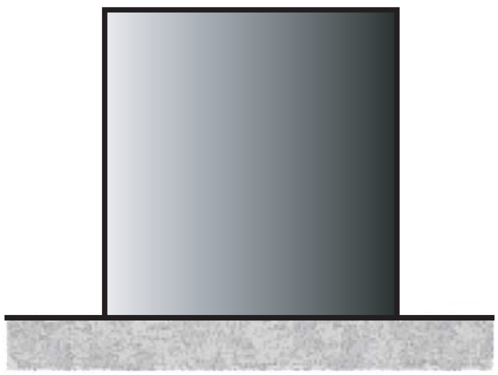
Miscellaneous:

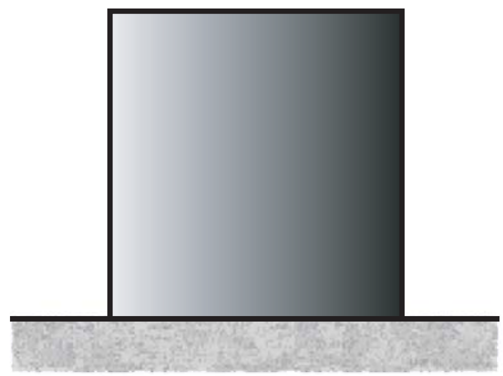
-Check your work by noting that you always start and end at zero.

-Always use the original loading and not the equivalent.

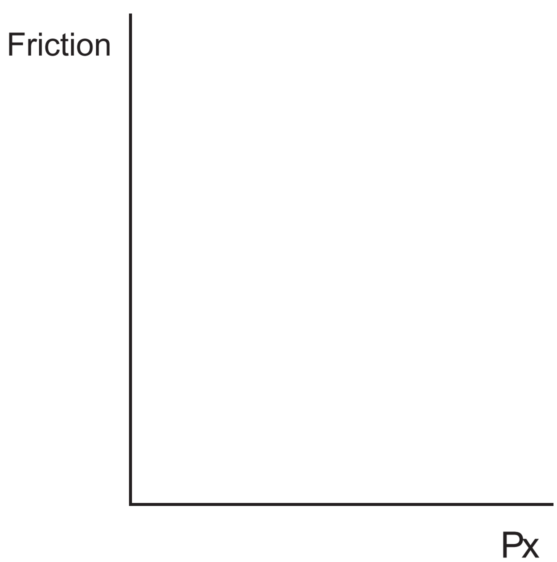
Chapter 8
Friction

Introduction





$$F = {}_{s}N$$

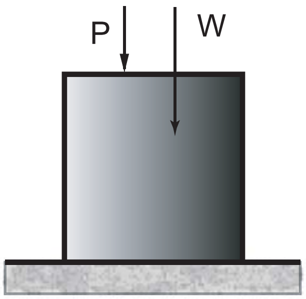


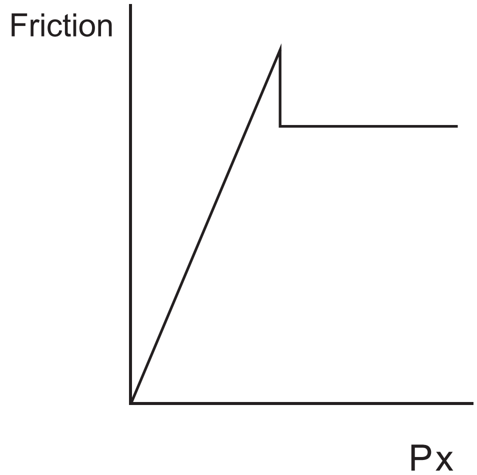
Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15-0.60
Metal on wood	0.20-0.60
Metal on stone	0.30-0.70
Metal on leather	0.30-0.60
Wood on wood	0.25-0.50
Wood on leather	0.25-0.50
Stone on stone	0.40-0.70
Earth on earth	0.20-1.00

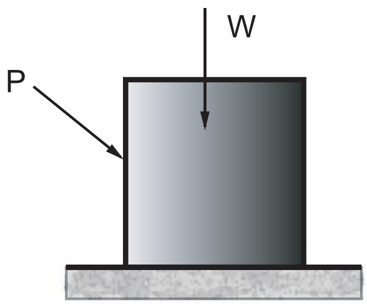
States of Friction

No Friction

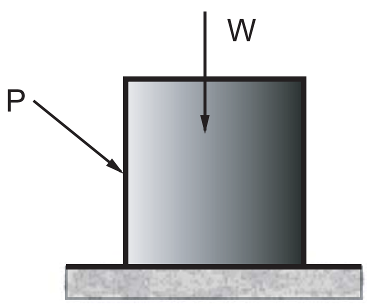




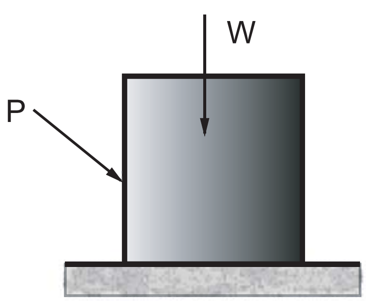
No Motion



Motion Impending

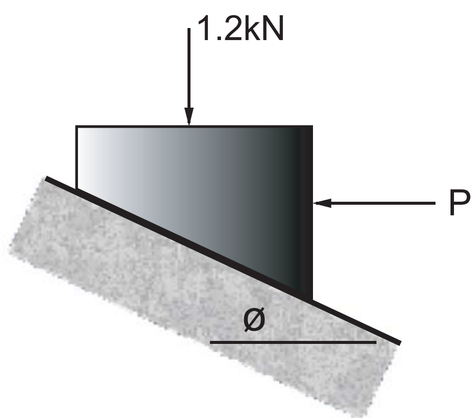


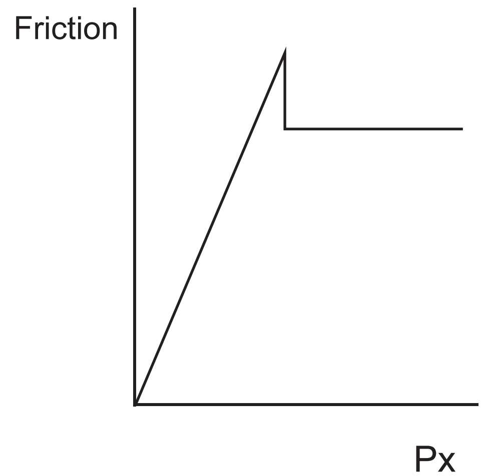
Motion

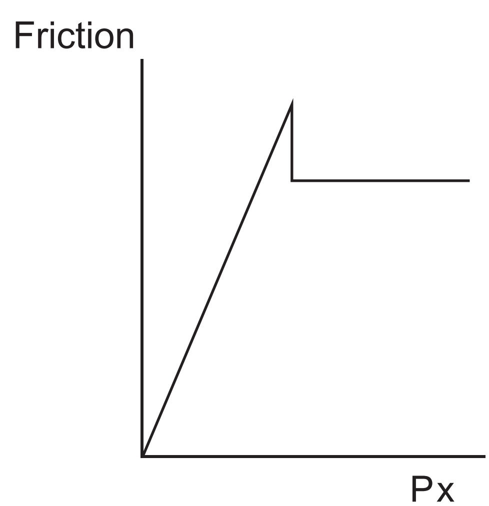


Example

The static and dynamic coefficients of friction between the block and the incline are 0.35 and 0.25 respectively. Determine whether the block is in equilibrium and find the magnitude and direction of the friction force when ø= 25° and P= 750N. Units: N.

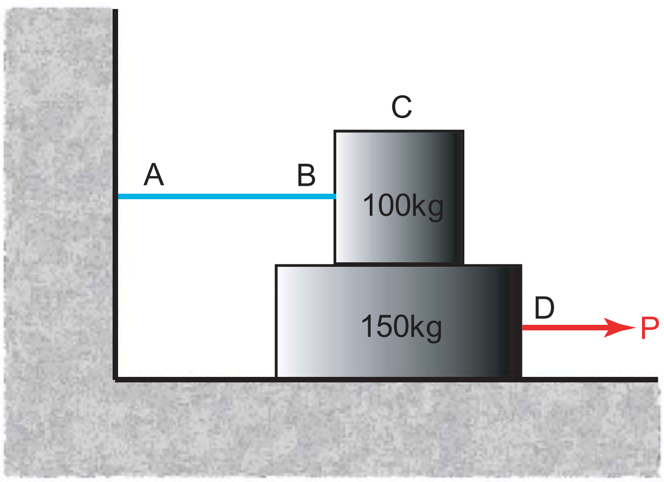




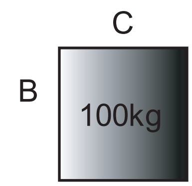


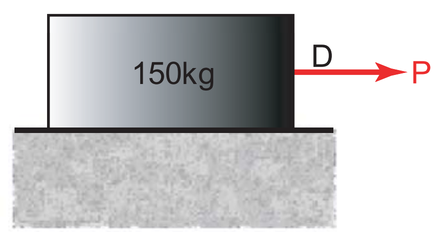
Example

The static and dynamic coefficients of friction between all surfaces are 0.30 and 0.25 respectively. Determine the smallest force P required to start block D moving if (a) block C is restrained by cable AB as shown, (b) cable AB is removed. Units: N.

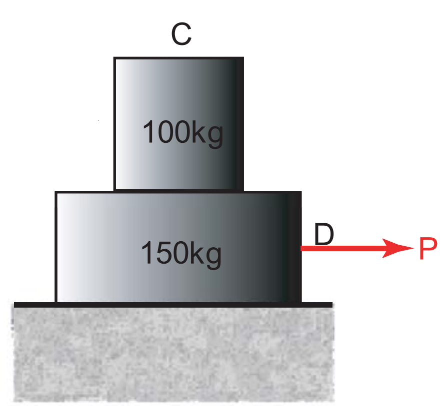


A)



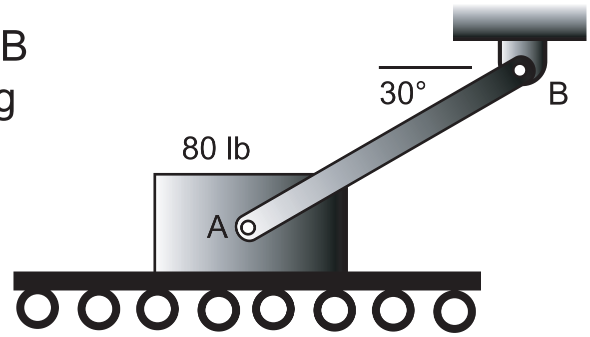


B)



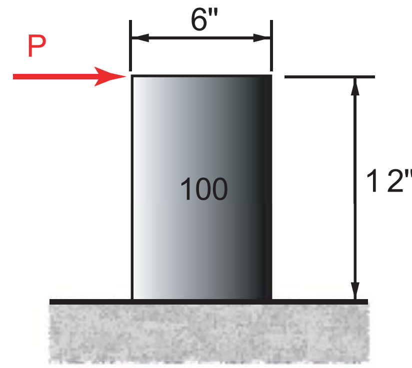
Example

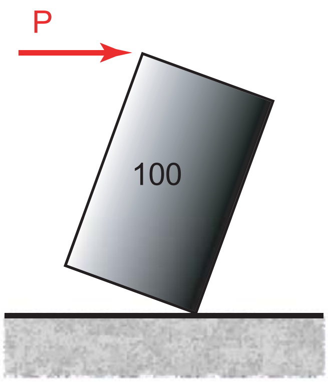
The 80 lb block is attached to link AB and rests on a moving belt. Knowing that the static and dynamic coefficients are 0.25 and 0.20, determine the magnitude of the horizontal force P which should be applied to the belt to maintain its motion (a) to the right, (b) to the left. Units: Lb.



Example

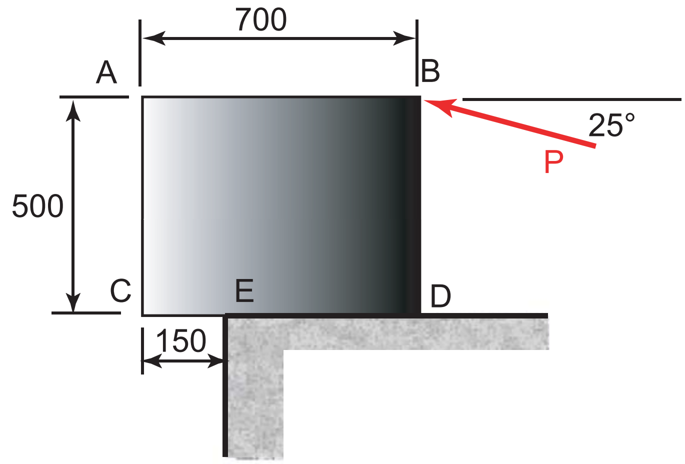
Determine whether the 100 lb block will tip before it has a chance to slide. The static coefficient is 0.30.





Example

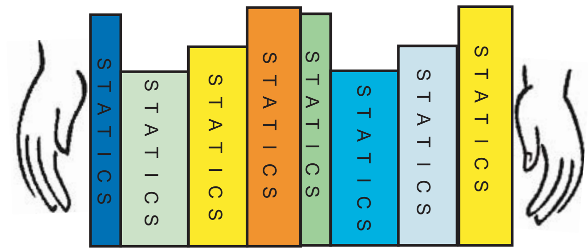
Knowing that the 100 kg crate starts to tip as it slides, determine (a) the magnitude of the force P, (b) the coefficient of kinetic friction.
Units: N, mm.

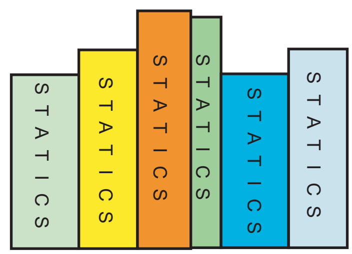


Example

Calculate the force that each hand must apply to support (8) 1 lb books. The coefficient of static friction between the hands and the books is 0.50 and 0.35 between each book.

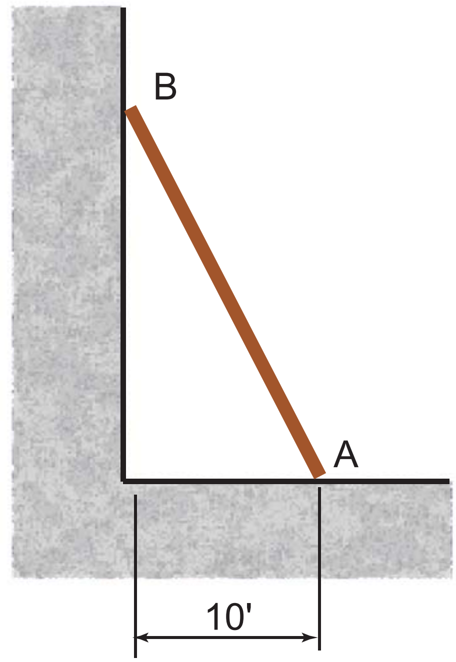
Units: lb.





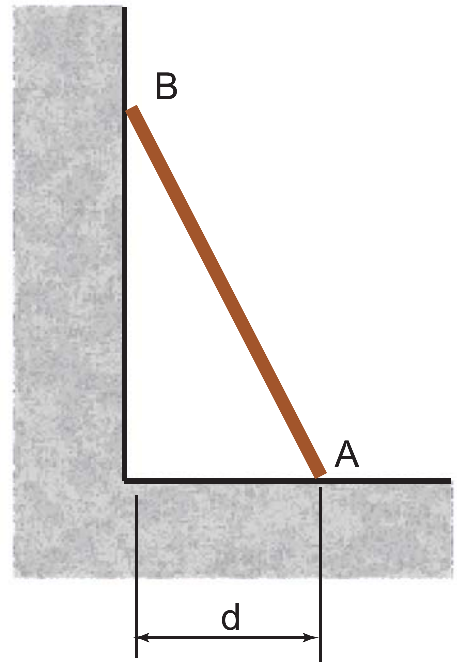
Example

Determine the reactions at A and B. A is rough and B is smooth. The ladder has a length of 25 ft and weighs 25 lb. The coefficient of static friction is 0.3. Units: Lb, ft.



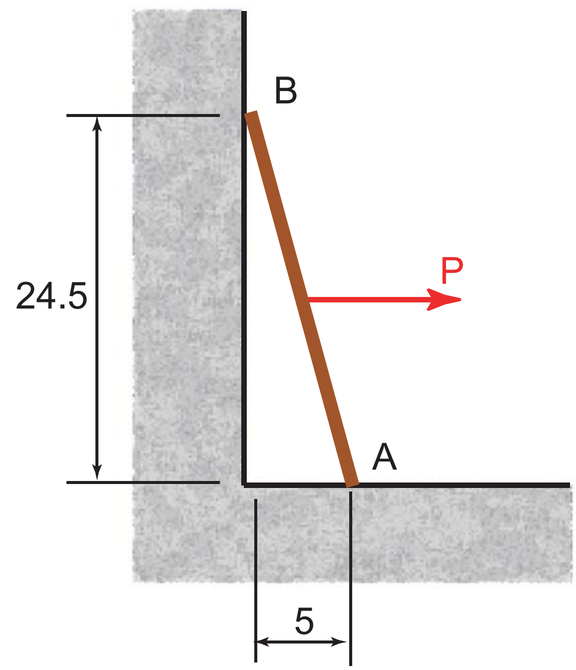
Example

Determine the distance d at which the ladder will start to slide. A is rough and B is smooth. The ladder has a length of 25 ft and weighs 25 lb. The coefficient of static friction is 0.3. Units: Lb, ft.



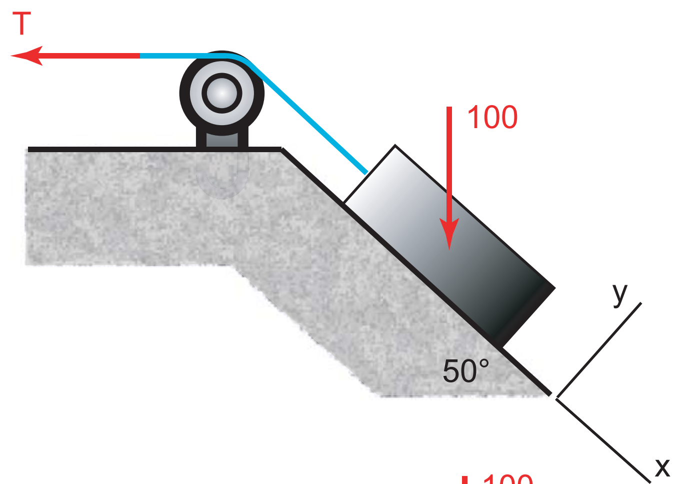
Example

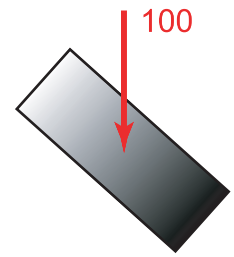
Determine the force P required to start the ladder to slide or tip. A is rough and B is smooth. P is applied to the middle of the ladder. The ladder has a length of 25 ft and weighs 25 lb. The coefficient of static friction is 0.3. Units: Lb, ft.

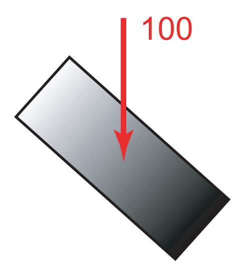


Example

Find the range of T for which the block is in equilibrium. The coefficient of friction is 0.3. Units: N.

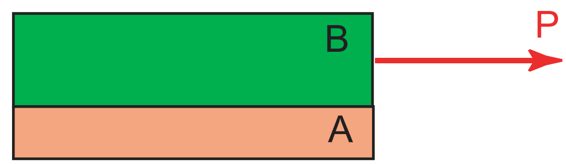






Example

Determine the smallest force P required to move the blocks. Block A and B weigh 20 and 40 lb respectively.
Units: Lb.



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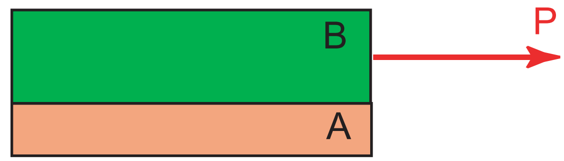
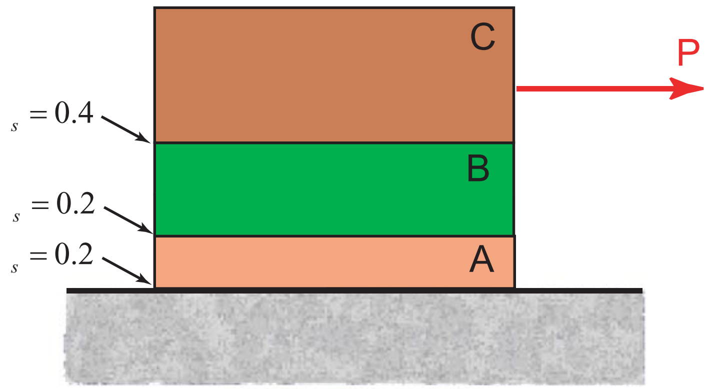
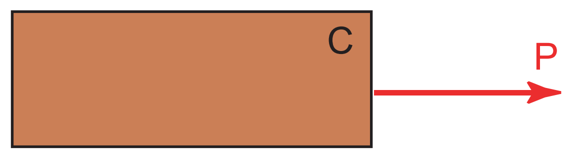
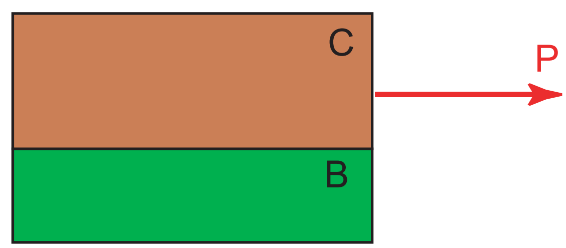


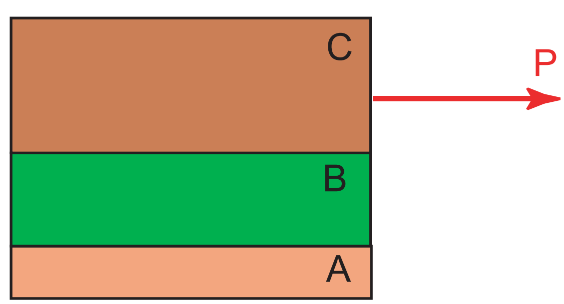
Example

Determine the smallest force P required to move the blocks. Block A, B, C weigh 20, 40, and 60 Lb respectively.
Units: Lb.

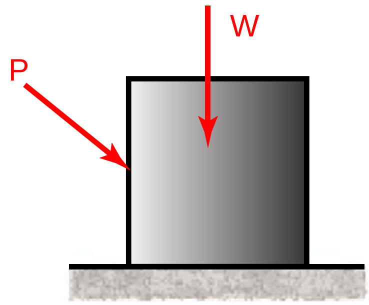




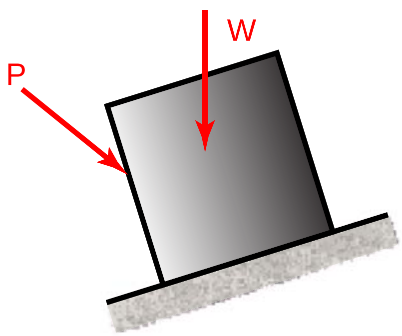




Angles of Friction



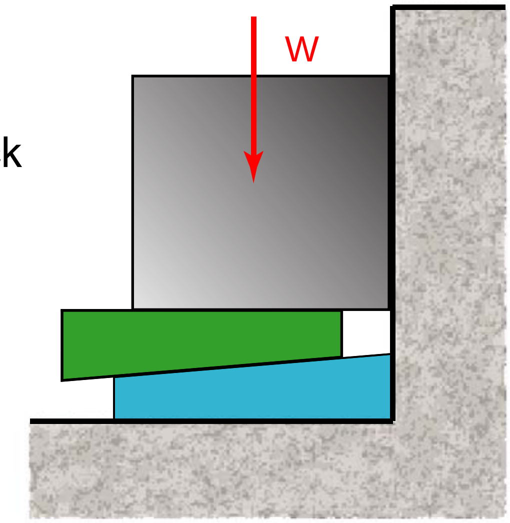
$$\tan \phi_s = \mu_s$$

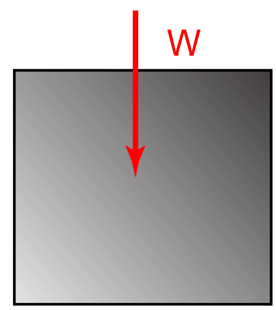


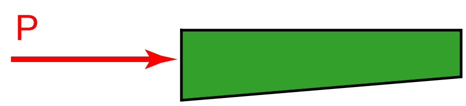
Wedges

Example

Find the force P required to move the block up. W= 500 lb, Static coefficient= 0.30, a 15° wedge. Units: Lb.

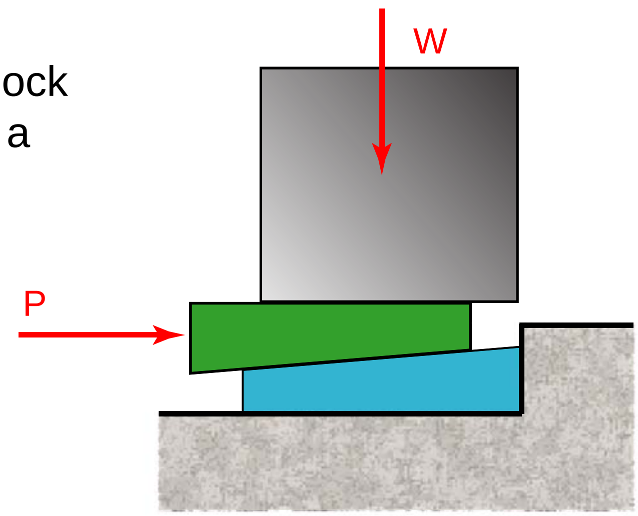


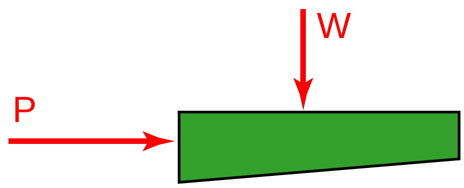




Example

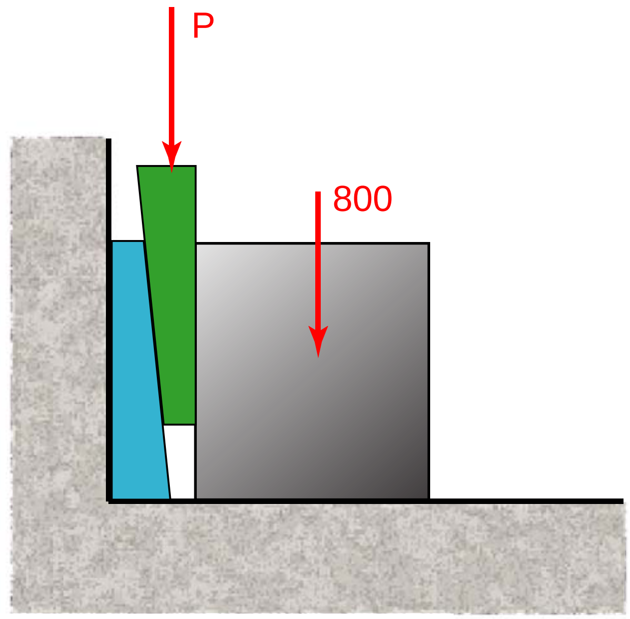
Find the force P required to move the block up. W= 500 lb, Static coefficient= 0.30, a 15° wedge. Units: Lb.

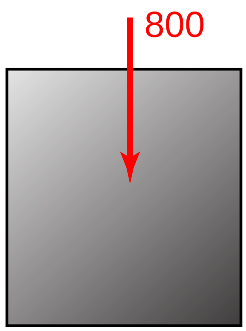


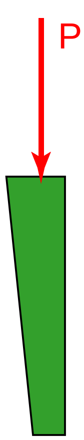


Example

Find the minimum force P required to move the 800 N block. Static coefficient= 0.30, 15° wedges. Units: N.

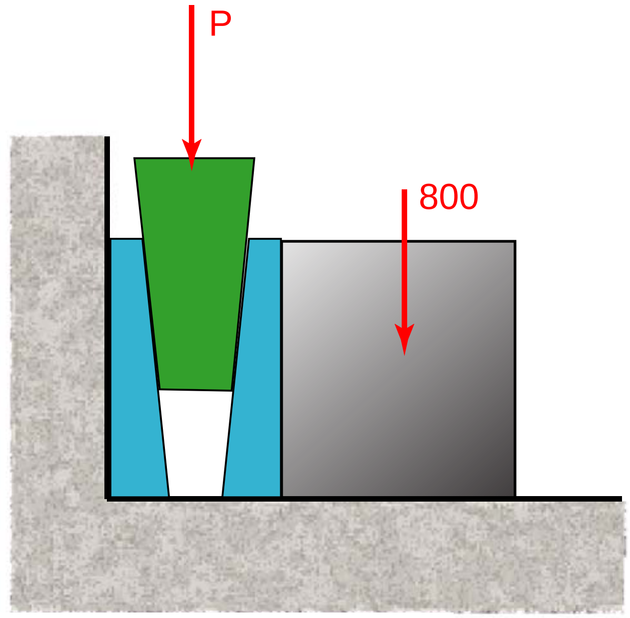


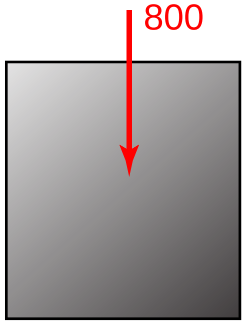


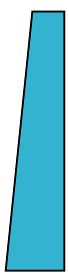


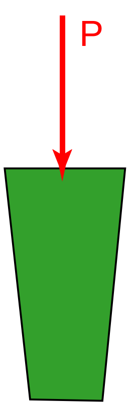
Example

Find the minimum force P required to move the 800 N block. Static coefficient= 0.30, 15° wedges. Units: N.



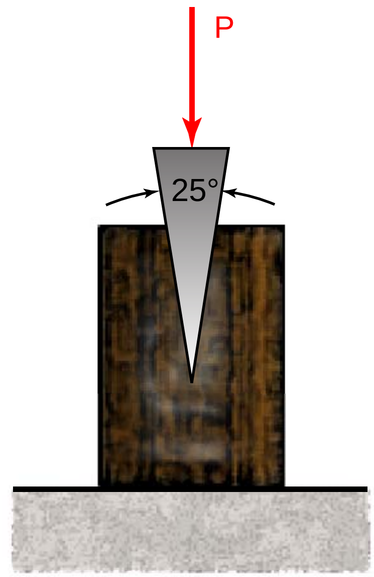


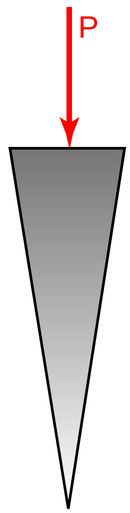




Example

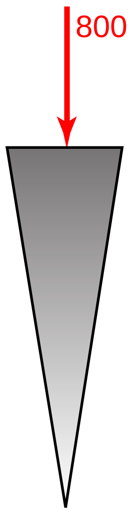
Knowing that it takes 800 N to insert the 25° wedge, find the forces exerted on the log. Kinetic coefficient= 0.26. Units: N.

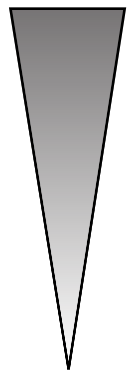


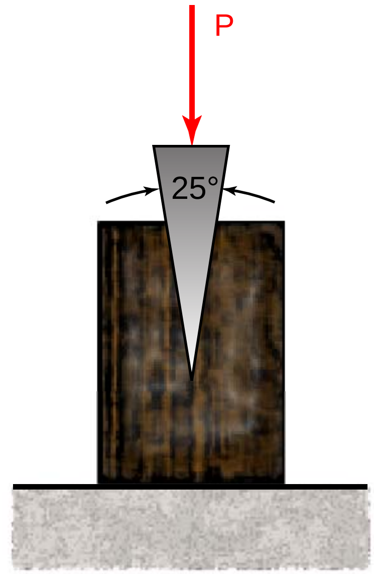


Example

Knowing that it takes 800 N to insert the 25° wedge, will the wedge remain in place after P is removed? Static coefficient= 0.30. Units: N.

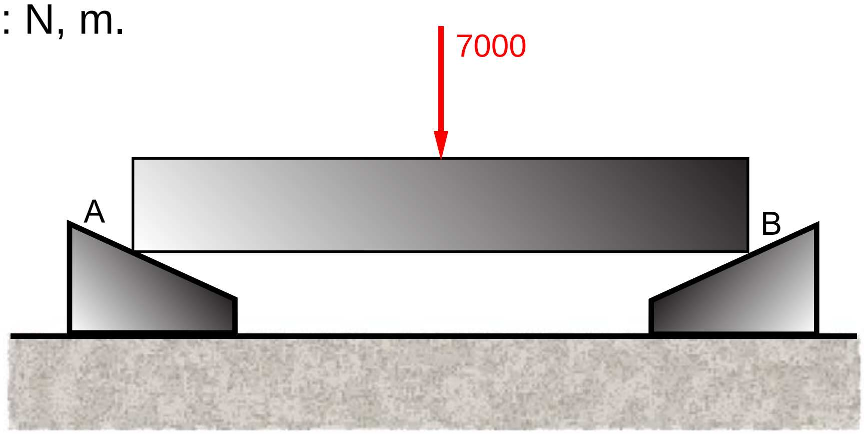


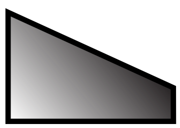


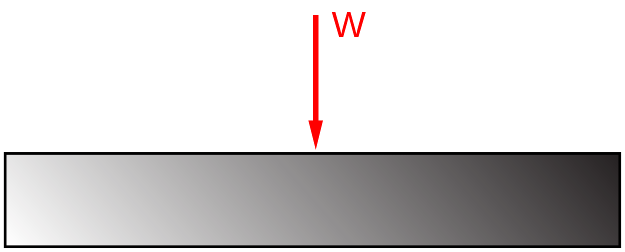


Example

Wedge B is rough and the right end of the beam may be considered as fixed. Determine the horizontal force P which should be applied to the 15° wedge A to raise the left end of the beam. The beam is 7 m long. Static coefficient= 0.20. Units: N, m.

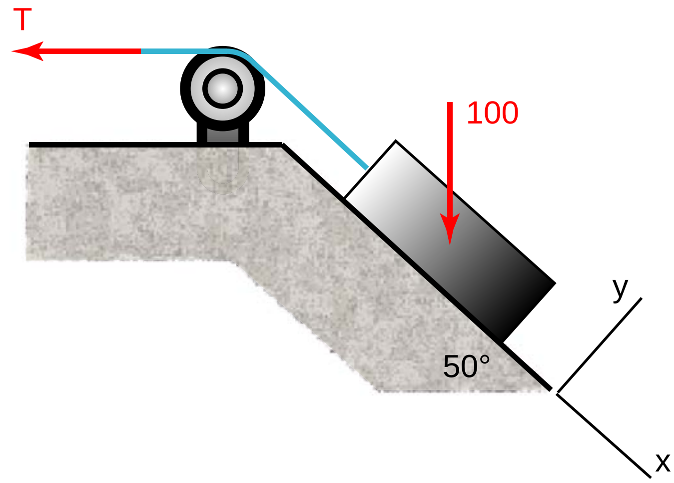


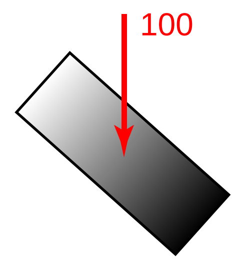


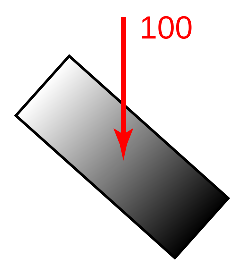


Example

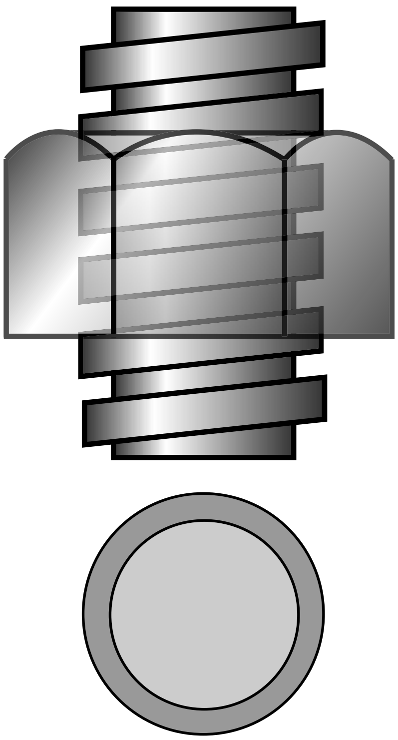
Find the range of T for which the block is in equilibrium. The static coefficient of friction is 0.3. Units: N.

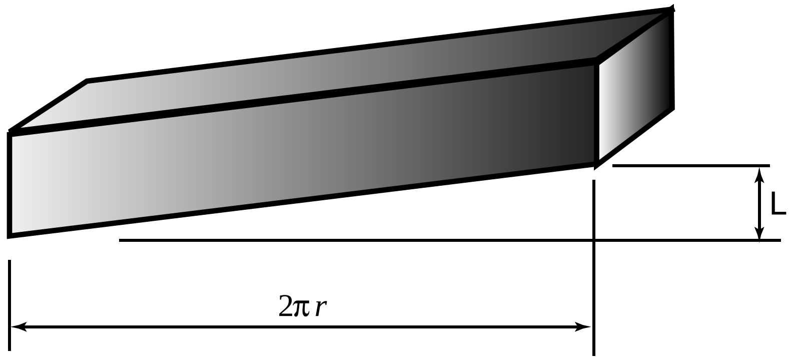


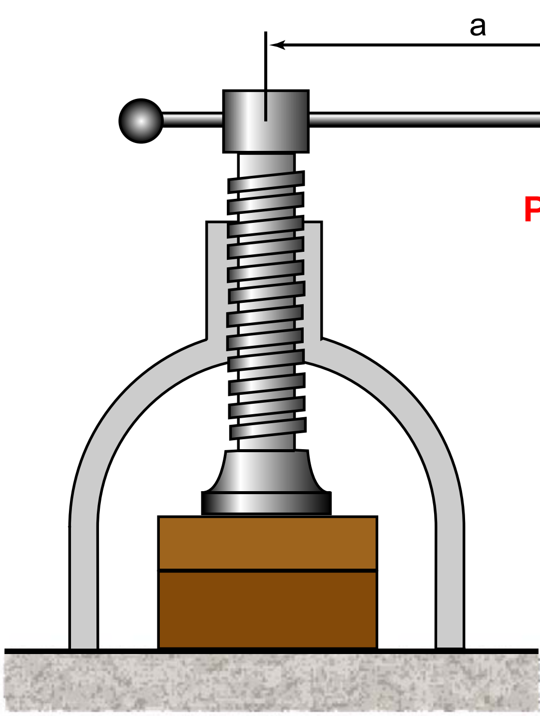


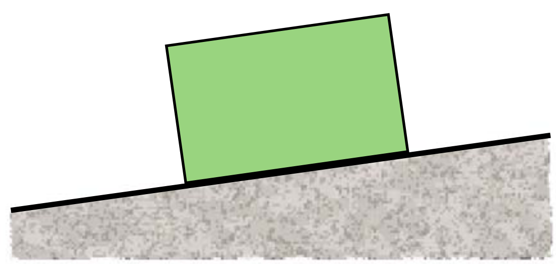


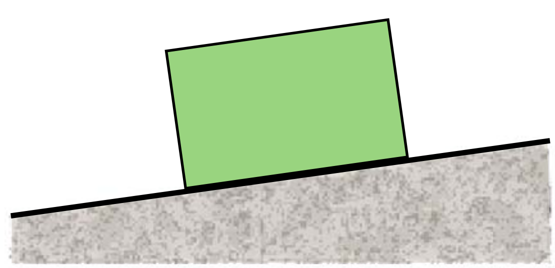
Square-Threaded Screws



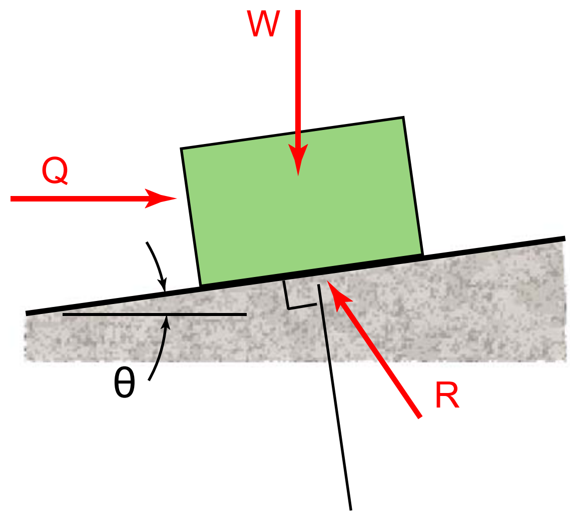






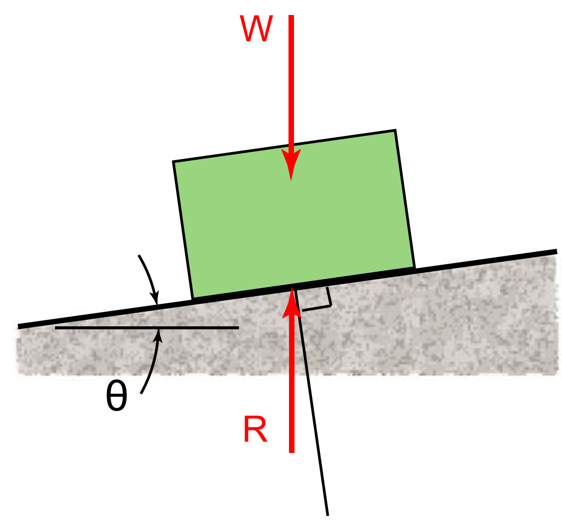


Tightening the Screw

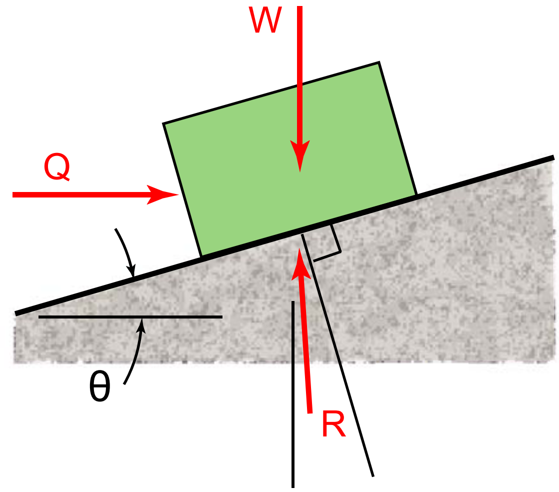


$$M = Wr\tan(\theta + \phi_s)$$

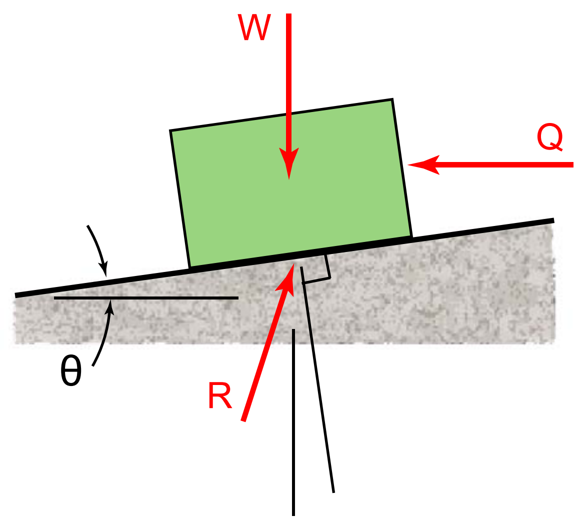
Self-Locking Screw



Loosening the Screw



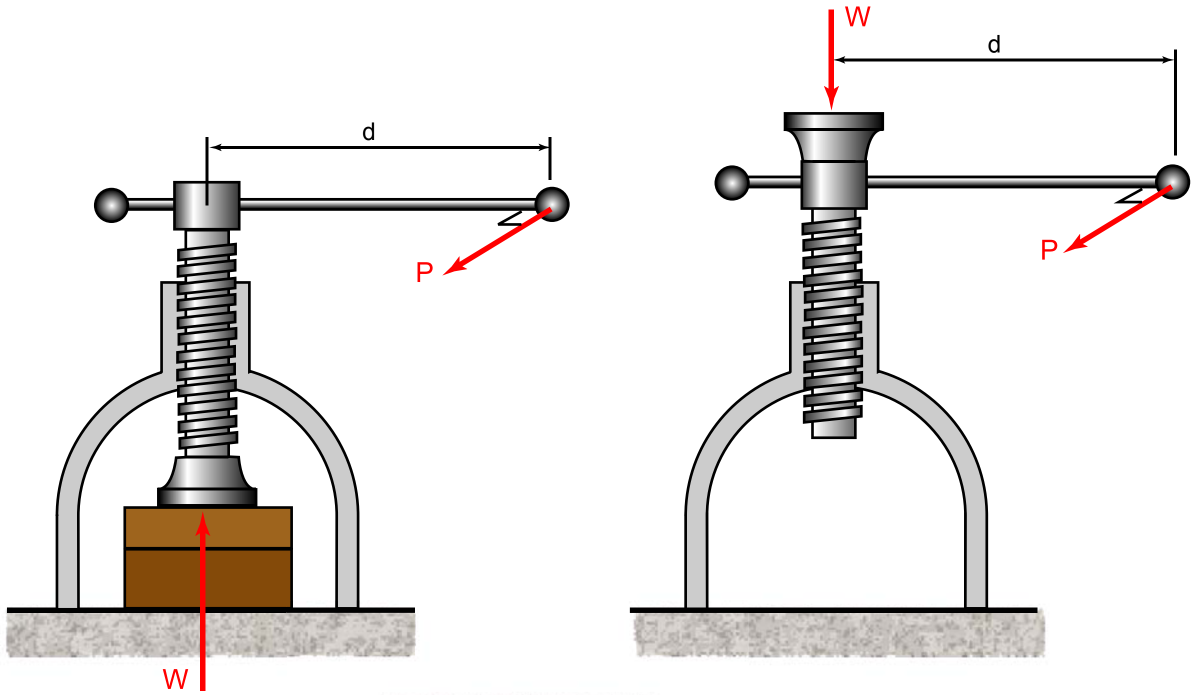
$$M = Wr\tan(\theta - \phi_s)$$

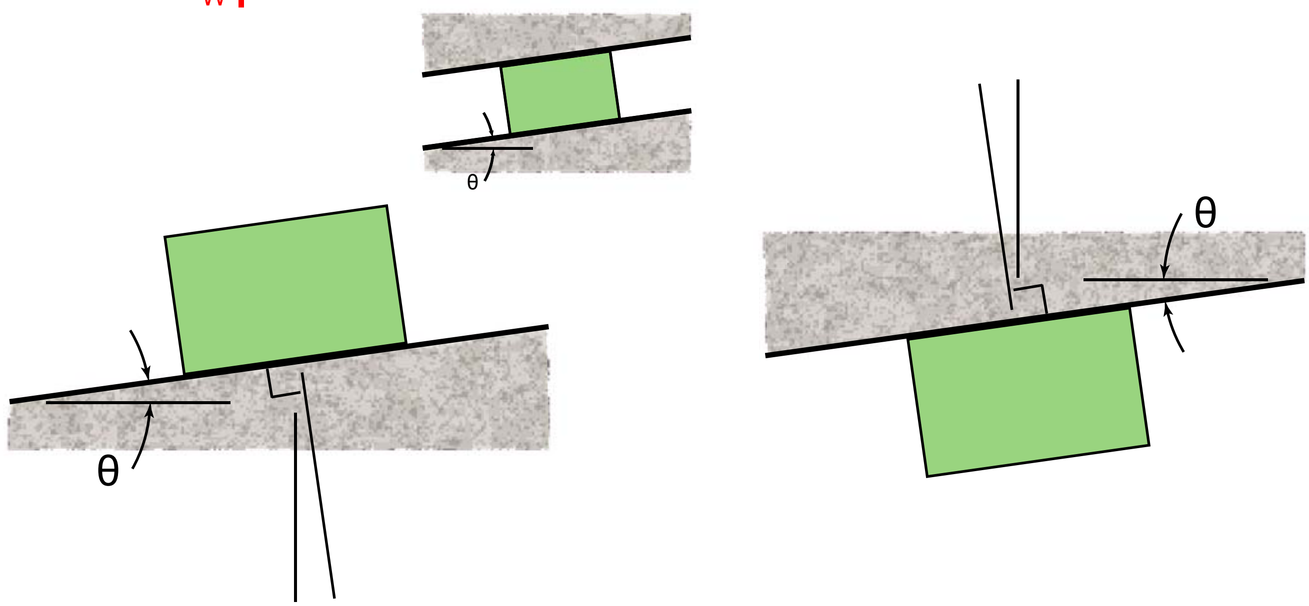


$$M = Wr\tan(\phi_s - \theta)$$

Example

Analyze the amount of torque required to move the screws downward. The two systems are identical except for the placement of the load W.

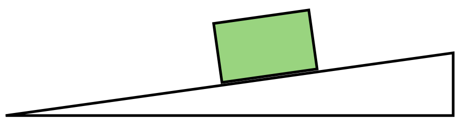


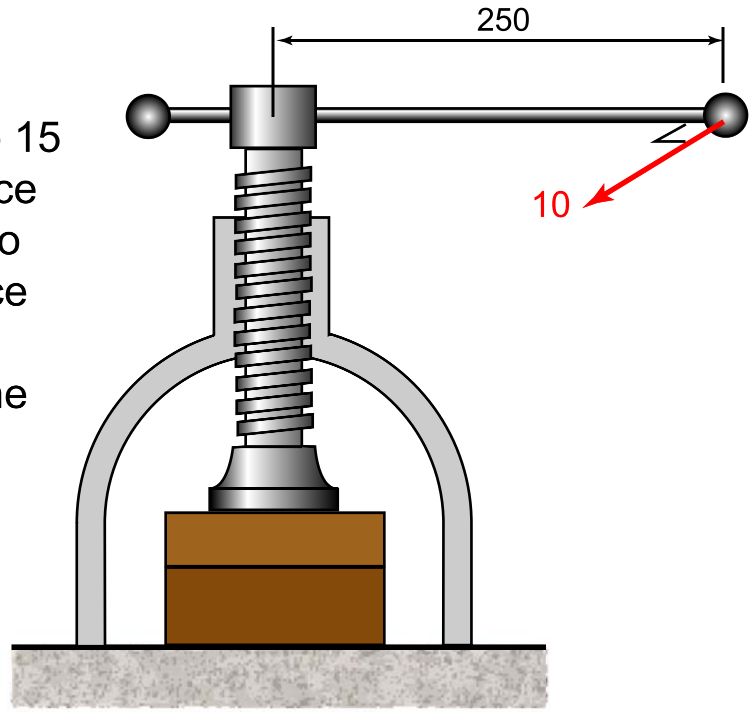


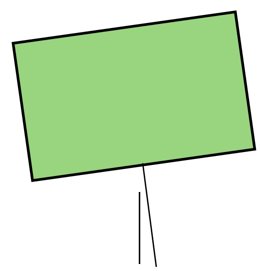
Common Sense: If you are turning it to where you know that it will be harder to turn, then use the equation on the left. If looser, then use the equation on the right.

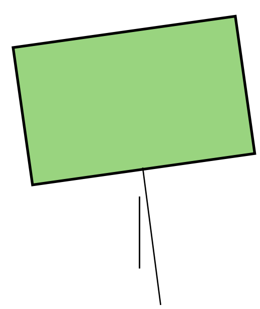
Example

The clamp has a single square thread of mean diameter equal to 15 mm with a pitch of 3 mm. If a force of 10 N is applied perpendicular to the handle, determine (a) the force exerted on the pieces of wood, (b) the force required to loosen the clamp. The static coefficient of friction is 0.3. Units: N, mm.



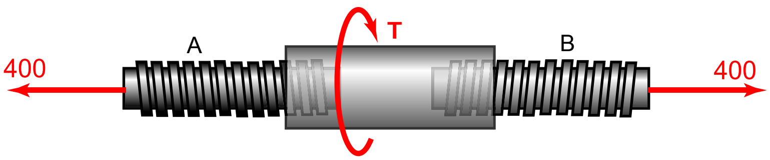


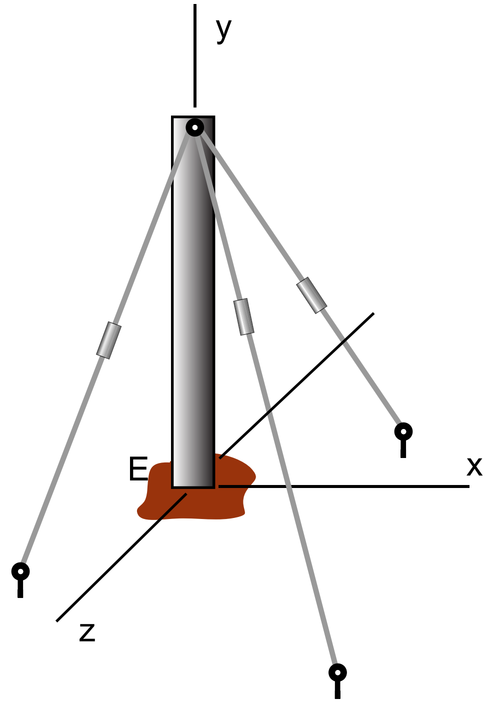




Example

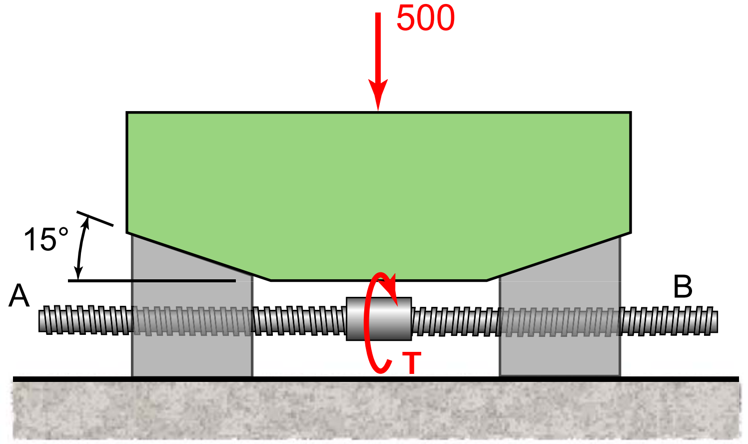
Rod A has a right-handed thread and rod B has a left-handed thread. Both rods are single-threaded with a mean radius of 0.3 in. and a pitch of 0.1 in. Determine the torque T that must be applied to the sleeve in order for the (a) rods to tighten, (b) rods to loosen. The static coefficient of friction is 0.30. Units: lb.

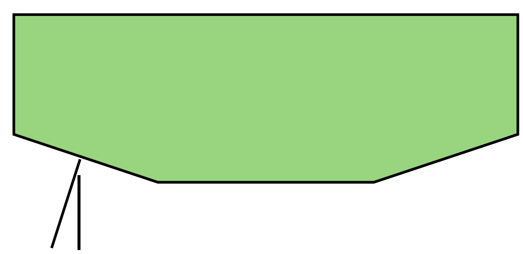


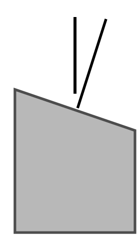


Example

The two sliding 15° wedges support the 25 lb block above them and the 500 lb load. Rod A has a right-handed thread and rod B has a left-handed thread. Both rods are single-threaded with a mean radius of 0.3 in. and a pitch of 0.1 in. Determine the torque T that must be applied to the sleeve in order for the blocks to come together. The static coefficient of friction between all surfaces is 0.3. Units: lb.

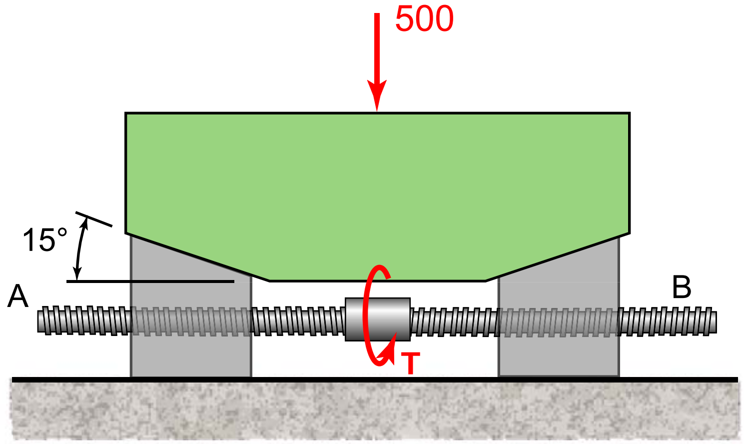


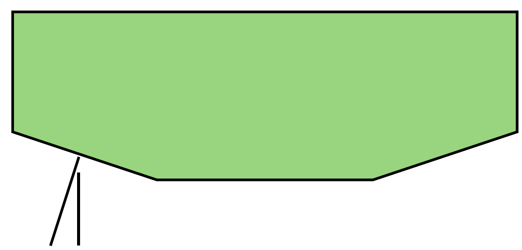


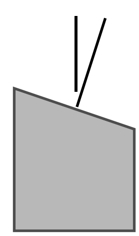


Example

The two sliding 15° wedges support the 25 lb block above them and the 500 lb load. Rod A has a right-handed thread and rod B has a left-handed thread. Both rods are single-threaded with a mean radius of 0.3 in. and a pitch of 0.1 in. Determine the torque T that must be applied to the sleeve in order for the blocks to move apart. The static coefficient of friction between all surfaces is 0.3. Units: lb.

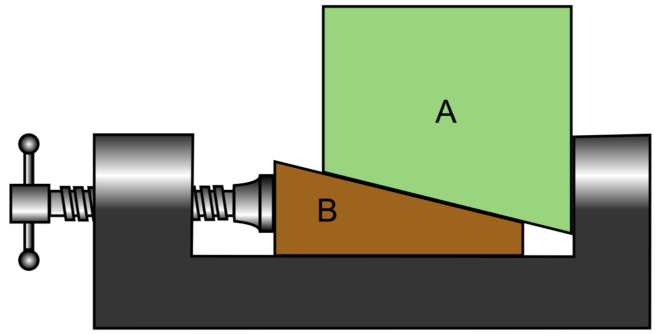


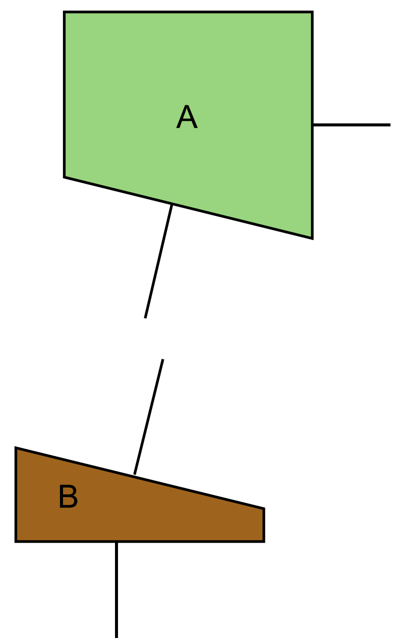




Example

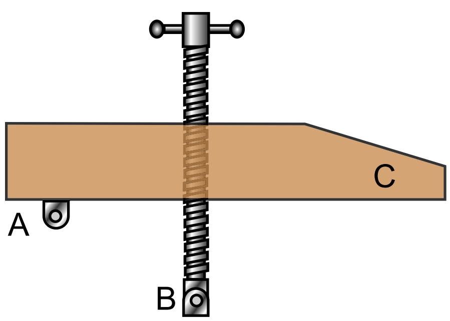
The clamp has a single square thread of mean diameter equal to 0.3 in. with a pitch of 0.1 in. The screw has a static coefficient of friction of 0.3, whereas it is 0.5 between the two blocks and the clamp. Ignore the weight of the 12° wedge B. Determine the torque required at the clamp's handle to lift the 200 lb block A.

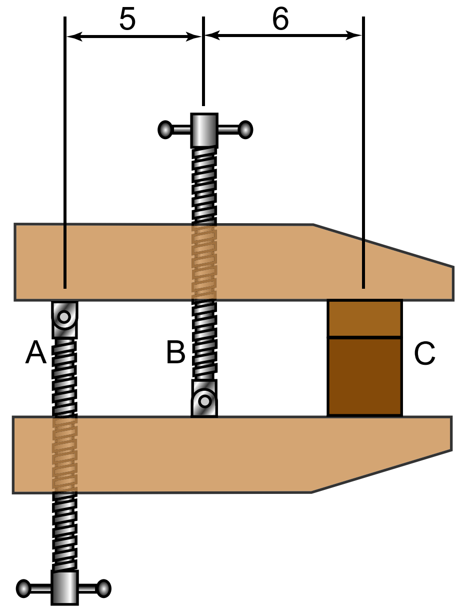




Example

The wood clamp has a single square thread of mean diameter equal to 0.3 in. with a pitch of 0.1 in. The screws have a static coefficient of friction of 0.3. The clamp exerts a force of 200 lbs on the blocks at C. Determine the torque required to loosen the clamp if a) the torque is applied to screw A, b) the torque is applied to screw B.
Units: Inches.





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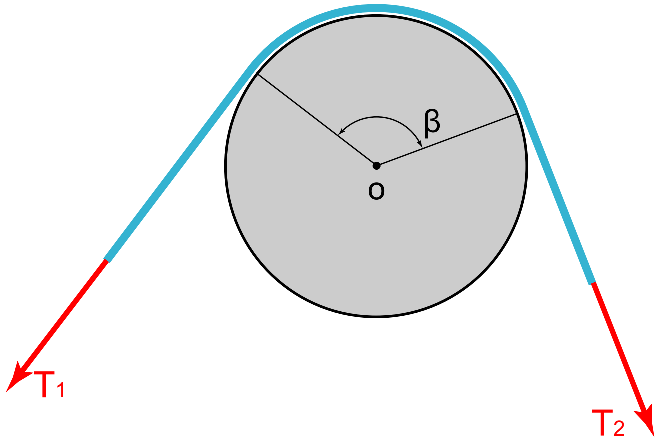
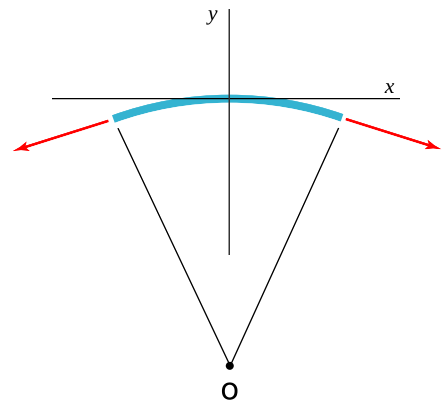
Belt Friction





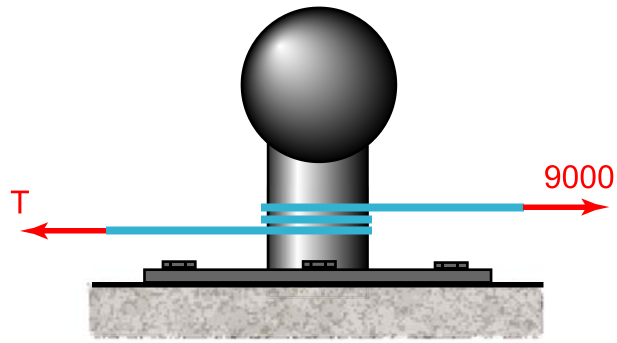
$$\sum F_x = 0: \qquad (T + \Delta T)\cos\frac{\Delta\theta}{2} - T\cos\frac{\Delta\theta}{2} - \mu_s \Delta N = 0$$

$$\sum F_y = 0: \qquad \Delta N - (T + \Delta T)\sin\frac{\Delta\theta}{2} - T\sin\frac{\Delta\theta}{2} = 0$$

$$\ln\frac{T_2}{T_1} = \mu_s \beta \qquad \frac{T_2}{T_1} = e^{\mu_s \beta}$$

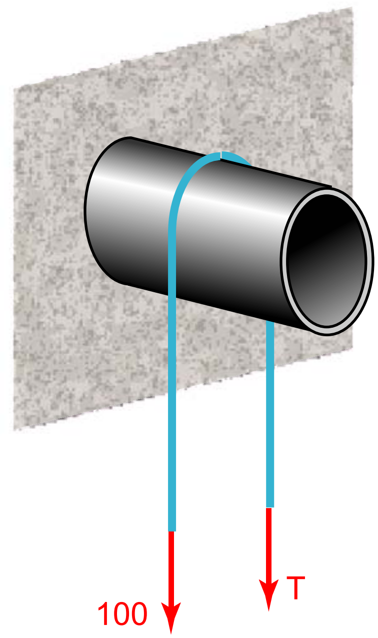
Example

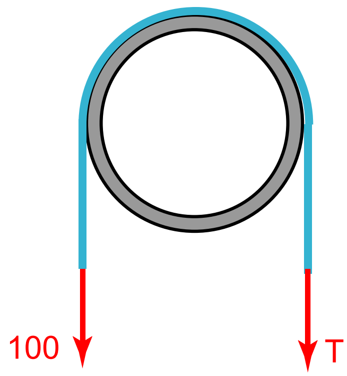
A ship tied up to the dock develops 9000 N in the rope that is wrapped two times around the bollard. Determine the minimum force required to keep the boat in place. The static coefficient is 0.3. Units: N.

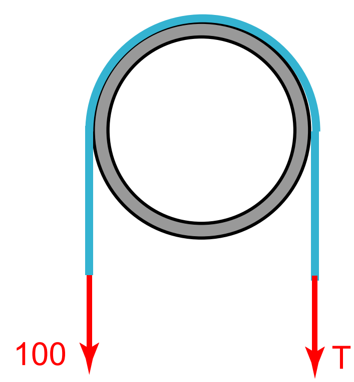


Example

A rope is thrown around a 250 mm diameter pipe to raise a 100 N weight. What is the minimum tension T that must be applied to a) raise the weight, b) maintain this position? c) What effect will replacing the 250 mm pipe with a 350 mm pipe have? The static coefficient is 0.3. Units: N.

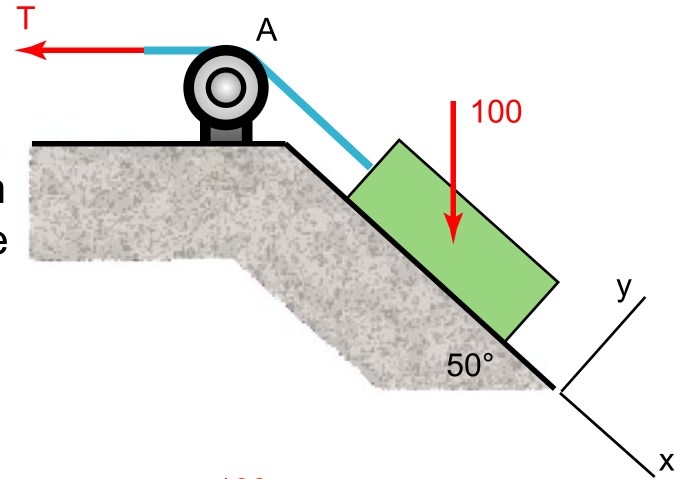


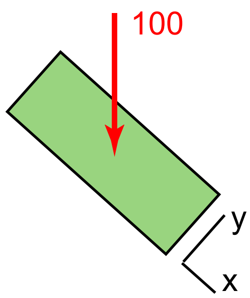


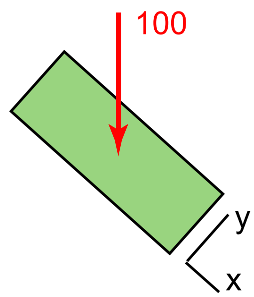


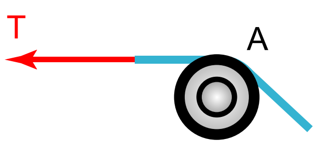
Example

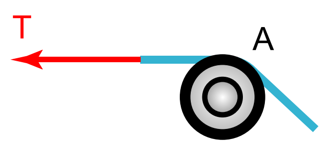
The pulley at A has rusted and is no longer able to rotate freely. Determine the (a) minimum force required to keep the block from sliding down the incline, (b) minimum force required to pull the block up the incline. The static coefficient is 0.3 between all surfaces. Units: N.





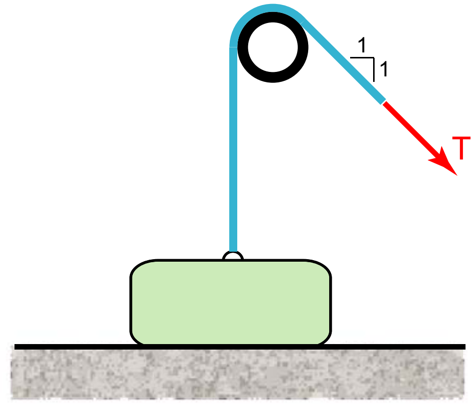


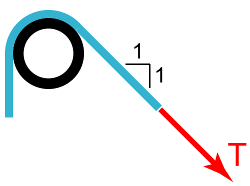


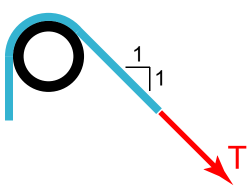


Example

A rope is thrown around a pipe to raise a 100 N weight. a) What is the minimum tension T that must be applied to lift the weight?
b) What is the minimum tension T that must be applied to then lower the weight?
The static coefficient is 0.3. Units: N.

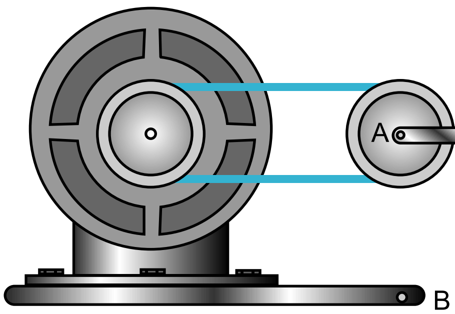


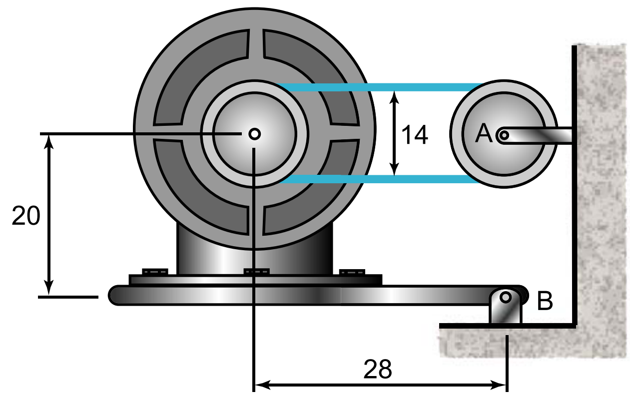


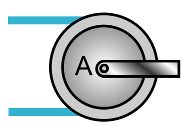


Example

The 200 lb motor's weight is used to maintain tension in the flat belts. Determine the largest torque that can be transmitted to pulley A when the motor is rotating clockwise. The static coefficient is 0.3. Units: Lb, inches.

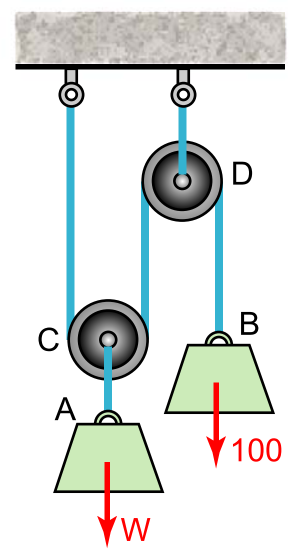






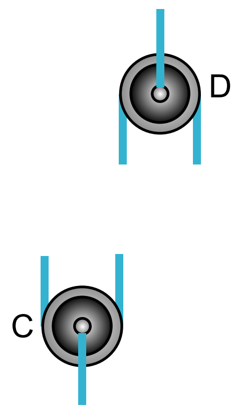
Example

Determine the minimum weight of A to maintain equilibrium if a) pulley D is locked, b) pulley C is locked, c) pulleys C and D are locked. The static coefficient is 0.3. Units: Lb.

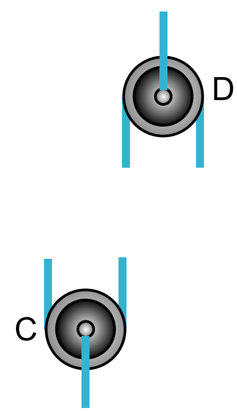


$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

a) D is locked: $\frac{T_2}{T_1} = e^{\mu_s \beta}$

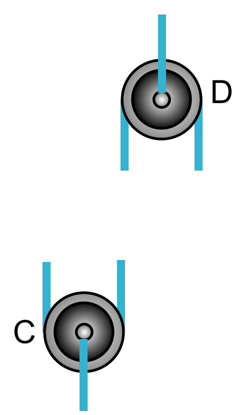


b) C is locked:



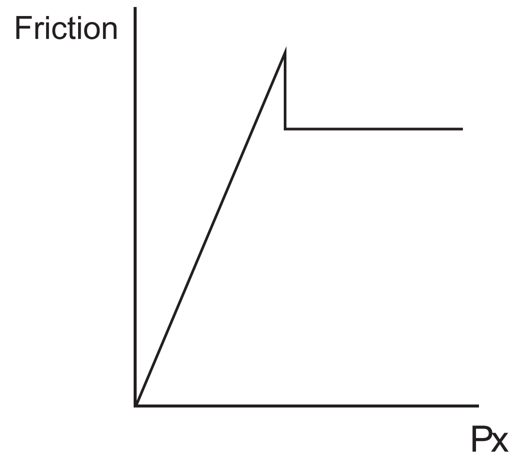
$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

a) C and D are locked:



$$\frac{T_2}{T_1} = e^{\mu_s \beta}$$

Summary

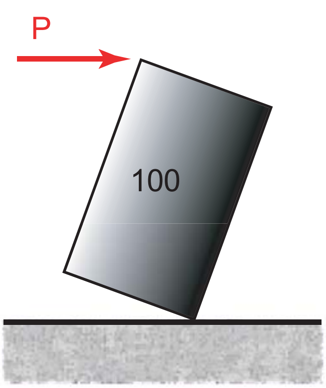


If Equilibrium

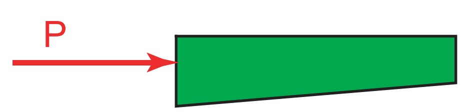
If Sliding

If Motion, but no Acceleration

If Tipping



Wedges

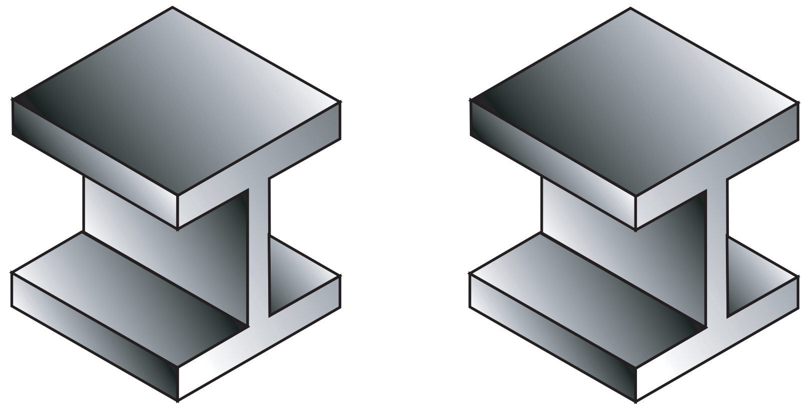


Chapter 9
Moments of Inertia of Areas

Introduction
Second Moment, or Moment of Inertia, of an Area

$$\sigma = \frac{My}{I}$$

$$\tau = \frac{VQ}{Ib}$$

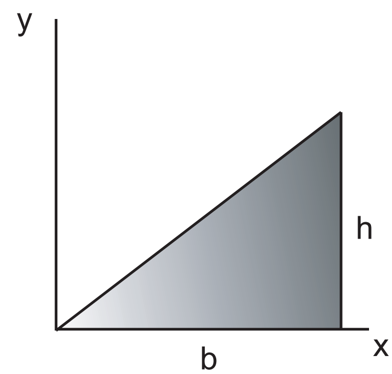


Determination of the Moment of Inertia of an Area by Integration

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$

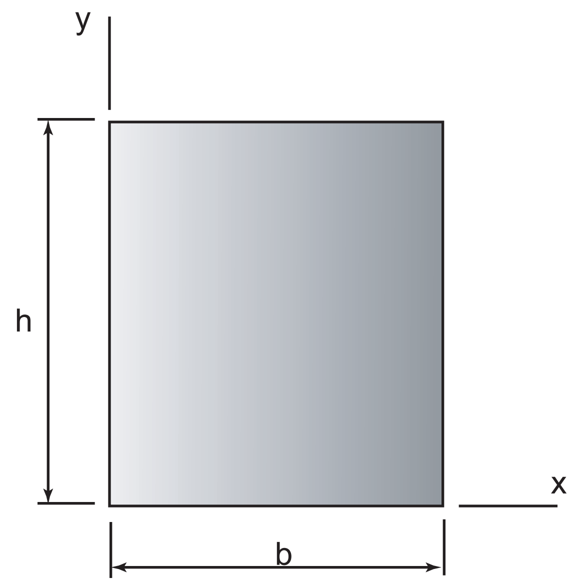
Example

Determine the moment of inertia about the x-axis of the area below. Use integration.

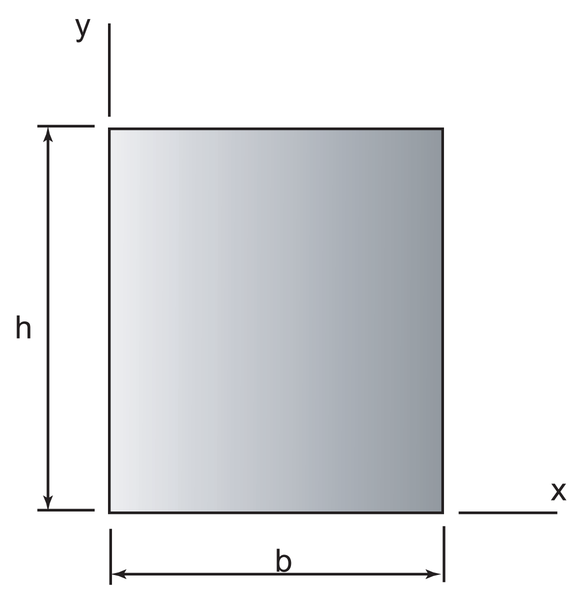


Example

Determine the moment of inertia about the axes of the area below. Use integration.



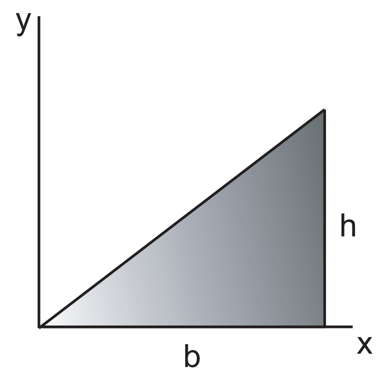
Alternative Solution



Example

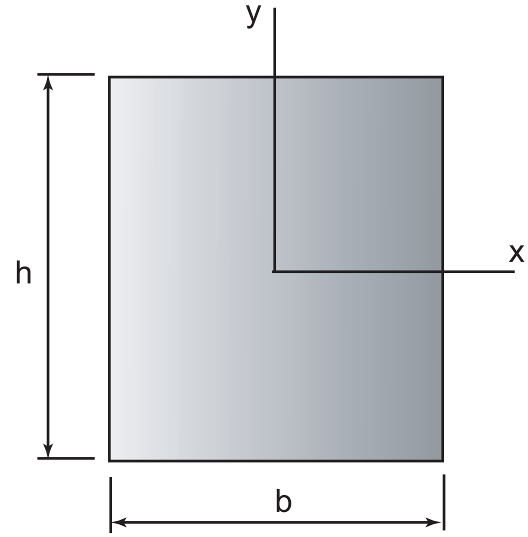
Determine the moment of inertia of the area below. Use integration.

Alternative Solution



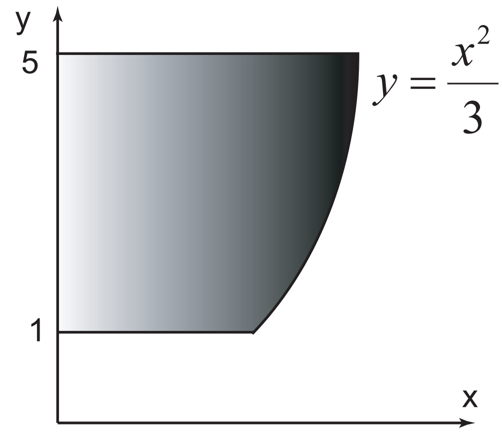
Example

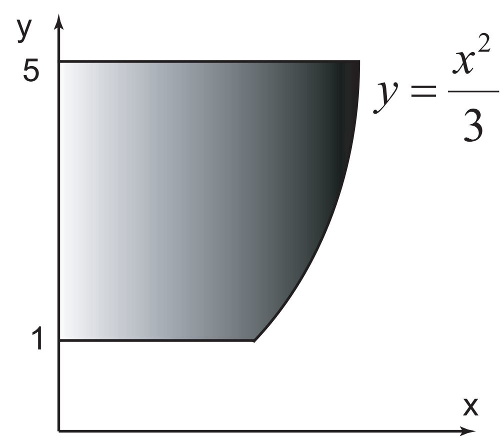
Determine the moment of inertia about the centroidal axes of the area below. Use integration.



Example

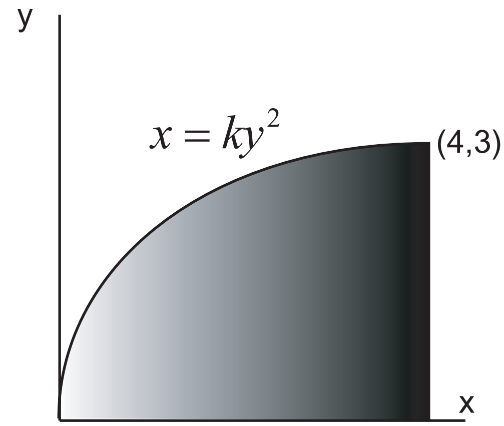
Determine the moment of inertia about the y-axis of the area below. Use integration.





Example

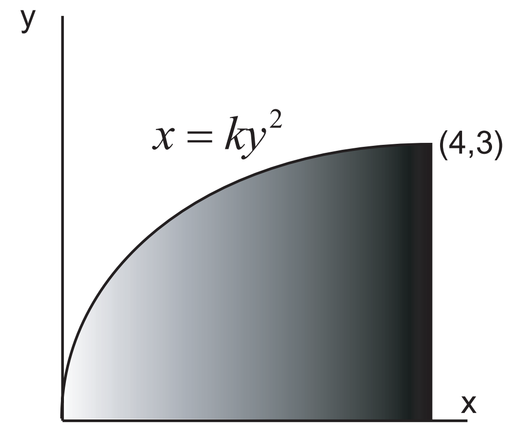
Determine the moment of inertia about the x-axis of the area below. Use integration.



Example

Determine the moment of inertia about the x-axis of the area below. Use integration.

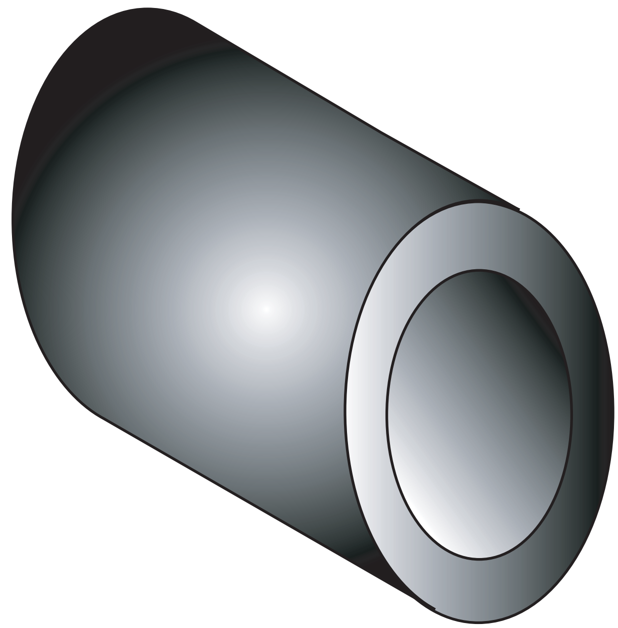
Alternative Solution



Polar Moment of Inertia

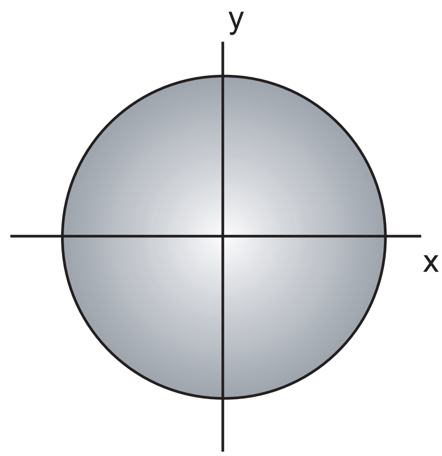
$$\tau = \frac{Tr}{J_o}$$

$$J_o = I_p = \int \rho^2 dA = \int (x^2 + y^2) dA$$

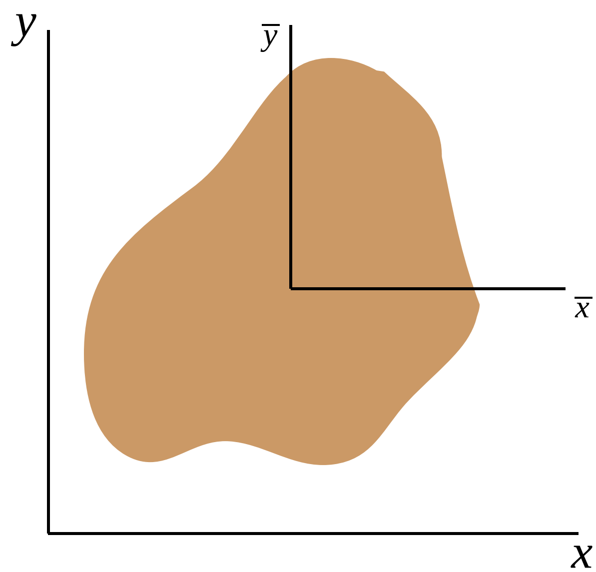


Example

Determine the polar moment of inertia of the area below. Use integration.



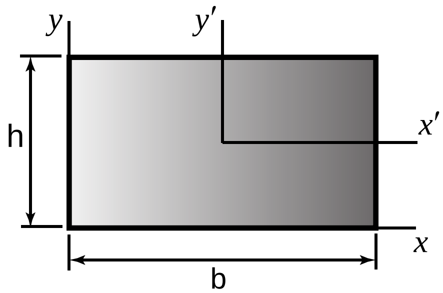
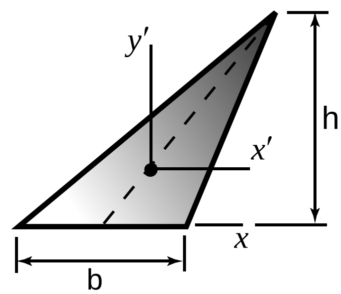
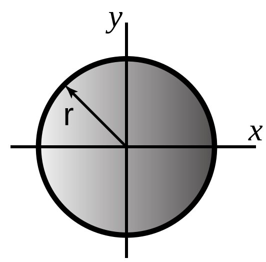
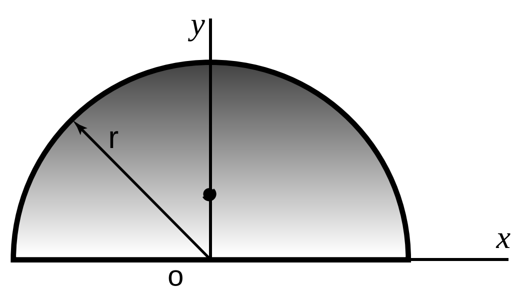
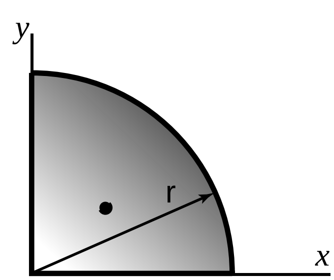
Parallel-Axis Theorem



$$I_x = \sum\left(\bar{I}_x + Ad_y^2\right)$$

$$I_y = \sum\left(\bar{I}_y + Ad_x^2\right)$$

Moments of Inertia of Composite Areas

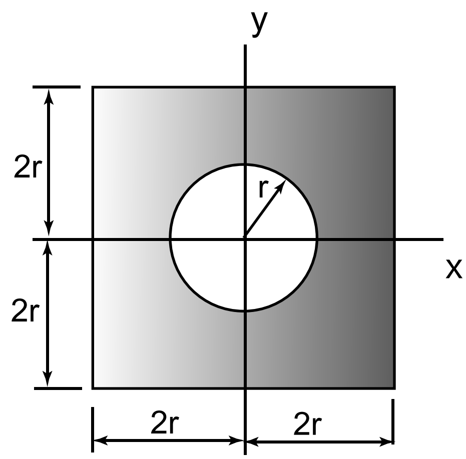
Shape		
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}hb^3$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}hb^3$ $J_c = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_o = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_o = \frac{1}{4}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_o = \frac{1}{8}\pi r^4$

Example

Determine the moment of inertia about the centrodial axes of the area below.

$$I_x = \sum \left(I_x + A d_y^2 \right)$$

$$I_y = \sum \left(I_y + A d_x^2 \right)$$



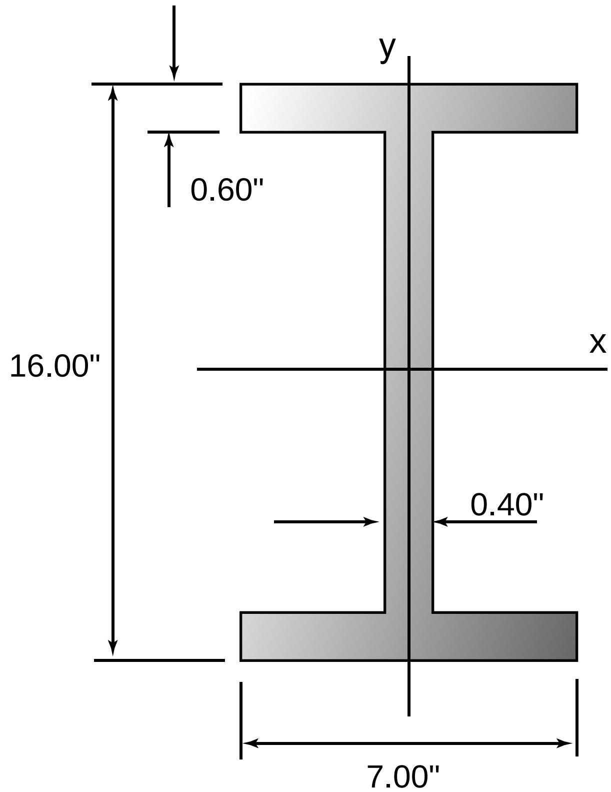
Part	I	Area	d	Ad^2

Example

Determine the moment of inertia about the centrodial axes of the area below. Units: in.

$$I_x = \sum\left(I_x + Ad_y^2\right)$$

$$I_y = \sum\left(I_y + Ad_x^2\right)$$



Note:
d is the distance between the property of the part to the new axis.
b is always parallel to the axis you want to find I about.

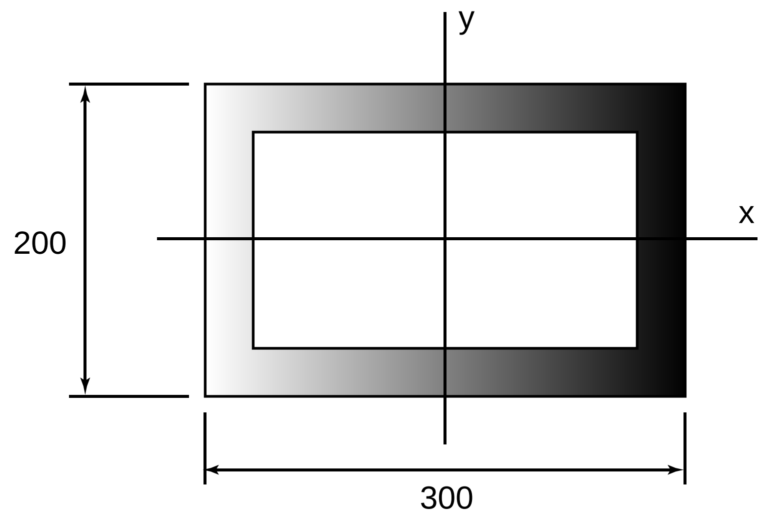
Part	I	d	Area	Ad^2

Example

Determine the moment of inertia about the centrodial axes of the area below. Uniform thickness t= 20 mm. Units: mm.

$$I_x = \sum \left(I_x + A d_y^2 \right)$$

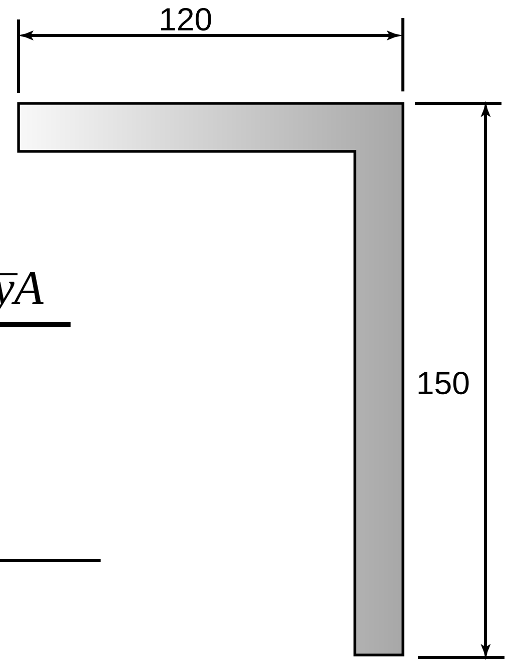
$$I_y = \sum \left(I_y + A d_x^2 \right)$$



Part	I_x	I_y	Area	d_x	d_y	Ad_x^2	Ad_y^2

Example

Determine the moment of inertia about the centrodial axes of the area below. The section has a uniform width of 20 mm. Units: mm.



Part	Area	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$

Part	I_x	I_y	Area	d_x	d_y	Ad_x^2	Ad_y^2

$$I_x = \sum \left(I_x + Ad_y^2 \right)$$

$$I_y = \sum \left(I_y + Ad_x^2 \right)$$

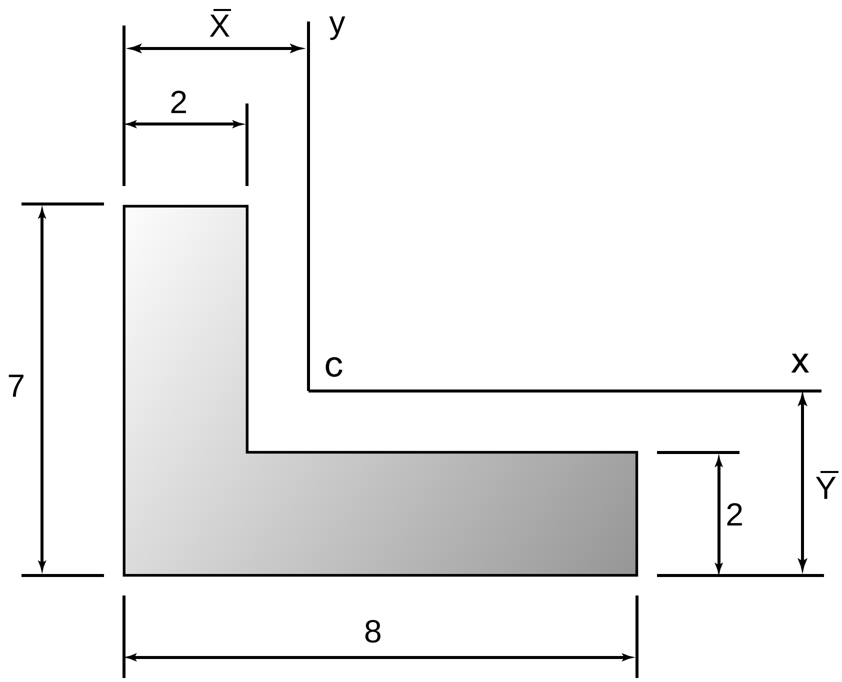
Example

Determine the moment of inertia about the x axis of the area below.
Units: in.

Given:

$$\bar{X} = 2.85$$

$$\bar{Y} = 2.35$$



Part	I	Area	d	Ad^2

$$I_x = \sum \left(I_x + Ad_y^2 \right)$$

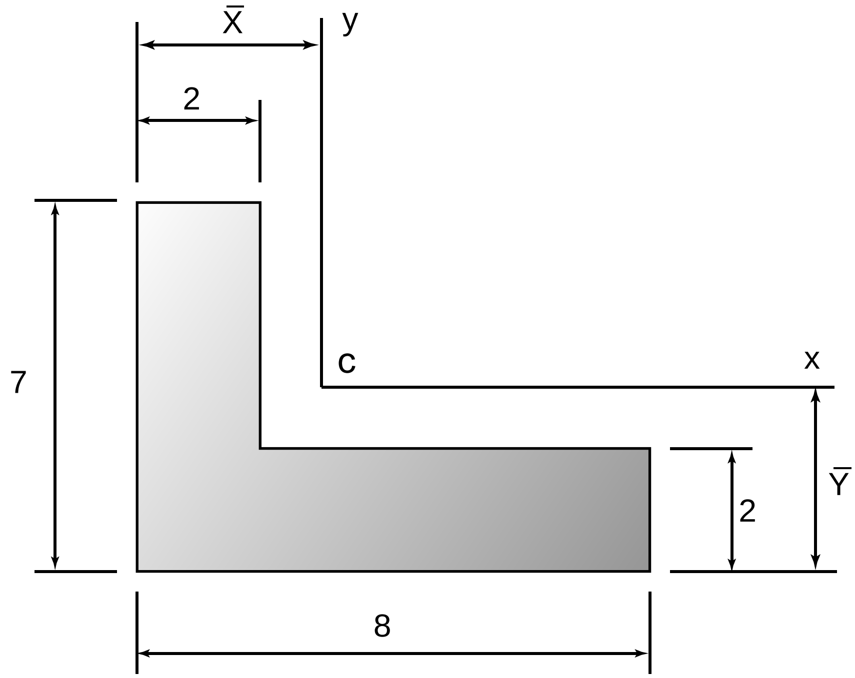
Example

Determine the moment of inertia about the y axis of the area below.
Units: in.

Given:

$$\bar{X} = 2.85$$

$$\bar{Y} = 2.35$$

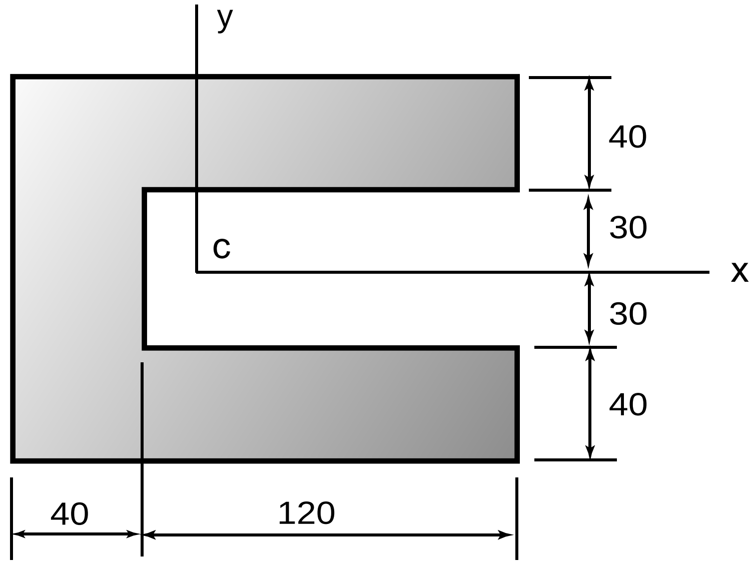


Part	I	Area	d	Ad^2

$$I_y = \sum \left(I_y + Ad_x^2 \right)$$

Example

Determine the moment of inertia about the x axis of the area below.
Units: mm.



Part	I	Area	d	Ad^2

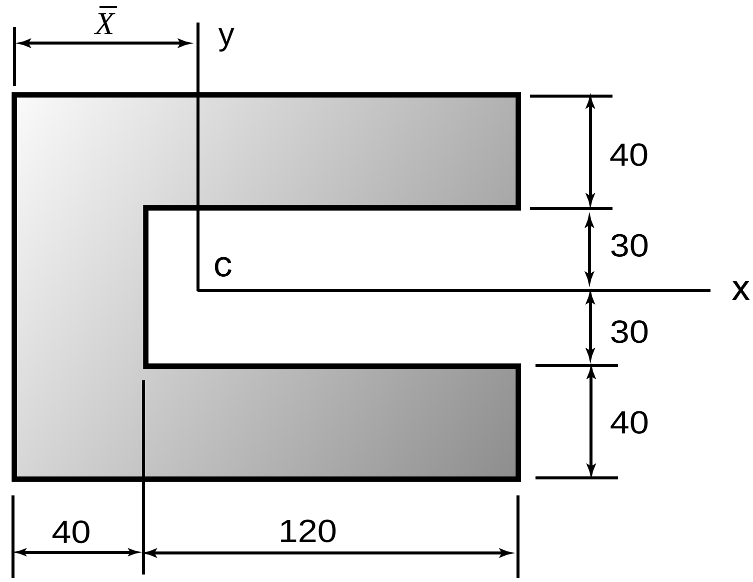
$$I_x = \sum \left(I_x + A d_y^2 \right)$$

Example

Determine the moment of inertia about the y-axis of the area below.
Units: mm.

Given:

$\bar{X} = 70.5$

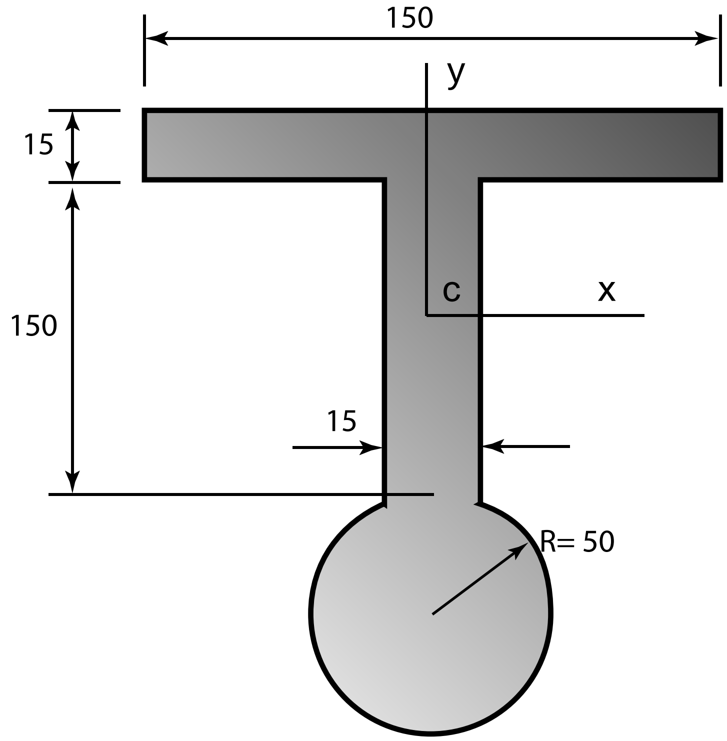


Part	I	Area	d	Ad^2

$$I_y = \sum \left(I_y + Ad_x^2 \right)$$

Example

Determine the moment of inertia about the centroidal y axis of the area below. Units: mm.



Part	I	Area	d	Ad^2

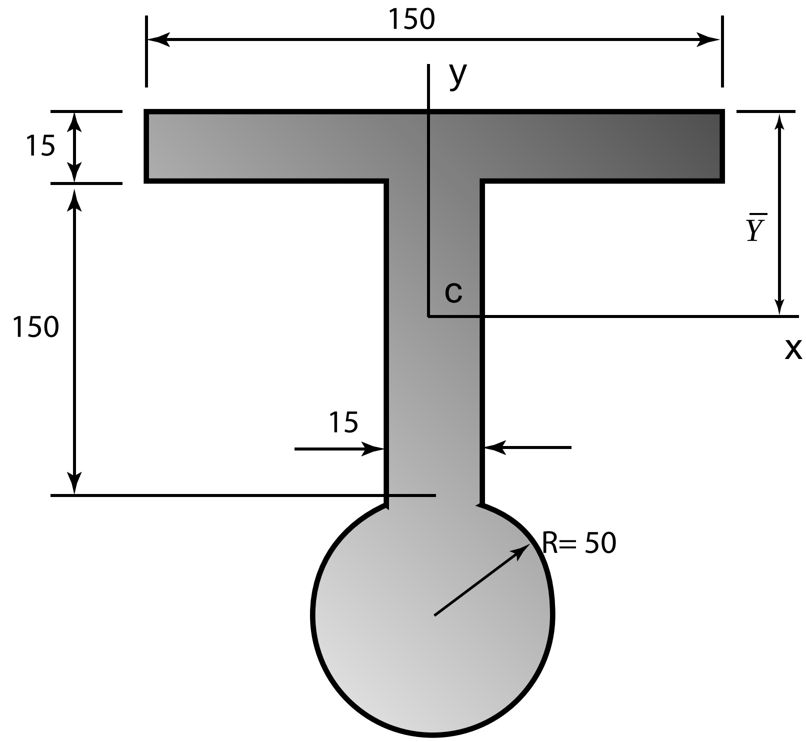
$$I_y = \sum \left(I_y + Ad_x^2 \right)$$

Example

Determine the moment of inertia about the centroidal x axis of the area below. Units: mm.

Given:

$\bar{Y} = 154$

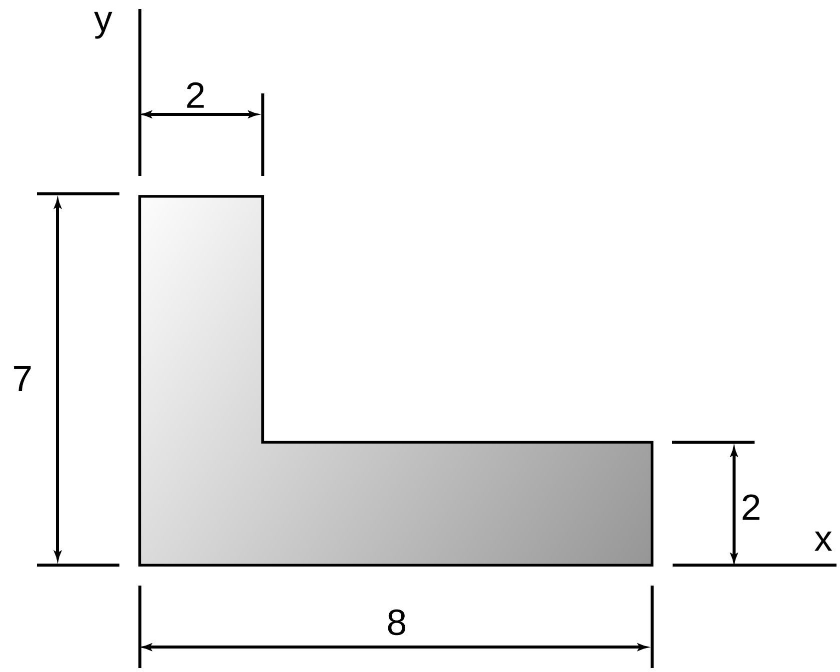


Part	I	Area	d	Ad^2

$$I_x = \sum \left(I_x + A d_y^2 \right)$$

Example

Determine the moment of inertia about the x axis of the area below.
Units: in.



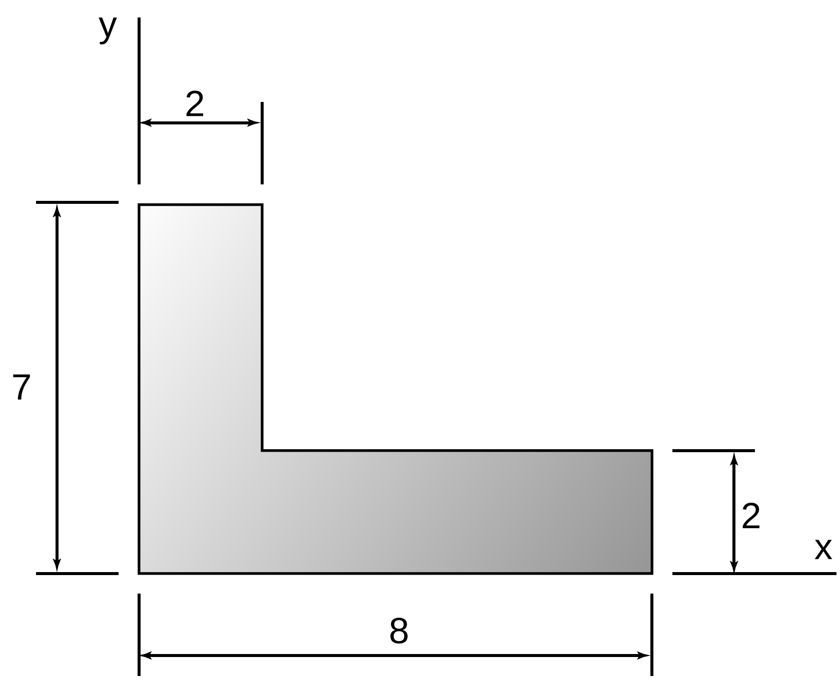
Part	I	Area	d	Ad^2

$$I_x = \sum \left(I_x + A d_y^2 \right)$$

Example

Determine the moment of inertia about the x axis of the area below.
Units: in.

Alternative Solution



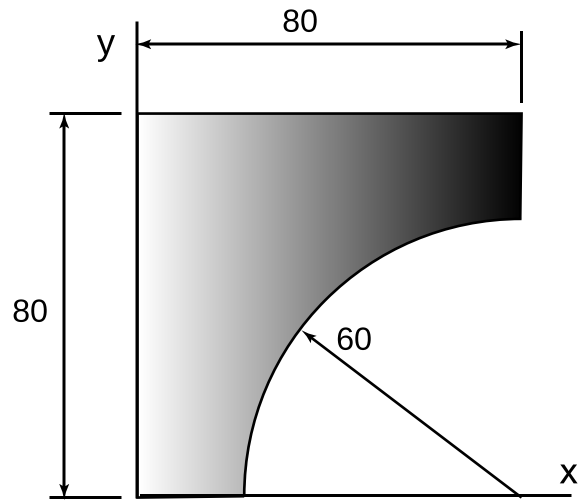
Part	I	Area	d	Ad^2

$$I_x = \sum \left(I_x + A d_y^2 \right)$$

Example

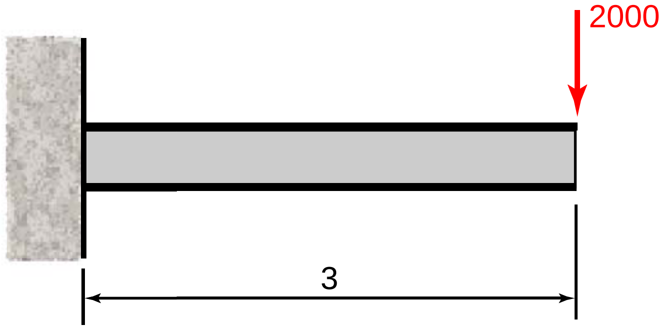
Determine the moment of inertia about the x axis of the area below. Units: mm.

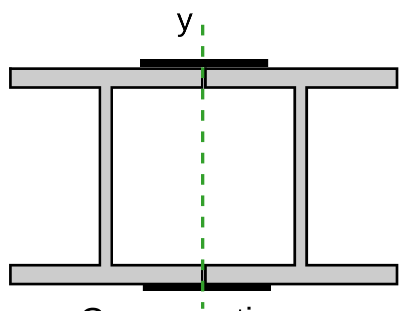
$$I_x = \sum \left(\bar{I}_x + A d_y^2 \right)$$



Example

The two beams are connected by a thin rigid plate on the top and bottom side of the flanges. Determine the moment of inertia about the y axis for the W6x20 beam. Units: lb, ft





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

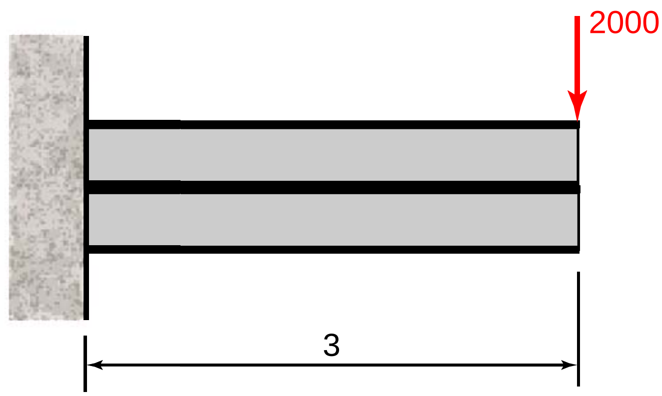
$I_y = 13.3^{in^4}$

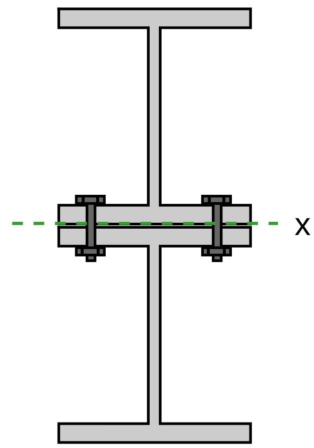
$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

The two beams are connected by bolts through the flanges. Determine the moment of inertia about the x axis for the W6x20 beam. Units: lb, ft





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

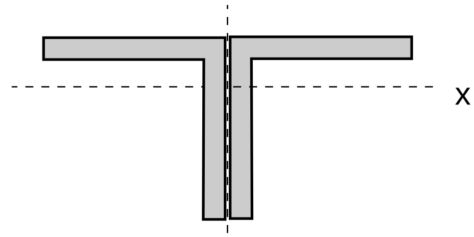
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

Two L76x76x12.7 angle sections are welded back-to-back. Determine the moment of inertia about the centroidal x axis.



L76x76x12.7

Area, $A = 1770^{mm^2}$

$d = b = 76^{mm}$

$\overline{x} = \overline{y} = 23.6^{mm}$

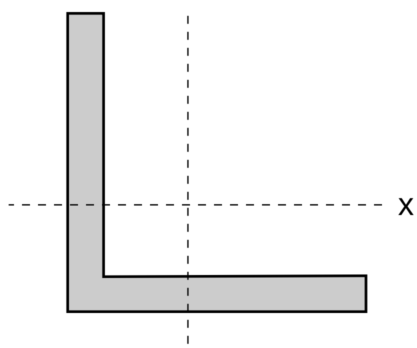
Thickness, $t = 12.7^{mm}$

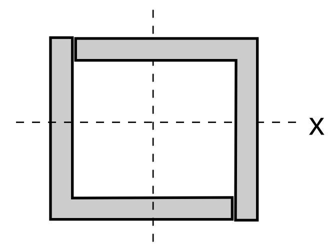
$I_x = I_y = 0.915x10^{6mm^4}$

$r_z = 14.8^{mm}$

Example

Two L76x76x12.7 angle sections are welded together as shown. Determine the moment of inertia about the centroidal x axis.





L76x76x12.7

Area, $A = 1770^{mm^2}$

$d = b = 76^{mm}$

$\overline{x} = \overline{y} = 23.6^{mm}$

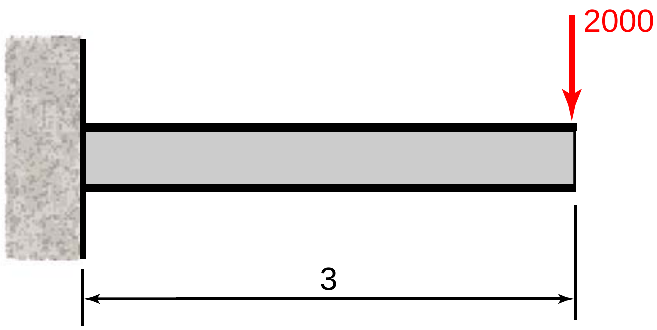
Thickness, $t = 12.7^{mm}$

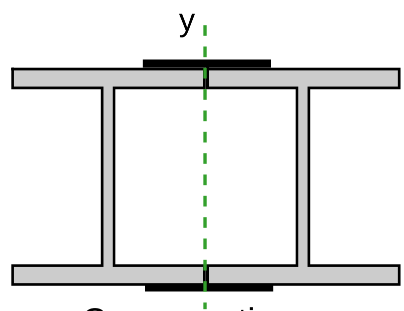
$I_x = I_y = 0.915x10^{6mm^4}$

$r_z = 14.8^{mm}$

Example

The two beams are connected by a thin rigid plate on the top and bottom side of the flanges. Determine the radius of gyration about the y axis for the W6x20 beam. Units: ft





Cross-section

W6x20

$\text{Area, A} = 5.87^{in^2}$

$\text{Depth, d} = 6.20^{in}$

$\text{Flange Width, } b_f = 6.02^{in}$

$\text{Flange Thickness, } t_f = 0.365^{in}$

$\text{Web Thickness, } t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

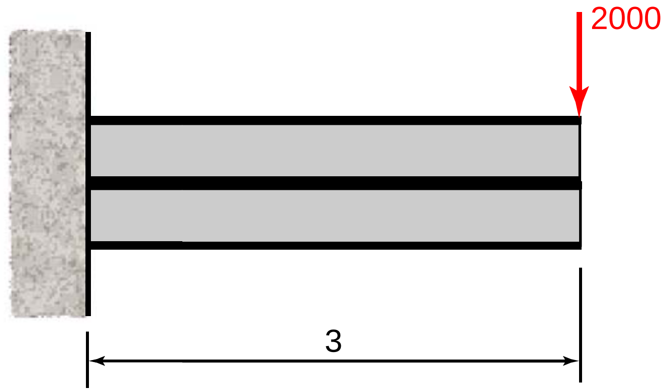
$I_y = 13.3^{in^4}$

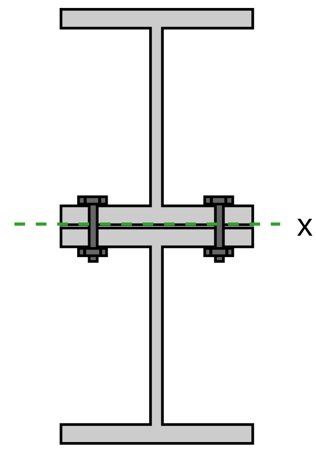
$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

The two beams are connected by bolts through the flanges. Determine the radius of gyration about the x axis for the W6x20 beam.
Units: lb, ft





Cross-section

W6x20

Area, $A = 5.87^{in^2}$

Depth, $d = 6.20^{in}$

Flange Width, $b_f = 6.02^{in}$

Flange Thickness, $t_f = 0.365^{in}$

Web Thickness, $t_w = 0.260^{in}$

$I_x = 41.4^{in^4}$

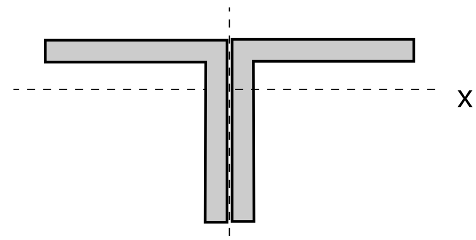
$I_y = 13.3^{in^4}$

$S_x = 13.4^{in^3}$

$S_y = 4.41^{in^3}$

Example

Two L76x76x12.7 angle sections are welded back-to-back. Determine the radius of gyration about the centroidal x axis.



L76x76x12.7

$\text{Area, A} = 1770^{mm^2}$

$\text{d} = b = 76^{mm}$

$\overline{x} = \overline{y} = 23.6^{mm}$

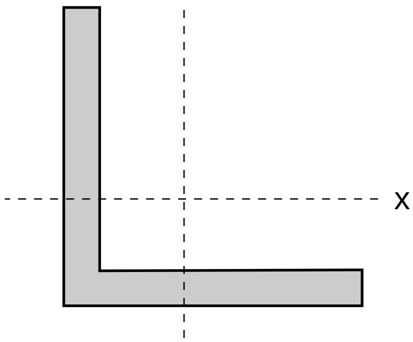
$\text{Thickness, t} = 12.7^{mm}$

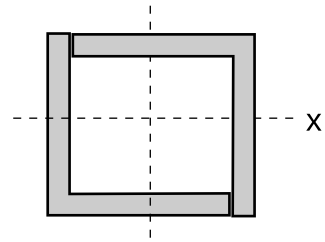
$\text{I}_x = \text{I}_y = 0.915x10^{6 mm^4}$

$r_z = 14.8^{mm}$

Example

Two L76x76x12.7 angle sections are welded together as shown. Determine the radius of gyration about the centroidal x axis.





L76x76x12.7

Area, A $= 1770^{mm^2}$

$\mathrm{d} = b = 76^{mm}$

$\overline{x} = \overline{y} = 23.6^{mm}$

Thickness, t $= 12.7^{mm}$

$\mathrm{I_x} = \mathrm{I_y} = 0.915x10^{6 mm^4}$

$r_z = 14.8^{mm}$

Example

Determine the radius of gyration about the x axis of the area below. Units: mm.