LINES AND ANGLES

Introduction

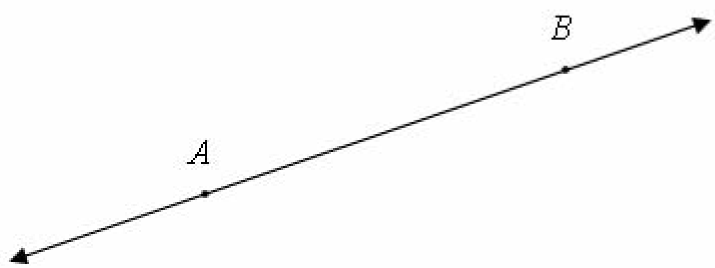
Geometry builds off of three objects: **points**, **lines** and **planes**. This lesson reviews these objects and explains some basic principles of two-dimensional geometric figures.

LINES AND ANGLES

A **point** is a specific location that has no width, no length, and no height. A point is shown as a dot and named with a single capital letter. Below you can see point P.

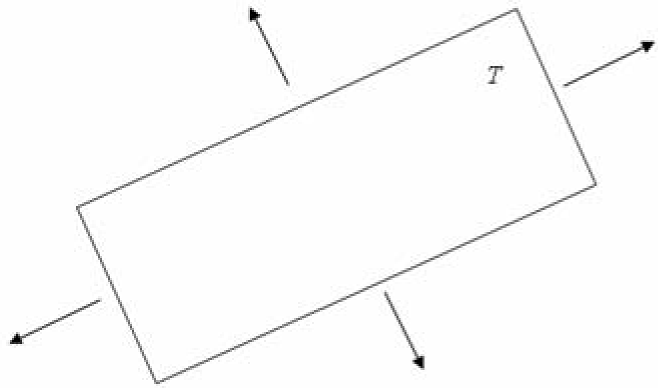
$\cdot\ P$

A **line** is made of an infinite number of points and stretches infinitely in opposite directions. Lines have length but no width. A line is represented by any two points on it and is referred to by these two points. Look at line AB below. You can write this line as $\overleftrightarrow{AB}$; the arrow heads on either end mean that the line continues in both directions.

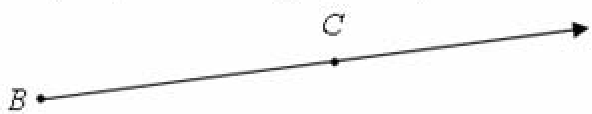


LINES AND ANGLES

A **plane** is a flat surface that extends infinitely in all directions. You can think of a plane as a sheet of paper that stretches forever on all sides. A plane has width and length but no depth. You can refer to a plane by a single capital letter. For example, the figure below represents plane T .



Another important geometrical object is a **ray**. A **ray** is part of a line that has one endpoint and continues infinitely in only <u>one</u> direction. The ray BC starts at point B (the endpoint) and passes through point C , and is written as $\overrightarrow{BC}$.



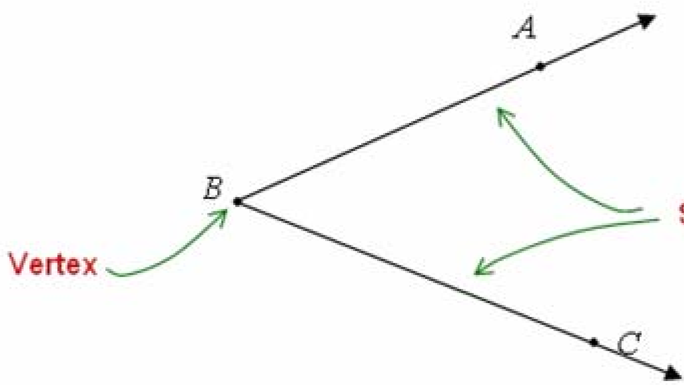
A **line segment** is a part of a line that is limited at both ends. A line segment is named by its two endpoints. $\overline{MN}$ is a line segment whose endpoints are the points M and N .

Angles

An **angle** is a geometric figure that is formed by two rays with a common endpoint. The two rays are called the **sides** of the angle, and the common endpoint is called the **vertex**. The symbol for angle is $\angle$.

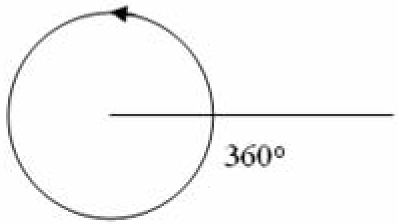
An angle can be named in more than one way. The angle below has the point A on one side, the point C on the other side, and the point B as its vertex. This angle can be referred to as angle ABC ($\angle ABC$), angle B ($\angle B$) or angle CBA ($\angle CBA$). Notice that the vertex is always named in the middle or alone.

$\angle ABC$ or $\angle CBA$ or $\angle B$

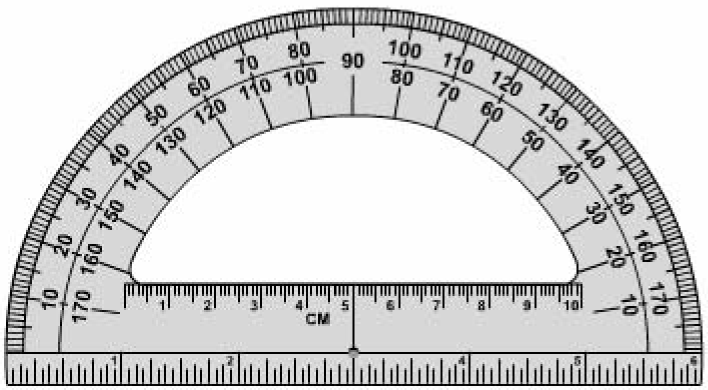


LINES AND ANGLES

An angle is measured in **degrees**. There are 360 degrees in one full revolution around a circle. The degree is denoted as a raised circle above and to the right of the number. For example, 360 degrees can be written as 360°.



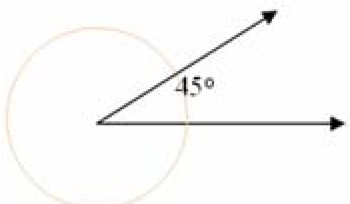
To measure an angle, you can use a **protractor** (shown below). A protractor is a device that measures the size of an angle in degrees. Play the animation below to see how a protractor works.



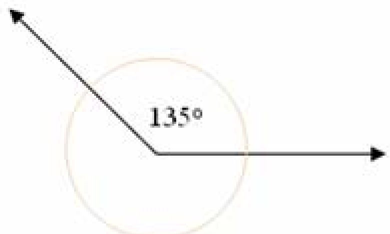
LINES AND ANGLES

Angles are categorized by their size (in degrees):

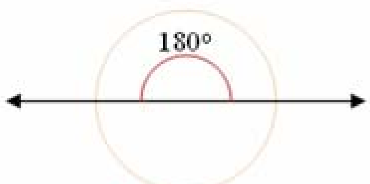
- If an angle is between $0°$ and $90°$, then it is called an **acute angle**.
- If an angle is equal to $90°$, then it is called a **right angle**. A right angle is equal to one-fourth of a revolution around a circle. A right angle is indicated by a small box (▫) at its vertex.
- If an angle measures between $90°$ and $180°$, then it is called an **obtuse angle**.
- If an angle is $180°$, then it is called a **straight angle**. The two sides of a straight angle together make a straight line.



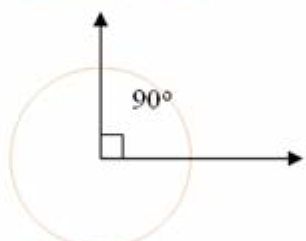
Acute Angle



Obtuse Angle



Straight Angle



Right Angle

LINES AND ANGLES

Complementary and Supplementary Angles

When the sum of two angles is $90°$, the two angles are called **complementary angles**. The following are examples of pairs of complementary angles:

$45°$ and $45°$ because $45° + 45° = 90°$

$30°$ and $60°$ because $30° + 60° = 90°$

$25°$ and $65°$ because $25° + 65° = 90°$

EXAMPLE A

What is the complementary angle of a $10°$ angle?

The answer is $80°$ because: $90° - 10° = 80°$.

When the sum of two angles is $180°$, the two angles are called **supplementary angles**. Some examples of pairs of supplementary angles follow:

$145°$ and $35°$ because $145° + 35° = 180°$

$90°$ and $90°$ because $90° + 90° = 180°$

$110°$ and $70°$ because $110° + 70° = 180°$

EXAMPLE B

What is the supplementary angle of a $60°$ angle ?

The answer is $120°$ because: $180° - 60° = 120°$.

LINES AND ANGLES

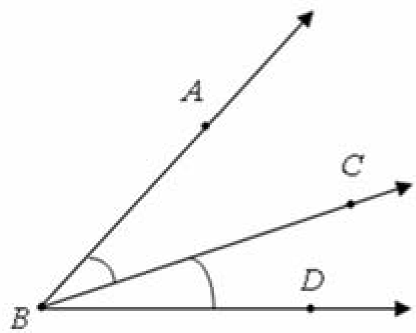
Extended Example 1a

What is the complementary angle of a 59° angle?

LINES AND ANGLES

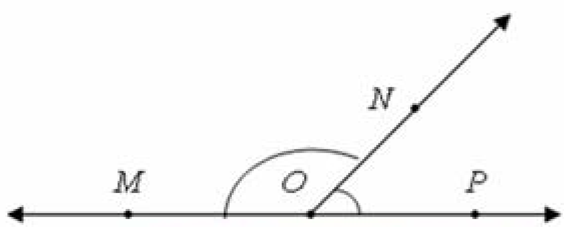
Adjacent Angles

Two angles that share a side are called **adjacent angles**. In the example below, the side BC is shared by $\angle ABC$ and $\angle CBD$. Therefore, $\angle ABC$ and $\angle CBD$ are adjacent angles.



$\angle ABC$ is adjacent to $\angle CBD$

In the following example, the side ON is a common side to both $\angle MON$ and $\angle NOP$, so these two angles are adjacent angles.



$\angle MON$ is adjacent to $\angle NOP$

EXAMPLE C

The sum of two adjacent angles is 175°. If one angle is 45°, what is the measure of the other angle?

Since we are told these angles are adjacent, we know that one angle plus the other angle equals 175°. We are given one angle, 45°, and the other is unknown, x. Write an equation for this sum:

$$x+45=175$$

Solve for x by isolating the variable term.

$$x+45=175$$
$$x=175-45=130^\circ$$

The other angle is 130°.

EXAMPLE D

The sum of two adjacent angles is 80°. If one angle is 23°, what is the measure of the other angle?

One angle (23°) plus the other angle (unknown, x) equals 80°. Write an equation for this sum:

$$x+23=80$$

Solve for x by isolating the variable term.

$$x+23=80$$
$$x=80-23=57^\circ$$

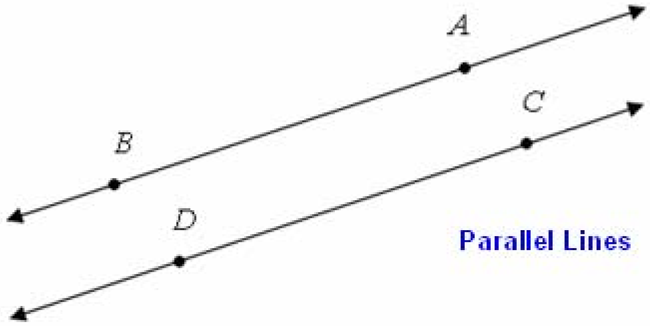
The other angle is 57°.

Extended Example 2a

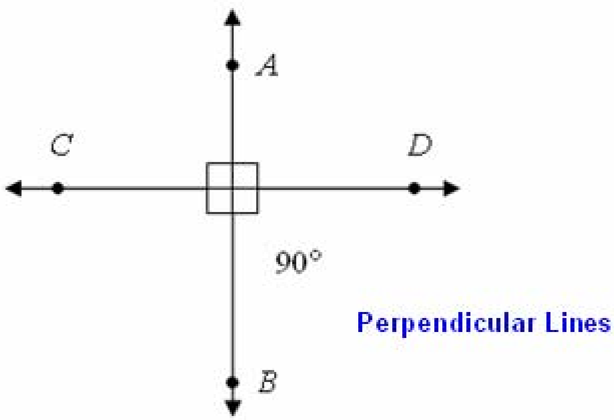
The sum of two adjacent angles is 153°. If one angle is 98°, what is the measure of the other angle?

LINES AND ANGLES

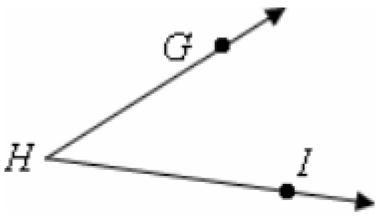
When two lines exist together in the same plane they may or may not intersect. Two lines in the same plane that do not intersect are called **parallel lines**.



Two lines that intersect to form right angles are called **perpendicular lines**.



Name the figure.



Find a) the complementary and b) the supplementary angle.

$11.4°$

Find a) the complementary and b) the supplementary angle.

$22°$

Use the information to find the measure of the second angle.

The sum of two adjacent angles is 33°. One angle is 22°.

Use the information to find the measure of the second angle.

The sum of two adjacent angles is 101°. One angle is 14°.

PERIMETER

Introduction

In this lesson, you'll learn the formulas for finding the perimeter of a few common geometric figures such as rectangles, squares, triangles, and circles.

Recall that the **perimeter** of any plane figure is the total distance around that figure. This means that the perimeter is the sum of the lengths of its sides.

PERIMETER

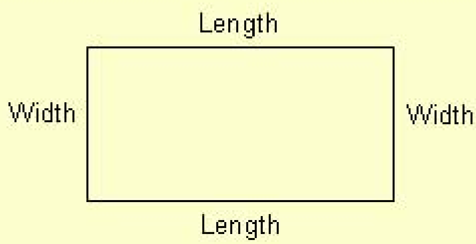
Rectangles

A **rectangle** has four sides, and its opposite sides are equal. Each of the longer sides is the **length** of the rectangle, and each of the shorter sides is the **width.** In general, the formula for the perimeter of a rectangle is as follows:

Perimeter of a rectangle $= 2 \cdot \text{length} + 2 \cdot \text{width}$

Using the variables P for perimeter, l for length and w for width, you can write the formula for the perimeter of a rectangle as:

$$P = 2l + 2w$$



EXAMPLE A

Find the perimeter of a rectangle with a length of 14.5 inches and a width of 9.3 inches.

$P = 2l + 2w$ Use the perimeter formula.

$P = 2(14.5) + 2(9.3)$ Substitute the values for length and width in the formula.

$P = 29 + 18.6$ Calculate the perimeter.

$P = 47.6 \text{ in}$ Don't forget to include the unit of measurement.

PERIMETER

EXAMPLE B

The perimeter of a rectangle is 184 inches. Its length is three times its width. Find the length and the width of this rectangle.

Width = w Since length is described in terms of the width, label the width w.

Length = $3w$ The length is three times the width.

$P = 2l + 2w = 184$ The perimeter of the rectangle is 184, so set the perimeter formula equal to 184.

$P = 2(3w) + 2(w) = 184$ Substitute the values for width and length into the perimeter formula.

$6x + 2w = 184$ Solve the equation for w.

$8w = 184$

$w = 23$

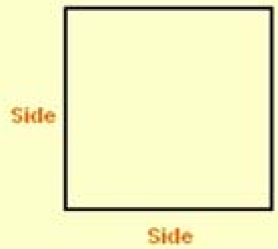
Width = 23 in In the equation w represents the width.

Length = $3(23) = 69 \text{ in}$ Substitute the value of w in the equation to find the length. Length = $3w$. Don't forget the unit of measurement in the final answer.

PERIMETER

Squares

Perimeter of a square = 4 · length of side
Using P for perimeter and s for the length of a side, since all four sides of a square are equal, you can write the formula for the perimeter of a square as: $$P = 4s$$



EXAMPLE C

Find the perimeter of a square tabletop if each side is $\frac{7}{16}$ feet long.

$P = 4s$ Use the perimeter formula.

$P = \cancel{4}^{1} \cdot \frac{7}{\cancel{16}^{4}}$ Substitute the value for the length of a side into the perimeter formula. Simplify.

$P = 1 \cdot \frac{7}{4}$

$P = \frac{7}{4}$ ft or $1\frac{3}{4}$ ft Don't forget to include the unit of measurement.

PERIMETER

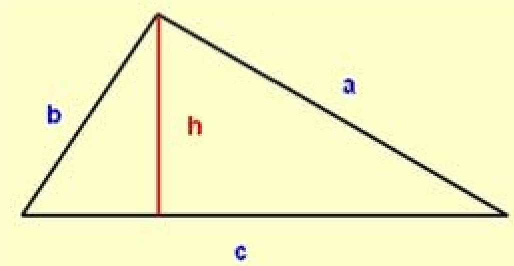
Triangles

Calculating the perimeter of a **triangle** is similar to calculating the perimeter of a rectangle or a square: Add the lengths of all its sides.

Perimeter of a triangle = sum of all the sides
= side a + side b + side c

Using P for perimeter and a, b, and c for the length of each side, you can write the formula for the perimeter of a triangle as:

$$P = a + b + c$$



EXAMPLE D

Find the perimeter of a triangle with side lengths 11cm, $9\frac{1}{2}\text{cm}$, and $13\frac{1}{4}\text{cm}$.

$P = a + b + c$ Use the perimeter formula for a triangle.

$P = 11 + 9\frac{1}{2} + 13\frac{1}{4}$ Substitute the values for the length of each side.

$P = 11 + 9\frac{2}{4} + 13\frac{1}{4}$ Use the LCD to add all three terms. Find the sum.

$P = 33\frac{3}{4}$ cm Don't forget to include the unit of measurement.

5 of 11

PERIMETER

Extended Example 1a

Find the perimeter of a rectangle with length of 12cm and a width of 8cm.

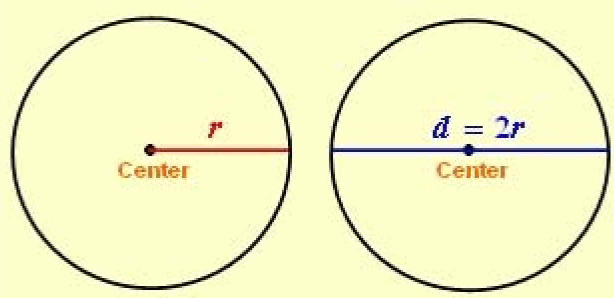
6 of 11

PERIMETER

Circles

Every point on a **circle** is an equal distance from a point in the **center** of the circle. This distance is called the **radius** (r). The distance across the circle through the center is the **diameter** (d). The diameter is twice the radius. The distance around the circle is called the **circumference** (C). The circumference of a circle is its perimeter.

Circumference of a circle $= 2 \cdot \pi \cdot$ **radius**
Using C for circumference and r for the radius, you can write the formula for the circumference of a circle as: $C = 2\pi r$.



PERIMETER

In the formula $C = 2\pi r$, the symbol π is the Greek letter **pi**. Pi is a fixed number that is approximately equal to 3.14 or $\frac{22}{7}$. Since early history it was known that if you divide the circumference of any circle by its diameter, the result is always the number pi. Most calculators have a button for π , and you can use it for calculating the circumference and area of a circle.

EXAMPLE E

Find the circumference of a circle with a radius of 5 centimeters.

$C = 2\pi r$	Use the formula for circumference of a circle.
$C = 2 \cdot \pi \cdot 5$	Substitute the value for the radius.
$C = 10 \cdot \pi$	Find the product of 2 and 5.
$C = 10(3.14)$	Substitute 3.14 for pi.
$C = 31.4 \text{ cm}$	Recall that when you multiply by 10, you move the decimal point one place to the right. Don't forget to include the unit of measurement.

8 of 11

PERIMETER

Extended Example 2a

Find the circumference of a circle with a radius of 6 feet.

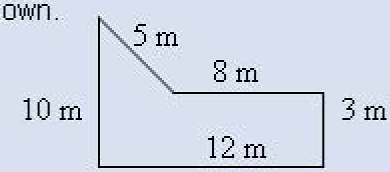
PERIMETER

Other Shapes

To find the perimeter of other polygons, simply find the sum of the length of all of the sides of the polygon.

EXAMPLE F

Find the perimeter of the shape shown.



$P = \textit{sum of all sides}$

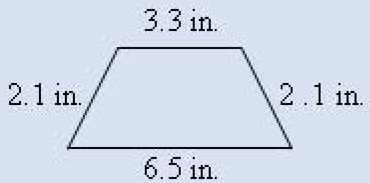
$P = 5 + 8 + 3 + 12 + 10$ Add the lengths of all sides of the shape.

$P = 38 \text{ m}$ Don't forget to include the unit of measurement.

10 of 11

Extended Example 3a

Find the perimeter of the shape shown below.



END OF LESSON

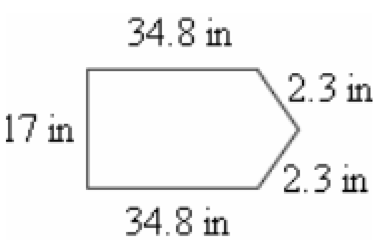
Use the measurements given to find the perimeter of each rectangle.

length = 23.5 m
width = 9.75 m

Use the measurements given to find the circumference of each circle. Use 3.14 for π. Round to the nearest hundredth.

$r = 2\frac{3}{8}$ m

Find the perimeter of the polygon.



The perimeter of a rectangle is 72 ft. Its length is two more than its width. Find the length and width of the rectangle.

The perimeter of a triangle is 32 cm. One side is twice the length of the shortest side. The third side is four more than the shortest side. Find the length of each side of the triangle.

AREA

Introduction

The **area** (A) of a figure refers to the amount of amount of space covered by that figure. Area is always measured in square units. We will learn the formulas to find the area of several common shapes in this lesson.

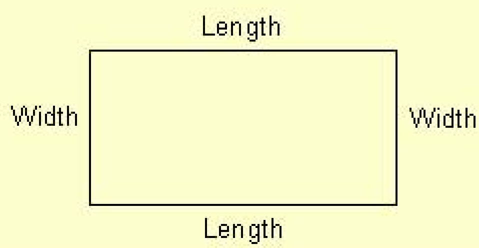
AREA

Rectangles

To find the area, A, of a rectangle, you need to know the measurements of its length, l, and its width, w.

Area of a Rectangle = Length · Width

$$A = l \cdot w$$



EXAMPLE A

Find the area of a rectangular wall with a length of 10 ft and a width of 5.75 ft.

$A = l \cdot w$ Use the equation for the area of a rectangle.

$A = 10(5.75)$ Substitute the values of length and width into the equation.

$A = 57.5 \text{ ft}^2$ Note that the units of measurement are squared.

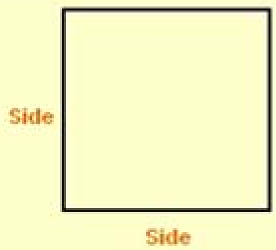
AREA

Squares

To find the area, A, of a square, you need to know the length of one of its sides, s.

Area of a Square = Length of side · Length of side

$$A = s \cdot s = s^2$$



EXAMPLE B

Find the area of a square carpet with a side length of $\frac{3}{4}$ feet.

$A = s^2$ Use the equation for the area of a square.

$A = \left(\frac{3}{4}\right)^2$ Substitute the value of the length of a side into the equation.

$A = \frac{9}{16}\ \text{ft}^2$ Don't forget the units of measurement are squared.

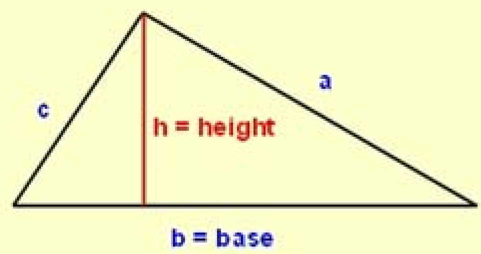
AREA

Triangles

To find the area of a triangle, you must know the height (h) and the base (b) of the triangle. The height of a triangle always refers to the length of the line segment from the vertex opposite the base and perpendicular to the base. That is, the height is a line that is perpendicular to the base.

Area of a Triangle = half of height × base

$$A = \frac{1}{2} \cdot h \cdot b$$



EXAMPLE C

Find the area of a triangle with a base of 10 feet and height of 15 feet.

$A = \frac{1}{2} \cdot h \cdot b$ Use the equation for the area of a triangle.

$A = \frac{1}{2} \cdot 15 \cdot 10$ Substitute the values for the base and height into the equation.

$A = \frac{1}{2} \cdot 150$

$A = 75$

$A = 75\text{ ft}^2$ Don't forget the units of measurement are squared.

AREA

EXAMPLE D

Find the area of a triangle with a base of 12 meters and height of $\frac{6}{15}$ meter.

$A = \frac{1}{2} \cdot h \cdot b$ Use the equation for the area of a triangle.

$A = \frac{1}{2} \cdot \frac{6}{15} \cdot 12$ Substitute the values for the base and height into the equation.

$A = \frac{1}{\cancel{2}^{1}} \cdot \frac{\cancel{6}^{3}}{\cancel{15}^{5}} \cdot \cancel{12}^{4}$

$A = \frac{3}{5} \cdot 4$

$A = \frac{12}{5}\ \mathrm{m}^2$ or $2\frac{2}{5}\,\mathrm{m}^2$ Don't forget the units of measurement are squared.

AREA

Extended Example 1a

Find the area of a rectangular carpet with a length of 12 feet and a width of 8.25 feet.

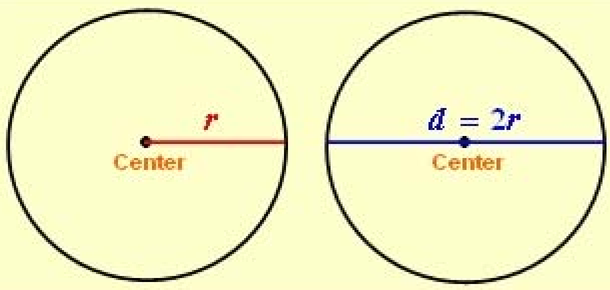
AREA

Circles

The formula for the area of a circle uses the radius, r , and π .

Area of a Circle = π · radius · radius

$$A = \pi r^2$$



EXAMPLE E

Find the area of a circle with a radius of 7 centimeters.

$A = \pi r^2$ Use the equation for the area of a circle.

$A = \pi \cdot 7^2$ Substitute the values for radius and pi into the equation.

$A = 3.14(49)$

$A = 153.86 \text{ cm}^2$ Don't forget the units of measurement are squared.

EXAMPLE F

Find the area of a circle with the radius of 6.23 centimeters.

$A = \pi r^2$ Use the equation for the area of a circle.

$A = \pi \cdot 6.23^2$ Substitute the values for the length of the base and the height into the equation.

$A = 3.14(38.8129)$

$A = 121.872506 \text{ cm}^2$

$A \approx 121.87 \text{ cm}^2$ Don't forget the units of measurement are squared.

Notice that in Example F the answer was rounded to two digits after the decimal. The symbol $\approx$ means "approximately equal to." As it was explained above, the value of $\pi \approx 3.14$. For a more accurate calculation of pi, you can use a value such as 3.1416.

AREA

Extended Example 2a

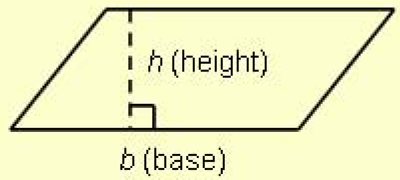
Find the area of a circle with a radius of 6 inches.

AREA

Other Shapes

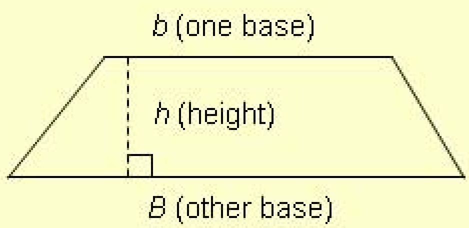
Area of a Parallelogram = base × height

$$A = b \cdot h$$



Area of a Trapezoid = half of the sum of the bases × height

$$A = \frac{1}{2}\left(b + B\right) \cdot h$$



EXAMPLE G

Find the area of a parallelogram with a base of 6 millimeters and height of 4.5 millimeters.

$A = b \cdot h$ Use the equation for the area of a parallelogram.

$A = 6 \cdot 4.5$ Substitute the values for the length of the base and the height into the equation.

$A = 27$

$A = 27 \text{ mm}^2$ Don't forget the units of measurement are squared.

EXAMPLE H

Find the area of a trapezoid with bases of 8.52 yards and 15.72 yards, and height of 1.33 yards.

$A = \frac{1}{2} \cdot (b + B) \cdot h$ Use the equation for the area of a trapezoid.

$A = \frac{1}{2} \cdot (8.52 + 15.72) \cdot 1.33$ Substitute the values for the length of the bases and the height into the equation.

$A = \frac{1}{2} \cdot 24.24 \cdot 1.33$

$A = 12.12 \cdot 1.33$

$A = 16.1196 \text{ yd}^2$ Don't forget the units of measurement are squared.

Extended Example 3a

Find the area of a parallelogram with a base of 7.5 meters and a height of 3.2 meters.

Use the measurements given to find the area of the rectangle.
length = 15 ft
width = 7 ft

Use the measurements given to find the area of the circle. Use 3.14 for π. Round to the nearest hundredth, if necessary.

r = 0.36 m

Use the measurements given to find the area of each parallelogram.

base = 8 yd
height = 9 yd

Use the measurements given to find the area of each trapezoid.

one base = 5 in
other base = 12 in
height = 3 in

Use the measurements given to find the area of each trapezoid.

one base = 4.5 cm
other base = 9.2 cm
height = 4.1 cm

SURFACE AREA AND VOLUME

Introduction

In previous sections, we explored the area and perimeter of geometric figures. In this section, you'll learn about the volume and surface area of geometric figures. The **surface area** (SA) of a three-dimensional figure is the area of the outside surface of the figure, and is given in square units. **Volume** (V) refers to the capacity, or the amount of the space inside a three-dimensional figure; it is given in cubic units.

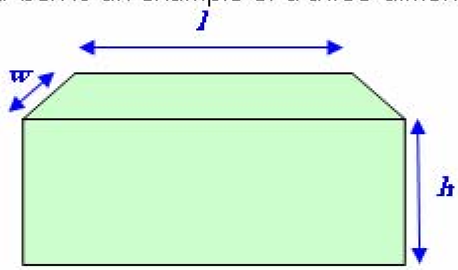
Rectangular Figures

The square and the rectangle are examples of two-dimensional rectangular figures. They both are four-sided figures. You learned that two-dimensional figures have length and width. You also learned how to find the area of rectangles and squares (reviewed below).

Area of a rectangle = length · width = $l \cdot w = lw$

Area of a square = side · side = s^2

Three-dimensional rectangular figures are figures whose faces are four-sided plane figures. A rectangular box is an example of a three-dimensional figure.



Rectangular Solid

The **surface area** of a rectangular solid is the sum of the areas of its faces and can be calculated with the following formula:

Surface Area = 2 · length · width + 2 · length · height + 2 · width · height

$$SA = 2lw + 2lh + 2wh$$

To calculate the **volume** of a rectangular solid, multiply the area of the base by the height of the box. Volume is always in cubic units.

Volume = length · width · height

$$V = l \cdot w \cdot h$$

SURFACE AREA AND VOLUME

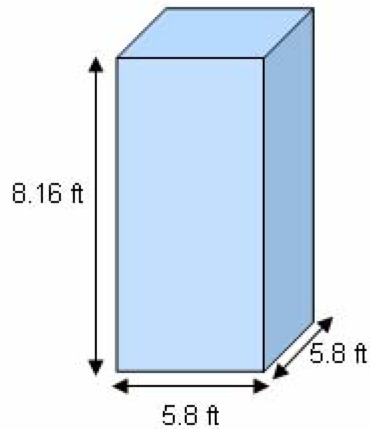
EXAMPLE A

Find the surface area and volume of a box with a height of 8.16 feet and a square base with side lengths of 5.8 feet.

Substitute the values for length, width, and height into the surface area equation.

$$
\begin{aligned}
SA &= 2lw + 2lh + 2wh \\
&= 2(5.8)(5.8) + 2(5.8)(8.16) + 2(5.8)(8.16) \\
&= 67.28 + 94.656 + 94.656 \\
&= 256.592 \text{ ft}^2
\end{aligned}
$$

Don't forget to include units squared in the answer.



Substitute the values for length, width, and height into the volume equation, and simplify.

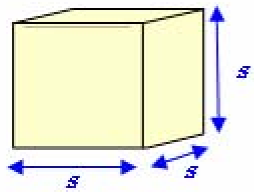
$$
\begin{aligned}
V &= l \cdot w \cdot h \\
&= 5.8(5.8)(8.16) \\
&= 274.5024 \text{ ft}^3
\end{aligned}
$$

Don't forget to include units cubed in the answer.

Cube

The **surface area** of a cube (another example of a rectangular solid) is the sum of the areas of its faces and can be calculated with the formula below.

Surface Area = 6 · side · side
or
$SA = 6s^2$



Cube

To calculate the **volume** of a cube, multiply the area of the base by the height of the box. Volume is always in cubic units.

Volume = side · side · side
or
$V = s^3$

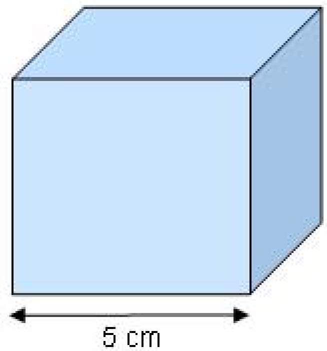
SURFACE AREA AND VOLUME

EXAMPLE B

Find the surface area and volume of a cube with a side length of 5 cm.

Substitute the values for the length of the side into the surface area equation.

$$\begin{aligned} SA &= 6s^2 \\ &= 6(5)^2 \\ &= 6(25) \\ &= 150 \text{ cm}^2 \end{aligned}$$



Substitute the values for the length of the side into the volume equation.

$$\begin{aligned} V &= l \cdot w \cdot h \\ &= 5^3 = 5 \cdot 5 \cdot 5 \\ &= 125 \text{ cm}^3 \end{aligned}$$

SURFACE AREA AND VOLUME

Extended Example 1a

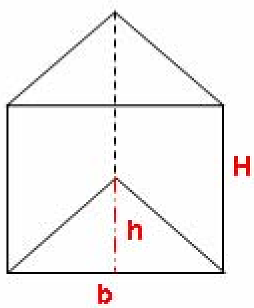
Find the surface area of a rectangular solid whose length is 5.6 mm, width is 10.49 mm, and height is 25.34 mm.

Triangular Figures

In the previous section, you learned to find the area of a two-dimensional triangle with measures of base and height.

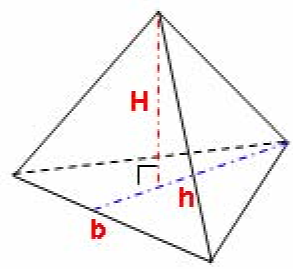
$$\text{Area of a triangle} = \frac{1}{2} \cdot b \cdot h$$

Examples of three-dimensional triangular figures are **prisms** and **pyramids** with triangular bases (below). For this course, we will look at the volume of these figures only. Notice the formula for the area of a triangle $\left(\frac{1}{2} \cdot b \cdot h\right)$ in each of the volume equations below—as with rectangular solids, the area of the base figures into the volume formulas, and in each of these cases the base is a triangle.



Prism

Volume$= \frac{1}{2} \cdot b \cdot h \cdot H$



Pyramid

Volume$= \frac{1}{3}\left(\frac{1}{2} \cdot b \cdot h\right) \cdot H$

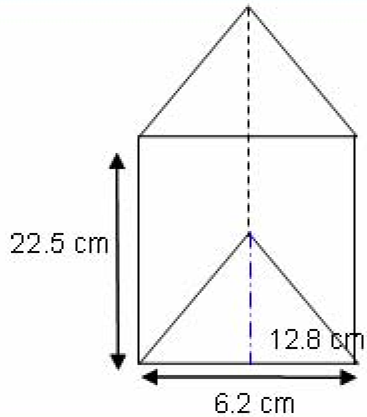
SURFACE AREA AND VOLUME

EXAMPLE C

Find the volume of a prism with a height of 22.5 cm and with a triangular base whose height is 12.8 cm and whose base is 6.2 cm.

Substitute the values into the volume equation.

$$V = \frac{1}{2} \cdot b \cdot h \cdot H$$
$$= \frac{1}{2} \cdot 6.2 \cdot 12.8 \cdot 22.5$$
$$= 892.8\,\text{cm}^3$$

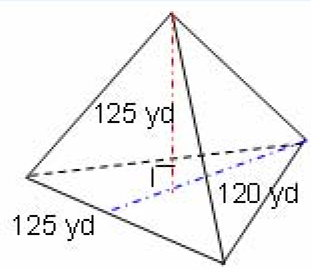


EXAMPLE D

Find the volume of a pyramid with a height of 125 yd and a triangular base whose height is 120 yd and base is 125 yd.

Substitute the values into the volume equation.

$$V = \frac{1}{3}\left(\frac{1}{2} \cdot b \cdot h\right) \cdot H$$
$$= \frac{1}{3}\left(\frac{1}{2} \cdot 125 \cdot 120\right) \cdot 125$$
$$= 312{,}500\,\text{yd}^3$$



8 of 13

Extended Example 2a

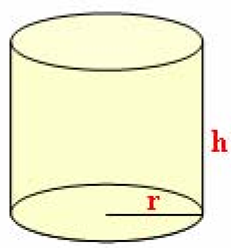
Find the volume of a prism with a height of 7.8 cm and with a triangular base whose height is 6.8 cm and whose base is 9.5 cm.

Circular Figures

Previously, you learned to find the area of a two-dimensional circle using radius and π .

Area of a circle = πr^2

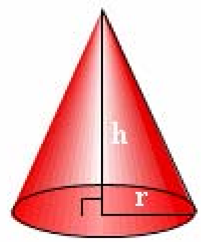
Some three-dimensional circular figures are shown here.



Cylinder

$$SA = 2 \cdot \pi \left(r \cdot h + r^2\right)$$

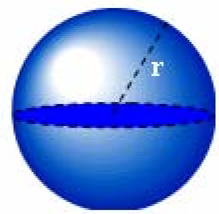
$$V = \pi \cdot r^2 \cdot h$$



Cone

$$SA = \pi \cdot r\left(l + r\right)$$

$$V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$$



Sphere

$$SA = 4 \cdot \pi \cdot r^2$$

$$V = \frac{4}{3} \cdot \pi \cdot r^3$$

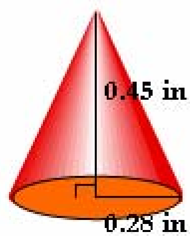
SURFACE AREA AND VOLUME

EXAMPLE E

Find the volume of a cone with a height of 0.45 in and a circular base whose radius is 0.28 in.

Substitute the values into the volume equation for a cone. Use 3.14 for π.

$$
\begin{aligned}
V &= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \\
&= \frac{1}{3} \cdot \pi \cdot (0.28)^2 \cdot 0.45 \\
&= 0.01176 \cdot \pi \\
&= 0.0369264 \\
&\approx 0.04 \text{in}^3
\end{aligned}
$$



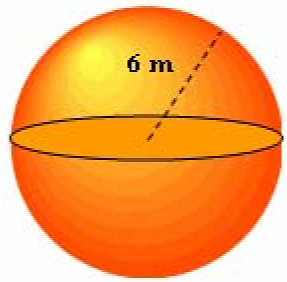
SURFACE AREA AND VOLUME

EXAMPLE F

Find the surface area and volume of a sphere with a radius of 6 m.

Substitute the values into the surface area equation for a sphere. Use 3.14 for π .

$$\begin{aligned} SA &= 4 \cdot \pi \cdot r^2 \\ &= 4 \cdot \pi \cdot (6)^2 \\ &= 144 \cdot \pi \\ &= 452.16\,\text{m}^2 \end{aligned}$$



Substitute the values into the volume equation for a sphere. Use 3.14 for π .

$$\begin{aligned} V &= \frac{4}{3} \cdot \pi \cdot r^3 \\ &= \frac{4}{3} \cdot \pi \cdot (6)^3 \\ &= 288 \cdot \pi \\ &= 904.32\ \text{m}^3 \end{aligned}$$

12 of 13

Extended Example 3a

Find the volume of a cylinder with a height of 5 mm and a radius of 9.9 mm.

Find the a) surface area and b) volume of the rectangular solid or cube. Round answers to the nearest hundredth, if necessary.

Rectangular solid
length = 12.5 in
width = 4.9 in
height = 4.9 in

Find the volume of the triangular prism. Round answers to the nearest hundredth, if necessary.

base = 12 in
height of base = 5 in
Height = 17 in

Find the volume of the triangular pyramid. Round answers to the nearest hundredth, if necessary.

base = 4 in
height of base = 4 in
Height = 9 in

Find the a) surface area and b) volume of the cylinder. Use 3.14 for π. Round answers to the nearest hundredth, if necessary.

h = 5 cm
r = 16 cm

Find the a) surface area and b) volume of the cone. Use 3.14 for π. Round answers to the nearest hundredth, if necessary.

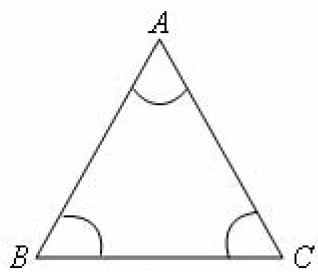
h = 4 cm
l = 5 cm
r = 3 cm

SIMILAR AND CONGRUENT TRIANGLES

Introduction

In previous sections you learned how to calculate the perimeter and area of triangles. Triangles have some important characteristics that are specific to them. In this lesson, we'll deepen our understanding of triangles.

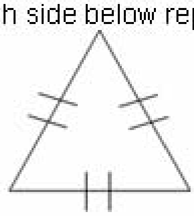
Recall that all triangles have three sides and three angles. The triangle below can be written as ΔABC . It is made up of three line segments: $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, and three angles: $\angle A$, $\angle B$, and $\angle C$.



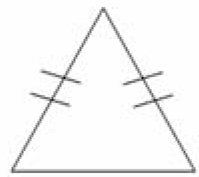
SIMILAR AND CONGRUENT TRIANGLES

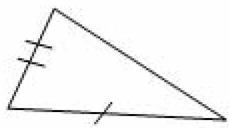
Types of Triangles

If all three sides of a triangle are equal, then the triangle is called **equilateral**. The identical marks on each side below represent equal measurements.



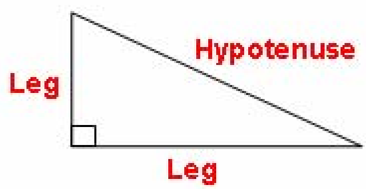
If two sides of a triangle are equal, the triangle is called **isosceles** (see example at right).





Triangles with three different side lengths are called **scalene** (see example at left).

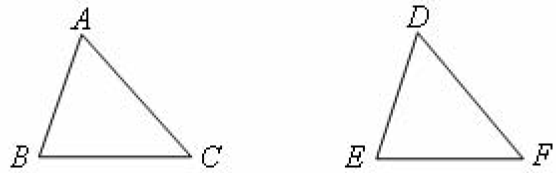
A triangle in which one of the angles is equal to 90° is a **right triangle**. The two sides that form the 90° angle are called **legs.** The side that is opposite the 90° angle is called the **hypotenuse.**



SIMILAR AND CONGRUENT TRIANGLES

Congruent Triangles

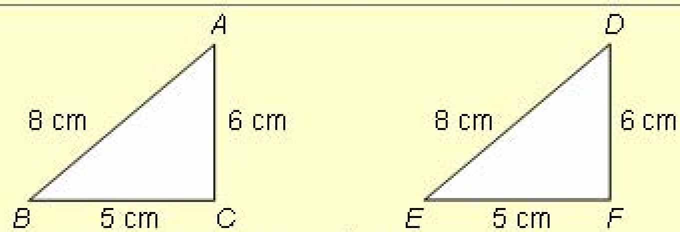
Congruent triangles are triangles that have the same area and same shape.



Two triangles that are congruent are made up of **corresponding** parts. In the triangles above, $\angle A$ and $\angle D$ are corresponding angles and have equal measures. Also, $\angle B$ and $\angle E$, and $\angle C$ and $\angle F$, are corresponding angles. The corresponding sides, $\overline{AB}$ and $\overline{DE}$, $\overline{BC}$ and $\overline{EF}$, and $\overline{AC}$ and $\overline{DF}$, have equal lengths. When two triangles are congruent it can be written this way: $\triangle ABC \cong \triangle DEF$.

We will study three different cases of congruent triangles: **side-side-side** (SSS), **side-angle-side** (SAS), and **angle-side-angle** (ASA).

Side-Side-Side (SSS)
If the lengths of the sides of one triangle are equal to the lengths of the corresponding sides of another triangle, then the triangles are congruent.

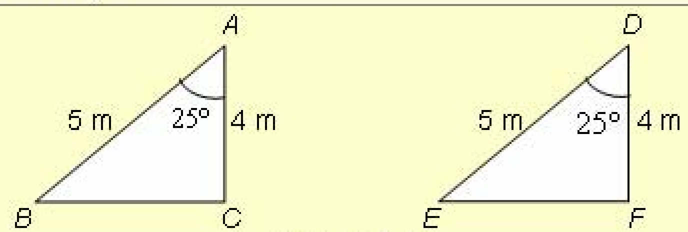


$\triangle ABC \cong \triangle DEF$

SIMILAR AND CONGRUENT TRIANGLES

Side-Angle-Side (SAS)

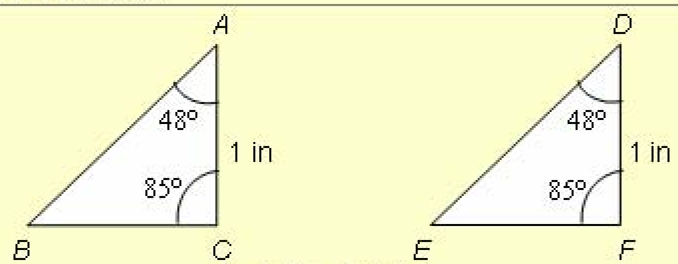
If the lengths of two sides of one triangle are equal to the lengths of the corresponding two sides of another triangle, and the measure of the corresponding angles between those two sides are equal, then the triangles are congruent.



$\triangle ABC \cong \triangle DEF$

Angle-Side-Angle (ASA)

If the measures of two angles of one triangle are equal to the measures of the corresponding two angles of another triangle, and the length of the corresponding sides between those two angles are equal, then the triangles are congruent.

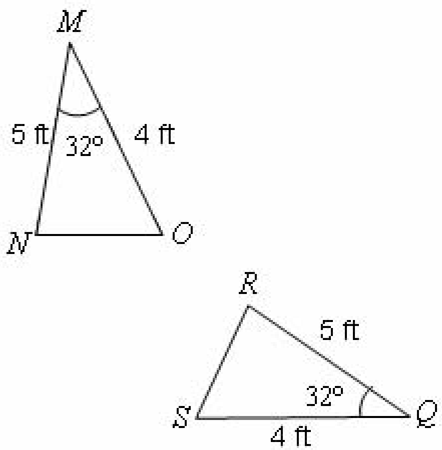


$\triangle ABC \cong \triangle DEF$

SIMILAR AND CONGRUENT TRIANGLES

EXAMPLE A

Is $\triangle MNO$ congruent to $\triangle QRS$?



$MN = 5$ ft is congruent to $QR = 5$ ft.

$\angle M$ is congruent to $\angle Q$ (both are 32°).

$MO = 4$ ft is congruent to $QS = 4$ ft.

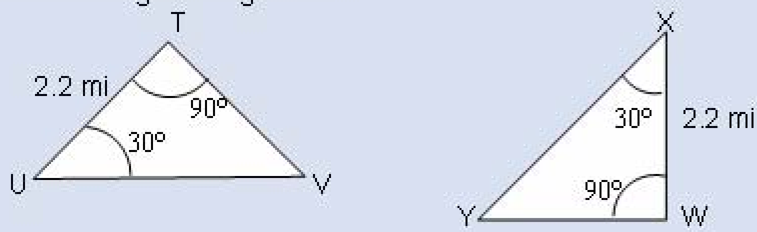
This fits the SAS format; therefore,

$$\triangle ABC \cong \triangle DEF.$$

SIMILAR AND CONGRUENT TRIANGLES

Extended Example 1a

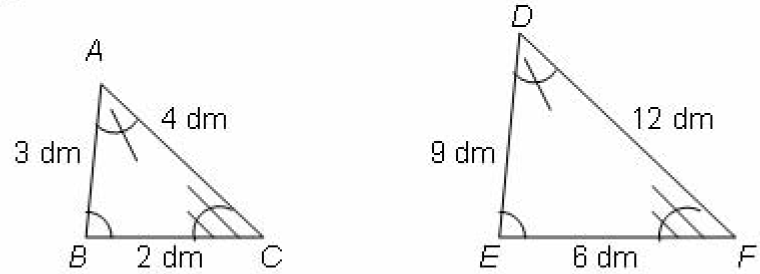
Are the two triangles congruent?



SIMILAR AND CONGRUENT TRIANGLES

Similar Triangles

Similar triangles are triangles with the same shape, but not necessarily the same size.



Two triangles that are similar have corresponding angles with equal measures and corresponding sides that are **proportional**. For corresponding sides to be proportional means that the ratio of the length of each set of corresponding sides is equal. In the triangles above, for example:

$$\frac{AB}{DE} = \frac{3\ \text{dm}}{9\ \text{dm}} = \frac{1}{3}\ ; \quad \frac{BC}{EF} = \frac{2\ \text{dm}}{6\ \text{dm}} = \frac{1}{3}\ ; \quad \frac{AC}{DF} = \frac{4\ \text{dm}}{12\ \text{dm}} = \frac{1}{3}\ .$$

Therefore,

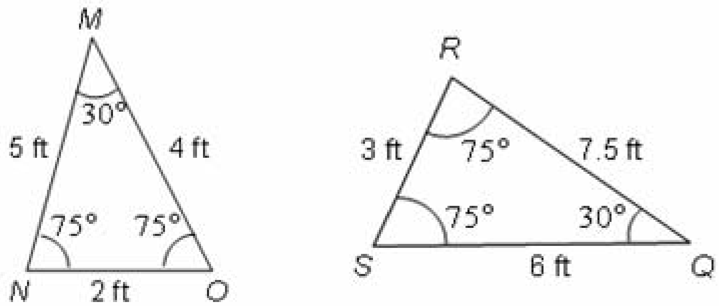
$$\frac{AB}{DE} = \frac{BC}{EF}\ ; \quad \frac{BC}{EF} = \frac{AC}{DF}\ ; \quad \frac{AC}{DF} = \frac{AB}{DE}\ ,$$

so $\triangle ABC$ and $\triangle DEF$ are similar.

SIMILAR AND CONGRUENT TRIANGLES

EXAMPLE B

Are $\triangle MNO$ and $\triangle QRS$ similar?



$$\frac{MN}{QR} = \frac{5\text{ ft}}{7.5\text{ ft}} = \frac{2}{3}$$

Find the ratio of each set of corresponding sides and simplify.

$$\frac{NO}{RS} = \frac{2\text{ ft}}{3\text{ ft}} = \frac{2}{3}$$

$$\frac{MO}{QS} = \frac{4\text{ ft}}{6\text{ ft}} = \frac{2}{3}$$

All three ratios are equal so, the triangles are similar.

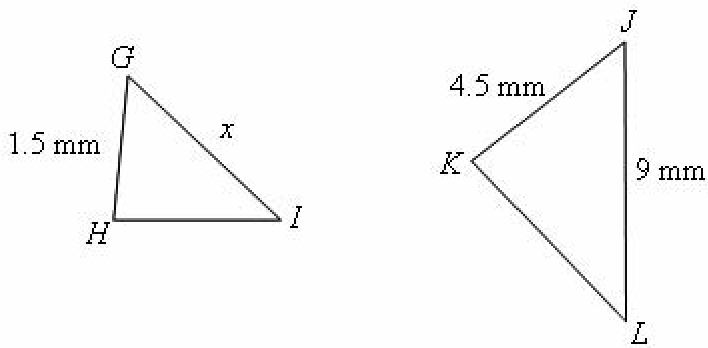
$\triangle MNO$ and $\triangle QRS$ are similar.

8 of 10

SIMILAR AND CONGRUENT TRIANGLES

EXAMPLE C

Given that $\triangle GHI$ and $\triangle JKL$ are similar, find the value of x.



$$\frac{GH}{JK} = \frac{GI}{JL}$$

$$\frac{1.5\text{ mm}}{4.5\text{ mm}} = \frac{x}{9\text{ mm}}$$

Since the triangles are similar, corresponding sides are proportional. Find the ratio of each set of corresponding sides and set them equal.

Use the property of proportions from Chapter 9 to solve for x.

$$4.5x = 1.5(9)$$

$$\frac{\cancel{4.5}x}{\cancel{4.5}} = \frac{13.5}{4.5}$$

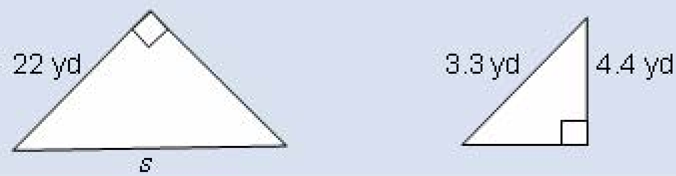
$$x = 3\text{ mm}$$

Don't forget to include the unit of measurement.

SIMILAR AND CONGRUENT TRIANGLES

Extended Example 2a

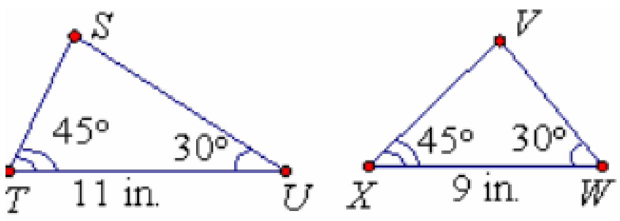
Given that the two triangles are similar, what is the value of s?



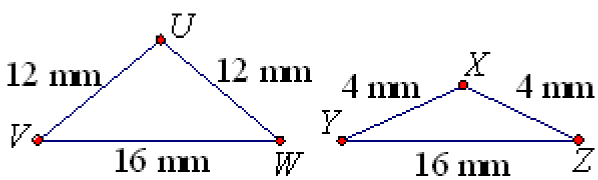
END OF LESSON

10 of 10

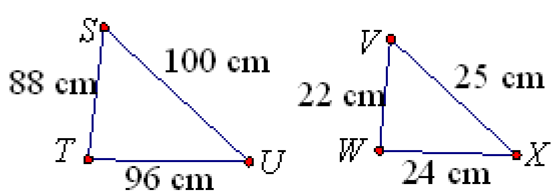
Find whether or not the two triangles are congruent. If so, by which form: SSS, SAS, or ASA.



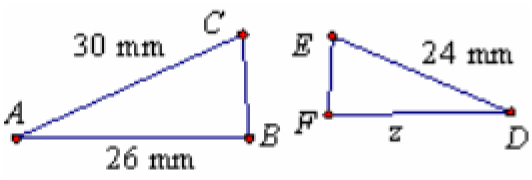
Find whether or not the two triangles are similar.



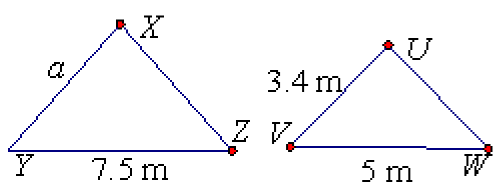
Find whether or not the two triangles are similar.



Given that the two triangles are similar, find the value of the unknown.



Given that the two triangles are similar, find the value of the unknown.



SQUARE ROOTS AND THE PYTHAGOREAN THEOREM

Introduction

Recall the definition of **exponent**. The square of a number means the number multiplied by itself: $b^2 = b \cdot b$. For example, the square of 3 is 9 because $3^2 = 3 \cdot 3 = 9$.

The **square root** of a number a is a number b whose square is equal to a.

$$\boxed{\sqrt{a} = b \text{ where } b^2 = a}$$

For example: $\sqrt{9} = 3$ because $3^2 = 9$

1 of 8

SQUARE ROOTS AND THE PYTHAGOREAN THEOREM

Square Root

The symbol $\sqrt{\ }$ is called a **radical**. You read the expression $\sqrt{9}$ as "the square root of 9." Also, $\sqrt{9}$ is equal to $\sqrt[2]{9}$. The little 2 above the radical is called the **index**. If the index is 2, you don't need to write it. Therefore, when you see $\sqrt{\ }$, it is understood that the index is 2. The following are examples of square roots whose answers are whole numbers. When the answer to a square root problem is a whole number, that number is called a **perfect square**. As you can see below, the numbers 36, 100, 4, and 64 are perfect squares. Can you think of three more perfect squares?

a. $\sqrt{36} = 6$ because $6 \cdot 6 = 36$

b. $\sqrt{100} = 10$ because $10 \cdot 10 = 100$

c. $\sqrt{4} = 2$ because $2 \cdot 2 = 4$

d. $\sqrt{64} = 8$ because $8 \cdot 8 = 64$

If the number under the radical is not a perfect square, then you can find an approximate answer using a calculator. For example, to find $\sqrt{22}$, because 22 is not a perfect square (there is no whole number that can be squared to result in 22), you should use a calculator to find the answer. Most calculators have a buttons for $\sqrt{\ }$.

$$\sqrt{22} = 4.69041575\ldots \approx 4.69$$

If the rounding is not specified, the answer is usually rounded to two decimal places. Recall that the symbol $\approx$ means "approximately equal to."

Use a calculator to try to find the square roots of the numbers in the examples below. See if your answers match the answers given. All answers are rounded to two decimal places. Click on $= ?$ to reveal each answer.

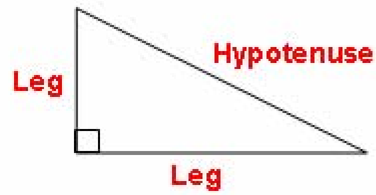
a. $\sqrt{38} = ?$ **b.** $\sqrt{55} = ?$ **c.** $\sqrt{20} = ?$ **d.** $\sqrt{15} = ?$

2 of 8

SQUARE ROOTS AND THE PYTHAGOREAN THEOREM

Pythagorean Theorem

Recall that a right triangle is a triangle in which one of the angles is equal to $90°$. The following relationship always exists between the three sides of a right triangle: The square of the hypotenuse is equal to the sum of the squares of the legs.



$$\text{hypotenuse}^2 = \text{leg}^2 + \text{other leg}^2$$

This relationship is known as the **Pythagorean Theorem**. If you call the length of one of the legs a, the length of the other leg b and the length of the hypotenuse c, then the Pythagorean formula can be written as follows.

Pythagorean Formula	$c^2 = a^2 + b^2$

If the length of one of the sides of a right triangle is unknown, you can use this formula to find it. Because all of the sides are squared in this formula, you have to find a square root to calculate it. The following versions of the Pythagorean formula can be used to find the length of an unknown side.

If the **hypotenuse is unknown** use:	$c = \sqrt{a^2 + b^2}$
If a **leg is unknown** use:	$a = \sqrt{c^2 - b^2}$ or $b = \sqrt{c^2 - a^2}$

SQUARE ROOTS AND THE PYTHAGOREAN THEOREM

EXAMPLE A

Find the hypotenuse of a right triangle if the lengths of the legs are 3 and 4 inches.

$c = \sqrt{a^2 + b^2}$ You know that $a = 3$ inches and $b = 4$ inches and c is unknown. To find c use the equation to find the hypotenuse.

$c = \sqrt{3^2 + 4^2}$ Substitute each of these values into the formula.

$c = \sqrt{9 + 16}$

$c = \sqrt{25}$

$c = 5$ in Don't forget to include the units of measurement.

EXAMPLE B

Find the hypotenuse of a right triangle if the lengths of the legs are 11 and 7 cm.

$c = \sqrt{a^2 + b^2}$ You know that $a = 11$ cm and $b = 7$ cm. To find c use the equation to find the hypotenuse.

$c = \sqrt{11^2 + 7^2}$ Substitute each of these values into the formula.

$c = \sqrt{121 + 49}$

$c = \sqrt{170}$

$c = 13.038$ ft

$c = 13.04$ ft Round to two decimal places.

4 of 8

EXAMPLE C

Find the unknown leg of a right triangle with a hypotenuse of 12 feet and a leg of 9 feet.

$b = \sqrt{c^2 - a^2}$	You know that $c = 12$ feet and $a = 9$ feet and b is unknown. To find b use the equation to find a leg.
$b = \sqrt{12^2 - 9^2}$	Substitute each of these values into the formula.
$b = \sqrt{144 - 81}$	
$b = \sqrt{63}$	
$b \approx 7.94$ ft	Round to two decimal places.

EXAMPLE D

Find the unknown leg of a right triangle with a hypotenuse of 9.6 feet and a leg of 4.2 feet.

$b = \sqrt{c^2 - a^2}$	You know that $c = 9.6$ feet and $a = 4.2$ feet. To find b use the equation to find a leg.
$b = \sqrt{(9.6)^2 - (4.2)^2}$	Substitute each of these values into the formula.
$b = \sqrt{92.16 - 17.64}$	
$b = \sqrt{74.52}$	
$b \approx 8.63$ ft	Round to two decimal places.

Note: For Examples C and D, we could have chosen to call the known leg b and the unknown a, and used the formula $a = \sqrt{c^2 - b^2}$ with the same result.

SQUARE ROOTS AND THE PYTHAGOREAN THEOREM

Extended Example 1a

Find the hypotenuse of a right triangle if the lengths of the legs are 6 and 8 feet.

SQUARE ROOTS AND THE PYTHAGOREAN THEOREM

If a triangle is a right triangle, then the Pythagorean Theorem ($c^2 = a^2 + b^2$) applies to it. This implies that if the Pythagorean formula is true for the given sides of a triangle, then the triangle is a right triangle. Study the examples below.

EXAMPLE E

Is a triangle with sides equal to 8, 7 and 5 a right triangle?

$c^2 = a^2 + b^2$

To find if this is a right triangle, substitute the given values into the Pythagorean formula. **Because the hypotenuse is always the longest side, substitute the longest side for c.** Then $c = 8$, $a = 7$ and $b = 5$.

$8^2 \stackrel{?}{=} 7^2 + 5^2$

$64 \stackrel{?}{=} 49 + 25$

$64 \neq 74$

If the two sides of the equation are equal, then the triangle is a right triangle.

Therefore, this is not a right triangle.

EXAMPLE F

Is a triangle with sides equal to 3, 4 and 5 a right triangle or not?

$c^2 = a^2 + b^2$

Let $c = 5$, $a = 3$ and $b = 4$. Substitute the given values into the Pythagorean formula.

$5^2 \stackrel{?}{=} 3^2 + 4^2$

$25 \stackrel{?}{=} 9 + 16$

$25 = 25$

If the two sides of the equation are equal, then the triangle is a right triangle.

Therefore, this is a right triangle.

7 of 8

Extended Example 2a

Is a triangle with sides equal to 7, 8 and 11 a right triangle?

Use the measurements given to find the unknown side of the right triangle. Round answers to the nearest hundredth, if necessary.

leg = 9 cm
leg = 3 cm

Use the measurements given to find the unknown side of the right triangle. Round answers to the nearest hundredth, if necessary.

leg = 5 in
leg = 6 in

Use the measurements given to find the unknown side of the right triangle. Round answers to the nearest hundredth, if necessary.

leg = 6 m
hypotenuse = 7.5 m

Use the measurements given to find whether or not it is a right triangle.

12 in, 16 in, and 20 in

Use the measurements given to find whether or not it is a right triangle.

20 in, 50 in, and 40 in