COMBINING LIKE TERMS WITH INTEGERS

Introduction

Chapter 1 introduced "like terms" and explained how to combine like terms involving whole numbers. In this section, you will learn how to combine like terms with integers. There are two steps to combining like terms: first you identify the like terms, and then you combine them.

COMBINING LIKE TERMS WITH INTEGERS

Identifying Like Terms

In order to simplify an expression by combining like terms, you must be able to recognize the like terms in an expression.

Terms have to meet both of the following conditions in order to be considered **like terms**.

1. The terms must have the same variable.
2. The variables in the terms must have the same exponent.

For example,

x, $2x$, $-x$, and $\frac{1}{2}x$ are all like terms.

However, x and x^2 are not like terms; though both terms have the same variable, their exponents are different.

In an expression, non-variable terms (**constants**) are also considered like terms.

COMBINING LIKE TERMS WITH INTEGERS

Combining Like Terms with Integers

Once you identify the like terms, combine them. To do this, add the coefficients of the like terms. For example, to combine $3x - 5x$, combine 3 and -5, following the rules for combining integers: $3x - 5x = (3 - 5)x = -2x$.

To simplify an expression:

- Identify the like terms.
- Rearrange the terms and group like terms together.
- Combine like terms by combining their coefficients.
- Combine the constants (non-variable terms).

EXAMPLE A

Simplify: $5m - 9m$

$5m - 9m$ These are like terms (same variable).

$= (5 - 9)m$ Combine like terms by combining their coefficients.

$= (-4)m$

$= -4m$

COMBINING LIKE TERMS WITH INTEGERS

EXAMPLE B

Simplify: $3y+17-18y-12+5y$

$3y+17-18y-12+5y$ Identify the like terms.

$=3y-18y+5y+17-12$ Rearrange the terms to group like terms together.

$=(3-18+5)y+(17-12)$ Combine like terms.

$=-10y+5$

EXAMPLE C

Simplify: $5a+8-11a+17$

$5a+8-11a+17$ Identify the like terms.

$=5a-11a+8+17$ Gather like terms together.

$=(5-11)a+(8+17)$ Combine like terms.

$=-6a+25$

EXAMPLE D

Simplify: $-33x+6y-17x-18y$

$-33x+6y-17x-18y$ Identify the like terms.

$=-33x-17x+6y-18y$ Gather like terms together.

$=(-33-17)x+(6-18)y$ Combine like terms.

$=-50x+(-12)y$

$=-50x-12y$

COMBINING LIKE TERMS WITH INTEGERS

Extended Example 1a

Simplify: $5s - 2 + 7s + 6$

COMBINING LIKE TERMS WITH INTEGERS

Evaluating Expressions with Integers

To **evaluate** an expression means to find the numerical value of that expression when a given value is substituted for the variable in the expression. The final answers to all evaluation problems are just numbers without any variables. If the substituted value is a negative number, then you should put it inside parentheses to keep the signs clear. Then follow the correct order of operations. Study the examples that follow.

EXAMPLE E

Evaluate $3x + 8$ when $x = -2$.

$3x + 8$

$3(-2) + 8$

$= -6 + 8$

$= 2$

Substitute -2 for x in the given expression enclosing -2 in parentheses.

Then follow the order of operations, from left to right: first multiplication, then addition.

EXAMPLE F

Evaluate $2x^2 + 3x + 5$ when $x = -3$.

$2x^2 + 3x + 5$

$2(-3)^2 + 3(-3) + 5$

$= 2(9) + 3(-3) + 5$

$= 18 - 9 + 5$

$= 14$

Substitute -3 for x in the given expression enclosing -3 in parentheses.

Follow the order of operations: first the exponent; then, from left to right: multiplication, and finally addition.

COMBINING LIKE TERMS WITH INTEGERS

Extended Example 2a

Evaluate $-21m+14$ when $m=-2$.

END OF LESSON

Identify each of the following: a) the variable terms, b) the variables, c) the constants, and d) the coefficients.

$m + 2n - 4m - n$

Simplify.

$r - 2s - 9r + 7r - r + 8s$

Simplify.

$-19p + 12r - 4q + 11p - 21q - 8$

Evaluate $3t^2 - 4t + 6$ *for the given values of* t.

$t = -3$

Evaluate the expression when $x = -2$.

$x^3 + 2x^2 - 6$

SOLVING EQUATIONS WITH INTEGERS

Introduction

As mentioned in Chapter 1, mathematical problems can be in the form of an **expression** or an **equation**. Recall that equations are distinguished from expressions by an equal sign (**=**).

SOLVING EQUATIONS WITH INTEGERS

Since $3x + 11 = 36$ contains an equal sign, it is considered an **equation**. The presence of an equal sign tells us that the two sides of the equation have the same numerical value—they are equal. To **solve** an equation means to find the numerical value of the variable (x, y, etc.) in that equation. This numerical value is called the **solution** of the equation.

When the variable in the original equation is replaced with the solution and both sides of the equal sign are simplified, the same number will result on both sides.

Solving Equations

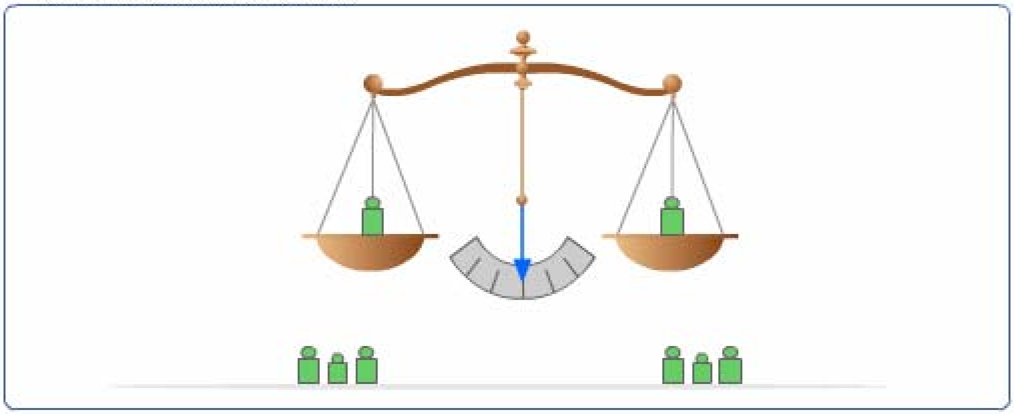
To solve an equation, a series of manipulations must be performed in order to isolate the variable term. The goal of these manipulations is to obtain the variable term on one side of the equal sign and the constant term on the other side of the equal sign. To isolate the variable, we "undo" the operations that occur around it.

Therefore, to isolate the variable in an equation:

- If the variable is added to a number, then subtract that number.
- If the variable is subtracted by or from a number, then add that number.
- If the variable is multiplied by a number, then divide by that number.
- If the variable is divided by a number, then multiply by that number.

SOLVING EQUATIONS WITH INTEGERS

View the animation below.



To keep an equation balanced, any operation performed on one side of the equation must also be performed on the other side of the equation. In an equation:

- When a number is added to one side of the equation, the same number must be added to the other side.
- When a number is subtracted from one side of the equation, the same number must be subtracted from the other side.
- When one side of an equation is multiplied by a number, the other side must be multiplied by the same number.
- When one side of the equation is divided by a number, the other side must be divided by the same number.

If you add or subtract a number on both sides of an equation, you are using the **Addition Property of Equality**. If you multiply or divide both sides of an equation by a number, you are using **Multiplication Property of Equality**.

SOLVING EQUATIONS WITH INTEGERS

To solve an equation, isolate the variable on one side of the equation. Study the following examples and their explanations.

EXAMPLE A

Solve: $m + 21 = -49$

$$\begin{array}{rl} m + 21 = -49 & \text{To undo adding 21, subtract 21 from both sides of the equation independently.} \\ \underline{-21 \quad -21} & \\ m + 0 \ = -70 & \text{Notice that } -21 + 21 \text{ is equal to 0 and } m + 0 = m. \\ m = -70 & \text{The solution is } m = -70. \end{array}$$

EXAMPLE B

Solve: $n - 8 = -34$

$$\begin{array}{rl} n - 8 = -34 & \text{To undo subtracting 8, add 8 to both sides of the equation.} \\ \underline{+8 \quad +8} & \text{Combine each side of the equation independently.} \\ n + 0 = -26 & \text{The solution is } n = -26. \\ n = -26 & \end{array}$$

EXAMPLE C

Solve: $\frac{y}{5} = 2$

$$\frac{y}{5} = 2$$

$$5 \cdot \frac{y}{5} = 2 \cdot 5$$ To undo dividing by 5, multiply both sides of the equation by 5.

$$\frac{\cancel{5}y}{\cancel{5}} = 10$$ Notice that on the left side, the multiplication of 5 cancels out the division of 5, leaving $1y$. On the right side, 2 times 5 is 10.

$$y = 10$$ The solution is $y = 10$.

When the instructions to a problem are "Solve," you are expected to decide what and how many steps are needed to isolate the variable and find the solution.

SOLVING EQUATIONS WITH INTEGERS

Extended Example 1a

Solve: $x - 41 = -5$

SOLVING EQUATIONS WITH INTEGERS

Checking the Solutions of Equations

After solving an equation, you should always check the solution you find. To check a solution, substitute it in place of the variable in the original equation. If both sides of the equation simplify to the same number, then the solution is correct. The problems below show how you can use substitution to check the solutions to Examples B and C above.

EXAMPLE D

Check to see if $n = -26$ is the correct solution to the equation $n - 8 = -34$.

$n - 8 = -34$

$(-26) - 8 = -34$ Replace n with -26.

$-34 = -34$ ✓ Evaluate the left side of the equation. Both sides are the same so $n = -26$ is the correct solution.

EXAMPLE E

Check to see if $y = 10$ is the correct solution to the equation $\frac{y}{5} = 2$.

$\frac{y}{5} = 2$

$\frac{10}{5} = 2$ Replace y with 10.

$2 = 2$ ✓ Simplify the left side of the equation. Both sides are the same so $y = 10$ is the correct solution.

SOLVING EQUATIONS WITH INTEGERS

You can use the same method to check if a given number is a solution to a given equation. Substitute the number in place of the variable. If the numerical values of both sides of the equation are equal, then the number is the solution to the equation.

EXAMPLE F

Is $x = 5$ the solution to $x - 11 = -6$?

$x - 11 = -6$	
$5 - 11 = -6$	Substitute 5 in place of x.
$-6 = -6$ ✓	Simplify the left side. Both sides of the equation equal −6, so $x = 5$ is the correct solution to this equation.

EXAMPLE G

Is $x = 30$ the solution to $x - 25 = 4$?

$x - 25 = 4$	
$30 - 25 = 4$	Substitute 30 in place of x.
$5 \neq 4$ ✗	Simplify the left side. The left side of the equation simplified to 5, but the right side is 4, so $x = 30$ <u>is not</u> the correct solution to this equation.

Extended Example 2a

Is $x = 3$ the solution to $-12 + x = 9$?

Solve.

$p + 11 = -8$

Solve.

$r + 19 = 11$

Solve.

$-7 = \frac{x}{-5}$

Indicate whether the value given is a solution to the equation by answering True *or* False.

$k - 21 = -30$; $k = -9$

Indicate whether the value given is a solution to the equation by answering True *or* False.

$-14 = \frac{m}{-14}$; $m = 28$

DISTRIBUTING AND COMBINING WITH INTEGERS

Introduction

In Chapter 1, you learned how to use the **distributive property** of multiplication over addition with whole numbers. Distributing allows you to remove parentheses in an expression and simplify it. This section deals with the distributive property and integers. The process is similar to using the distributive property with whole numbers.

DISTRIBUTING AND COMBINING WITH INTEGERS

Recall that when using the distributive property, we multiply every term inside the parentheses by the number on the outside of the parentheses. When distributing with integers, **be careful of the signs**! Remember, the product of two numbers with the same sign is positive, and the product of two numbers with different signs is negative. Now study the following examples.

EXAMPLE A

Simplify: $4(3x-7)$

$4(3x-7)=4(3x)+4(-7)$ Distribute 4 to $3x$ and -7.

$=12x+(-28)$ Multiply.

$=12x-28$

EXAMPLE B

Simplify: $-5(x+7)$

$-5(x+7)=-5(x)+(-5)(7)$ Distribute -5 to x and 7.

$=-5x+(-35)$ Multiply.

$=-5x-35$

EXAMPLE C

Simplify: $-3(2a-7)$

$-3(2a-7)=-3(2a)+(-3)(-7)$

$=-6a+21$

DISTRIBUTING AND COMBINING WITH INTEGERS

EXAMPLE D

Simplify: $-6(-2x+1)$

$$\begin{aligned}-6(-2x+1) &= -6(-2x)+(-6)(1)\\ &= 12x+(-6)\\ &= 12x-6\end{aligned}$$

EXAMPLE E

Simplify: $7(-x-8)$

$$\begin{aligned}7(-x-8) &= 7(-x)+7(-8)\\ &= -7x+(-56)\\ &= -7x-56\end{aligned}$$

EXAMPLE F

Simplify: $-(2x-5)$

$$\begin{aligned}-(2x-5) &= -1(2x-5)\\ &= (-1)(2x)+(-1)(-5)\\ &= -2x+5\end{aligned}$$

Note that the negative sign $= -1$.

DISTRIBUTING AND COMBINING WITH INTEGERS

The following problems use the distributive property more than once. After distributing, the like terms are combined.

EXAMPLE G

Simplify: $7(x+2)+2(2x+1)$

$$7(x+2)+2(2x+1) = 7(x)+7(2)+2(2x)+2(1)$$ Distribute through both sets of parentheses.

$$= 7x+14+4x+2$$ Multiply, then identify like terms.

$$= 11x+16$$ Combine like terms.

EXAMPLE H

Simplify: $5(3x-4)+3(2x-5)$

$$5(3x-4)+3(2x-5) = 5(3x)+5(-4)+3(2x)+3(-5)$$

$$= 15x-20+6x-15$$

$$= 21x-35$$

DISTRIBUTING AND COMBINING WITH INTEGERS

EXAMPLE I

Simplify: $-2(x-4)+3(5x-8)$

$$\begin{aligned} -2(x-4)+3(5x-8) &= (-2)(x)+(-2)(-4)+3(5x)+3(-8) \\ &= \underline{-2x}+\underline{\underline{8}}+\underline{15x}-\underline{\underline{24}} \\ &= 13x-16 \end{aligned}$$

EXAMPLE J

Simplify: $-(4x-4)-6(x-3)$

$$\begin{aligned} -(4x-4)-6(x-3) &= (-1)(4x)+(-1)(-4)+(-6)(x)+(-6)(-3) \\ &= \underline{-4x}+\underline{\underline{4}}-\underline{6x}+\underline{\underline{18}} \\ &= -10x+22 \end{aligned}$$

DISTRIBUTING AND COMBINING WITH INTEGERS

Extended Example 1a

Simplify: $-9(3x+4)$

END OF LESSON

Simplify.

$-(4y - 1)$

Simplify.

$-2(a + 7b - 3)$

Simplify.

$4a + 7b - (a + 3b)$

Simplify.

$$6(2m + 1) - (7m + 15)$$

Simplify.

$$2(q - r + s) - (5r - 1)$$

SOLVING EQUATIONS OF THE TYPE *ax* = *b*

Introduction

In Section 2 of this chapter you learned how to solve equations by isolating the variable term. In an equation of type $ax = b$ (where a and b represent integers), the variable term is ax, and it is already isolated. In this section, you will learn to isolate just the variable. In order to change ax to just x, you need to divide both sides of the equation by a.

SOLVING EQUATIONS OF THE TYPE $ax = b$

In each of the examples below, in order to isolate the variable, it is necessary to divide both sides of the equation by the coefficient of the variable.

EXAMPLE A

Solve: $8x = 16$

$8x = 16$

$\frac{\cancel{8}x}{\cancel{8}} = \frac{16}{8}$ Divide both sides of the equation by the coefficient, 8.

$1x = 2$ 8 divided by 8 is equal to 1, which simplifies the left side of the equation to $1x$ or just x. 16 divided by 8 is equal to 2.

$x = 2$ So, x is equal to 2.

EXAMPLE B

Solve: $-7s = 42$

$-7s = 42$

$\frac{\cancel{-7}s}{\cancel{-7}} = \frac{42}{-7}$ Divide both sides of the equation by the coefficient, –7.

$s = -6$ Because 42 and –7 have different signs, the solution is negative.

SOLVING EQUATIONS OF THE TYPE $ax = b$

EXAMPLE C

Solve: $-5n = -105$

$$-5n = -105$$

$$\frac{\cancel{-5}n}{\cancel{-5}} = \frac{-105}{-5}$$ Divide both sides of the equation by the coefficient, –5.

$$n = 21$$ Because both –105 and –5 have the same sign, the solution is positive.

EXAMPLE D

Solve: $3k = -81$

$$3k = -81$$

$$\frac{\cancel{3}k}{\cancel{3}} = \frac{-81}{3}$$ Divide both sides of the equation by the coefficient, 3.

$$k = -27$$ Because –81 and 3 have different signs, the solution is negative.

Notice how in Examples A through D it took only one operation to isolate the variable; equations such as these are considered "one-step" equations.

SOLVING EQUATIONS OF THE TYPE *ax* = *b*

Extended Example 1a

Solve: $6m = 96$

END OF LESSON

Solve.

$-q = 8$

Solve.

$-8f = -56$

Solve.

$-b = -59$

Solve.

$12x = -72$

SOLVING EQUATIONS OF THE TYPE $ax + b = c$

Introduction

In equations of the type $ax + b = c$, it is necessary to "undo" two operations to isolate the variable and solve the equation. This type of equation is a "two-step equation." To solve equations of the type $ax + b = c$, follow these steps:

1. First isolate the variable term, ax. To isolate the variable term, add the opposite of b to both sides of the equation.
2. Then isolate the variable, x. To isolate the variable, divide both sides of the equation by a.

SOLVING EQUATIONS OF THE TYPE $ax + b = c$

Study the following examples.

EXAMPLE A

Solve: $9x+4=31$

$$\begin{array}{rl} 9x+4 &= 31 \\ -4 & -4 \\ \hline 9x &= 27 \end{array}$$

To isolate $9x$, subtract 4 from both sides of the equation to "undo" the addition of 4.

Notice that $9x$ is now isolated, since $4 - 4 = 0$.

$$\frac{\cancel{9}x}{\cancel{9}} = \frac{27}{9}$$

Divide both sides of the equation by the coefficient, 9.

$$x = 3$$

The coefficient of x is now 1, so the variable is isolated. Simplifying the right side, we get $x = 3$.

EXAMPLE B

Solve: $6y-7=35$

$$\begin{array}{rl} 6y-7 &= 35 \\ +7 & +7 \\ \hline 6y &= 42 \end{array}$$

To isolate $6y$, add 7 to both sides of the equation to "undo" the subtraction of 7.

Notice that $6y$ is now isolated, since $7 + -7 = 0$.

$$\frac{\cancel{6}y}{\cancel{6}} = \frac{42}{6}$$

Divide both sides of the equation by the coefficient, 6.

$$y = 7$$

The coefficient of y is now 1, so the variable is isolated. Simplifying the right side we get $y = 7$.

SOLVING EQUATIONS OF THE TYPE $ax + b = c$

EXAMPLE C

Solve: $11y - 18 = -62$

$$\begin{aligned} 11y - 18 &= -62 \\ +18 \quad & \quad +18 \\ \hline 11y \quad &= -44 \end{aligned}$$

To isolate $11y$, add 18 to both sides to "undo" subtraction.

$$\frac{\cancel{11}y}{\cancel{11}} = \frac{-44}{11}$$

To isolate the variable, y, divide both sides by the coefficient 11 to "undo" multiplication.

$$y = -4$$

The solution is $y = -4$.

EXAMPLE D

Solve: $-x - 9 = -11$

$$\begin{aligned} -x - 9 &= -11 \\ +9 \quad & \quad +9 \\ \hline -x \quad &= -2 \end{aligned}$$

Note that $-x = -1x$. To isolate $-1x$, add 9 to both sides to "undo" subtraction.

$$\frac{\cancel{-1}x}{\cancel{-1}} = \frac{-2}{-1}$$

To isolate the variable, x, divide both sides by the coefficient -1 to "undo" multiplication.

$$x = 2$$

The solution is $x = 2$.

SOLVING EQUATIONS OF THE TYPE $ax + b = c$

EXAMPLE E

Solve: $17 = 2m - 5$

$$\begin{array}{rcl} 17 &=& 2m - 5 \\ +5 & & \quad +5 \\ \hline 22 &=& 2m \end{array}$$

To isolate $2m$ on the right side of the equation, add 5 to both sides to "undo" subtraction.

$$\frac{22}{2} = \frac{\cancel{2}m}{\cancel{2}}$$

To isolate m, divide both sides by the coefficient 2 to "undo" multiplication.

$11 = m$ or $m = 11$ The solution is $m = 11$.

EXAMPLE F

Solve: $-12 = -2x + 8$

$$\begin{array}{rcl} -12 &=& -2x + 8 \\ -8 & & \quad -8 \\ \hline -20 &=& -2x \end{array}$$

To isolate $-2x$ on the right side of the equation, subtract 8 from both sides to "undo" addition.

$$\frac{-20}{-2} = \frac{\cancel{-2}x}{\cancel{-2}}$$

To isolate x, divide both sides by the coefficient -2 to "undo" multiplication.

$10 = x$ or $x = 10$ The solution is $x = 10$.

SOLVING EQUATIONS OF THE TYPE $ax + b = c$

Extended Example 1a

Solve: $6t - 16 = 14$

END OF LESSON

Simplify.

$4k - 16 = -36$

Simplify.

$-t + 23 = -14$

Simplify.

$6m - 4 = 20$

Simplify.

$2y + 8 = 4$

Simplify.

$-4s - 26 = 70$

MORE ON SOLVING EQUATIONS

Introduction

You have already learned the main steps for solving equations. In this section, you will learn how to solve equations that need to be simplified first. One or both sides of the equation will need to be simplified as a first step, either by combining like terms or by using the distributive property.

MORE ON SOLVING EQUATIONS

Examples of Simplifying by Combining Like Terms

EXAMPLE A

Solve: $4x - x + 7 = 28$

$4x - x + 7 = 28$ Combine $4x$ and $-x$ first, then solve.

$3x + 7 = 28$ Isolate $3x$ by subtracting 7 from both sides.

$\underline{\quad -7 \quad -7}$

$3x \quad = 21$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{21}{3}$ Divide both sides by 3 to "undo" multiplication.

$x = 7$ The solution is $x = 7$

EXAMPLE B

Solve: $3y - 8 - 7y = -33 + 1$

$3y - 8 - 7y = -33 + 1$ On the left side of the equation, combine $3y$ and $-7y$, and on the right side of the equation, combine -33 and 1. Then solve as before.

$-4y - 8 = -32$

$\underline{\quad +8 \quad +8}$

$-4y \quad = -24$

$\frac{\cancel{-4}y}{\cancel{-4}} = \frac{-24}{-4}$

$y = 6$

MORE ON SOLVING EQUATIONS

EXAMPLE C

Solve: $6m+7-17=-28$

$$6m+7-17=-28$$
$$6m-10=-28$$
$$\underline{\quad +10 \quad +10}$$
$$6m \quad =-18$$
$$\frac{\cancel{6}m}{\cancel{6}}=\frac{-18}{6}$$
$$m=-3$$

EXAMPLE D

Solve: $12x-8x=-16+8$

$$12x-8x=-16+8$$
$$4x=-8$$
$$\frac{\cancel{4}x}{\cancel{4}}=\frac{-8}{4}$$
$$x=-2$$

MORE ON SOLVING EQUATIONS

Examples of Simplifying by Using the Distributive Property

EXAMPLE E

Solve: $2(4x-3)+8=50$

$$
\begin{aligned}
2(4x-3)+8&=50\\
8x-6+8&=50 \quad \text{First distribute 2 on the left side.}\\
8x+2&=50 \quad \text{Then combine like terms and solve.}\\
-2\ \ &\ \ -2\\
8x&=48\\
\frac{\cancel{8}x}{\cancel{8}}&=\frac{48}{8}\\
x&=6
\end{aligned}
$$

EXAMPLE F

Solve: $-(2y+9)=-7$

$$
\begin{aligned}
-(2y+9)&=-7\\
-2y-9&=-7 \quad \text{First distribute } -1 \text{ to the terms in parentheses on the left side. Then solve.}\\
+9\ \ &\ \ +9\\
-2y&=2\\
\frac{\cancel{-2}y}{\cancel{-2}}&=\frac{2}{-2}\\
y&=-1
\end{aligned}
$$

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Extended Example 1a

Solve: $3x - 6x - 9 = -3$

Solve.

$9m - 15 - 13m = -43$

Solve.

$-q + 19 - 13q = -51$

Solve.

$-(9m - 5) = 95$

Solve.

$$5(3g - 1) + 15 = -5$$

Solve.

$$7(3d + 5) + 7d = -77$$