

TRANSLATING WORDS INTO ALGEBRA

Review the definitions in the table below.

The answer to an addition problem is called the sum.

The answer to a subtraction problem is called the difference.

The answer to a multiplication problem is called the product.

The answer to a <u>division</u> problem is called the **quotient**.

Besides the list above, there are other words or short statements that suggest which basic operation needs to be used to solve a problem. The following table lists some common words or phrases that indicate each operation.

Addition	Subtraction	Multiplication	Division
sum	difference	product	quotient
increased by	decreased by	times	divided by
added	less than	twice	
more than	subtracted from	triple	
added to		of	
total			

TRANSLATING WORDS INTO ALGEBRA

The missing item in a word problem (such as a number, an amount of money, a length, etc.) is called the **unknown**. The unknown in a mathematical expression or an equation is represented by a **variable** (for example, x, n, t). When translating words into algebra, you first have to recognize the unknown and assign it a variable. Then set up an expression or an equation using the definitions from the tables above to determine the appropriate operation. Carefully study the examples below and on the next screen.

Phrase	Expression	Notes
a. Five more than a number 5 + n	5+n or n+5	Addition is commutative; the order of the terms makes no difference.
b. Seven times a number x	7x	
c. A number, decreased by six	n – 6 not 6 – n	Subtraction is not commutative; you need to be careful about the order of the terms.
d. The difference of a number and 2 ★ 2	x - 2 not 2 - x	Subtraction is not commutative; you need to be careful about the order of the terms.
e. Triple a number	3n	

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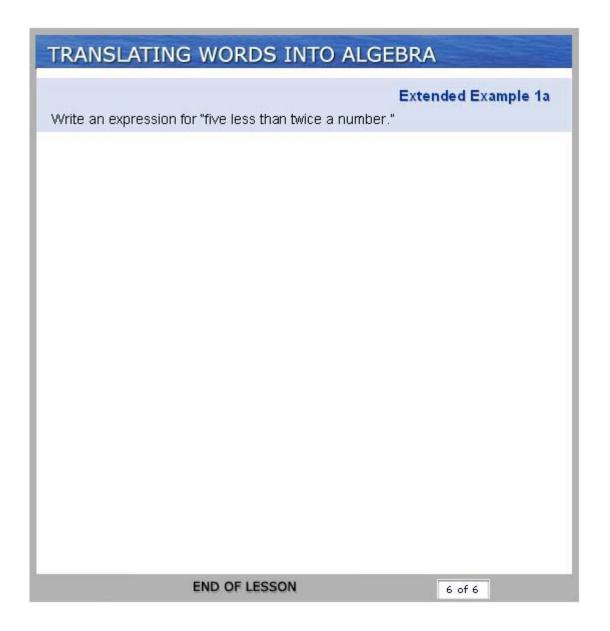
Phrase	Expression	Notes
f. 10 decreased by a number x	$ \begin{array}{c c} 10 - x \\ \underline{\text{not } x - 10} \end{array} $	Subtraction is <u>not</u> commutative; you need to be careful about the order o the terms.
g. 15 less than twice a number 15 – 2. ກ	2n – 15	With "less than," the second term must be written first in the expression.
h. The quotient of a number and 5 ້ ກັ່	$n \div 5$ or $\frac{n}{5}$ $not \ 5 \div n$ $not \ \frac{5}{n}$	Division is <u>not</u> commutative; you need to be careful about the order of the terms.
i. The product of two numbers x and y	ху	In this case you need two variables.
j. Twice the total of x and 2y 2v x 2y	2(x+2y)	Note that the x and $2y$ are inside parentheses because we need to multiply by a sum: the total of $x+2y$. The order of operations says that we simplify within grouping symbols first, so the parentheses give us the

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Translation Practice

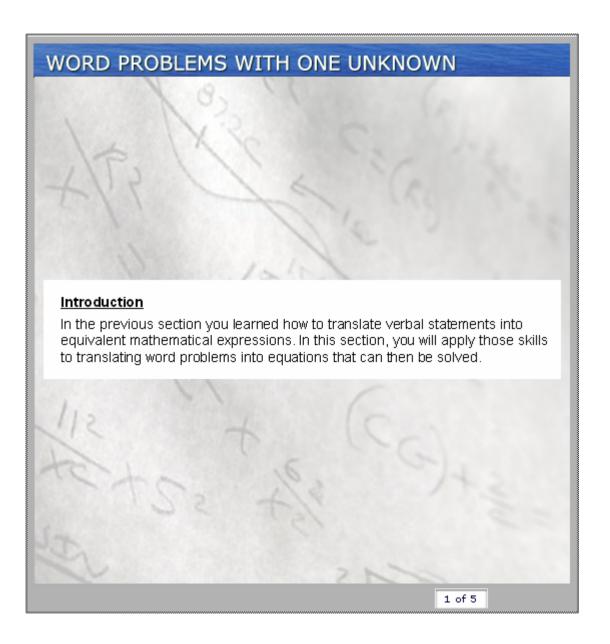
On your own, determine the mathematical expression for each phrase below. Remember that to write equivalent mathematical expressions for the following phrases, you have to first recognize the part that is unknown and assign it a variable. Then, use the vocabulary learned above to write the mathematical expression that represents the words. After you are finished, click the question marks to check your answers.

Mathematical Expression	
2	
š.	
š.	
2	
•	



Write an equivalent mathematical expression for the statement.
A number and 4 decreased by 8.
Write an equivalent mathematical expression for the statement.
The difference of a number and 23.
Write an equivalent mathematical expression for the statement.
Jolanta's new job pays her \$15,000 more a year.

Write an equivalent mathematical expression for the statement.
The quotient of 12 and a number increased by 8.
Muita an acción alant mathamatica l'accusación familia atatamant
Write an equivalent mathematical expression for the statement.
The difference of a number and the sum of 15 and 5.



Equations have equal signs. Below are some of the words and phrases that indicate that an equal sign should be used.

Results in | Equal to | Is | The result is | Is the same as |

The following word problems have only one unknown. To solve them, start by assigning a variable to the unknown. Then set up an equation and solve it. The numerical value that you find for the variable is the answer to the problem.

EXAMPLE A

The sum of a number and 5 is 12. Find the number.

A number = x

The sum of a number and 5 is 12. x + 5

x + 5 = 12

<u>-5 -5</u>

x = 7 So, the number is 7.

EXAMPLE B

The sum of twice a number and 6 is equal to 22. What is the number?

The number = x.

The sum of twice a number and 6 is equal to $\frac{22}{2x}$.

$$2x + 6 = 22$$

$$\frac{-6}{2x = 16}$$

$$x = 8$$

x = 8 So, the number is 8.

EXAMPLE C

Sherry paid \$78 for a pair of shoes after receiving a \$14 discount. What was the original price of the shoes?

Price of the shoes = p.

$$p - 14 = 78$$

$$+14 + 14$$

$$p = 92$$

The original price of the shoes was \$92.

EXAMPLE D

Each month, Amy spends \$450 on food and other expenses, which is \$220 less than what she pays in rent. How much does she pay in rent?

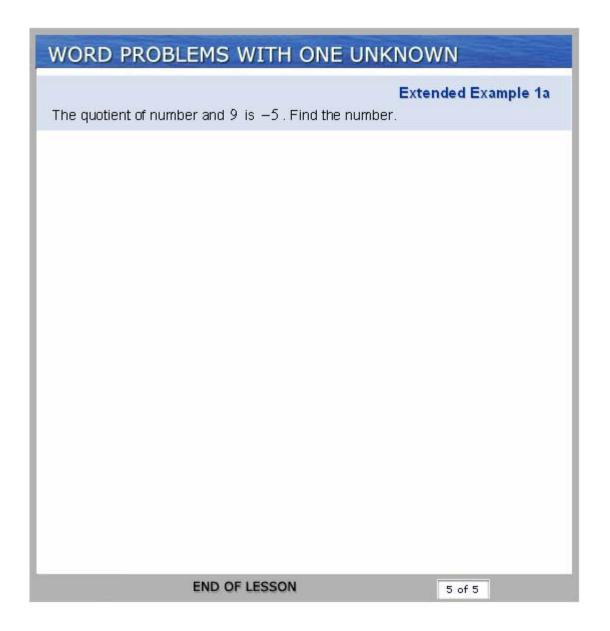
Amount of rent = r.

$$r = \frac{1}{2}$$

r - 220 = 450

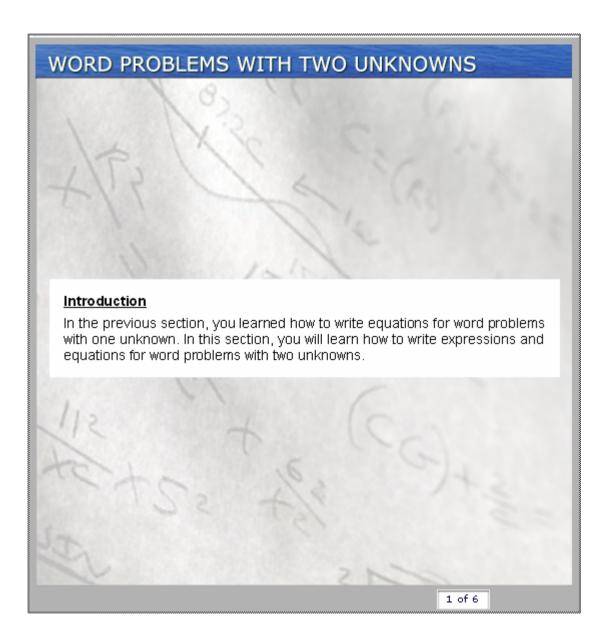
$$r = 670$$

The rent is \$670.



Write an equation for the problem and solve for the unknown.
A number increased by -16 is 9.
Write an equation for the problem and solve for the unknown.
Twice a number and 5 is equal to 33.
Write an equation for the problem and solve for the unknown.
Three times the sum of twice a number and 4 is 36.

Write an equation for the problem and solve for the unknown.
A 38 foot log is cut into two pieces. If one piece is 29 feet long, what is the length of the other piece?
Write an equation for the problem and solve for the unknown.
Mary spent \$78 on two books. She received a \$22 discount for buying two books. How much was the price of each book originally?
,
,
,
,
,
,



When two pieces of information are missing in a word problem, first assign a variable to one of the unknowns. Then, write an expression for the second unknown in terms of the first unknown that you defined. Translate the information in the problem into an equation. The solution to the equation is the value of the first unknown. Substitute this value into the expression for the second unknown.

How to Solve a Word Problem with Two Unknowns

- Read the problem carefully (reread it if you have to).
- Identify the two unknowns.
- Assign one of them a variable such as x.
- 4. Write an expression for the second unknown in terms of x based on the description provided in the problem.
- Write an equation based on the phrase or sentence provided in the problem.
- Solve the equation for x; this is the value of the first unknown.
- 7. Substitute the answer to x (from step 6) into the expression you wrote in step 4 to find the value of the second unknown.

This might seem complicated at first, but it will make more sense as you study the examples in this lesson.

Study the following examples carefully.

EXAMPLE A

One number is twice the other number. Their sum is 21. What are the two numbers?

Translate "one number":	First number = x
Translate "twice the other number":	Second number = $2x$
Translate "Their sum is 21":	x + 2x = 21
Combine like terms and solve the equation for x :	3x = 21 $x = 7$
First number = x:	First number = 7
Second number = $2x$:	Second number = $2(7) = 14$
Check:	7 + 14 = 21 ✓

EXAMPLE B

The distance from Las Vegas to San Francisco is 30 miles more than twice the distance from Las Vegas to Los Angeles. If the total of the distances is 855 miles, find the distance from Las Vegas to Los Angeles and the distance from Las Vegas to San Francisco.

EXAMPLE C

James paid \$325 for 3 books and a briefcase. Each book was the same price and the briefcase was \$130 more than the total price of the three books. Find the price of each book and the price of the briefcase.

Translate "price of each book":	Price of each book = x
Translate "The briefcase was \$130 more than the total price of the three books":	Price of all 3 books = $3x$ Price of the briefcase = $3x+130$
Translate "James paid \$325 for 3 books and a briefcase": Combine like terms:	3 books and $1 briefcase = 325$ $3x + 3x + 130 = 325$ $6x + 130 = 325$
Solve the equation for x:	6x + 130 = 325 $6x = 325 - 130$ $6x = 195$ $x = 32.5$
Price of one book = x:	Price of one book = \$32.50
Price of the briefcase = $3x + 130$:	Price of the briefcase = 3(32.5) + 130 = 97.5 + 130 = 227.5

Each book cost \$32.50 and the briefcase cost \$227.50.

EXAMPLE D

Twice the sum of Mary's age and her mom's age is 160 years. If her mom's age is 4 years less than twice Mary's age, how old is each woman?

The mom's age is given in terms of Mary's age, so let x represent Mary's age. Mary's age = x

Translate "mom's age is 4 years less than twice Mary's age":

Mom's age =
$$2x-4$$

Translate "Twice the sum of Mary's age and her mom's age is 160 years":

Twice the sum of Mary's age and her mom's age is 160.

$$2(x+2x-4)=160$$

Combine like terms and solve the equation for x:

$$2(x + 2x - 4) = 160$$

$$2(3x-4)=160$$

$$6x - 8 = 160$$

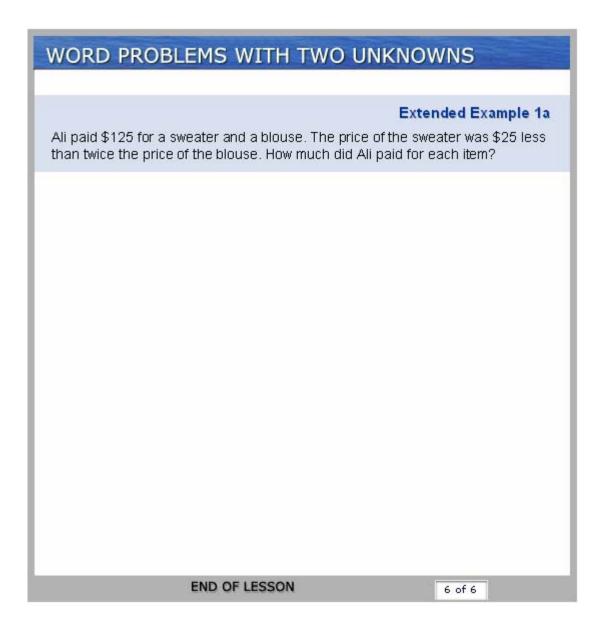
$$6x = 168$$

$$x = 28$$

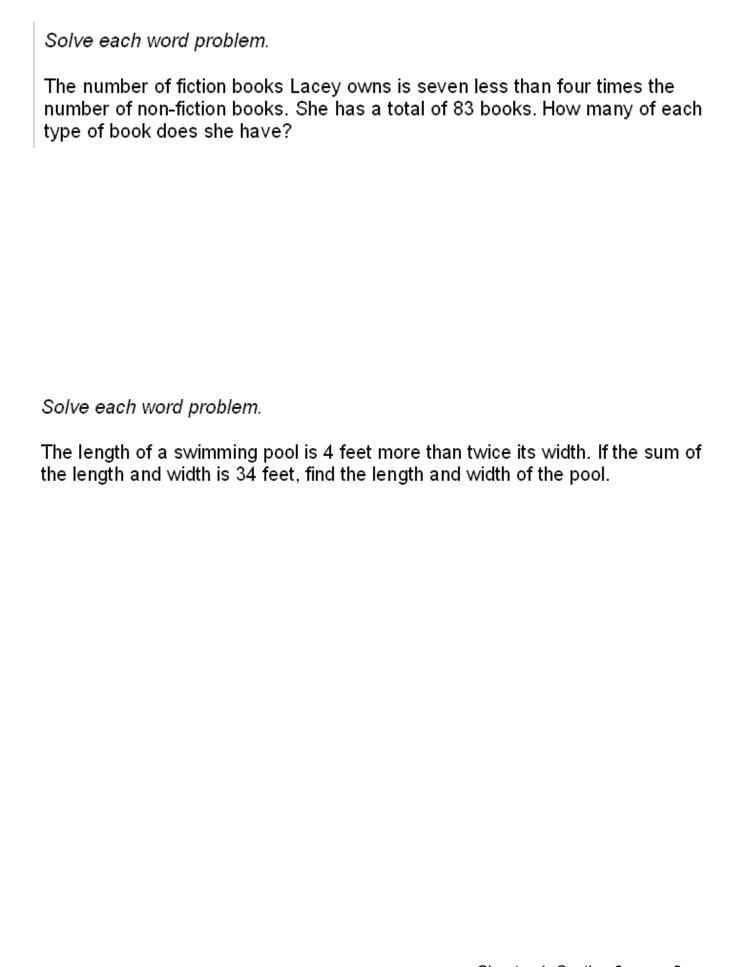
Mary's age = x: Mary's age = 28

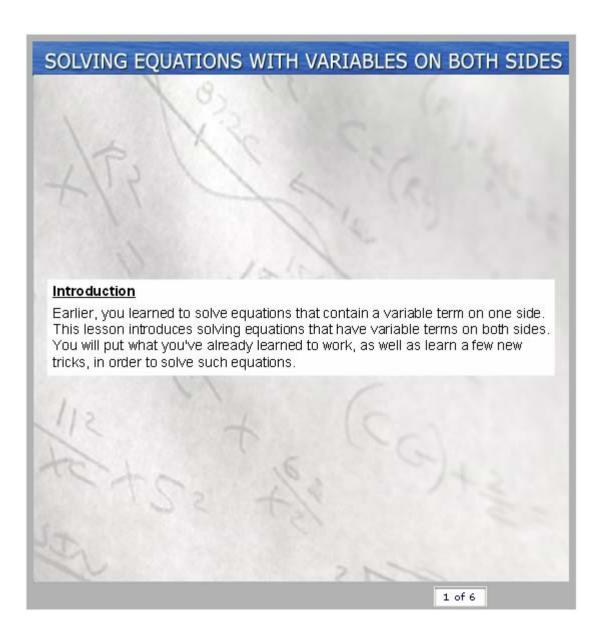
Mom's age = 2x - 4: Mom's age = 2(28) - 4 = 56 - 4 = 52

Mary is 28 and her mother is 52 years old.



Write an equation for the problem and solve for the two unknowns.
A number is 9 less than three times another number. Their sum is 55.
Write an equation for the problem and solve for the two unknowns.
One number is twice the sum of another number and 12. The sum of the two numbers is 3.
Solve each word problem.
The price of an mp3 player is \$120 more than three memory sticks. Sherry paid \$864 for one mp3 player and three memory sticks. What was the cost of each?





Recall the concept of opposite numbers: The sum of a number and its opposite is always equal to zero (-3+3=0). The same concept can be extended to variable terms: the sum of a term and its opposite is always equal to zero. For example:

Term	Opposite of the Term	Property of Opposite Numbers
×	-x	x + (-x) = 0
-4 <i>y</i>	47	-4y + 4y = 0
$\frac{1}{3}x^2$	$-\frac{1}{3}x^2$	$\frac{1}{3}x^2 + \left(-\frac{1}{3}x^2\right) = 0$

To solve equations of the type ax + b = cx + d, with variable terms on both sides, follow these steps:

- 1. Bring all variable terms to one side of the equation. First decide which variable term you want to move, and then add the opposite of that term to both sides of the equation. (Remember that the sum of opposite terms is zero).
- 2. Bring all non-variable terms (constants) to the other side of the equation. To move a constant term to the other side of the equation, add its opposite to both sides of the equation.
- **3. Completely isolate the variable.** Finish solving the equation by performing the operations necessary to isolate the variable.

EXAMPLE A

Solve: 2x - 5 = x + 12

2x - 5 = x + 12 Choose which variable term to move: x. To eliminate x from the right side of the equation, add -x to both sides.

$$x - 5 = 12$$

$$+5 + 5$$

To eliminate the -5 from the left side of the equation, add 5 to both sides.

x = 17

Note: You can choose to move either term; we used x to end up with a positive x-value on the left side.

EXAMPLE B

3 of 6

Solve: 7y + 11 = 4y - 34

7y + 11 = 4y - 34 Choose which variable term to move: 4y. Add -4y to both sides.

$$\frac{-4y}{3y + 11 = 0 - 34}$$

$$3y + 11 = -34$$
$$-11 - 11$$

To move 11 to right side of the equation, add -11 to both sides.

$$3y + 0 = -45$$

$$\frac{\cancel{5}y}{\cancel{5}} = \frac{-45}{3}$$

y = -15

To completely isolate $\mathcal Y$, divide both sides by the coefficient of $\mathcal Y$, which is 3.

EXAMPLE C

Solve:
$$-3m - 24 = -m + 18$$

$$-3m - 24 = -m + 18$$

$$+m + m$$

$$-2m - 24 = 0 + 18$$

$$-2m - 24 = 18$$

$$-2m+0=42$$

$$\frac{\cancel{2m}}{\cancel{2}} = \frac{42}{-2}$$

$$m = -21$$

To solve some equations, you may have to first simplify each side of the equation by combining like terms.

EXAMPLE D

Solve: 3x + 12 - 6x = -5x - 4

3x + 12 - 6x = -5x - 4 First combine like terms on the left.

$$-3x + 12 = -5x - 4$$
 Choose which variable term to move: $-3x$.

$$+3x$$
 +3x Add 3x to both sides.

$$0 + 12 = -2x - 4$$
$$12 = -2x - 4$$

$$8 = -2x$$

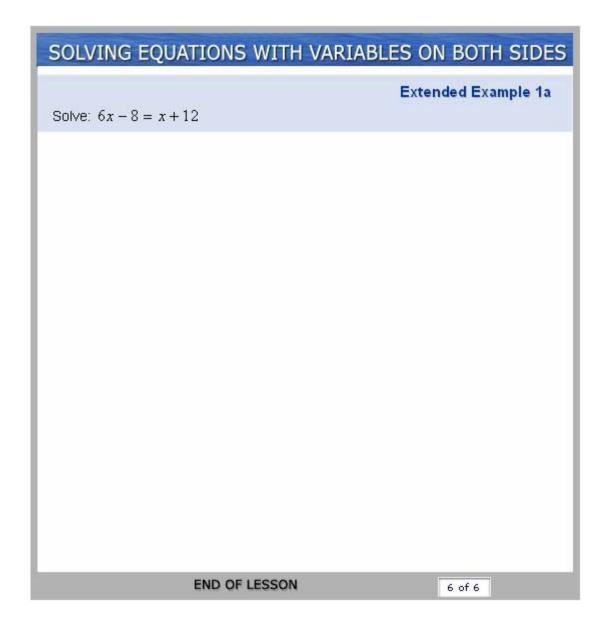
$$\frac{8}{-2} = \frac{\cancel{2}x}{\cancel{2}}$$

 $\frac{8}{-2} = \frac{2x}{2}$ To completely isolate x, divide both sides by -2.

-4 = x

EXAMPLE E

Solve: 7t + 13 - t + 16 = 11t + 9 + 3t + 12



Solve.

$$-7m - 16 = -3m - 24$$

Solve.

$$x + 6 = -6 - 4x - 8$$

Solve.

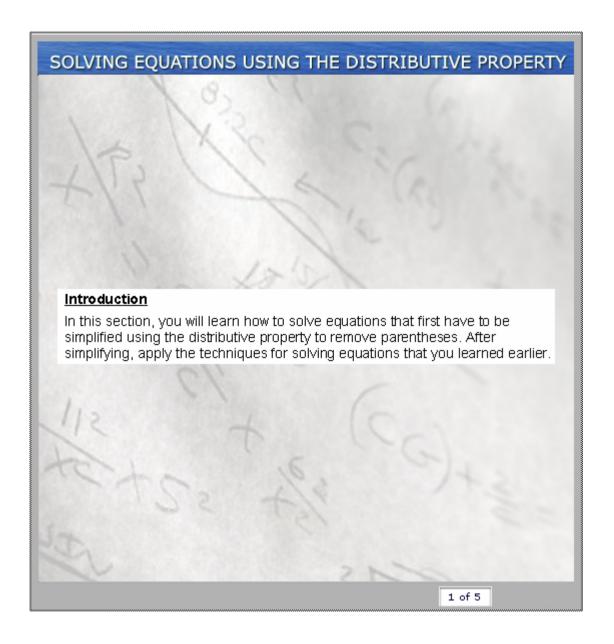
$$a + 6 - 17 = -5a - 10 - 19$$

Solve.

$$2x - 80 + 17x + 9 = -21 + x + 5x + 2$$

Solve.

$$r - 25 - 32 + 9 = 18r - 6 - 19r + 5r$$



SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

EXAMPLE A

Solve:
$$8(2y + 5) = -24$$

$$8(2y+5) = -24$$

$$16y + 40 = -24$$

$$16y = -64$$

$$\frac{\cancel{16}y}{\cancel{16}} = \frac{-64}{16}$$

$$y = -4$$

EXAMPLE B

Solve:
$$-3(m-6) = 60$$

$$-3(m-6) = 60$$

$$-3m + 18 = 60$$

$$-18 - 18$$

$$-3m = 42$$

$$\frac{-3m}{m} = \frac{42}{m}$$

$$m = -14$$

SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

EXAMPLE C

Solve:
$$-3(-m+9) = 72$$

$$-3(-m+9) = 72$$

$$3m - 27 = 72$$

$$3m = 99$$

$$\frac{\cancel{3}m}{\cancel{3}} = \frac{99}{3}$$

$$m = 33$$

EXAMPLE D

Solve:
$$2(7n-8) = 5n-52$$

$$2(7n - 8) = 5n - 52$$

$$14n - 16 = 5n - 52$$

$$9n - 16 = -52$$

+ 16 + 16

$$9n = -36$$

$$\frac{9n}{2} = \frac{-36}{2}$$

$$n = -4$$

SOLVING EQUATIONS USING THE DISTRIBUTIVE PROPERTY

EXAMPLE E

Solve: -(x+5) = 6(-2x+1)

$$-(x+5) = 6(-2x+1)$$

$$-x-5 = -12x+6$$

$$+12x +12x$$

$$11x-5 = 0+6$$

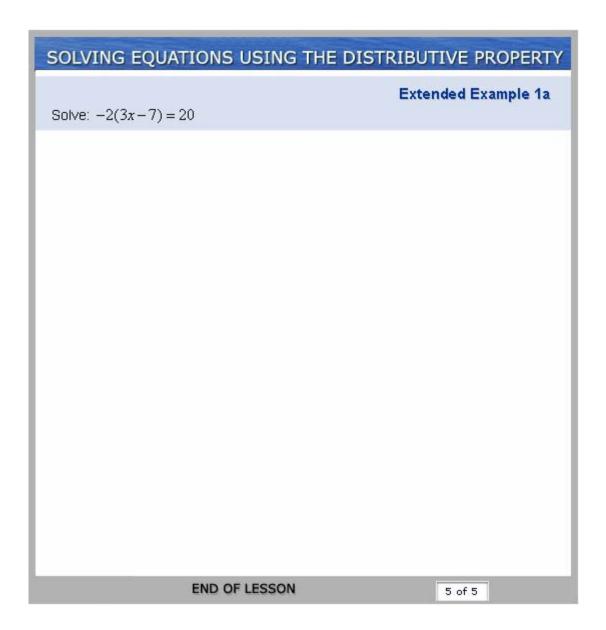
$$11x-5 = 6$$

$$+5 +5$$

$$11x = 11$$

$$\frac{1/x}{1/1} = \frac{11}{11}$$

$$x = 1$$



Solve.

$$-(10m - 7) = -33$$

Solve.

$$4n = 5(n - 16) + 3n$$

Solve.

$$4(f+3) - 5(2f-7) = 29$$

Solve.

$$3(2x-5) - (4x+6) = -71$$

Solve.

$$3(5-6y) - 2(2y+3) = -7(y+18)$$