FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Introduction

The ability to perform basic operations with whole numbers (addition, subtraction, multiplication, and division) is fundamental for success in all levels of mathematics. Therefore, we will briefly review these operations in this section.

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Place Value

Recall that in the number 555, for example, each of the 5s represents a different value. The rightmost 5 represents five ones. The middle 5 represents five tens or 50, and the leftmost 5 represents five hundreds or 500. This concept can be represented by thinking of 555 in terms of dollars. The rightmost 5 means $5, the middle 5 means $50, and the leftmost 5 means $500. This is why it is read as "five hundred fifty-five dollars."

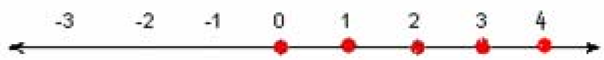
The position of a digit in a number determines its value. The position is called **place value**. The following table shows the place value of each digit in 8,789,605,134.

8,	7	8	9,	6	0	5,	1	3	4
Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Adding Whole Numbers

Whole numbers are a family of numbers that starts with zero and builds up one at a time $\{0, 1, 2, 3, 4, \ldots\}$. These numbers can be shown on a number line. Each red dot on the number line below represents a whole number.



All whole numbers are on or to the right of zero on the number line. The numbers to the right of zero are also called **positive numbers**.

The "**+**" symbol is used to show addition. The number line can be used to add two whole numbers. For example, to add $1+3$, start at point 1 on the number line and move 3 units to the right. You will land on point 4, which is the answer to this problem. This addition can be written as $1+3=4$. The numbers 1 and 3 are called the **addends** and 4 is called the **sum**.

Note the examples of addition below to see the addition process on the number line.

$9+3=12$ $\qquad$ $4+3=7$

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Vertical Addition

Vertical addition can be used when adding numbers with more than one digit. For example, to add $24+56$, write each number vertically so that the digits align by place value. If the sum of the digits in a column is greater than 9 , carry all but the ones value of that sum to the next column on the left.

$$\begin{array}{r} \color{red}{1} \\ 24 \\ \underline{+56} \\ 80 \end{array}$$

Adding 4 and 6 in the ones column results in 10.

Write 0 under the ones digits, but since the "1" in "10" represents a ten, carry it to the tens column (this 1 is written in red above the tens column at left).

Then add this 1 with the other numbers in the tens column, the numbers 2 and 5, which is 8.

EXAMPLE A

Add $7,689+1,479+308$ vertically.

$$\begin{array}{r} \color{red}{1\ 1\ 2} \\ 7,689 \\ 1,476 \\ \underline{+\ \ 308} \\ 9,473 \end{array}$$

Add the digits in the ones column first: they add up to 23.

Write 3 under the ones digits, and 2 above the tens column (shown in red at left).

Add the digits in the tens column including the 2 that was carried over from the first sum: we get 17.

Write 7 in the tens place and carry the 1 to the hundreds column. Add the hundreds column: 14.

Write 4 in the hundreds place, carry the 1 to the thousands column and add this column: 9.

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Subtracting Whole Numbers

If a sandwich is $5 and a drink is $1, what is the total cost of these two items? To find the answer, you would add the two prices.

$$5 + 1 = 6$$

Now, if you have only $9 in your pocket, after paying $6 for the food, how much money would you have left? To answer this question, you would subtract or find the **difference** between the amount you have and the amount you pay. The "–" symbol is used to show subtraction.

$$9 - 6 = 3$$

This shows that you have $3 left.

Addition and subtraction are closely related. For example, in the problem above if we add $3 and $6, the sum is $9.

$$9 - 6 = 3 \quad \text{because} \quad 3 + 6 = 9$$

Vertical Subtraction

Like addition, if the numbers that are being subtracted have more than one digit, then it is more convenient to subtract vertically. The first number, or the number being subtracted from, is called the **minuend**, and the second number, or the number being subtracted, is called the **subtrahend**. The answer to subtraction is called the **difference**. To check if the difference is correct, add the difference and the subtrahend. If the sum is equal to the minuend, the answer is correct.

EXAMPLE B

Subtract 48 – 25 vertically.

48	⟵ minuend	Subtract the ones digit first.
−25	⟵ subtrahend	Subtract the tens digit.
23	⟵ difference	Find the difference.

Add to check the answer:

$$\begin{array}{r} 23 \\ +25 \\ \hline 48 \end{array} \checkmark$$

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Subtracting with Borrowing

In a subtraction problem, if the digit of the second number (subtrahend) in any column is larger than the digit of the first number (minuend) in that column, then borrowing can be used to find the difference—borrow from the column to the left.

Method I: A common method of borrowing follows the steps below.

1. Borrow a 1 from the tens column of the minuend. Since this is 1 ten being borrowed, add 10 to the ones digit in the minuend.
2. Subtract 1 (since it was borrowed) from the tens column of the minuend. From this difference subtract the tens value of the subtrahend.
3. Repeat the same procedure for any other columns that require borrowing.

EXAMPLE C

Subtract: $92 - 56$.

$$\begin{array}{r} 92 \\ -56 \\ \hline \end{array}$$

Since 6 is greater than 2, borrow 1 from the tens digit of the minuend and change the 2 to 12. Since 1 is borrowed from 9, cross out the 9 and change it to 8.

$$\begin{array}{rr} 8 & 12 \\ \cancel{9} & \cancel{2} \\ -5 & 6 \\ \hline \end{array}$$

Subtract each column:

$$\begin{array}{rr} 8 & 12 \\ \cancel{9} & \cancel{2} \\ -5 & 6 \\ \hline 3 & 6 \end{array}$$

EXAMPLE D

Subtract: $2,021 - 457$.

$$\begin{array}{r} 2,021 \\ -\ \ 457 \\ \hline \end{array}$$

Notice that we need to use borrowing for the ones, tens, and hundreds columns. Borrow 1 from the column to the left and continue borrowing as needed.

$$\begin{array}{r} \scriptstyle 1\ 11 \\ 2,0\cancel{2}\cancel{1} \\ -457 \\ \hline 4 \end{array} \Rightarrow \begin{array}{r} \scriptstyle 1\ 10\ 1\ 11 \\ \cancel{2},\cancel{0}\cancel{2}\cancel{1} \\ -457 \\ \hline 4 \end{array} \Rightarrow \begin{array}{r} \scriptstyle 9\ 11 \\ \scriptstyle 1\ \cancel{10}\ \cancel{1}\ 11 \\ \cancel{2},\cancel{0}\cancel{2}\cancel{1} \\ -457 \\ \hline 1,564 \end{array}$$

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Method II: Another method of borrowing follows these steps:

1. Write a 1 before the ones digit in the minuend. This 1 is borrowed from the tens column.
2. Add 1 (since it was borrowed) to the tens column of the subtrahend, and subtract this sum from the first number.
3. Repeat the same procedure for any other columns that require borrowing.

EXAMPLE E

Subtract: $92 - 56$.

$$\begin{array}{r} 92 \\ -56 \\ \hline \end{array}$$

Since 6 is greater than 2, write a 1 before the 2. This changes it to 12.

$$\begin{array}{r} 9\,^{1}2 \\ -5\,6 \\ \hline \end{array}$$

Now subtract 6 from 12. This results in 6.

$$\begin{array}{r} 9\,^{1}2 \\ -5\,6 \\ \hline 6 \end{array}$$

To subtract the tens remember that a 1 has been borrowed from this column. Mentally add 1 to the 5 of the subtrahend; which changes it to 6.

$$\begin{array}{r} 9\;2 \\ {}^{+1}\;\; \\ -\,5\;6 \\ \hline 6 \end{array}$$

Now subtract 6 from 9, and write the difference.

$$\begin{array}{r} 9\;2 \\ {}^{+1}\;\; \\ -\,5\;6 \\ \hline 36 \end{array}$$

EXAMPLE F

Subtract: $73{,}609 - 25{,}164$.

$$\begin{array}{r} 73{,}609 \\ -25{,}164 \\ \hline \end{array}$$

Notice that the tens and thousands columns need borrowing. Write 1 before these digits in the minuend. Also add 1 to the digits of the subtrahend in the column to the left of each.

$$\begin{array}{r} 7\,^{1}3{,}6\,^{1}09 \\ {}^{+1}\quad{}^{+1}\;\;\;\; \\ -\,2\,5{,}1\,64 \\ \hline \end{array}$$

Now subtract each column from right to left $9 - 4$, $10 - 6$, $6 - 2$, $13 - 5$, and $7 - 3$.

$$\begin{array}{r} 7\,^{1}3{,}6\,^{1}09 \\ {}^{+1}\quad{}^{+1}\;\;\;\; \\ -\,2\,5{,}1\,64 \\ \hline 4\,8{,}4\,45 \end{array}$$

Practice both methods of subtracting with borrowing and decide which method you prefer.

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Multiplying Whole Numbers

Multiplication is a short way of adding the same number several times. The "**×**" symbol is used to show multiplication. For example:

$$\underbrace{3+3+3+3+3}_{5\text{ of them}} \text{ can be writen as } 3\times 5$$

$$3+3+3+3+3=15 \quad \text{and} \quad 3\times 5=15$$

The answer to a multiplication problem is called a **product** and the numbers being multiplied are called **factors**. To find the answer to a multiplication problem, either write each multiplication problem in the form of addition (not recommended), or use products you've memorized from the Multiplication Table.

EXAMPLE G

Multiply using the multiplication table: 34×7.

$$\begin{array}{r} {\scriptstyle 2} \\ 34 \\ \times\ \ 7 \\ \hline 8 \end{array}$$

Multiply 4 by 7. The product is 28. Write 8 in the ones column of the answer and carry 2 (in red) to the tens column.

$$\begin{array}{r} {\scriptstyle 2} \\ 34 \\ \times\ \ 7 \\ \hline 238 \end{array}$$

Multiply 3 by 7. The product is 21. Add the 2 that was carried to the 21 to get 23. Write 3 in the tens column and 2 in the hundreds column of the answer.

EXAMPLE H

Multiply using the multiplication table: 563×79.

This example requires multiplication with multi-digit numbers. To multiply 79 by 563, first multiply 9 (the ones column of 79) by each digit of 563, then multiply 7 (the tens column of 79) by each digit of 563.

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

Dividing Whole Numbers

To write a division problem such as "a divided by b," three different symbols can be used.

Division symbols: $b\overline{)a}$, $a \div b$, $\frac{a}{b}$

In these division problems, a is called the **dividend**, b is called the **divisor**, and the answer is called the **quotient**. Multiplication and division are closely related. To check the answer to a division problem, multiply the quotient by the divisor. The product should equal the dividend.

Long division is a common way of dividing two whole numbers such as $345 \div 5$. The answer to this problem is how many times 5 goes into 345.

EXAMPLE I

Divide using long division: $345 \div 5$.

$$\begin{array}{r} ? \leftarrow \textit{quotient} \\ \textit{divisor} \rightarrow 5\overline{)345} \rightarrow \textit{dividend} \end{array}$$

First arrange the numbers as shown here.

FOUR BASIC OPERATIONS USING WHOLE NUMBERS

EXAMPLE J

Divide using long division: $2,470 \div 21$.

$\text{divisor} \rightarrow 21\overline{)2,470} \leftarrow \text{dividend}$, $? \leftarrow \text{quotient}$	First arrange the numbers as shown.
$\begin{array}{r} 1 \\ 21\overline{)2470} \\ -21\downarrow \\ \hline 37 \end{array}$	Think of how many times 21 goes into 24. The answer is 1. Write 1 in the quotient space above the 4. Multiply 1 by 21 and subtract this product from 24. The difference is 3. Bring down the 7 (blue arrow) to the right of 3 and start the process again.
$\begin{array}{r} 11 \\ 21\overline{)2470} \\ -21\downarrow\downarrow \\ \hline 37\downarrow \\ -21\downarrow \\ \hline 160 \end{array}$	Think of how many times 21 goes into 37. The answer is 1. Write 1 in the quotient space. Multiply 1 by 21 and subtract the product from 37. The difference is 16. Bring down the 0 (blue arrow) and start the process again.
$\begin{array}{r} 117 \\ 21\overline{)2470} \\ -21\downarrow\downarrow \\ \hline 37\downarrow \\ -21\downarrow \\ \hline 160 \\ -147 \\ \hline 13 \leftarrow \text{remainder} \end{array}$	Think of how many times 21 goes into 160. The easiest way is to guess this number by thinking how many times 2 goes into 16. The answer is 8 times so guess 7 and write it in quotient space. Multiply 7 by 21 and subtract this product from 160. The difference is 13. NOTE: Always make sure that the remainder is smaller than the divisor.

END OF LESSON

Which of these are whole numbers: 6, 33, –11, $\frac{1}{2}$, –42, 100, 0.5, 0?

Perform the indicated operation without using a calculator.

$$\begin{array}{r} 2{,}469 \\ 301 \\ +\quad 785 \\ \hline \end{array}$$

Perform the indicated operation without using a calculator.

$$\begin{array}{r} 10{,}023 \\ -\quad 875 \\ \hline \end{array}$$

Perform the indicated operation without using a calculator.

$$\begin{array}{r} 1{,}009 \\ \times \quad 1{,}009 \\ \hline \end{array}$$

Perform the indicated operation without using a calculator.

$369 \div 21$

MATH VOCABULARY

Introduction

Becoming familiar with mathematical terminology is an important first step in learning algebra. We will discuss some of that terminology in this lesson.

MATH VOCABULARY

A **variable** is a letter that stands for a number, such as x, y, z, or k.

Variables can be combined with numbers and the arithmetic operations addition, subtraction, multiplication, and division to form mathematical "phrases" called **expressions**. An expression does not contain an equal sign and can be **simplified** or **evaluated**. There are many different types of expressions and multiple techniques for simplifying. We will look at expressions in greater detail throughout this course.

An **equation** is a mathematical "sentence" that contains an equal sign. An equation can also be thought of as two expressions that are equal to each other. Equations can be solved. To **solve** an equation means to find the numerical value of the variable that is in the equation.

Expressions	Equations
$2x+5$	$x+1=9$
$3x-10+y$	$y+5=-6$

Terms and Coefficients

A **term** is a number or the product of a number and a variable. Terms are separated from one another by addition or subtraction symbols. The expression $2x+5$ has two terms. One term is $2x$, and the other term is 5. In the expression $3x+10+y$, there are three terms. These terms are $3x$, 10, and y. The equations $x+1=9$ and $y+5=-6$ each have two terms on the left side of the equation and one term on the right side.

MATH VOCABULARY

A term can be either a **variable term** or a **non-variable term**.

- If a term contains a variable, then it is considered a variable term. Example: $3x$, $5x$, and x .
- If a term does not contain a variable, then it is considered a non-variable term. Example: 3 , 4 , and $\frac{1}{2}$. A non-variable term is also called a **constant**.

All variable terms have **coefficients**. In a variable term, the coefficient is located in front of the variable, and the coefficient and variable are multiplied by each other. The term $5x$ means 5 times x , where 5 is the coefficient and x is the variable. $5x$ also means:

$$\underbrace{x+x+x+x+x}_{5 \text{ of them}}.$$

In the terms $6x$, $2y$, and k , the coefficients are 6 , 2 , and 1 , respectively. In the case of k , the coefficient is 1; however, we do not write the 1 since multiplication by 1 does not change the value.

A constant is always a number without a variable. In the expression $2+3z$, the constant is 2 , the variable term is $3z$, the coefficient is 3 , and the variable is z .

MATH VOCABULARY

EXAMPLE A

In the expression $3x+5$, how many terms are there?

Two terms: $3x$ and 5, which are separated by the "$+$" sign.

EXAMPLE B

In the expression $3x+5$, what is the variable term? What is the variable?

The only variable term is $3x$ and x is the variable.

EXAMPLE C

In the expression $3x+5$, what is the coefficient?

3 is the coefficient, because in the variable term $3x$, 3 is the number that is multiplied by the variable x.

EXAMPLE D

Is there any constant term in $3x+5$?

Yes: 5 is the constant or non-variable term.

MATH VOCABULARY

Extended Example 1a

In the expression $8k + m + 7n$, how many terms are there? List the terms.

Factors

Recall from the previous section that **factors** are numbers or variables that are multiplied together. In the term $6x$, the 6 is the coefficient that is being multiplied by x to make the expression $6x$. Therefore, 6 and x are factors of $6x$. The expression $32x+9$ has two terms. One term is $32x$, in which 32 is the coefficient and x is the variable, and the other term is 9. Since 32 and x are multiplied together, they are factors of $32x$.

Note: There is more than one way to show multiplication in algebra. For example, "5 times 3" can be written as

$$5 \cdot 3,\quad 5(3),\quad (5)(3),\quad 3 \cdot 5,\quad 3(5),\quad \text{or } (3)(5).$$

When multiplying by a variable the same rules apply. For example, "7 times x" can be written as $7 \cdot x$, $7(x)$, $(7)(x)$, $x \cdot 7$, $x(7)$, or $(x)(7)$, but the most common way is to write it as $7x$.

In algebra, division is often shown in fraction format. For example,

"$4x$ divided by 7" is commonly written as $\dfrac{4x}{7}$.

MATH VOCABULARY

EXAMPLE E

What are the factors of $5x$?

The factors are 5 and x . Note that the number 1 is also a factor of $5x$; even though it is not shown, we know that $1 \cdot 5x = 5x$.

EXAMPLE F

What are all of the factors of $12kt$?

We can see right away that four factors are 1 , 12 , k , and t . However, notice that there are also several other factors of 12 :

$$6 \cdot 2 = 12 \text{ and } 4 \cdot 3 = 12 .$$

So, all of the factors of $12kt$ are: 1 , 2 , 3 , 4 , 6 , 12 , k , and t .

Question: What are all of the factors of $24x$?

END OF LESSON

Decide whether the following is an expression or an equation.

$3x - 2y + 6$

Tell how many terms are in the expression.

$4x + y - 9 + 2z$

Identify each of the following: a) the variable terms, b) the variables, c) the constants, and d) the coefficients.

$8m + 2n$

Identify each of the following: a) the variable terms, b) the variables, c) the constants, and d) the coefficients.

$$4x + y + 9 + 2z$$

List all of the factors for the expression.

$$15mn$$

COMBINING LIKE TERMS

Introduction

As mentioned in the previous section, expressions can be either **simplified** or **evaluated**. Simplifying is a procedure that makes the expression look simpler and shorter. Simplifying does not change the value of the expression; it only changes the appearance of the expression.

The following are procedures used to simplify an expression:

- Combining like terms
- Distribution

We'll discuss **combining like terms** in this lesson, and we'll review the commutative and associative properties of addition and multiplication.

COMBINING LIKE TERMS

To combine items that are alike means to combine the number of things or ideas belonging to the same category.

For example, if you counted 6 apples, 1 orange, 3 apples, and 4 oranges in a fruit bowl, you would probably say that there are 9 apples and 5 oranges in the bowl. To get this, you add 6 apples + 3 apples to get 9 apples, and 1 orange + 4 oranges to get 5 oranges. You did this because those items were alike. Algebra works much the same way, except that numbers and variables are used instead of words.

In mathematics, items that are alike are called **like terms**. The example of fruit in a fruit bowl can be expressed in algebraic form if we think of apples as x and oranges as y:

$6x + y + 3x + 4y$

$6x + 3x + y + 4y$ Gather like terms so they are next to each other.

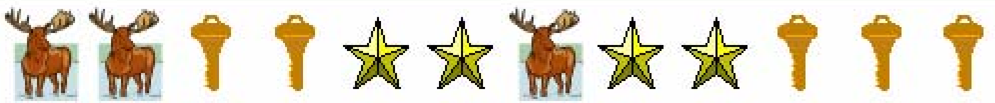
$9x + 5y$ Combine like terms.

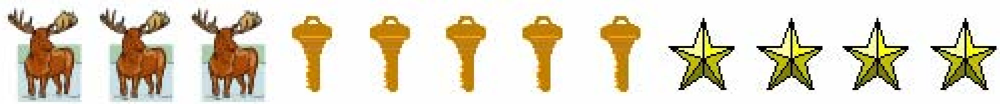
To **combine like terms** is to add or subtract those terms that are alike. The coefficients of the terms with the same variable were added: the coefficient of the first x-term with the coefficient of the second x-term, and the coefficient of first y-term with the coefficient of the second y-term.

COMBINING LIKE TERMS

EXAMPLE A

Combine the like objects shown below.





2 moose + **2 keys** + **2 stars** + **1 moose** + **2 stars** + **3 keys** =
2 moose + **1 moose** + **2 keys** + **3 keys** + **2 stars** + **2 stars** =
3 moose + **5 keys** + **4 stars**

Even though the order of the items was "scrambled" in the original problem, you still combine moose with moose, keys with keys, and stars with stars to find the total number of each object. Algebra works in a similar way.

Example B on the next screen is an algebraic form of Example A with x representing keys, y representing moose, and stars as constants.

EXAMPLE B

Simplify the expression $2y+2x+2+y+2+3x$.

$$
\begin{aligned}
&2y+2x+2+y+2+3x \\
&= 2x+3x+2y+y+2+2 \quad \text{Gather like terms so they are next to each other.} \\
&= 5x+3y+4 \quad \text{Combine like terms.}
\end{aligned}
$$

The standard order for writing an expression in algebra is to write the variable terms first (in the order they appear in alphabet) with the constant at the end. The expression $5x+3y+4$ cannot be simplified or combined any further because the remaining three terms are not alike.

EXAMPLE C

Simplify: $x+4+2x+6$.

$$
\begin{aligned}
x+4+2x+6 &= 1x+4+2x+6 \quad \text{Write } x \text{ as } 1x. \\
&= 1x+2x+4+6 \quad \text{Gather like terms.} \\
&= 3x+10 \quad \text{Combine like terms.}
\end{aligned}
$$

In many cases, you will simply be asked to simplify an expression. It is up to you to recognize if and how an expression is to be simplified.

Extended Example 1a

Simplify: $5+3y+7+2y+y+1$.

COMBINING LIKE TERMS

Addition and multiplication are both commutative and associative operations.

Commutative Property of Addition and Multiplication

The **Commutative Property** allows the order of two numbers that are being added or multiplied to be reversed without changing the final result.

Examples:

Addition	Multiplication
$5+4=9$	$5(4)=20$
$4+5=9$	$4(5)=20$

For both examples above notice that even though the order of the terms was reversed the results remained the same.

Associative Property of Addition and Multiplication

The **Associative Property** states that changing the grouping in an addition or multiplication problem will not change the final result. For example, compare the following:

$1+(7+5)$	is the same as	$(1+7)+5$
$=1+12$		$=8+5$
$=13$		$=13$

$2(3(5))$	is the same as	$(2(3))(5)$
$=2(15)$		$=(6)(5)$
$=30$		$=30$

Since both addition and multiplication are commutative and associative, numbers can be added or multiplied in any order. The commutative property of addition is the property that allows us to rearrange the order of terms in an expression when we are combining like terms. Notice this in the following example:

$$x+5+20x+8 = x+20x+5+8$$ Use the commutative property to rearrange the terms.

$$= 21x+13$$

END OF LESSON

Simplify.

$$8x + 9y + 2x + y$$

Simplify.

$$17m + 5n + 3 + 8m + 4$$

Simplify.

$$6m + 10n + 17p + 4m + 2q + 8p + 3n + 9q$$

Determine which property is used: commutative property of addition, commutative property of multiplication, associative property of addition, or associative property of multiplication.

$$(9 + 3) + m = (3 + 9) + m$$

THE DISTRIBUTIVE PROPERTY

Introduction

Another property used to simplify expressions is the **Distributive Property**, which we explore in this lesson. Remember, simplifying does not change the value of the expression. It only changes its appearance.

THE DISTRIBUTIVE PROPERTY

Consider this: Three groups of musicians are coming to town to perform in a show. Each group consists of 5 women and 4 men. How many total women are there? How many total men?

To find out, multiply 5 women and 4 men in each group by 3 groups. There will be a total of 15 women and 12 men. This can be shown algebraically as follows:

3 (5 women + 4 men) = 3 (5 women) + 3 (4 men) = 15 women + 12 men

This is an example of the **distributive property** of multiplication over addition. The distributive property is used in the multiplication of a number by a sum of two or more numbers.

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THE DISTRIBUTIVE PROPERTY

It is possible for there to be more than two terms inside the parentheses, as shown in the following examples. You simply distribute to each term.

EXAMPLE A

Simplify: $6(x+4y+1)$.

$6(x+4y+1) = 6(x)+6(4y)+6(1)$ Distribute 6 to x , $4y$, and 1 .

$= 6x+24y+6$ Multiply.

EXAMPLE B

Simplify: $3(7x+2y+7)$.

$3(7x+2y+7) = 3(7x)+3(2y)+3(7)$ Distribute 3 to $7x$, $2y$, and 7 .

$= 21x+6y+21$ Multiply.

You should now be able to recognize when to use the distributive property to help simplify an expression.

THE DISTRIBUTIVE PROPERTY

Extended Example 1a

Simplify: $5(4n+3)$

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THE DISTRIBUTIVE PROPERTY

Distributing and Combining

Now that you have learned to combine like terms and to use the distributive property, you are ready to use a combination of these procedures to simplify more complex expressions.

In a problem that requires distribution, multiply each term inside the parentheses by the number in front of the parentheses. Then remove the parentheses. Once you have completed all distribution(s) in the problem, then combine like terms. (Recall: to combine like terms means to rearrange terms that are alike so that they are next to each other. Then add/subtract the coefficients of the like terms.) Study the examples below carefully.

EXAMPLE C

Simplify: $2(3m+4)+7(2m+5)$.

$2(3m+4)+7(2m+5) = 6m+8+14m+35$ Distribute the 2 and the 7.

$= 6m+14m+8+35$ Gather like terms.

$= 20m+43$ Combine like terms.

EXAMPLE D

Simplify: $6(y+3)+2(2y+9)$.

$6(y+3)+2(2y+9) = 6y+18+4y+18$ Distribute the 6 and the 2.

$= 10y+36$ Mentally gather and combine like terms.

When asked to simplify an expression you will be expected to know, by looking at the expression, how to distribute to remove the parentheses and whether or not there are like terms to combine to finish simplifying the problem.

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THE DISTRIBUTIVE PROPERTY

Extended Example 2a

Simplify: $4(3x+7)+6(2x+7)$.

END OF LESSON

6 of 6

Simplify.

$3(4t + 8)$

Simplify.

$8(m + 5n + 2)$

Simplify.

$7k + 7(2k + 3)$

Simplify.

$9(2t + 4) + (6t + 2)$

Simplify.

$11(x + 6) + (4y + 1) + 3x + y$

EXPONENTS AND THE ORDER OF OPERATIONS

Introduction

In this section, we'll look at **exponents** and exponential notation. We will also learn the standard **order of operations** that is used to simplify expressions that contain multiple operations.

EXPONENTS AND THE ORDER OF OPERATIONS

Exponents

When we have to perform repeated multiplication, we can use exponents. For example, in $(6)(6)(6)(6)(6)$ we use 6 as a factor 5 times. This can be written using **exponential notation** as 6^5, which is read as "6 to the fifth power." Exponential notation provides a compact way of writing repeated multiplication. In the exponential expression 6^5, the number 6 is the **base** and the number 5 is the **exponent.**

Examples:

3^4 means $(3)(3)(3)(3)$ which is equal to 81.

$(7)(7)(7)(7)(7)$ can be written as 7^5 and is equal to $16,807$.

In the first example above, the base is 3 and the exponent is 4. It can be read as "three to the fourth power" or "three to the power of four." In the second example, the base is 7 and the exponent is 5. It can be read as "seven to the fifth power" or "seven to the power of five."

Additional examples:

$$5^2 = (5)(5) = 25 \qquad 3^5 = (3)(3)(3)(3)(3) = 243$$

$$2^3 = (2)(2)(2) = 8 \qquad 1^4 = (1)(1)(1)(1) = 1$$

The base of an exponential expression can be a variable. For example, x^4 means $(x)(x)(x)(x)$, which can be read as "x to the fourth power" or "x to the power of 4."

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EXPONENTS AND THE ORDER OF OPERATIONS

Order of Operations

When working with an expression that has multiple operations, there is an order in which the operations must be performed to get the correct answer.

Here is the standard **Order of Operations**:

1. First, simplify inside **grouping symbols**, if there are any. Grouping symbols include parentheses **( )**, brackets **[ ]**, and braces **{ }**. If more than one set of grouping symbols is present, work from within the innermost grouping symbols to the outermost. **Note:** In the expression $7(5)$ the parentheses are not considered grouping symbols; instead, they indicate multiplication. To be considered grouping symbols, the ( ), [ ], or { }, need to contain an operation inside them. For example, in $7(5+2)$, the parentheses are considered grouping symbols.
2. Second, simplify expressions with **exponents**, if there are any.
3. Third, perform all **multiplication** and/or **division**, from left to right.
4. Finally, perform all **addition** and/or **subtraction**, from left to right.

The only way to become comfortable with the correct order of operations is to work through several examples, followed by a lot of practice! Carefully study the examples that follow.

EXPONENTS AND THE ORDER OF OPERATIONS

EXAMPLE A

Simplify: $9-4+3$.

$9-4+3$ The only operations present are addition and subtraction, which we perform from left to right.

$=5+3$ First simplify 9 - 4.

$=8$ Then add 5 and 3.

EXAMPLE B

Simplify: $12 \div 2(3)$.

$12 \div 2(3)$ There are two operations here--division and multiplication.

$=6(3)$ First divide 12 by 2 since division comes first from left to right.

$=18$ Then multiply 6 by 3.

EXPONENTS AND THE ORDER OF OPERATIONS

EXAMPLE C

Simplify: $8+4(9-3)$.

$8+4(9-3)$ Note that there are grouping symbols this time, so...

$=8+4(6)$ First, simplify inside the parentheses.

$=8+24$ Then, multiply.

$=32$ Finally, add.

EXAMPLE D

Simplify: $25-3(5+2)$.

$25-3(5+2)$ Note the grouping symbols and operations.

$=25-3(7)$ First, simplify inside the parentheses.

$=25-21$ Second, multiply.

$=4$ Finally, subtract.

EXPONENTS AND THE ORDER OF OPERATIONS

EXAMPLE E

Simplify: $36 \div 2(3) - 3(6-1)$.

$36 \div 2(3) - 3(6-1)$ Note the grouping symbols and operations.

$= 36 \div 2(3) - 3(5)$ First, simplify inside the parentheses.

$= 18(3) - 3(5)$ Second, divide (the division is to the left of the multiplication).

$= 54 - 15$ Third, multiply from left to right.

$= 39$ Finally, subtract.

EXAMPLE F

Simplify: $(3+2)^2 - 2^3$.

$(3+2)^2 - 2^3$

$= (5)^2 - 2^3$ First, simplify inside the parentheses.

$= 25 - 8$ Second, simplify the exponents. Recall, $5 \times 5 = 25$, and $2 \times 2 \times 2 = 8$.

$= 17$ Finally, subtract.

EXPONENTS AND THE ORDER OF OPERATIONS

If there are multiple operations within a grouping symbol, follow the order of operations within that grouping symbol also.

EXAMPLE G

Simplify: $18 + 3[70 - 3(5)]$.

$18 + 3[70 - 3(5)]$

$= 18 + 3[70 - 15]$ First, multiply 3 and 5 inside the brackets.

$= 18 + 3[55]$ Second, subtract the numbers inside the brackets.

$= 18 + 165$ Third, multiply 3 and 55.

$= 183$ Finally, add.

You may find it necessary to work through the examples in this lesson several times to become comfortable with this process. When you encounter problems that are not exactly like the ones given here, remember that following the order of operations will guide you to the answer.

In summary, the order of operations is a method of simplifying that guarantees a consistent result. First, simplify inside grouping symbols; second, simplify exponents; third, simplify all multiplication and division in order from left to right; and finally, simplify all addition and subtraction in order from left to right. When the instructions to a problem say "Simplify," you should be able to recognize which steps to take to find the correct answer.

EXPONENTS AND THE ORDER OF OPERATIONS

Extended Example 1a

Simplify: $(8-3)^3 + 4^2$.

END OF LESSON

Simplify.

4^4

Simplify.

$10 - 4 \cdot 2$

Simplify.

$8 \cdot 4 \div 2 \div 4$

Simplify.

$$4 \cdot 3^2 - 10$$

Simplify.

$$12 + 2(4 + 5(7))$$

EVALUATING EXPRESSIONS

Introduction

As mentioned earlier in this chapter, expressions can either be simplified or evaluated. We have been looking at how to simplify expressions in the last few sections, and now we will see how to **evaluate** them.

EVALUATING EXPRESSIONS

To **evaluate** an expression is to find the numerical value of that expression when a given value is substituted for the variable(s). To substitute means to replace each variable with the specified number. Study the following examples.

EXAMPLE A

Evaluate $x+6$ when $x=5$.

$x+6$

$(5)+6$ Substitute 5 for x in the expression.

$=11$ Simplify.

EXAMPLE B

Evaluate $3x+8$ when $x=2$.

$3x+8$

$3(2)+8$ Substitute 2 for x in the expression.

$=6+8$ Simplify.

$=14$

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EVALUATING EXPRESSIONS

EXAMPLE C

Evaluate $4x + y$ when $x = 1$ and $y = 9$.

In this case there are two variables and we are given two values. It's important to substitute the right number for each variable to find the correct answer!

$4x + y$

$4(1) + 9$ Substitute 1 for x and 9 for y in the expression.

$= 4 + 9$ Simplify.

$= 13$

EXAMPLE D

Evaluate $2x + 4y + 5$ when $x = 3$ and $y = 2$.

$2x + 4y + 5$

$2(3) + 4(2) + 5$ Substitute 3 for x and 2 for y.

$= 6 + 8 + 5$ Simplify.

$= 14 + 5$

$= 19$

EVALUATING EXPRESSIONS

Extended Example 1a

Evaluate $4x+10$ when $x=5$.

END OF LESSON

Evaluate the expression for the given value.

$2(3k - 7)$; $k = 4$

Evaluate the expression for the given value.

$5(3t^2 - 7t)$; $t = 3$

Evaluate 8r + 5s for the given values of r and s.

$r = 5$ and $s = 7$

Evaluate the expression when $x = 2$ and $y = 3$.

$5x^4 - 7y^2$

Evaluate the expression when $m = 4$ and $n = 8$.

$3m^3 \div 2$