

INTRODUCTION TO PERCENTS

Understanding Percents

The word **percent** means **per 100**. The symbol used to indicate percent is %. For example, 35% means 35 per 100.

Every percent can be written as a fraction. For example:

$$35\% = \frac{35}{100}$$
.

Write a Percent as a Decimal

Percents can also be written as decimal numbers. To write a percent as a decimal you can first write the percent as a fraction, and then convert the fraction to a decimal. Recall that to convert from a fraction to a decimal we divide the numerator (the dividend) by the denominator (the divisor).

$$\frac{35}{100} \rightarrow 35 \div 100 = 0.35$$

You also learned earlier that when dividing by a multiple of ten we can move the decimal point of the dividend to the left as many places as there are zeros in the divisor. In the case of percents, the divisor is always 100, so the conversion is a simple case of dropping the % symbol and moving the decimal point two places to the left (recall that in a whole number the decimal point is to the right of the ones digit).

For example:

$$12\% \rightarrow 12. \rightarrow 0.12$$
 There are two zeros in 100, so we move the decimal point two places to the left.

INTRODUCTION TO PERCENTS

Study the examples below.

Write 23% as a decimal.

23% ightarrow 23. ightarrow 0.23 Drop the percent symbol and move the decimal point two places to the left.

EXAMPLE B

EXAMPLE A

Write 4.9% as a decimal.

Drop the percent symbol and move the decimal point two $4.9\% \rightarrow 4.9 \rightarrow 0.049$ places to the left. Use zeros as placeholders when needed.

EXAMPLE C

Write 0.78% as a decimal.

Drop the percent symbol and move the decimal point 0.78% \rightarrow 0.78 \rightarrow 0.0078 two places to the left. Again, use zeros as placeholders when needed.

INTRODUCTION TO PERCENTS

Write a Decimal as a Percent

To write a decimal as a percent, simply reverse the procedure for writing percents as decimals. Move the decimal point in the decimal number two places to the <u>right</u> and add the % symbol.

EXAMPLE D

Write 0.67 as a percent.

 $0.67 \rightarrow 67. \rightarrow 67\%$ Move the decimal point two places to the right and add the % symbol.

EXAMPLE E

Write 1.24 as a percent.

 $1.24 \rightarrow 124. \rightarrow 124\%$ Move the decimal point two places to the right and add the % symbol.

EXAMPLE F

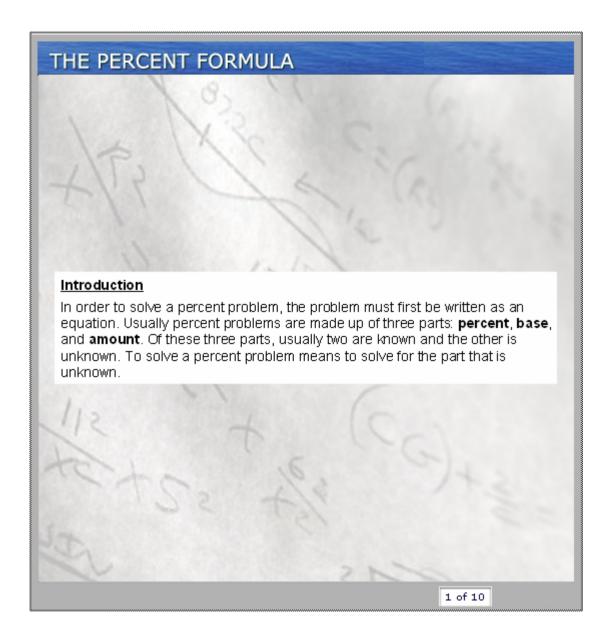
Write 0.058 as a percent.

 $0.058 \rightarrow 5.8 \rightarrow 5.8\%$ Move the decimal point two places to the right and add the % symbol.

INTRODUCTION TO PERCENTS	
Question: Write 1.3% as a decimal.	
Question: Write 154% as a decimal.	
Question: Write 0.0099 as a percent.	
END OF LESSON	5 of 5

	109%	
147.7		
VVrite	e each percent as a decimal. 0.39%	
Write	e the decimal as a percent.	
	0.133	

Write the percent as a fraction.



Percent Formula

Percent • Base = Amount

Or

Amount = Percent • Base

To find the unknown, first translate each part of the problem into an equation. In any percent problem, you have to recognize which part is unknown and replace it with a variable, such as x. Then change the percent to a decimal, change "of" to multiplication, change "is" to an equal sign and solve for the unknown, x.

Hint: The word of represents multiplication, and the word is represents equals.

Remember--change the percent to a decimal for the calculations.

18% of 50 is what number?

11

U U U U

Percent • Base = Amount

18% of 50 = x

 $0.18 \bullet 50 = x$

So we have 0.18(50) = 9. That is, 18% of 50 is 9.

Solving for the Amount

Recall that Amount = Percent • Base. Study the following examples of finding what value (the "amount") is a certain percent of a given value (the "base").

EXAMPLE A

What is 35% of 60?

What number is 35% of 60?

$$x = 21$$

Replace "What number" with x. Change "is" to equals. = 0.35 · 60 Change the percent to a decimal. Change "of" to multiplication.

To find x, multiply 0.35 by 60.

That is, 35% of 60 is 21.

EXAMPLE B

What is 85% of 300?

What is 85% of 300?

$$\Downarrow \Downarrow \ \Downarrow \ \Downarrow \ \Downarrow$$

$$x = 0.85 \cdot 300$$

x = 255

Change "is" to equals. Change the percent to a decimal. Change "of" to multiplication. To find x, multiply 0.85 by 300.

Replace "What is" with x.

That is, 85% of 300 is 255.

THE PERCENT FORMULA	
What is 65% of 50?	Extended Example 1a
	4 of 10

Solving for the Base

In Examples A and B above, we found the value (the "amount") that was a certain percent of a given value (the "base"). Sometimes you are given an amount and are told what percent that amount is of an unknown value that you are asked to find (the "base"). See the examples below.

EXAMPLE C

13 is $6\frac{1}{2}\%$ of what number?

13 is
$$6\frac{1}{2}\%$$
 of what number?

U U U U U Change "is" to equals. Change the percent to Change "of" to multiplic Replace "what number $13 = 0.065 \times x$

Recall that in order to set to divide both sides of $\frac{13}{0.065} = \frac{0.065x}{0.065}$

Use a calculator to find 0.065 is equal to 200.

Change the percent to a decimal. Change "of" to multiplication. Replace "what number" with x.

Recall that in order to solve for x, you need to divide both sides of the equation by 0.065.

Use a calculator to find that 13 divided by 0.065 is equal to 200.

That is, $6\frac{1}{2}\%$ of 200 is 13.

EXAMPLE D

12% of what number is 0.6?

12% of what number is 0.6?

$$0.12 \cdot x = 0.6$$

 $0.12x = 0.6$

$$\frac{0.12x}{0.12} = \frac{0.6}{0.12}$$

$$x = 5$$

Change the percent to a decimal. Change "of" to multiplication. $0.12 \cdot x = 0.6$ Replace "what number" with x. Change "is" to equals.

> Recall that in order to solve for x, you need to divide both sides of the equation by 0.12.

> Use a calculator to find that 0.6 divided by 0.12 is equal to 5.

That is, 12% of 5 is 0.6.

THE PERCENT FORMULA	
	Extended Example 2a
72 is 25% of what number?	
	7 of 10

Solving for the Percent

When you are solving for the percent value in a problem, there is an additional step to creating the equation. When solving for an unknown percent, the unknown must be changed into a percent value. To accomplish this, recall that all percents can be written as fractions with a denominator of 100. Think of the unknown percent, p, as $\frac{p}{100}$, which can be written as the decimal 0.01p.

EXAMPLE E

What percent of 12 is 9?

What percent of 12 is 9? Replace "What percent" with the decimal $\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \quad \text{expression for } \frac{x}{100} \text{ , which is equal to } 0.01x.$

0.01x + 12 = 9 Change "of" to multiplication; change "is" to equals.

 $\frac{0.12x}{0.12} = \frac{9}{0.12}$ Solve for x: divide both sides by 0.12. x = 75

The answer is a percent, so be sure to include a percent symbol!

EXAMPLE F

78 is what percent of 65?

78 is what percent of 65? Change "is" to equals. Replace "What percent" $\Downarrow \; \Downarrow \; \Downarrow \; \Downarrow \; \Downarrow \; \Downarrow \; \Downarrow \;$ with the decimal expression for $\frac{x}{100}$, which is

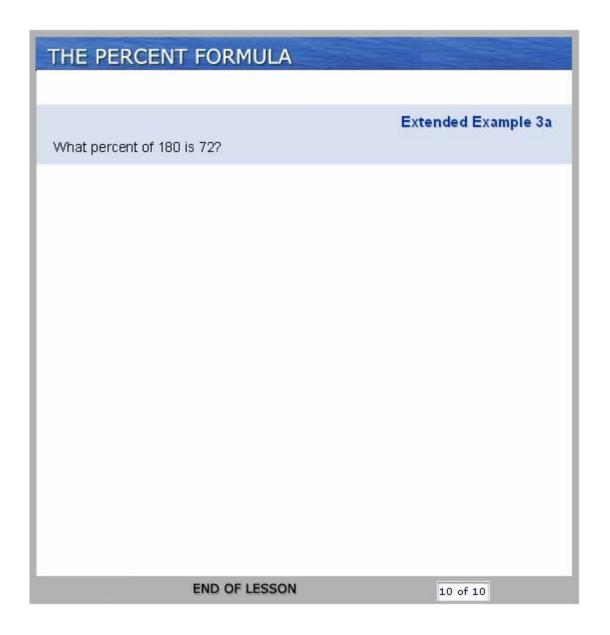
78 = 0.01x - 65 equal to 0.01x. Change "of" to multiplication.

78 = 0.65x

 $\frac{78}{0.65} = \frac{0.65x}{0.65}$ Divide both sides by 0.65.

120 = x

The answer is a percent, so be sure to include a percent symbol.



1.9 is 40% of what number?
Solve for the unknown.
What percent of 342 is 10.944?
Solve for the unknown.
What percent of 30 is 0.198?

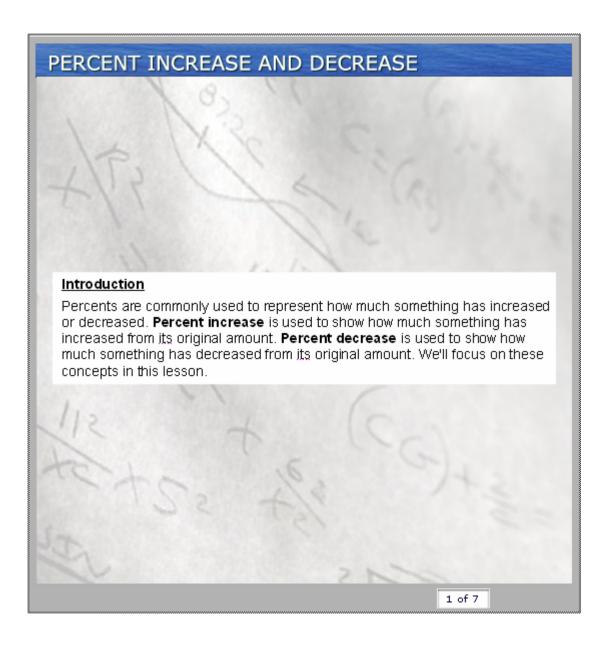
Solve for the unknown.

Solve for the unknown.

7.32 is
$$30\frac{1}{2}\%$$
 of what number?

Solve for the unknown.

What percent of 44 is 121?



Percent Increase

Percent increase is used to show how much something has increased from its original amount.

Percent Increase Formula

Percent • Original Amount = Amount of Increase

Percent increase can be found using the formula above along with the steps outlined in Section 2 of this chapter for solving for the percent. The amount of increase can be found by subtracting the smaller value from the larger value.

EXAMPLE A

Maria's tuition increased from \$540 per credit hour to \$567 per credit hour over the last year. What is the percent increase in tuition per credit hour?

\$567 - \$540 = \$27 First, find the amount of increase by finding the difference in the two tuition rates.

Percent : 540 = 27 Put the original amount and the amount of increase into the percent increase formula.

 $0.01x \cdot 540 = 27$ Recall that when solving for a percent, replace the unknown percent with a decimal representation: $\frac{x}{100} = 0.01x$.

 $\frac{5.4x}{5.4} = \frac{27}{5.4}$ Solve for x: divide both sides by 5.4. x = 5

 $x = 5 \rightarrow 5\%$ The answer is a percent so be sure to include a percent symbol.

Tuition increased by 5% per credit hour over the last year.

EXAMPLE B

A company originally employed 15 people. Two years later there were 75 employees. What is the percent increase in the number of employees?

75 - 15 = 60

First, find the amount of increase by finding the difference in the number of employees.

Percent .15 = 60 Put the original amount and the amount of increase into the percent increase formula.

 $0.01x \cdot 15 = 60$

Replace the unknown percent with a decimal representation.

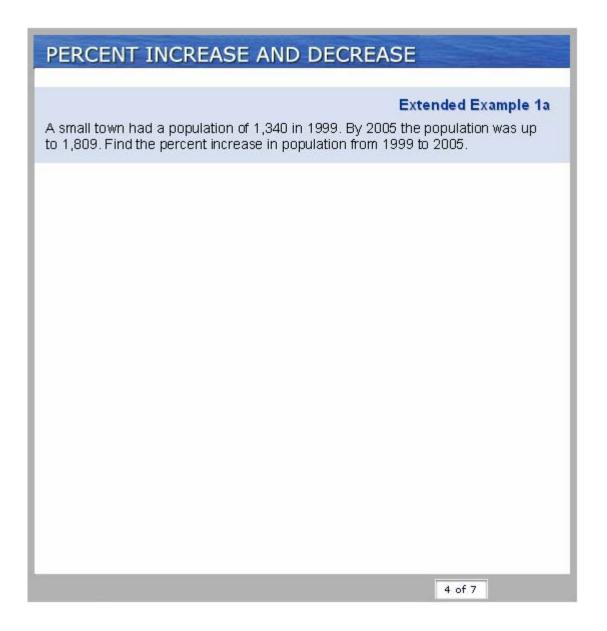
0.15x = 60

x = 400

 $\frac{0.15x}{0.15} = \frac{60}{0.15}$ Divide both sides by 0.15.

 $x = 400 \rightarrow 400\%$ The answer is a percent so include a percent symbol.

The number of employees increased by 400% over the last two years.



Percent Decrease

Percent decrease is used to show how much something has decreased from its original amount.

Percent Decrease Formula

Percent • Original Amount = Amount of Decrease

Percent decrease can be found using the above formula along with the steps outlined in Section 2 of this chapter for solving for the percent. The amount of decrease can be found by subtracting the smaller value from the larger value.

EXAMPLE C

After four months of training, Karl's weight dropped from 180 pounds to 166.5 pounds. What is the percent decrease of Karl's weight?

180 - 166.5 = 13.5

First, find the amount of decrease by finding the difference in the two weights.

Percent .180 = 13.5

Put the original amount and the amount of decrease into the percent decrease formula.

 $0.01x \cdot 180 = 13.5$

Replace the unknown percent with a decimal representation.

1.8x = 13.5

1.8x 13.5 Divide both sides by 1.8.

1.8 x = 7.5

 $x = 7.5 \rightarrow 7.5\%$ The answer is a percent so include a percent symbol.

Karl's weight decreased by 7.5%.

EXAMPLE D

Toby's ice cream shop averages 200 customers a day in June and 50 customers a day in September. What is the percent decrease of average number of customers from June to September?

$$200 - 50 = 150$$

First, find the amount of decrease by finding the difference in the average number of customers.

Percent $\cdot 200 = 150$

Put the original amount and the amount of decrease into the percent decrease formula.

 $0.01x \cdot 200 = 150$

Replace the unknown percent with a decimal representation.

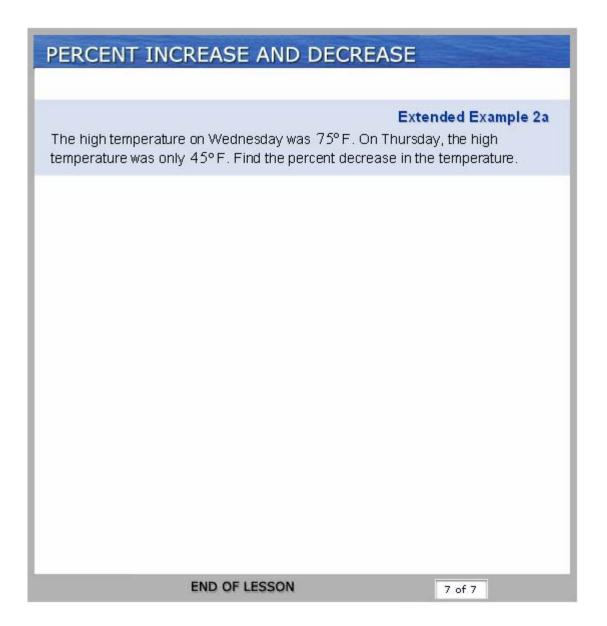
$$2x = 150$$

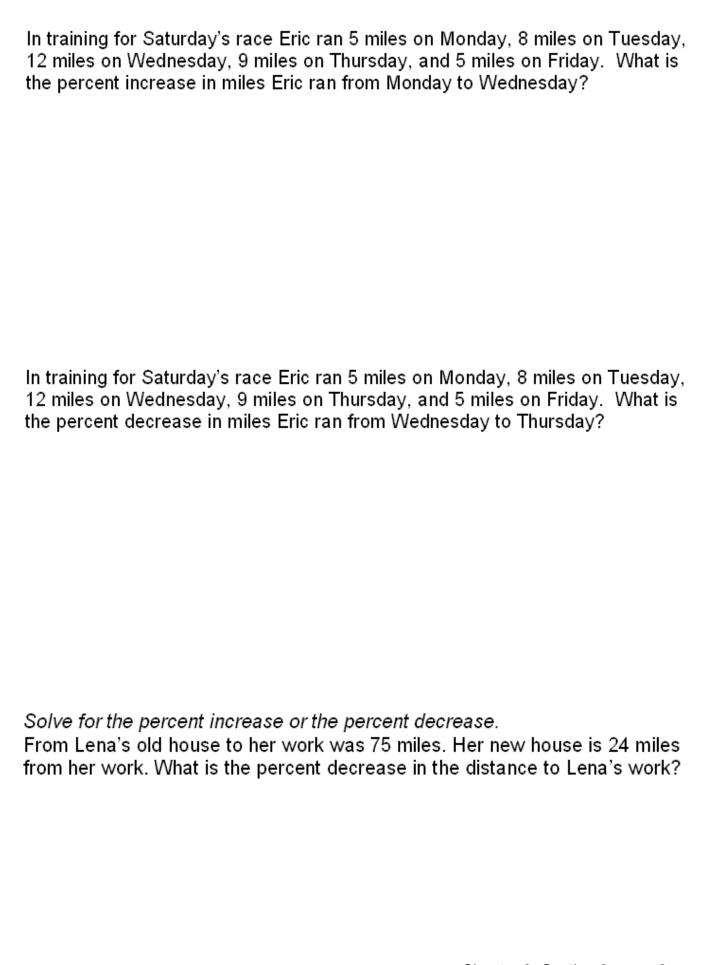
2x = 150 Divide both sides by 2.

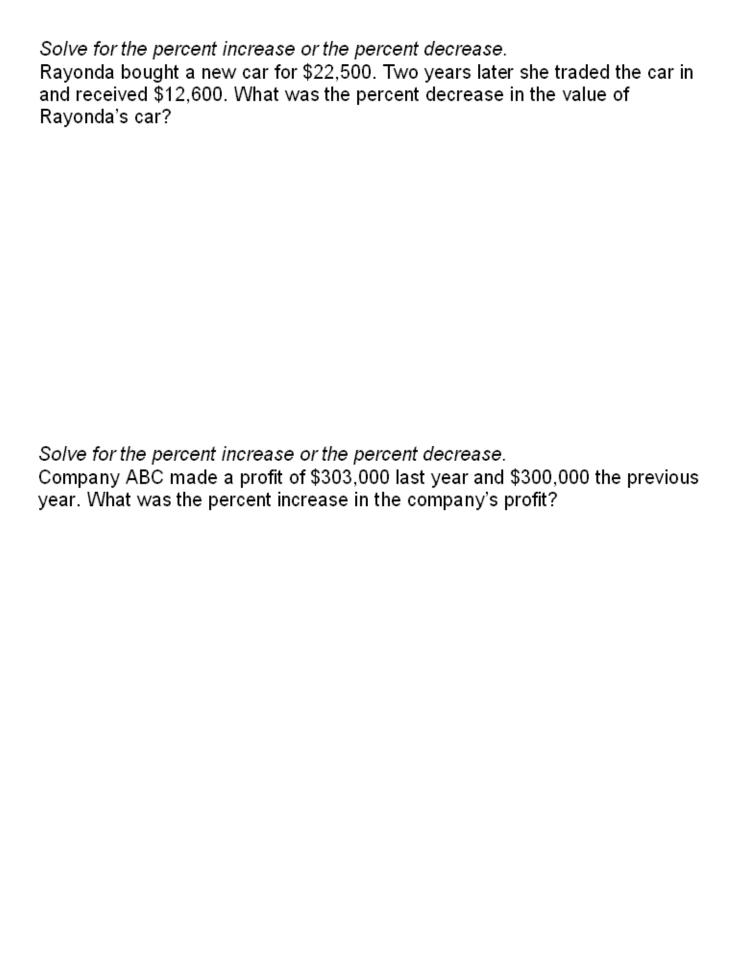
$$x = 75$$

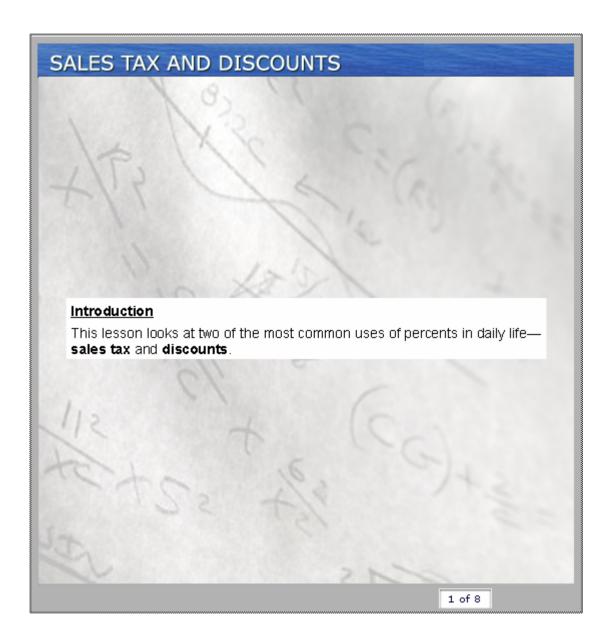
 $x = 75 \rightarrow 75\%$ The answer is a percent so include a percent symbol.

The average number of customers decreased by 75%.









Sales Tax

Many states in America collect **sales tax**, which is applied to certain items you purchase. Most advertised prices do not include sales tax, so you must calculate it yourself to determine the total cost you will pay for an item. A modified form of the percent formula can be used for sales tax problems.

Sales Tax Formula

Tax Rate • Original Cost = Sales Tax

Tax rate is the percentage of taxes a state charges for certain items. **Original cost** is the original price of an item. **Sales tax** is the additional amount you must pay based on the tax rate. Thus, the **total cost** of an item can be found by adding the original cost to the amount paid in sales tax.

Total Cost

Total Cost = Original Cost + Sales Tax

EXAMPLE A

If the tax rate in your state is 4% and you buy a book for \$30.00, how much is the sales tax and what is the total cost?

4% · \$30.00 = Sales Tax Substitute the appropriate values into the sales tax formula.

 $0.04 \cdot 30 = 1.2$ Don't forget to change the percent into a decimal.

Sales tax is \$1.20.

Total cost = \$30.00 + \$1.20
= \$31.20

The sales tax can be used to find the total cost. Substitute the appropriate values into the total cost equation.

The total cost of the book is \$31.20.

EXAMPLE B

A CD costs \$12.99. The tax rate is 6.5%. How much is the sales tax and what is the total cost?

6.5% - \$12.99 = **Sales Tax**Substitute the appropriate values into the formula. Don't forget to change the percent into a decimal.

0.065 · 12.99 = 0.84435 When working with money, always round to the Sales tax is \$0.84. nearest cent (two decimal places).

Total cost = \$12.99 + \$0.84 The sales tax can be used to find the total cost.

Substitute the appropriate values into the total cost equation.

The total cost of the CD is \$13.83.

Some questions may call for you to solve for the tax rate. To find the tax rate, use the sales tax formula and procedure from Section 2 on "Solving for the Percent."

EXAMPLE C

The original cost of a model airplane is \$120.00. If the amount of sales tax is \$9.60, what is the tax rate?

Tax Rate · \$120.00 = \$9.60 Substitute the appropriate values into the sales tax formula.

 $0.01x \cdot 120 = 9.60$ Replace the unknown percent with a decimal representation. Solve for x.

1.2x = 9.6

 $\frac{1.2x}{1.2} = \frac{9.6}{1.2}$

Divide both sides of the equation by 1.2.

x = 8

 $x = 8 \rightarrow 8\%$ The answer is a percent so be sure to include a percent symbol.

SALES TAX AND DISCOUNTS
Extended Example 1a A computer game costs \$35.95. If the tax rate is 5%, what is the total cost of the computer game?
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Discounts

When items in a store are on sale, the **discount price** is commonly found using a **discount rate**. Discount rates are represented by percentages. To find a discount and discount price you can use a form of the percent formula.

Discount Formula

Discount Rate • Original Price = Discount

The **Discount Rate** is the percentage off the original cost, **Original Price** is the original cost of the item, and **Discount** is the amount that will be taken off of the original price. The new price of the item is the **Sale Price**.

Sale Price

Sale Price = Original Price — Discount

EXAMPLE D

A shoe store is having a 25% off sale on all sneakers. You find a pair of sneakers you like that originally cost \$59.99. What is the sale price of the sneakers?

25% · \$59.99 = **Discount** Substitute the appropriate values into the discount formula.

 $0.25 \cdot 59.99 = 14.9975$ Don't forget to change the percent into a decimal.

The discount is \$15.00. Round to the nearest cent.

Sale Price = \$59.99 - \$15.00 The discount can be used to find the sale price. Substitute the appropriate values into the sale price equation.

The sale price of the sneakers is \$44.99.

EXAMPLE E

A pet store is advertising 60% off all glass aquariums. If an aquarium originally costs \$130.00, what is the sale price?

60% · \$130 = **Discount** Substitute the appropriate values into the discount formula.

0.60 · 130 = 78 Don't forget to change the percent into a decimal.

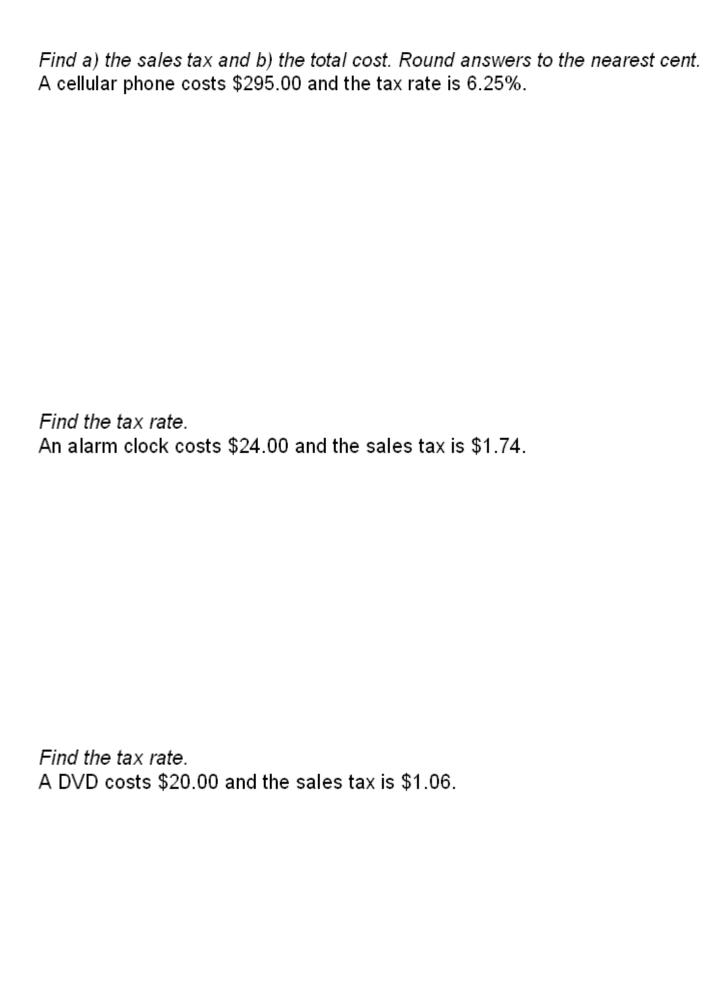
The discount is \$78.00.

Sale Price = \$130.00 - \$78.00
= \$52.00

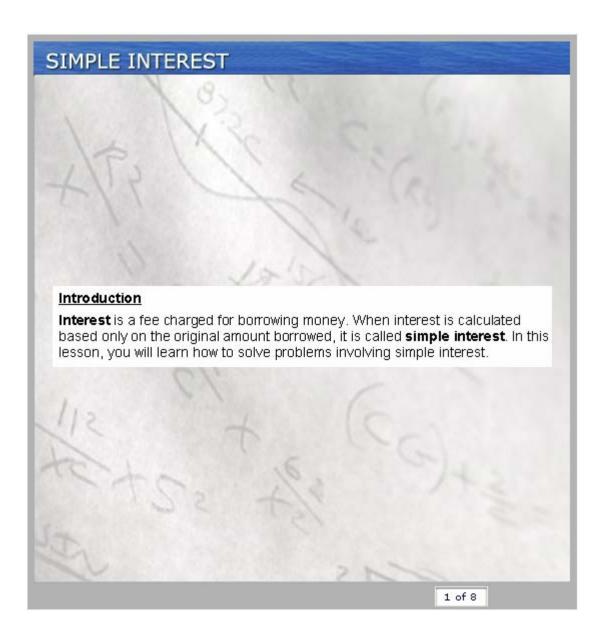
The discount can be used to find the sale price. Substitute the appropriate values into the sale price equation.

The sale price of the aquarium is \$52.00.

SALES TAX AND DISCOUNTS
Extended Example 2a If digital cameras are on sale for 15% off, what would the sale price be for a digital camera that regularly costs \$345.50?
END OF LESSON 8 of 8



Find a) the discount and b) the sale price. Round answers to the nearest central The original cost of a tire is \$186.82. It is on sale for 10% off.
Find a) the discount and b) the sale price. Round answers to the nearest center A couch originally costs \$499.99. It is on sale for 70% off.



In the simple interest formula below, the original amount borrowed is the **principal amount** (or just **principal**). The interest **rate** is the rate used to calculate the interest and is usually given as a percent. Unless otherwise stated, the given interest rate is a yearly rate. **Time** is the amount of time in years.

Simple Interest Formula

Simple Interest = Principal • Rate • Time

or $I = P \bullet r \bullet t$

The total amount of a loan can be calculated by adding the principal and the interest.

Total Amount

Total Amount = Principal + Interest

EXAMPLE A

Anja borrows \$3,000.00 from her bank to build a new deck for her house. The loan has an interest rate of 5.2% for 3 years. What is the simple interest on Anja's loan?

I=prt Substitute the appropriate values into the simple interest formula.

= \$3,000 \cdot 0.052 \cdot 3 years

= \$468 Simplify.

The simple interest is \$468.00.

What is the total amount Anja owes on the loan?

Total = \$3,000 + \$468.00
= \$3,468.00
The simple interest can be used to find the total amount. Substitute the appropriate values into the total amount equation.

The total amount of Anja's loan is \$3,468.00.

EXAMPLE B

Linus borrows \$900.00 to help pay for books this semester. The interest rate is 8.9% for 1 year. What is the simple interest on Linus' loan?

I = prt $I = $900 \cdot 8.9\% \cdot 1$ year

Substitute the appropriate values into the simple interest formula.

= \$900 · 0.089 · 1 year

Change the percent into a decimal.

= 80.1

Simplify.

The simple interest is \$80.10.

What is the total amount Linus owes on the loan?

= \$980.10

Total = \$900 + \$80.10 The simple interest can be used to find the total amount. Substitute the appropriate values into the total amount equation.

The total amount of Linus' loan is \$980.10.

SIMPLE INTEREST
Extended Example 1a What is the total amount owed on a loan of \$20,000.00, borrowed for 7 years at 7.25% simple interest?
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Monthly Payments

Borrowed money is often repaid on a monthly basis. These **monthly payments** are usually of equal amounts. To calculate the monthly payment on a loan, the following formula is used.

Monthly Payment

Monthly Payment = Total Amount ÷ Total Number of Payments

The **total number of payments** can be calculated by multiplying the number of years to pay off a loan by 12 months in a year. For example, if you want to pay a loan off in 5 years then that would be $5 \cdot 12 = 60$ months.

EXAMPLE C

Find the monthly payments on a \$9,000.00 loan for 3 years if the amount of interest is \$585.00.

Total = \$9,000 + \$585 First find the total amount of the loan. = \$9,585

Total Payments = $3 \cdot 12 = 36$ Then, find the total number of payments.

Monthly Payment = 9,585 ÷ 36 Now, substitute the appropriate values into the monthly payment equation.

The payments are \$266.25 per month.

EXAMPLE D

Find the monthly payments on a \$50,000.00 loan for 10 years at 3% simple interest.

$$I = 50,000 \cdot 0.03 \cdot 10$$

= 15,000

= \$65,000

Total Payments = $10 \cdot 12 = 120$

Then, find the total number of payments.

First find the total amount of the loan. To

find this you will first need to find the

amount of interest on the loan.

 $= 541.\overline{6} \rightarrow 541.66...$

\$541.67

Monthly Payment = 65,000 ÷ 120 Now, substitute the appropriate values into the monthly payment equation.

Round to nearest cent.

The payments are \$541.67 per month.

SIMPLE INTEREST
Extended Example 2a Find the monthly payments on a \$8,500.00 loan for 6 years if the amount of interest is \$161.50.
END OF LESSON 8 of 8

Find the simple interest. Round answers to the nearest cent.

Principal = \$500,000

Rate = 5.5%

Time = 30 years

Find the total amount. Round answer to the nearest cent.

Principal = \$1,000

Rate = 8%

Time = 3 years

Find the monthly payment. Round answer to the nearest cent.

Principal = \$50,000

Interest = \$16,500

Time = 11 years

