PRODUCT RULE OF EXPONENTS

Introduction

Earlier you learned that an exponent has the same meaning whether the base is a number or a variable. For example,

$$4^5 = \underbrace{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}_{\text{5 factors of 4}} \qquad x^5 = \underbrace{x \cdot x \cdot x \cdot x \cdot x}_{\text{5 factors of } x}$$

From this definition of exponents, additional properties can be developed that will help us simplify more complex exponential expressions. We'll look at one of those properties, the product rule of exponents, in this lesson.

PRODUCT RULE OF EXPONENTS

Using the definition of an exponent, let's find the product of m^2 and m^5.

$$\begin{aligned} m^2 \cdot m^5 &= (m \cdot m)(m \cdot m \cdot m \cdot m \cdot m) \\ &= \underbrace{m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m}_{\text{7 factors of } m} \\ &= m^7 \end{aligned}$$

Notice the resulting exponent, 7, is equal to the sum of the two original exponents, 5 and 2:

$$m^2 \cdot m^5 = m^{2+5} = m^7.$$

The above is an example of the product rule of exponents.

Product Rule of Exponents
If m and n are positive integers and x is a real number, then:
$x^m \cdot x^n = x^{m+n}$.

PRODUCT RULE OF EXPONENTS

The product rule of exponents says that when multiplying two exponential expressions with the same base, we keep the base and add the exponents. Study the following examples. Usually the directions for these types of problems will either ask you to "Multiply" or "Simplify."

EXAMPLE A

Simplify: $x^3 \cdot x^4$

$$x^3 \cdot x^4 = x^{3+4}$$
$$= x^7$$

EXAMPLE B

Simplify: $2y^5 \cdot 4y^3$

$$2y^5 \cdot 4y^3 = (2 \cdot 4)\left(y^5 \cdot y^3\right)$$ Multiply the coefficients.

$$= 8 \cdot y^{5+3}$$
$$= 8y^8$$

PRODUCT RULE OF EXPONENTS

If any of the coefficients are negative, the rules are the same as for multiplying integers.

EXAMPLE C

Simplify: $\left(-2a^3b^9\right)\left(9a^5b^7\right)$

$$\begin{aligned}\left(-2a^3b^9\right)\left(9a^5b^7\right) &= (-2\cdot 9)\left(a^3\cdot a^5\right)\left(b^9\cdot b^7\right)\\ &= (-18)\left(a^{3+5}\right)\left(b^{9+7}\right)\\ &= -18a^8b^{16}\end{aligned}$$

EXAMPLE D

Simplify: $-2x^4\cdot 5x^3\cdot -4x$

$$\begin{aligned}-2x^4\cdot 5x^3\cdot -4x &= (-2\cdot 5\cdot -4)\left(x^4\cdot x^3\cdot x^1\right)\\ &= (40)\left(x^{4+3+1}\right)\\ &= 40x^8\end{aligned}$$

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PRODUCT RULE OF EXPONENTS

Helpful Hint	
It is important to recognize the difference between adding and multiplying terms.	
Examples of Adding	**Examples of Multiplying**
$-9x^2+2x^2=(-9+2)x^2=-7x^2$	$(-9x^2)(2x^2)=(-18)x^{2+2}=-18x^4$
$2x^3+7x=2x^3+7x$	$(2x^3)(7x)=14x^{3+1}=14x^4$

PRODUCT RULE OF EXPONENTS

Extended Example 1a

Simplify: $m^4 \cdot m^8$

END OF LESSON

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Simplify.

$9x^2 \cdot x^8$

Simplify.

$r^2 s^3 \cdot rs^6$

Simplify.

$(-2xy^4)(-6x^3y^7)$

Simplify.

$$a(-4a^{11})(-3a^{5})$$

Simplify.

$$(-7x^{5})(-9y^{12})(x^{3}y^{4})$$

QUOTIENT RULE OF EXPONENTS

Introduction

This section introduces another helpful property of exponents: the quotient rule, and the related concepts of negative exponents and zero exponents.

QUOTIENT RULE OF EXPONENTS

The definition of an exponent allows you to find the quotient of two exponential expressions. For example, $m^5 \div m^2$ can be written as:

$$\frac{m^5}{m^2} = \frac{m \cdot m \cdot m \cdot \cancel{m} \cdot \cancel{m}}{\cancel{m} \cdot \cancel{m}}$$
$$= \underbrace{m \cdot m \cdot m}_{\text{3 factors of } m}$$
$$= m^3$$

Notice that 2 factors are taken away from 5 factors to get 3 factors:

$$\frac{m^5}{m^2} = m^{5-2} = m^3.$$

This is an example of the quotient property of exponents.

Quotient Rule of Exponents
If m and n are integers and x is a real number, then:
$\frac{x^m}{x^n} = x^{m-n}$

QUOTIENT RULE OF EXPONENTS

The quotient rule says that if you are dividing two exponential expressions with the same base, keep the base and subtract the exponent of the divisor from the exponent of the dividend. Study the following examples.

EXAMPLE A

Simplify: $\frac{x^4}{x^3}$

$\frac{x^4}{x^3} = x^{4-3}$ Use the quotient rule of exponents: subtract the exponent of the denominator from that of the numerator.

$= x^1$

$= x$

EXAMPLE B

Simplify: $\frac{4y^5}{2y^3}$

$\frac{4y^5}{2y^3} = \left(\frac{4}{2}\right)\left(\frac{y^5}{y^3}\right)$ Separate the coefficients from the variables.

$= 2 \cdot y^{5-3}$ Simplify the fraction at left, and use the quotient rule of exponents.

$= 2y^2$

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QUOTIENT RULE OF EXPONENTS

If any of the coefficients are negative, the sign of the answer will follow the rules for dividing integers.

EXAMPLE C

Simplify: $\dfrac{-2a^6b^9}{9a^3b^7}$

$$\frac{-2a^6b^9}{9a^3b^7} = \left(\frac{-2}{9}\right)\left(\frac{a^6}{a^3}\right)\left(\frac{b^9}{b^7}\right)$$ Separate the coefficients and like variables.

$$= \left(\frac{-2}{9}\right)\left(a^{6-3}\right)\left(b^{9-7}\right)$$ The fraction cannot be simplified. Use the quotient rule of exponents.

$$= \left(\frac{-2}{9}\right)a^3b^2$$

$$= \frac{-2a^3b^2}{9}$$ Simplify.

QUOTIENT RULE OF EXPONENTS

Extended Example 1a

Simplify: $\dfrac{x^7}{x^2}$

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QUOTIENT RULE OF EXPONENTS

Integer Exponents

Through the use of the quotient rule of exponents, an important concept arises. Notice in the following example that after applying the quotient property, what is left is a 0 power of x:

$$\frac{y^5}{y^5} = y^{5-5} = y^0.$$

We also know that any fraction with the same numerator and denominator is equal to 1:

$$\frac{y^5}{y^5} = 1.$$

This leads us to the rule for zero exponents.

Zero Exponents
If x is a real number not equal to zero, then:
$x^0 = 1$

Here are some examples of the zero exponent rule:

a. $5^0 = 1$ **b.** $3,258,647^0 = 1$

c. $(-21)^0 = 1$ **d.** $8b^0 = 8(1) = 8$

QUOTIENT RULE OF EXPONENTS

EXAMPLE D

Simplify: $\frac{3x^6}{9x^6}$

$$\frac{3x^6}{9x^6} = \left(\frac{3}{9}\right)\left(\frac{x^6}{x^6}\right)$$

$$= \left(\frac{1}{3}\right)\left(x^{6-6}\right)$$

$$= \left(\frac{1}{3}\right)\left(x^0\right)$$

$$= \left(\frac{1}{3}\right)(1)$$

$$= \frac{1}{3}$$

QUOTIENT RULE OF EXPONENTS

Another important concept arises from knowledge of the quotient rule—the concept of negative exponents. For example:

$$\frac{y^3}{y^5} = y^{3-5} = y^{-2}.$$

What does this negative exponent mean?

Definition of Negative Exponent
If n is an integer and x is a real number not equal to zero, then:
$x^{-n} = \frac{1}{x^n}$

Here are some examples of negative exponents:

a. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

b. $m^{-15} = \frac{1}{m^{15}}$

c. $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = -\frac{1}{8}$

d. $7b^{-4} = 7\left(\frac{1}{b^4}\right) = \frac{7}{b^4}$

QUOTIENT RULE OF EXPONENTS

EXAMPLE E

Simplify: $\dfrac{3x^6}{9x^9}$

$$\frac{3x^6}{9x^9} = \left(\frac{3}{9}\right)\left(\frac{x^6}{x^9}\right)$$

$$= \left(\frac{1}{3}\right)\left(x^{6-9}\right)$$

$$= \left(\frac{1}{3}\right)\left(x^{-3}\right)$$

$$= \left(\frac{1}{3}\right)\left(\frac{1}{x^3}\right)$$

$$= \frac{1}{3x^3}$$

QUOTIENT RULE OF EXPONENTS

More complex problems might require the use of multiple properties.

EXAMPLE E

Simplify: $\dfrac{-2x^4}{5x^3 \cdot -4x}$

$$\frac{-2x^4}{5x^3 \cdot -4x} = \left(\frac{-2}{5(-4)}\right)\left(\frac{x^4}{x^3 \cdot x}\right)$$ Separate the coefficients and variables.

$$= \left(\frac{-2}{-20}\right)\left(\frac{x^4}{x^{3+1}}\right)$$ Combine the coefficients, and use the product rule of exponents.

$$= \left(\frac{1}{10}\right)\left(\frac{x^4}{x^4}\right)$$ Simplify.

$$= \left(\frac{1}{10}\right)\left(x^{4-4}\right)$$ Use the quotient property of exponents.

$$= \left(\frac{1}{10}\right)\left(x^0\right)$$

$$= \left(\frac{1}{10}\right)(1) = \frac{1}{10}$$ Simplify.

QUOTIENT RULE OF EXPONENTS

Extended Example 2a

Simplify: $\dfrac{12x^8}{3x^5 \cdot -2x}$

END OF LESSON

Simplify.

$$\frac{-4n^{10}}{-6n^{22}} =$$

Simplify.

$$\frac{16y^{12}z^{12}}{-24yz^{12}} =$$

Simplify.

$$\frac{14k^{14}mn^{14}}{49k^{5}m^{9}n^{15}} =$$

Simplify.

$$\frac{20x^2y^4z}{-4x^6y^8z^5} =$$

Simplify.

$$\frac{-2p^6 \cdot 3p^6}{9p^8(-8p^9)} =$$

POWER RULE OF EXPONENTS

Introduction

So far, you have learned how to simplify multiplication and division of exponential terms with like bases.

Product Rule: $x^m \cdot x^n = x^{m+n}$ **Quotient Rule:** $\frac{x^m}{x^n} = x^{m-n}$

Suppose we want to simplify an exponential expression that is raised to a power, such as $\left(x^3\right)^2$. This lesson will explore the property that allows us to do this: the power rule of exponents.

POWER RULE OF EXPONENTS

Use the definition of exponents and the product rule of exponents that you learned in Section 1 of this chapter to simplify $\left(x^3\right)^2$:

$$\begin{aligned}\left(x^3\right)^2 &= \left(x^3\right)\left(x^3\right)\\ &= x^{3+3}\\ &= x^6\end{aligned}$$

This result is the same as that of multiplying the two exponents.

$$\left(x^3\right)^2 = x^{3\cdot 2} = x^6$$

This is an example of the power rule of exponents.

Power Rule of Exponents
If m and n are positive integers and x is a real number, then:
$\left(x^m\right)^n = x^{m\cdot n}$

POWER RULE OF EXPONENTS

EXAMPLE A

Simplify: $\left(y^{11}\right)^4$

$$\left(y^{11}\right)^4 = y^{11 \cdot 4}$$
$$= y^{44}$$

EXAMPLE B

Simplify: $\left(x^3\right)^5\left(x^2\right)^4$

$$\left(x^3\right)^5\left(x^2\right)^4 = \left(x^{3 \cdot 5}\right)\left(x^{2 \cdot 4}\right)$$
$$= x^{15} \cdot x^8$$
$$= x^{15+8}$$
$$= x^{23}$$

Before we look at more properties of exponents, let's make sure we understand the difference between the product rule (from Section 1 of this chapter) and the power rule. Carefully study the examples in the table below.

Product Rule → Add Exponents	**Power Rule → Multiply Exponents**
$z^4 \cdot z^6 = z^{4+6} = z^{10}$	$\left(z^4\right)^6 = z^{4 \cdot 6} = z^{24}$
$x^3 \cdot x^5 = x^{3+5} = x^8$	$\left(x^3\right)^5 = x^{3 \cdot 5} = x^{15}$

POWER RULE OF EXPONENTS

Now suppose we are simplifying the power of a product. For example, let's simplify $(2mn)^3$. Let's use the definition of an exponent:

$$\begin{aligned}(2mn)^3 &= (2mn)(2mn)(2mn)\\ &= (2\cdot 2\cdot 2)(m\cdot m\cdot m)(n\cdot n\cdot n)\\ &= 2^3\cdot m^3\cdot n^3\\ &= 8m^3n^3\end{aligned}$$

Notice that the power of a product can be written as the product of powers. For example, we could have written the same problem as follows:

$$\begin{aligned}(2mn)^3 &= 2^3\cdot m^3\cdot n^3\\ &= 8m^3n^3\end{aligned}$$

Power of Product Rule of Exponents
If n is a positive integer and x and y are real numbers, then:
$(xy)^n = x^n y^n$

In other words, to raise a product to a power, raise each factor to that power:

$$(3p)^4 = 3^4\cdot p^4 = 81p^4$$

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POWER RULE OF EXPONENTS

Armed with the power of product rule, you can simplify even complex-looking expressions, as shown in Examples C and D below.

EXAMPLE C

Simplify: $\left(4x^3y^7\right)^3$

$$\left(4x^3y^7\right)^3 = 4^3 \cdot \left(x^3\right)^3 \cdot \left(y^7\right)^3$$
$$= 64 \cdot x^{3 \cdot 3} \cdot y^{7 \cdot 3}$$
$$= 64x^9y^{21}$$

EXAMPLE D

Simplify: $\left(4k^2h^6\right)^3\left(-3k^6h^8\right)^2$

$$\left(4k^2h^6\right)^3\left(-3k^6h^8\right)^2 = \left[4^3\left(k^2\right)^3\left(h^6\right)^3\right] \cdot \left[(-3)^2\left(k^6\right)^2\left(h^8\right)^2\right]$$
$$= \left(64k^{2 \cdot 3}h^{6 \cdot 3}\right)\left(9k^{6 \cdot 2}h^{8 \cdot 2}\right)$$
$$= \left(64k^6h^{18}\right)\left(9k^{12}h^{16}\right)$$
$$= (64 \cdot 9)\left(k^6k^{12}\right)\left(h^{18}h^{16}\right)$$
$$= 576k^{18}h^{34}$$

POWER RULE OF EXPONENTS

Extended Example 1a

Simplify: $\left(-6m^4n^7\right)^4$

END OF LESSON

Simplify.

$$(m^2)^2(m^3)^4$$

Simplify.

$$(-3y)^2$$

Simplify.

$$(-6y^7)^2$$

Simplify.

$$(4q^8)^2(2q^4)^3$$

Simplify.

$$(2xy)^4(3x^4y^3)^3$$

POLYNOMIALS

Introduction

In Chapter 1 you learned some basic vocabulary of algebra, and learned what it means to evaluate an expression. We will add to that vocabulary in this lesson, and will learn to evaluate a type of expression called a **polynomial**.

POLYNOMIALS

In Chapter 1 you learned about **expressions**, which can be simplified or evaluated, and **equations**, which can be solved. Recall that a **term** is one of the addends of an algebraic expression. For example:

- $6x+8$ has two terms. $6x$ is a variable term and 8 is a **constant** or non-variable term.
- $5y^2-4y+9$ has three terms. $5y^2$ and $-4y$ are variable terms, and 9 is a constant.

Also recall that all variable terms have a **coefficient**—study the examples in the table below.

Variable Term	The Coefficient
$6x$	6
$5y^2$	5
$-4y$	-4
$-x$	-1
m	1

POLYNOMIALS

A **monomial** is a term that contains only positive exponents and that does not contain a variable in the denominator. Note that this means that not all terms are monomials—see the table below.

Monomials	Not Monomials
$7y^2$	$\frac{2}{y}$
$-\frac{8}{27}x$	$4x^{-5}$

Monomials or a sum of monomials are known as **polynomials**. Some examples of polynomials are:

a. $2x$

b. $5x^3+2x^2-3x+1$

c. $7x+4$

d. $8y^2+\frac{1}{2}y-6$

POLYNOMIALS

There is no limit to the number of terms a polynomial can have. Some common types of polynomials with a specific number of terms have special names.

- If a polynomial has **one** term, then it is called a **monomial**.
- If a polynomial has **two** terms, then it is called a **binomial**.
- If a polynomial has **three** terms, then it is called a **trinomial**.

There is no special name for polynomials with more than three terms.

The **degree of a polynomial** is equal to the largest exponent of all the terms in the polynomial. The chart below shows the details for the examples on the previous screen.

Polynomial	Number of Terms and Name	Degree
a. $2x$	1 term = monomial	1
b. $5x^3 + 2x^2 - 3x + 1$	4 terms = polynomial	3
c. $7x + 4$	2 terms = binomial	1
d. $8y^2 + \frac{1}{2}y - 6$	3 terms = trinomial	2

POLYNOMIALS

Evaluating Polynomials

Since polynomials are a type of expression, they can be evaluated. Evaluating a polynomial is done by replacing the variable of a polynomial with a specific given number, and then computing the result. If the given value is negative or a fraction, you should always use parentheses when substituting it in place of the variable.

EXAMPLE A

Evaluate $7x-5$ when $x=-1$.

$7x-5$

$7(-1)-5$ Replace x with –1, then simplify.

$=-7-5$

$=-12$

EXAMPLE B

Evaluate $-5x^2-x+12$ when $x=-2$.

$-5x^2-x+12$ Be sure to pay close attention to signs!

$-5(-2)^2-(-2)+12$

$=-5(4)+4+12$ Note: $(-2)^2=(-2)(-2)=4$ and $-(-2)=4$.

$=-20+4+12$

$=-4$

EXAMPLE C

Evaluate $x^2 - x + 6$ when $x = \frac{1}{3}$.

$$x^2 - x + 6$$

$$\left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right) + 6$$

Note: $\left(\frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \frac{1}{9}$

$$= \frac{1}{9} - \frac{1}{3} + 6$$

$$= \frac{1}{9} - \frac{3}{9} + \frac{54}{9}$$

Remember that you need to find a common denominator when adding or subtracting fractions.

$$= \frac{1 - 3 + 54}{9}$$

$$= \frac{52}{9}$$

POLYNOMIALS

Note that when evaluating a polynomial you will obtain different results when you substitute different values for the variable . Study the following example, where the same polynomial is evaluated for four different values—with four different results.

Evaluate $10x^2 - 3x + 7$ when $x = 1$, $x = 3$, $x = -3$, and $x = \frac{1}{2}$:

$x = 1$	$x = 3$	$x = -3$	$x = \frac{1}{2}$
$10x^2 - 3x + 7$ $10(1)^2 - 3(1) + 7$ $= 10(1) - 3 + 7$ $= 10 - 3 + 7$ $= 14$	$10x^2 - 3x + 7$ $10(3)^2 - 3(3) + 7$ $= 10(9) - 9 + 7$ $= 90 - 9 + 7$ $= 88$	$10x^2 - 3x + 7$ $10(-3)^2 - 3(-3) + 7$ $= 10(9) + 9 + 7$ $= 90 + 9 + 7$ $= 106$	$10x^2 - 3x + 7$ $10\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 7$ $= 10\left(\frac{1}{4}\right) - \frac{3}{2} + 7$ $= \frac{5}{2} - \frac{3}{2} + 7$ $= 8$

Extended Example 1a

Evaluate $7x^2 - 2x - 13$ when $x = -2$.

END OF LESSON

Identify the following:
a) number of terms,
b) type (monomial, binomial, trinomial, or polynomial),
c) degree.

$$x^2 - 4x + 16$$

Evaluate $x^2 + 3x$ *for the value of* x.

$$x = -\frac{1}{2}$$

Evaluate $y^3 + 7y^2 - 13$ *for the value of* y.

$$y = 3$$

Evaluate for the polynomial when $x = 5$.

$$\frac{3}{2}x^2 - 4$$

Evaluate for the polynomial when $x = -2$.

$$3x^2 - 2x + 10$$

ADDING AND SUBTRACTING POLYNOMIALS

Introduction

In this lesson, we revisit techniques you learned earlier for simplifying expressions and apply them to adding and subtracting polynomials.

ADDING AND SUBTRACTING POLYNOMIALS

Adding Polynomials

To add polynomials, first remove the parentheses and then follow the steps for simplifying expressions.

Earlier, you learned to combine **like terms**. Remember that to combine like terms, you combine their coefficients (following the rules for combining integers) and keep the variable as is. Examples:

a. $7x + 3x = (7+3)x = 10x$

b. $5y^2 - 4y^2 = (5-4)y^2 = (1)y^2 = y^2$

Remember, to simplify an expression:

- Identify like terms.
- Gather like terms together.
- Combine like variable terms by combining their coefficients.
- Combine any constants (non-variable terms).

EXAMPLE A

Add: $(3x-5)+(-6x+4)$

$(3x-5)+(-6x+4)$

$= 3x - 5 - 6x + 4$ Remove the parentheses and identify like terms.

$= 3x - 6x - 5 + 4$ Gather like terms together.

$= (-3x) + (-1)$ Combine like terms.

$= -3x - 1$

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EXAMPLE B

Add: $(8x^2 - 5x + 3) + (-2x - x + 4)$

$(8x^2 - 5x + 3) + (-2x^2 - x + 4)$

$= \underline{8x^2} - \underline{\underline{5x}} + 3 - \underline{2x^2} - \underline{\underline{x}} + 4$ Remove the parentheses; identify like terms.

$= (8x^2 - 2x^2) + (-5x - x) + (3 + 4)$ Gather like terms.

$= (6x^2) + (-6x) + (7)$ Combine like terms.

$= 6x^2 - 6x + 7$

EXAMPLE C

Add: $(-2x^2 - x + 11) + (6x - 3)$

$(-2x^2 - x + 11) + (6x - 3)$

$= \underline{-2x^2} - \underline{\underline{x}} + 11 + \underline{\underline{6x}} - 3$ Remove the parentheses; identify like terms.

$= (-2x^2) + (-x + 6x) + (11 - 3)$ Gather like terms.

$= (-2x^2) + (5x) + (8)$ Combine like terms.

$= -2x^2 + 5x + 8$

ADDING AND SUBTRACTING POLYNOMIALS

Subtracting Polynomials

When subtracting one polynomial from another, first remove any parentheses around the polynomials. When you remove the parentheses from a polynomial that follows a subtraction sign, you must change the sign of each term inside the parentheses.

This is the same as distributing a negative number (see example **a** below) or a negative sign (example **b** below) over each term within a set of parentheses.

a. $-2(x^2 - 7x + 1) = -2x^2 + 14x - 2$

b. $-(6x - 3) = -1(6x - 3) = -6x + 3$

After removing the parentheses and changing the sign of each term of the second polynomial, follow the steps for simplifying an expression.

Remember: the distribution of a negative number over parentheses will change the sign of each term inside the parentheses!

ADDING AND SUBTRACTING POLYNOMIALS

EXAMPLE D

Subtract: $(-4x^2 - 20x - 10) - (6x^2 - 3x + 5)$

$(-4x^2 - 20x - 10) - (6x^2 - 3x + 5)$

$= -4x^2 - 20x - 10 - 6x^2 + 3x - 5$ Remove the parentheses and change the sign of each term of the second polynomial.

$= (-4x^2 - 6x^2) + (-20x + 3x) + (-10 - 5)$ Group like terms. Notice that the operation between each set of parentheses is addition.

$= (-10x^2) + (-17x) + (-15)$ Combine like terms.

$= -10x^2 - 17x - 15$

EXAMPLE E

Subtract $(-2x - 1)$ from $(10x^2 - 3x + 4)$.

$(10x^2 - 3x + 4) - (-2x - 1)$ Write the problem. Subtract a binomial from a trinomial.

$= 10x^2 - 3x + 4 + 2x + 1$ Remove the parentheses and change the sign of each term of the second polynomial.

$= 10x^2 + (-3x + 2x) + (4 + 1)$ Group like terms. Notice that the operation between each set of parentheses is addition.

$= 10x^2 + (-x) + (5)$ Combine like terms.

$= 10x^2 - x + 5$

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ADDING AND SUBTRACTING POLYNOMIALS

Extended Example 1a

Simplify: $(2x-3)+(-9x+4)$

END OF LESSON

Add or subtract the polynomial.

$$(12y - 20) + (9y^2 + 13y - 20)$$

Add or subtract the polynomial.

$$(9n + 4) + (5n^6 + n^3 - 9n + 14)$$

Add or subtract the polynomial.

$$(14x^4 + 2x^3 - 8x^2 - x) + (7x^4 - 3x^2 + 1)$$

Add or subtract the polynomial.

$$(10y^2 - 7) - (20y^3 - 2y^2 - 3)$$

Add or subtract the polynomial.

$$(-11w^5 - w^4 + 33w - 7) - (9w^3 + 41w)$$

MULTIPLYING POLYNOMIALS

Introduction

In this lesson, we will look at how to multiply polynomials using the distributive property and an application of the distributive property called the **FOIL method**.

MULTIPLYING POLYNOMIALS

Multiplying Monomials

Recall that a monomial is a polynomial with one term. To multiply two monomials, multiply the coefficients together and multiply the variables together. Multiply the coefficients using the rules for multiplication of integers, and multiply the variables using the product rule for exponents.

EXAMPLE A

Multiply: $2x(9x)$

$2x(9x)$

$= (2 \cdot 9)(x \cdot x)$ Multiply the coefficients together and the variables together.

$= (18)(x^2)$

$= 18x^2$

EXAMPLE B

Multiply: $5y(-3y)$

$5y(-3y)$

$= (5 \cdot (-3))(y \cdot y)$ Multiply the coefficients together and the variables together.

$= (-15)(y^2)$

$= -15y^2$

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MULTIPLYING POLYNOMIALS

The **distributive property** is used to multiply a monomial by a polynomial. First, distribute the monomial to each term of the polynomial inside the parentheses. Then, simplify each term by following the procedure for multiplying two monomials.

EXAMPLE C

Multiply: $7y^2(-6y+11)$

$$7y^2(-6y+11)$$

$$= \underbrace{7y^2(-6y)}_{(7\cdot -6)(y^2\cdot y)} + \underbrace{7y^2(11)}_{(7\cdot 11)(y^2)}$$ Distribute the monomial to each term of the binomial.

$$= -42y^3 + 77y^2$$ Simplify each term.

EXAMPLE D

Multiply: $7y(9y^2-y+4)$

$$7y(9y^2-y+4)$$

$$= \underbrace{7y(9y^2)}_{(7\cdot 9)(y\cdot y^2)} + \underbrace{7y(-y)}_{(7\cdot -1)(y\cdot y)} + \underbrace{7y(4)}_{(7\cdot 4)(y)}$$ Distribute to each term of the trinomial.

$$= 63y^3 - 7y^2 + 28y$$ Simplify each term.

MULTIPLYING POLYNOMIALS

Multiplying Binomials

To multiply two binomials, we use a special version of the distributive property called the **FOIL Method**. FOIL stands for; **F**irst + **O**uter + **I**nner + **L**ast.

$$(a+b)(x+y) = ax + ay + bx + by$$

The steps in the FOIL process are also listed below, for your reference.

Step 1	**F**irst +	Multiply the first term of the first binomial and the first term of the second binomial.
Step 2	**O**uter +	Multiply the first term of the first binomial and the second term of the second binomial.
Step 3	**I**nner +	Multiply the second term of the first binomial and the first term of the second binomial.
Step 4	**L**ast	Multiply the second term of the first binomial and the second term of the second binomial.

MULTIPLYING POLYNOMIALS

EXAMPLE E

Multiply: $(x+5)(x-6)$

First		Outer		Inner		Last
$x \cdot x$	$+$	$x \cdot (-6)$	$+$	$5 \cdot x$	$+$	$5 \cdot (-6)$

$$= x^2 \underbrace{-6x + 5x}_{-1x} - 30$$

$$= x^2 - x - 30$$

EXAMPLE F

Multiply: $(3x+8)(2x-5)$

First		Outer		Inner		Last
$3x \cdot 2x$	$+$	$3x \cdot (-5)$	$+$	$8 \cdot 2x$	$+$	$8 \cdot (-5)$

$$= 6x^2 \underbrace{-15x + 16x}_{1x} - 40$$

$$= 6x^2 + x - 40$$

MULTIPLYING POLYNOMIALS

EXAMPLE G

Multiply: $(2x+5)(2x-5)$

First		**O**uter		**I**nner		**L**ast
$2x \cdot 2x$	+	$2x \cdot (-5)$	+	$5 \cdot 2x$	+	$5 \cdot (-5)$

$$= 4x^2 \underbrace{-10x + 10x}_{0} - 25$$

$$= 4x^2 - 25$$

Notice that in Example G the two middle terms canceled out. Can you see why the middle terms cancel? For better understanding, try multiplying $(x-4)(x+4)$.

$$(x-4)(x+4) = x^2 + \underbrace{4x + (-4x)}_{0} + (-16)$$

$$= x^2 - 16$$

Whenever you multiply two binomials with identical terms but opposite operations, the middle terms will cancel out.

MULTIPLYING POLYNOMIALS

Squaring a binomial is the same as multiplying two binomials. Use the definition of an exponent when squaring a binomial. The base, which in this case is the binomial, is multiplied by itself.

The same is true when squaring any polynomial.

EXAMPLE H

Multiply: $(5x-1)^2$

$(5x-1)^2 = (5x-1)(5x-1)$ Rewrite the problem. Then FOIL.

First		**O**uter		**I**nner		**L**ast
$5x \cdot 5x$	$+$	$5x \cdot (-1)$	$+$	$(-1) \cdot 5x$	$+$	$(-1) \cdot (-1)$

$$= 25x^2 \underbrace{-5x - 5x}_{-10x} + 1$$

$$= 25x^2 - 10x + 1$$

MULTIPLYING POLYNOMIALS

Multiplying Polynomials

To multiply larger polynomials, distribute each term in the first polynomial to each term in the second polynomial. Study Example I.

EXAMPLE I

Multiply: $(x-2)(x^2+3x+4)$

Distribute the first term and the second term separately to help keep things clear:

$$\begin{cases} (x-2)(x^2+3x+4) \rightarrow x(x^2)+x(3x)+x(4) = x^3+3x^2+4x \\ (x-2)(x^2+3x+4) \rightarrow -2(x^2)-2(3x)-2(4) = -2x^2-6x-8 \end{cases}$$

Then put those results together and combine like terms:

$$\begin{aligned}(x-2)(x^2+3x+4) &= x^3+\underline{3x^2}+\underline{\underline{4x}}-\underline{2x^2}-\underline{\underline{6x}}-8 \\ &= x^3+x^2-2x-8\end{aligned}$$

MULTIPLYING POLYNOMIALS

Extended Example 1a

Simplify: $-7x(8x-4)$

END OF LESSON

Multiply the polynomials.

$$(3a^3b^6)(12a^2b^9)$$

Multiply the polynomials.

$$10k^3(7k^9+9k^6+k^3)$$

Multiply the polynomials.

$$(y+7)(y-7)$$

Multiply the polynomials.

$$(5b+3)(4b+5)$$

Multiply the polynomials.

$$(2c^2 - 14)(c^4 - 12c^2 - 14)$$