#### **Introduction**

Earlier you learned that an exponent has the same meaning whether the base is a number or a variable. For example,

$$4^{5} = \underbrace{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}_{\text{5 factors of 4}} \qquad x^{5} = \underbrace{x \cdot x \cdot x \cdot x \cdot x}_{\text{5 factors of } x}$$

$$x^{5} = \underbrace{x \cdot x \cdot x \cdot x \cdot x}_{5 \text{ factors of } x}$$

From this definition of exponents, additional properties can be developed that will help us simplify more complex exponential expressions. We'll look at one of those properties, the product rule of exponents, in this lesson.

Using the definition of an exponent, let's find the product of  $m^2$  and  $m^5$ .

$$m^{2} \cdot m^{5} = (m \cdot m) (m \cdot m \cdot m \cdot m \cdot m)$$

$$= \underbrace{m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m}_{7 \text{ factors of } m}$$

$$= m^{7}$$

Notice the resulting exponent, 7 , is equal to the sum of the two original exponents,  $5\,$  and  $\,2\,$ 

$$m^2 \cdot m^5 = m^{2+5} = m^7$$
.

The above is an example of the product rule of exponents.

#### **Product Rule of Exponents**

If m and n are positive integers and x is a real number, then:

$$x^m \cdot x^n = x^{m+n}$$

The product rule of exponents says that when multiplying two exponential expressions with the same base, we keep the base and add the exponents. Study the following examples. Usually the directions for these types of problems will either ask you to "Multiply" or "Simplify."

**EXAMPLE A** 

Simplify:  $x^3 \cdot x^4$ 

$$x^3 \cdot x^4 = x^{3+4}$$
$$= x^7$$

**EXAMPLE B** 

Simplify:  $2y^5 \cdot 4y^3$ 

$$2y^5 \cdot 4y^3 = (2 \cdot 4)(y^5 \cdot y^3)$$
 Multiply the coefficients.  
=  $8 \cdot y^{5+3}$   
=  $8y^8$ 

If any of the coefficients are negative, the rules are the same as for multiplying integers.

**EXAMPLE C** 

Simplify:  $\left(-2a^3b^9\right)\left(9a^5b^7\right)$ 

$$(-2a^{3}b^{9})(9a^{5}b^{7}) = (-2 \cdot 9)(a^{3} \cdot a^{5})(b^{9} \cdot b^{7})$$
$$= (-18)(a^{3+5})(b^{9+7})$$
$$= -18a^{8}b^{16}$$

**EXAMPLE D** 

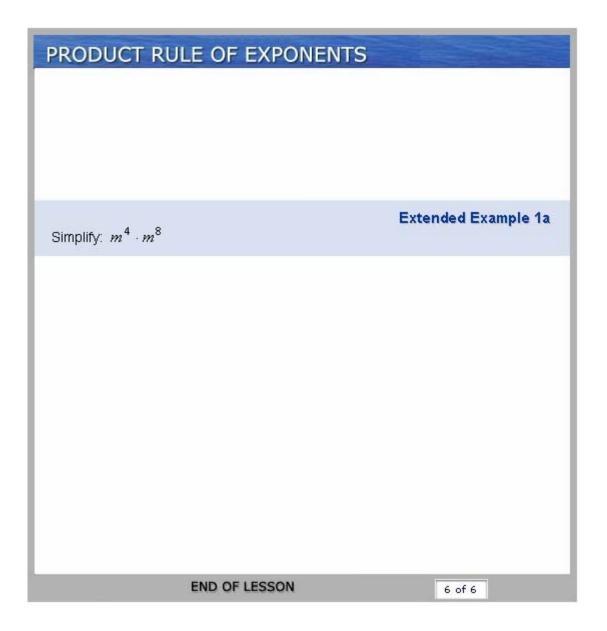
Simplify:  $-2x^4 \cdot 5x^3 \cdot -4x$ 

$$-2x^{4} \cdot 5x^{3} \cdot -4x = (-2 \cdot 5 \cdot -4)(x^{4} \cdot x^{3} \cdot x^{1})$$
$$= (40)(x^{4+3+1})$$
$$= 40x^{8}$$

### Helpful Hint

It is important to recognize the difference between adding and multiplying terms.

Examples of Adding	Examples of Multiplying	
$-9x^2 + 2x^2 = (-9 + 2)x^2 = -7x^2$	$(-9x^2)(2x^2) = (-18)x^{2+2} = -18x^4$	
$2x^3 + 7x = 2x^3 + 7x$	$(2x^3)(7x) = 14x^{3+1} = 14x^4$	



$$9x^2 \cdot x^8$$

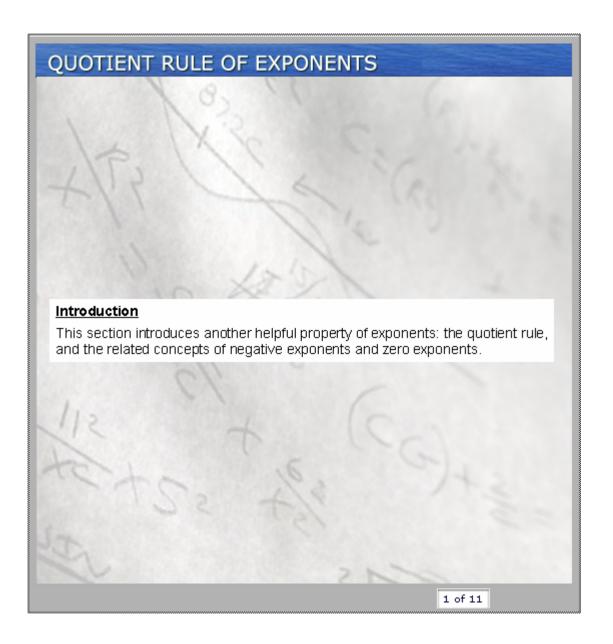
Simplify.

$$r^2s^3 \cdot rs^6$$

$$(-2xy^4)(-6x^3y^7)$$

$$a(-4a^{11})(-3a^5)$$

$$(-7x^5)(-9y^{12})(x^3y^4)$$



The definition of an exponent allows you to find the quotient of two exponential expressions. For example,  $m^5 \div m^2$  can be written as:

$$\frac{m^5}{m^2} = \frac{m \cdot m \cdot m \cdot m \cdot m \cdot m}{m \cdot m}$$

$$= \underbrace{m \cdot m \cdot m}_{3 \text{ factors of } m}$$

$$= m^3$$

Notice that 2 factors are taken away from 5 factors to get 3 factors:

$$\frac{m^5}{m^2} = m^{5-2} = m^3.$$

This is an example of the quotient property of exponents.

#### **Quotient Rule of Exponents**

If m and n are integers and x is a real number, then:

$$\frac{x^m}{x^n} = x^{m-n}$$

The quotient rule says that if you are dividing two exponential expressions with the same base, keep the base and subtract the exponent of the divisor from the exponent of the dividend. Study the following examples.

**EXAMPLE A** 

Simplify:  $\frac{x^4}{x^3}$ 

$$\frac{x^4}{x^3} = x^{4-3}$$

 $\frac{x^4}{x^3} = x^{4-3}$  Use the quotient rule of exponents: subtract the exponent of the denominator from that of the numerator.

$$= x^1$$

= x

**EXAMPLE B** 

Simplify:  $\frac{4y^5}{2y^3}$ 

$$\frac{4y^5}{2y^3} = \left(\frac{4}{2}\right) \left(\frac{y^5}{y^3}\right)$$

Separate the coefficients from the variables.

$$=2\cdot y^{5-3}$$

Simplify the fraction at left, and use the quotient rule of exponents of exponents.

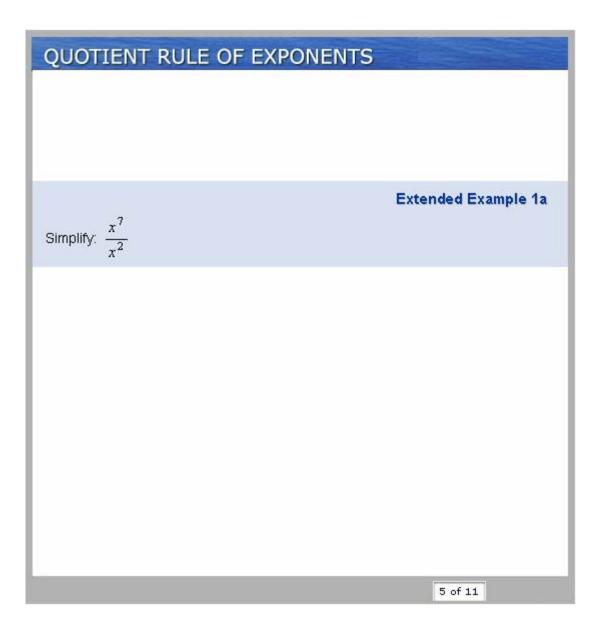
$$=2y^{2}$$

If any of the coefficients are negative, the sign of the answer will follow the rules for dividing integers.

**EXAMPLE C** 

Simplify: 
$$\frac{-2a^6b^9}{9a^3b^7}$$

$$\frac{-2a^6b^9}{9a^3b^7} = \left(\frac{-2}{9}\right) \left(\frac{a^6}{a^3}\right) \left(\frac{b^9}{b^7}\right)$$
 Separate the coefficients and like variables. 
$$= \left(\frac{-2}{9}\right) \left(a^{6-3}\right) \left(b^{9-7}\right)$$
 The fraction cannot be simplified. Use the quotient rule of exponents. 
$$= \left(\frac{-2}{9}\right) a^3b^2$$
 
$$= \frac{-2a^3b^2}{9}$$
 Simplify.



#### **Integer Exponents**

Through the use of the quotient rule of exponents, an important concept arises. Notice in the following example that after applying the quotient property, what is left is a 0 power of x:

$$\frac{y^{5}}{y^{5}} = y^{5-5} = y^{0}$$
.

We also know that any fraction with the same numerator and denominator is equal to 1:

$$\frac{y^5}{v^5} = 1.$$

This leads us to the rule for zero exponents.

#### Zero Exponents

If x is a real number not equal to zero, then:

$$x^0 = 1$$

Here are some examples of the zero exponent rule:

**a.** 
$$5^0 = 1$$

**a.** 
$$5^0 = 1$$
 **b.** 3, 258, 647<sup>0</sup> = 1

**c.** 
$$(-21)^0 = 1$$
 **d.**  $8b^0 = 8(1) = 8$ 

**d.** 
$$8b^0 = 8(1) = 8$$

Simplify:  $\frac{3x^6}{9x^6}$ 

$$\frac{3x^6}{9x^6} = \left(\frac{3}{9}\right) \left(\frac{x^6}{x^6}\right)$$
$$= \left(\frac{1}{3}\right) \left(x^{6-6}\right)$$
$$= \left(\frac{1}{3}\right) \left(x^0\right)$$
$$= \left(\frac{1}{3}\right) (1)$$

Another important concept arises from knowledge of the quotient rule—the concept of negative exponents. For example:

$$\frac{y^3}{y^5} = y^{3-5} = y^{-2}.$$

What does this negative exponent mean?

#### **Definition of Negative Exponent**

If n is an integer and x is a real number not equal to zero, then:

$$x^{-n} = \frac{1}{x^n}$$

Here are some examples of negative exponents:

**a.** 
$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

**b.** 
$$m^{-15} = \frac{1}{m^{15}}$$

**c.** 
$$(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$$
 **d.**  $7b^{-4} = 7\left(\frac{1}{b^4}\right) = \frac{7}{b^4}$ 

**d.** 
$$7b^{-4} = 7\left(\frac{1}{b^4}\right) = \frac{7}{b^4}$$

Simplify:  $\frac{3x^6}{9x^9}$ 

### **EXAMPLE E**

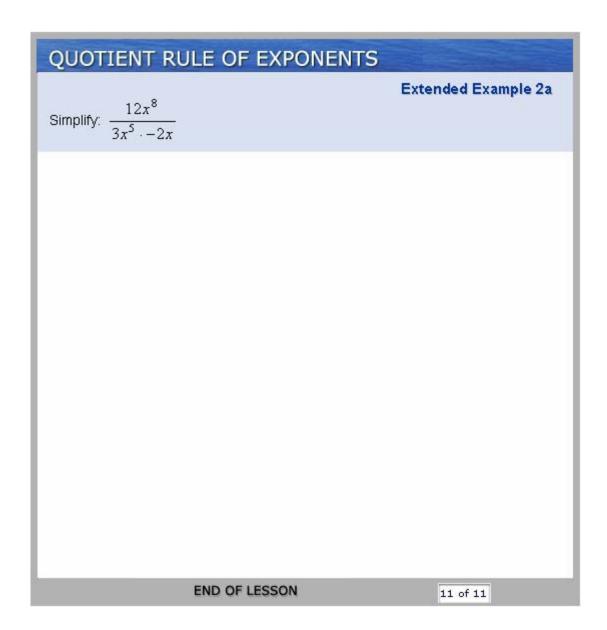
$$\frac{3x^6}{9x^9} = \left(\frac{3}{9}\right) \left(\frac{x^6}{x^9}\right)$$
$$= \left(\frac{1}{3}\right) \left(x^{6-9}\right)$$
$$= \left(\frac{1}{3}\right) \left(x^{-3}\right)$$
$$= \left(\frac{1}{3}\right) \left(\frac{1}{x^3}\right)$$
$$= \frac{1}{3x^3}$$

More complex problems might require the use of multiple properties.

**EXAMPLE E** 

Simplify: 
$$\frac{-2x^4}{5x^3 \cdot -4x}$$

$$\frac{-2x^4}{5x^3 \cdot -4x} = \left(\frac{-2}{5(-4)}\right) \left(\frac{x^4}{x^3 \cdot x}\right)$$
 Separate the coefficients and variables. 
$$= \left(\frac{-2}{-20}\right) \left(\frac{x^4}{x^{3+1}}\right)$$
 Combine the coefficients, and use the product rule of exponents. 
$$= \left(\frac{1}{10}\right) \left(\frac{x^4}{x^4}\right)$$
 Simplify. 
$$= \left(\frac{1}{10}\right) \left(x^{4-4}\right)$$
 Use the quotient property of exponents. 
$$= \left(\frac{1}{10}\right) \left(x^0\right)$$
 
$$= \left(\frac{1}{10}\right) (1) = \frac{1}{10}$$
 Simplify.



$$\frac{-4n^{10}}{-6n^{22}}$$

Simplify.

$$\frac{16y^{12}z^{12}}{-24yz^{12}} =$$

$$\frac{14k^{14}mn^{14}}{49k^5m^9n^{15}} =$$

$$\frac{20x^2y^4z}{-4x^6y^8z^5} =$$

$$\frac{-2p^6 \cdot 3p^6}{9p^8(-8p^9)} =$$

### Introduction

So far, you have learned how to simplify multiplication and division of exponential terms with like bases.

Product Rule:  $x^m \cdot x^n = x^{m+n}$  Quotient Rule:  $\frac{x^m}{x^n} = x^{m-n}$ 

Suppose we want to simplify an exponential expression that is raised to a power, such as  $\left(x^3\right)^2$ . This lesson will explore the property that allows us to do this: the power rule of exponents.

Use the definition of exponents and the product rule of exponents that you learned in Section 1 of this chapter to simplify  $\left(x^3\right)^2$ :

$$(x^3)^2 = (x^3)(x^3)$$
$$= x^{3+3}$$
$$= x^6$$

This result is the same as that of multiplying the two exponents.

$$\left(x^3\right)^2 = x^{3 \cdot 2} = x^6$$

This is an example of the power rule of exponents.

#### **Power Rule of Exponents**

If m and n are positive integers and x is a real number, then:

$$\left(x^{m}\right)^{n} = x^{m \cdot n}$$

Simplify:  $\left( \mathcal{Y}^{11} \right)^4$ 

**EXAMPLE A** 

$$\left(y^{11}\right)^4 = y^{11 \cdot 4}$$
$$= y^{44}$$

Simplify:  $(x^3)^5 (x^2)^4$ 

**EXAMPLE B** 

$$(x^3)^5 (x^2)^4 = (x^{3 \cdot 5})(x^{2 \cdot 4})$$
$$= x^{15} \cdot x^8$$
$$= x^{15+8}$$
$$= x^{23}$$

Before we look at more properties of exponents, let's make sure we understand the difference between the product rule (from Section 1 of this chapter) and the power rule. Carefully study the examples in the table below.

Product Rule → Add Exponents	Power Rule -> Multiply Exponents
$z^4 \cdot z^6 = z^{4+6} = z^{10}$	$\left(z^4\right)^6 = z^{4\cdot 6} = z^{24}$
$x^3 \cdot x^5 = x^{3+5} = x^8$	$\left(x^{3}\right)^{5} = x^{3 \cdot 5} = x^{15}$

Now suppose we are simplifying the power of a product. For example, let's simplify  $(2mn)^3$ . Let's use the definition of an exponent:

$$(2mn)^3 = (2mn)(2mn)(2mn)$$
$$= (2 \cdot 2 \cdot 2)(m \cdot m \cdot m)(n \cdot n \cdot n)$$
$$= 2^3 \cdot m^3 \cdot n^3$$
$$= 8m^3n^3$$

Notice that the power of a product can be written as the product of powers. For example, we could have written the same problem as follows:

$$(2mn)^3 = 2^3 \cdot m^3 \cdot n^3$$
$$= 8m^3n^3$$

#### **Power of Product Rule of Exponents**

If n is a positive integer and x and y are real numbers, then:

$$(xy)^n = x^n y^n$$

In other words, to raise a product to a power, raise each factor to that power:

$$(3p)^4 = 3^4 \cdot p^4 = 81p^4$$

Armed with the power of product rule, you can simplify even complex-looking expressions, as shown in Examples C and D below.

**EXAMPLE C** 

Simplify:  $\left(4x^3y^7\right)^3$ 

$$(4x^{3}y^{7})^{3} = 4^{3} \cdot (x^{3})^{3} \cdot (y^{7})^{3}$$
$$= 64 \cdot x^{3 \cdot 3} \cdot y^{7 \cdot 3}$$
$$= 64x^{9}y^{21}$$

Simplify:  $\left(4k^2h^6\right)^3\left(-3k^6h^8\right)^2$ 

**EXAMPLE D** 

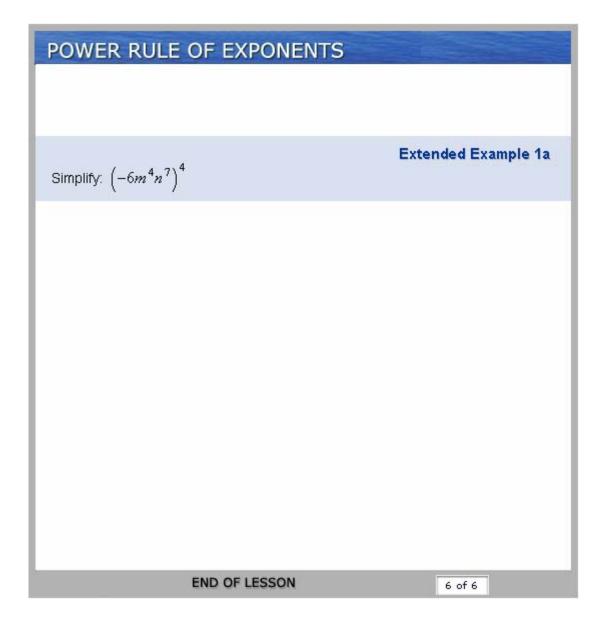
$$(4k^{2}h^{6})^{3} (-3k^{6}h^{8})^{2} = [4^{3}(k^{2})^{3}(h^{6})^{3}] \cdot [(-3)^{2}(k^{6})^{2}(h^{8})^{2}]$$

$$= (64k^{2+3}h^{6+3})(9k^{6+2}h^{8+2})$$

$$= (64k^{6}h^{18})(9k^{12}h^{16})$$

$$= (64\cdot 9)(k^{6}k^{12})(h^{18}h^{16})$$

$$= 576k^{18}h^{34}$$



$$(m^2)^2(m^3)^4$$

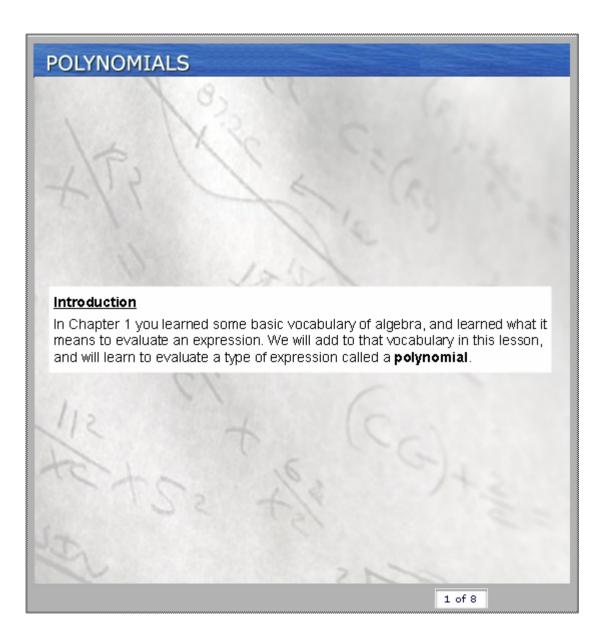
Simplify.

$$(-3y)^2$$

$$(-6y^7)^2$$

$$(4q^8)^2(2q^4)^3$$

$$(2xy)^4(3x^4y^3)^3$$



In Chapter 1 you learned about **expressions**, which can be simplified or evaluated, and **equations**, which can be solved. Recall that a **term** is one of the addends of an algebraic expression. For example:

- 6x + 8 has two terms. 6x is a variable term and 8 is a constant or non-variable term.
- $5y^2 4y + 9$  has three terms.  $5y^2$  and -4y are variable terms, and 9 is a constant.

Also recall that all variable terms have a **coefficient**—study the examples in the table below.

Variable Term	The Coefficient	
6 <i>x</i>	6	
5y <sup>2</sup>	5	
-4 <i>y</i>	-4	
-x	-1	
m	1	

A **monomial** is a term that contains only <u>positive</u> exponents and that does <u>not</u> contain a variable in the denominator. Note that this means that not all terms are monomials—see the table below.

Monomials	Not Monomials	
$7y^2$	$\frac{2}{y}$	
$-\frac{8}{27}x$	4 x -5	

Monomials or a sum of monomials are known as polynomials. Some examples of polynomials are:

- $\mathbf{a}$ . 2x
- **b.**  $5x^3 + 2x^2 3x + 1$
- **c.** 7x + 4**d.**  $8y^2 + \frac{1}{2}y 6$

There is no limit to the number of terms a polynomial can have. Some common types of polynomials with a specific number of terms have special names.

- If a polynomial has one term, then it is called a monomial.
- If a polynomial has two terms, then it is called a binomial.
- If a polynomial has three terms, then it is called a trinomial.

There is no special name for polynomials with more than three terms.

The **degree of a polynomial** is equal to the <u>largest exponent</u> of all the terms in the polynomial. The chart below shows the details for the examples on the previous screen.

Polynomial	Number of Terms and Name	Degree
<b>a.</b> 2 <i>x</i>	1 term = monomial	1
<b>b.</b> $5x^3 + 2x^2 - 3x + 1$	4 terms = polynomial	3
<b>c.</b> $7x + 4$	2 terms = binomial	1
<b>d.</b> $8y^2 + \frac{1}{2}y - 6$	3 terms = trinomial	2

#### **Evaluating Polynomials**

Since polynomials are a type of expression, they can be evaluated. Evaluating a polynomial is done by replacing the variable of a polynomial with a specific given number, and then computing the result. If the given value is negative or a fraction, you should always use parentheses when substituting it in place of the variable.

#### **EXAMPLE A**

Evaluate 7x - 5 when x = -1.

$$7x - 5$$

7(-1) - 5 Replace x with -1, then simplify.

$$= -7 - 5$$

$$= -12$$

#### **EXAMPLE B**

Evaluate  $-5x^2 - x + 12$  when x = -2.

$$-5x^2 - x + 12$$
 Be sure

Be sure to pay close attention to signs!

$$-5(-2)^2 - (-2) + 12$$

$$= -5(4) + 4 + 12$$
 Note:  $(-2)^2 = (-2)(-2) = 4$  and  $-(-2) = 4$ .

$$= -20 + 4 + 12$$

$$= -4$$

#### **EXAMPLE C**

Evaluate  $x^2 - x + 6$  when  $x = \frac{1}{3}$ .

$$x^{2} - x + 6$$

$$\left(\frac{1}{3}\right)^{2} - \left(\frac{1}{3}\right) + 6 \quad \text{Note: } \left(\frac{1}{3}\right)^{2} = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9}$$

$$= \frac{1}{9} - \frac{1}{3} + 6$$

$$= \frac{1}{9} - \frac{3}{9} + \frac{54}{9} \qquad \text{Remember that you need to find a common denominator when adding or subtracting fractions.}$$

$$= \frac{1 - 3 + 54}{9}$$

Note that when evaluating a polynomial you will obtain different results when you substitute different values for the variable . Study the following example, where the same polynomial is evaluated for four different values—with four different results.

Evaluate 
$$10x^2 - 3x + 7$$
 when  $x = 1$ ,  $x = 3$ ,  $x = -3$ , and  $x = \frac{1}{2}$ :

x = 1	x = 3	x = -3	$x = \frac{1}{2}$
		$10x^2 - 3x + 7$	$10x^2 - 3x + 7$
$10(1)^{2} - 3(1) + 7$ $= 10(1) - 3 + 7$	$10(3)^{2} - 3(3) + 7$ $= 10(9) - 9 + 7$	$10(-3)^2 - 3(-3) + 7$ $= 10(9) + 9 + 7$	$10\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 7$
= 10 - 3 + 7	= 90 - 9 + 7	= 90 + 9 + 7	$=10\left(\frac{1}{4}\right)-\frac{3}{2}+7$
= 14	= 88	= 106	A A
			$= \frac{5}{2} - \frac{3}{2} + 7$ $= 8$

POLYNOMIALS	
Evaluate $7x^2 - 2x - 13$ when $x = -2$ .	Extended Example 1a
END OF LESSON	8 of 8
END OF LESSON	8 01 8

Identify the following:

- a) number of terms,
- b) type (monomial, binomial, trinomial, or polynomial),
- c) degree.

$$x^2 - 4x + 16$$

Evaluate  $x^2 + 3x$  for the value of x.

$$x = -\frac{1}{2}$$

Evaluate  $y^3 + 7y^2 - 13$  for the value of y.

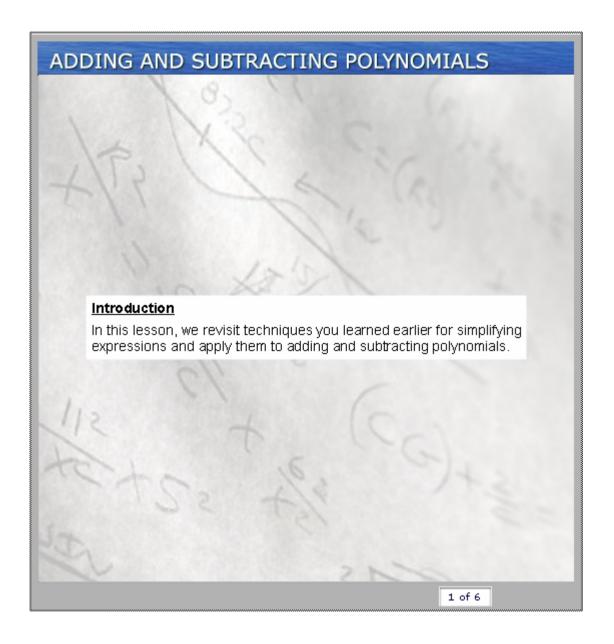
$$y = 3$$

Evaluate for the polynomial when x = 5.

$$\frac{3}{2}x^2 - 4$$

Evaluate for the polynomial when x = -2.

$$3x^2 - 2x + 10$$



#### **Adding Polynomials**

To add polynomials, first remove the parentheses and then follow the steps for simplifying expressions.

Earlier, you learned to combine **like terms**. Remember that to combine like terms, you combine their coefficients (following the rules for combining integers) and keep the variable as is. Examples:

**a.** 
$$7x + 3x = (7 + 3)x = 10x$$

**b.** 
$$5y^2 - 4y^2 = (5-4)y^2 = (1)y^2 = y^2$$

Remember, to simplify an expression:

- · Identify like terms.
- Gather like terms together.
- Combine like variable terms by combining their coefficients.
- Combine any constants (non-variable terms).

**EXAMPLE A** 

Add: 
$$(3x - 5) + (-6x + 4)$$

$$(3x-5)+(-6x+4)$$

$$=$$
  $3x - 5 - 6x + 4$  Remove the parentheses and identify like terms.

$$= 3x - 6x - 5 + 4$$
 Gather like terms together.

$$= (-3x) + (-1)$$
 Combine like terms.

$$= -3x - 1$$

**EXAMPLE B** 

Add:  $(8x^2 - 5x + 3) + (-2x - x + 4)$ 

$$(8x^2 - 5x + 3) + (-2x^2 - x + 4)$$

$$=$$
  $8x^2 - 5x + 3 - 2x^2 - x + 4$  Remove the parentheses; identify like terms.

$$=(8x^2-2x^2)+(-5x-x)+(3+4)$$
 Gather like terms.

$$= (6x^2) + (-6x) + (7)$$

Combine like terms.

$$=6x^2-6x+7$$

**EXAMPLE C** 

Add:  $(-2x^2 - x + 11) + (6x - 3)$ 

$$(-2x^2 - x + 11) + (6x - 3)$$

$$=$$
  $\frac{-2x^2}{x^2}$   $\frac{x}{x}$   $+$   $11$   $+$   $\frac{6x}{x}$   $3$  Remove the parentheses; identify like terms.

$$= (-2x^2) + (-x + 6x) + (11 - 3)$$
 Gather like terms.

$$= (-2x^2) + (5x) + (8)$$

Combine like terms.

$$=-2x^2+5x+8$$

#### **Subtracting Polynomials**

When subtracting one polynomial from another, first remove any parentheses around the polynomials. When you remove the parentheses from a polynomial that follows a subtraction sign, you <u>must</u> change the sign of each term inside the parentheses.

This is the same as distributing a negative number (see example **a** below) or a negative sign (example **b** below) over each term within a set of parentheses.

**a.** 
$$-2(x^2 - 7x + 1) = -2x^2 + 14x - 2$$

**b.** 
$$-(6x-3) = -1(6x-3) = -6x+3$$

After removing the parentheses and changing the sign of each term of the second polynomial, follow the steps for simplifying an expression.

**Remember:** the distribution of a negative number over parentheses will change the sign of each term inside the parentheses!

**EXAMPLE D** 

Subtract:  $(-4x^2 - 20x - 10) - (6x^2 - 3x + 5)$ 

 $(-4x^2 - 20x - 10) - (6x^2 - 3x + 5)$ 

Remove the parentheses and cl  
= 
$$-4x^2 - 20x - 10 - 6x^2 + 3x - 5$$
 sign of each term of the second

Remove the parentheses and change the polynomial.

$$= (-4x^2 - 6x^2) + (-20x + 3x) + (-10 - 5)$$
Group like terms. Notice that the operation between each set of parenthoses is addition.

Group like terms. Notice that of parentheses is addition.

 $= (-10x^2) + (-17x) + (-15)$  Combine like terms.

$$=-10x^2-17x-15$$

**EXAMPLE E** 

Subtract (-2x - 1) from  $(10x^2 - 3x + 4)$ .

 $(10x^2 - 3x + 4) - (-2x - 1)$  Write the problem. Subtract a binomial from a trinomial.

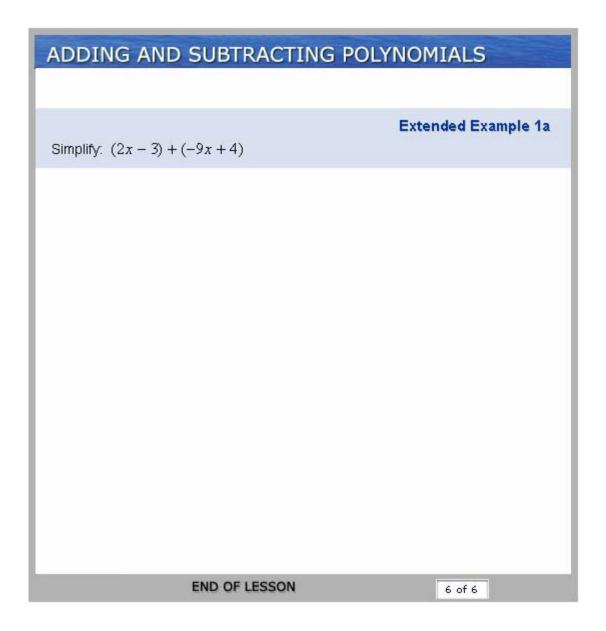
 $=10x^2-3x+4+2x+1$  Remove the parentheses and change the sign of each term of the second polynomial.

$$= 10x^2 + (-3x + 2x) + (4 + 1)$$

Group like terms. Notice that the operation  $=10x^2 + (-3x + 2x) + (4+1)$  between each set of parentheses is addition.

=  $10x^2 + (-x) + (5)$  Combine like terms.

$$=10x^2-x+5$$



Add or subtract the polynomial.

$$(12y-20)+(9y^2+13y-20)$$

Add or subtract the polynomial.

$$(9n+4)+(5n^6+n^3-9n+14)$$

Add or subtract the polynomial.

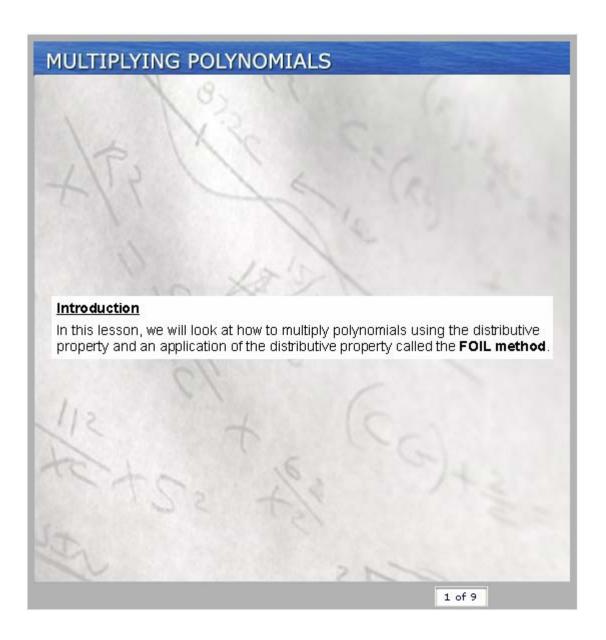
$$(14x^4 + 2x^3 - 8x^2 - x) + (7x^4 - 3x^2 + 1)$$

Add or subtract the polynomial.

$$(10y^2-7)-(20y^3-2y^2-3)$$

Add or subtract the polynomial.

$$(-11w^5 - w^4 + 33w - 7) - (9w^3 + 41w)$$



#### **Multiplying Monomials**

Recall that a monomial is a polynomial with one term. To multiply two monomials, multiply the coefficients together and multiply the variables together. Multiply the coefficients using the rules for multiplication of integers, and multiply the variables using the product rule for exponents.

#### **EXAMPLE A**

Multiply: 2x(9x)

$$2x(9x)$$
  
=  $(2 \cdot 9)(x \cdot x)$  Multiply the coefficients together and the variables together.

$$= (18) \left(x^2\right)$$

$$= 18x^{2}$$

#### **EXAMPLE B**

Multiply: 5y(-3y)

$$5y(-3y)$$
=  $(5 \cdot (-3))(y \cdot y)$  Multiply the coefficients together and the variables together.
=  $(-15)(y^2)$ 

 $=-15y^2$ 

The **distributive property** is used to multiply a monomial by a polynomial. First, distribute the monomial to each term of the polynomial inside the parentheses. Then, simplify each term by following the procedure for multiplying two monomials.

**EXAMPLE C** 

Multiply:  $7y^2(-6y+11)$ 

$$\overline{7y^2(-6y+11)}$$

 $=\underbrace{7y^2(-6y)}_{\left(7\cdot -6\right)\left(y^2\cdot y\right)} +\underbrace{7y^2(1\,1)}_{\left(7\cdot 1\,1\right)\left(y^2\right)}$  Distribute the monomial to each term of the binomial.

$$= -42y^3 + 77y^2$$
 Simplify each term.

**EXAMPLE D** 

Multiply:  $7y(9y^2 - y + 4)$ 

$$7y(9y^2 - y + 4)$$

 $= \underbrace{\frac{7y(9y^2)}{(7\cdot 9)(y\cdot y^2)}}_{\left(7\cdot -1\right)\left(y\cdot y\right)} + \underbrace{\frac{7y(4)}{(7\cdot 4)(y)}}_{\left(7\cdot 4\right)\left(y\right)}$  Distribute to each term of the trinomial.

$$= 63y^3 - 7y^2 + 28y$$

Simplify each term.

### **Multiplying Binomials**

To multiply two binomials, we use a special version of the distributive property called the **FOIL Method**. FOIL stands for; **First** + **O**uter + **I**nner + **L**ast.

$$(a + b)(x + y) = ax + ay + bx + by$$

The steps in the FOIL process are also listed below, for your reference.

Step 1	First +	Multiply the first term of the first binomial and the first term of the second binomial.
Step 2	outer +	Multiply the first term of the first binomial and the second term of the second binomial.
Step 3	Inner +	Multiply the second term of the first binomial and the first term of the second binomial.
Step 4	Last	Multiply the second term of the first binomial and the second term of the second binomial.

## **EXAMPLE E**

Multiply: (x+5)(x-6)

First	<b>O</b> uter			Inner		<b>L</b> ast	
$x \cdot x$	+	x · (-6)	+	5 · x	+	5 · (-6)	
	•	$=x^2$	$\frac{-6x+5}{-1x}$	<u>5x</u> – 30			
		$=x^2$	x - x - 3	0			

## **EXAMPLE F**

Multiply: (3x + 8)(2x - 5)

<b>F</b> irst		Outer		Inner		<b>L</b> ast	
$3x \cdot 2x$	+	3x · (-5)	+	8 · 2x	+	8 · (-5)	
		$= 6x^{2}$	$\frac{-15x + 1x}{1x}$	16x - 40	Tr. 10		
		$= 6x^2$	+ x - 4	0			

**EXAMPLE G** 

Multiply: (2x + 5)(2x - 5)

First		<b>O</b> uter		Inner		<b>L</b> ast
$2x \cdot 2x$	+	2x · (-5)	+	5 · 2x	+	5 · (-5)
	ė s	$=4x^{2}$	-10x +	10x - 25	*: 3	
		$=4x^{2}$	– 25			

Notice that in Example G the two middle terms canceled out. Can you see why the middle terms cancel? For better understanding, try multiplying (x - 4)(x + 4).

$$(x-4)(x+4) = x^{2} + \underbrace{4x + (-4x)}_{0} + (-16)$$
$$= x^{2} - 16$$

Whenever you multiply two binomials with identical terms but opposite operations, the middle terms will cancel out.

Squaring a binomial is the same as multiplying two binomials. Use the definition of an exponent when squaring a binomial. The base, which in this case is the binomial, is multiplied by itself.

The same is true when squaring any polynomial.

**EXAMPLE H** 

Multiply:  $(5x - 1)^2$ 

$$(5x-1)^2 = (5x-1)(5x-1)$$
 Rewrite the problem. Then FOIL.

First		<b>O</b> uter		Inner L		<b>L</b> ast	
5x · 5x	+	5x · (-1)	+	$(-1)\cdot 5x$	+	(-1) · (-1)	
	$= 25x^2 - 5x - 5x + 1$						
		= 2	$5x^2 - 1$				

#### **Multiplying Polynomials**

To multiply larger polynomials, distribute each term in the first polynomial to each term in the second polynomial. Study Example I.

**EXAMPLE I** 

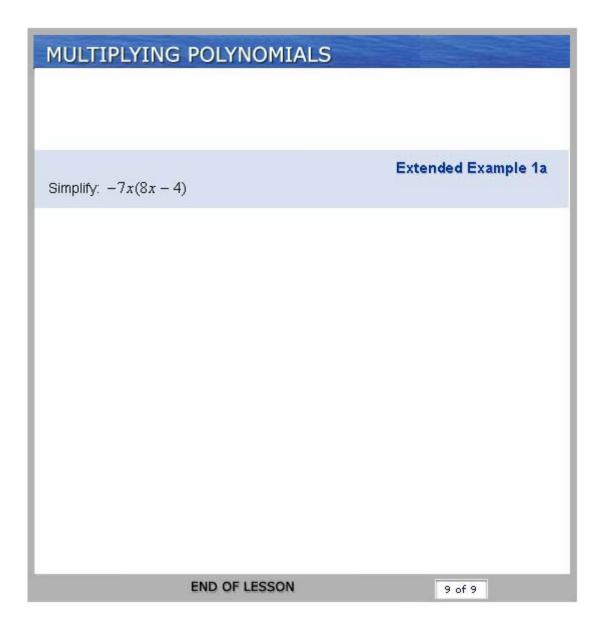
Multiply:  $(x-2)(x^2+3x+4)$ 

Distribute the first term and the second term separately to help keep things clear:

$$\begin{cases} (x-2)(x^2+3x+4) \rightarrow x(x^2) + x(3x) + x(4) = x^3 + 3x^2 + 4x \\ (x-2)(x^2+3x+4) \rightarrow -2(x^2) - 2(3x) - 2(4) = -2x^2 - 6x - 8 \end{cases}$$

Then put those results together and combine like terms:

$$(x-2)(x^2+3x+4) = x^3 + \underline{3x^2} + \underline{4x} - \underline{2x^2} - \underline{\underline{6x}} - 8$$
$$= x^3 + x^2 - 2x - 8$$



Multiply the polynomials.

$$(3a^3b^6)(12a^2b^9)$$

Multiply the polynomials.

$$10k^3(7k^9+9k^6+k^3)$$

Multiply the polynomials.

$$(y+7)(y-7)$$

Multiply the polynomials.

$$(5b+3)(4b+5)$$

Multiply the polynomials.

$$(2c^2-14)(c^4-12c^2-14)$$