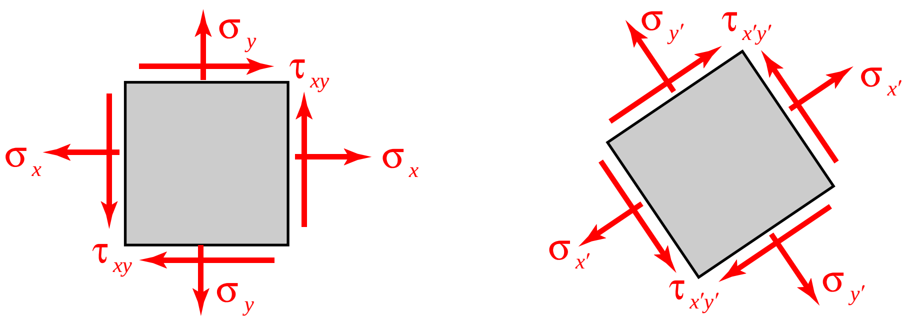
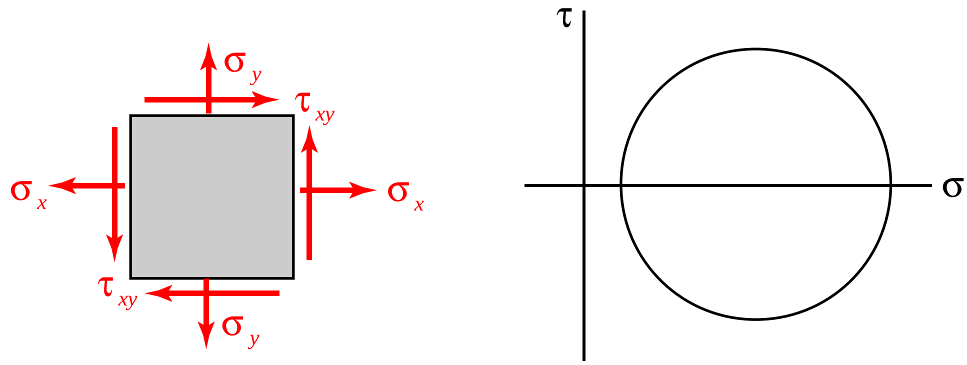
Chapter 7
Transformations of Stress and Strain

INTRODUCTION

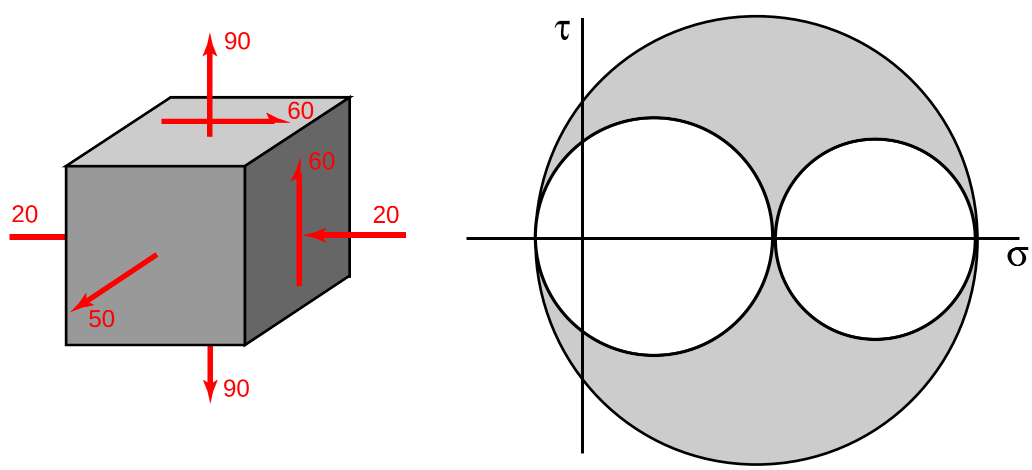
Transformation of Plane Stress



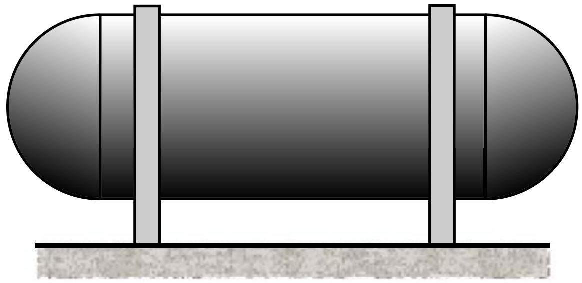
Mohr's Circle for Plane Stress



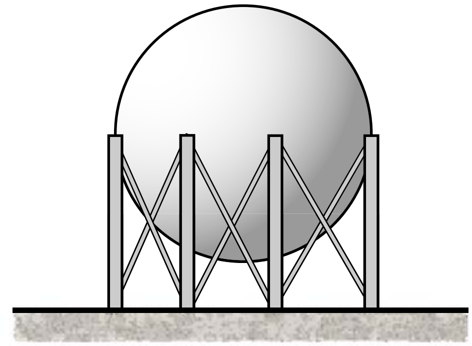
Application of Mohr's Circle to 3D Analysis



Stresses in Thin-Walled Pressure Vessels

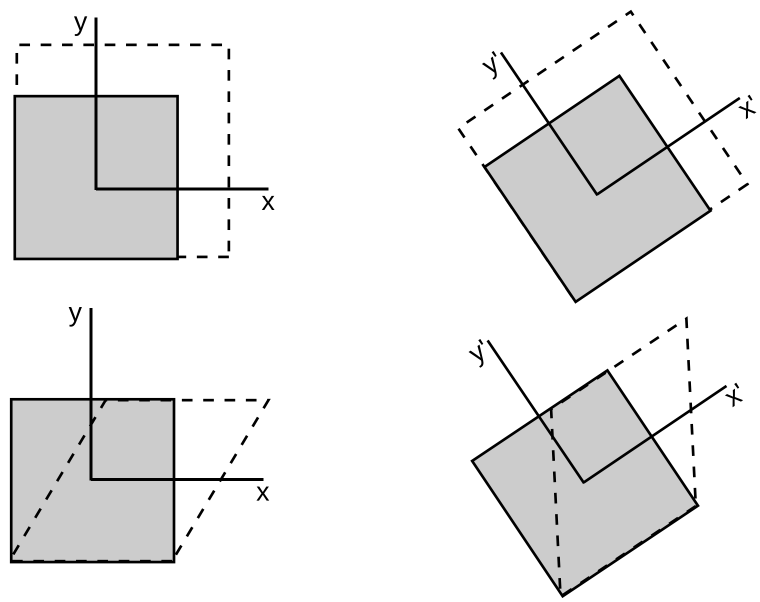


Cylindrical Pressure Vessel

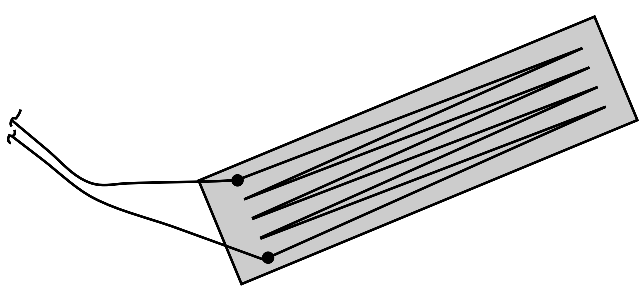


Spherical Pressure Vessel

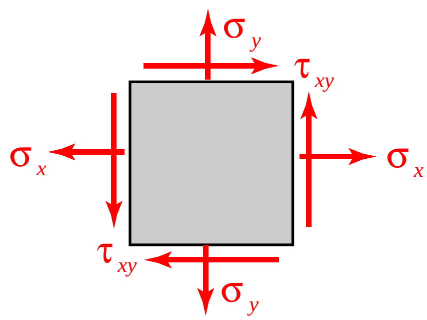
Transformation of Plane Strain

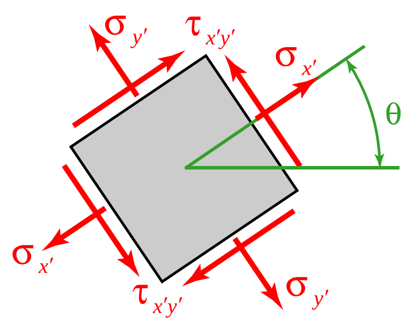


Measurements of Strain; Strain Rosette



TRANSFORMATION OF PLANE STRESS





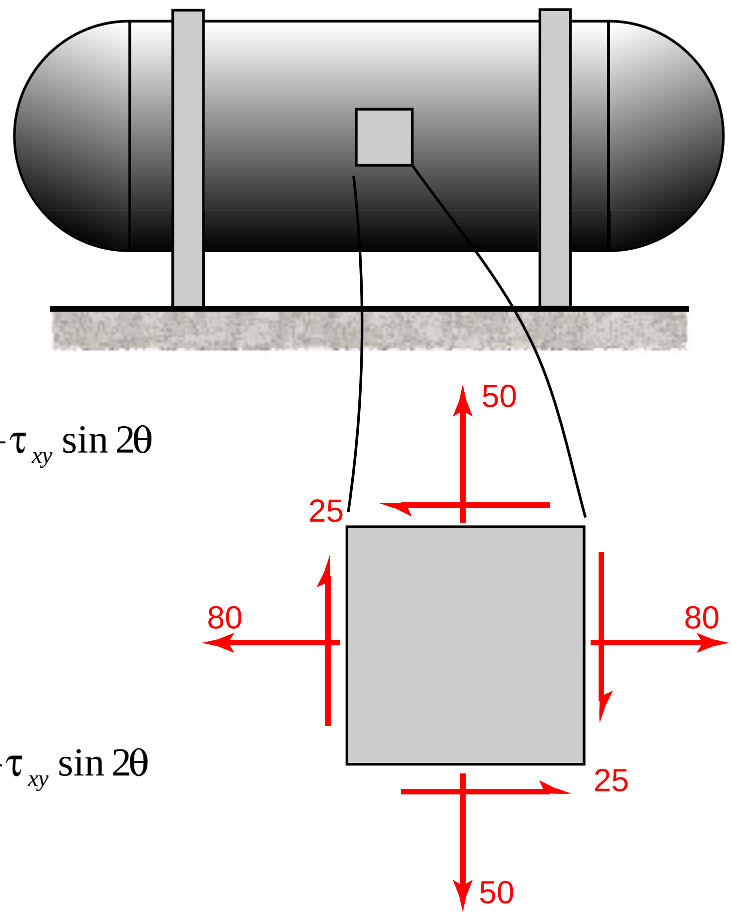
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Example

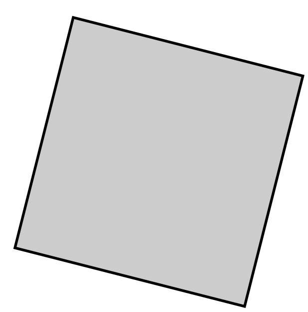
The state of stress at a point on the surface of a pressure vessel is represented on the element shown. Represent the state of stress at the point on another element that is orientated 30° clockwise from the position shown. Units: MPa.



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

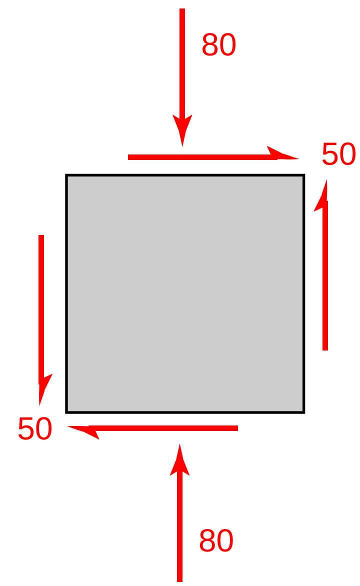
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



Example

Determine the stresses on a surface that is rotated (a) 30° clockwise, (b) 15° counterclockwise. Units: MPa



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

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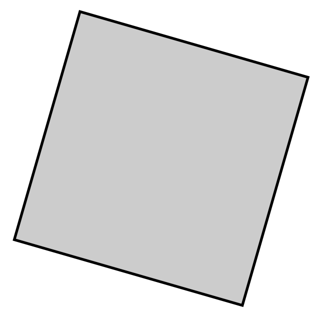
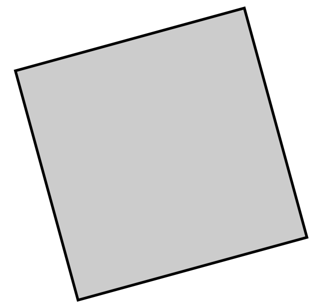


$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

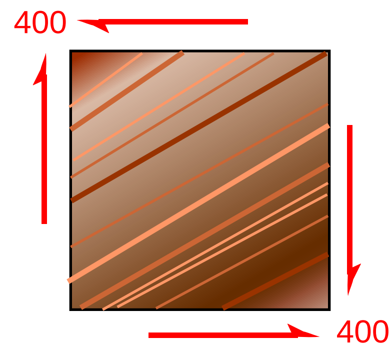


$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

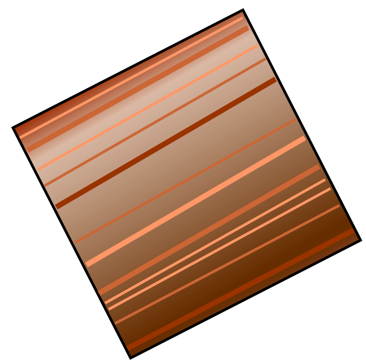
$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Example

For the piece of wood, determine the in-plane shear stress parallel to the grain, (b) the normal stress perpendicular to the grain. The grain is rotated 30° from the horizontal. Units: psi

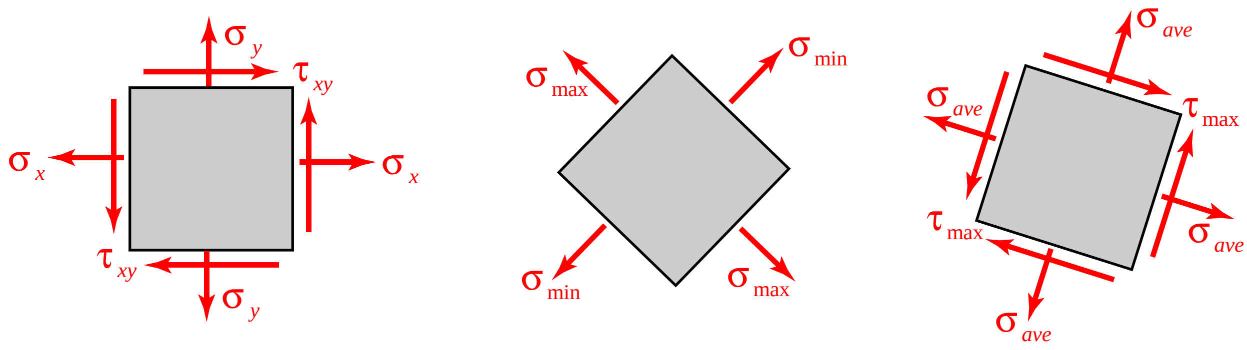


$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$



$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

PRINCIPAL STRESSES: MAXIMUM SHEARING STRESS



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

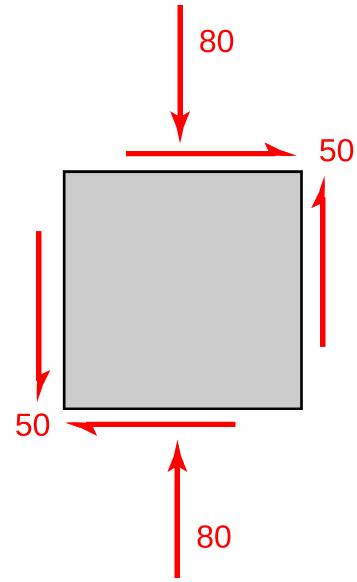
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\theta_s = \theta_p \pm 45°$$

$$\tau_{\max} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa



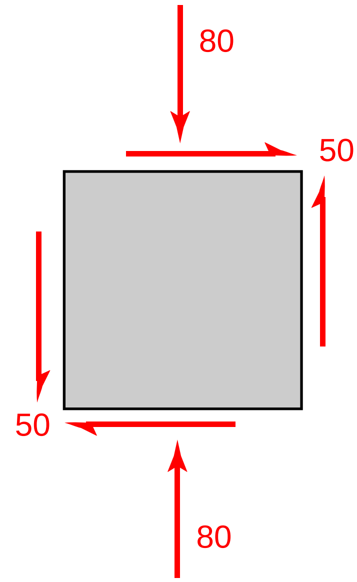
$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

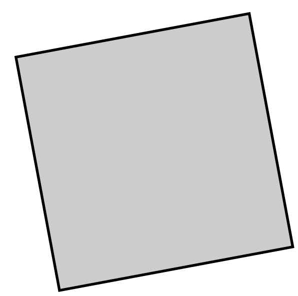
Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Sketch the resulting stresses on the element and the corresponding orientation. Units: MPa

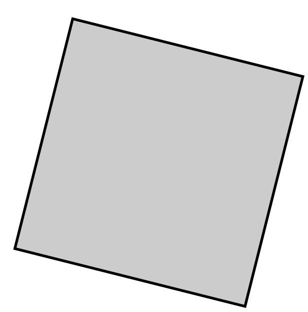


$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$



$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

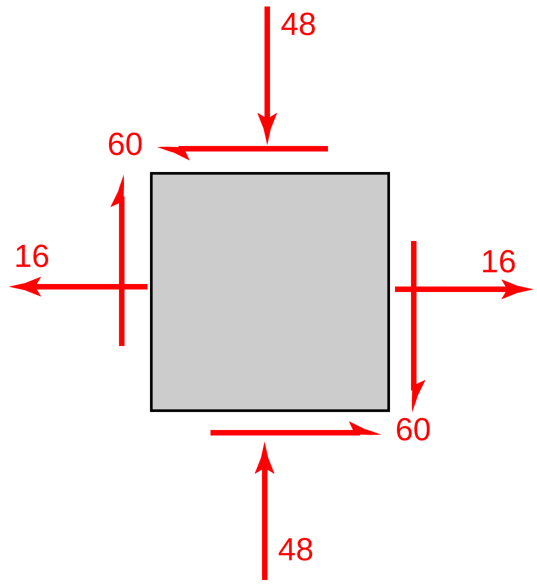


$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa



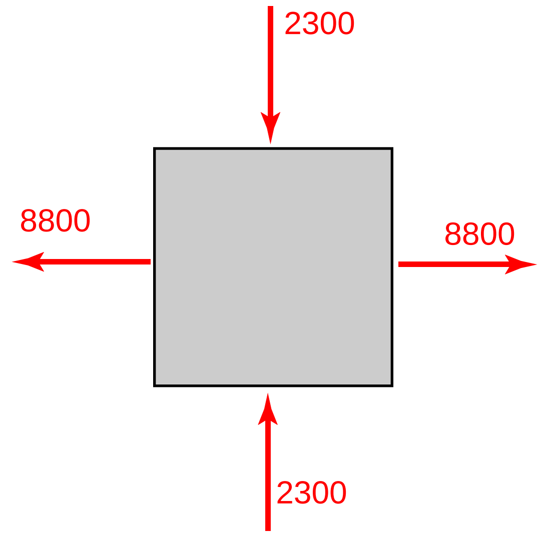
$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Example

Determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: psi



$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

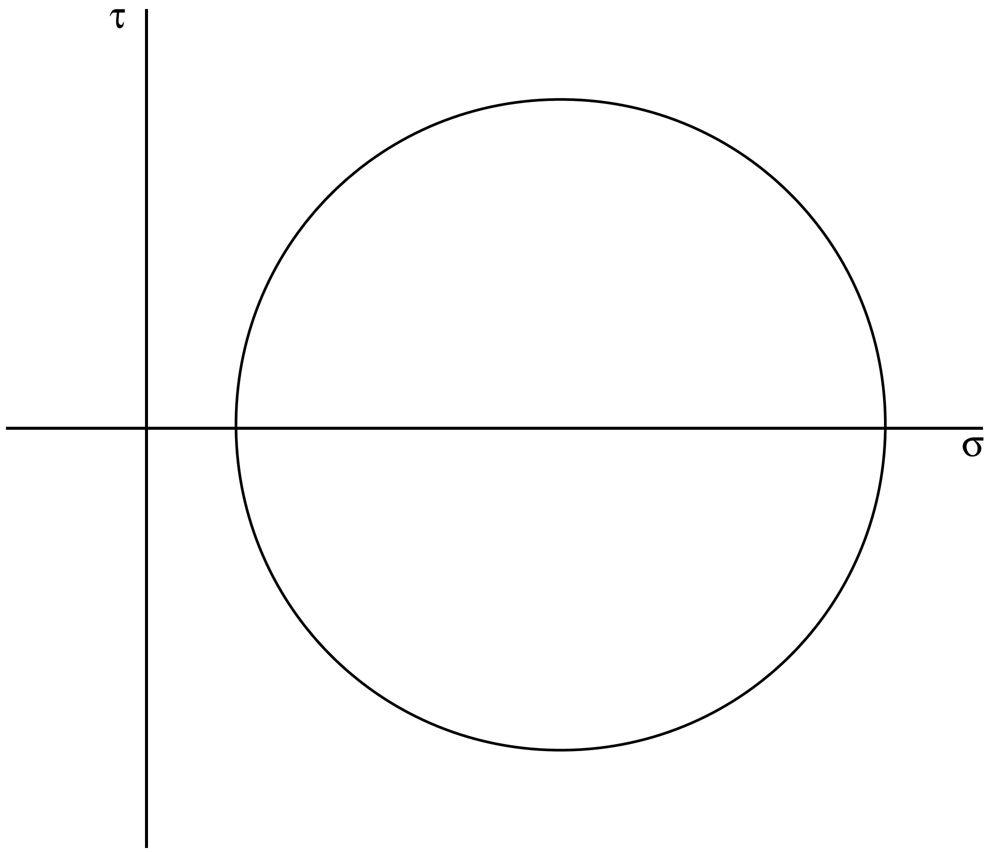
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

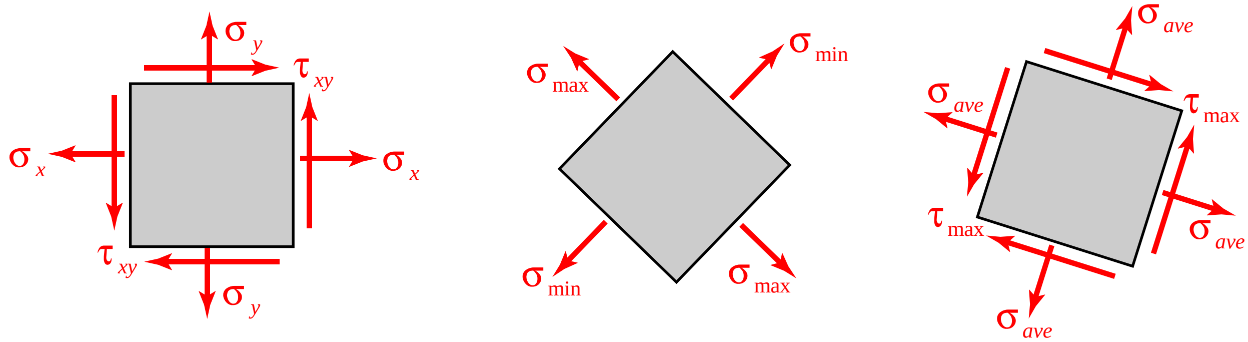
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

MOHR'S CIRCLE FOR PLANE STRESS

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

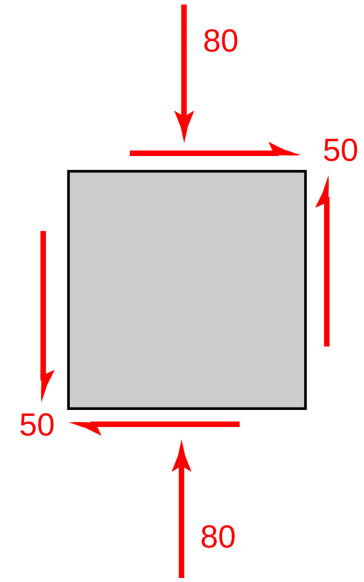
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$





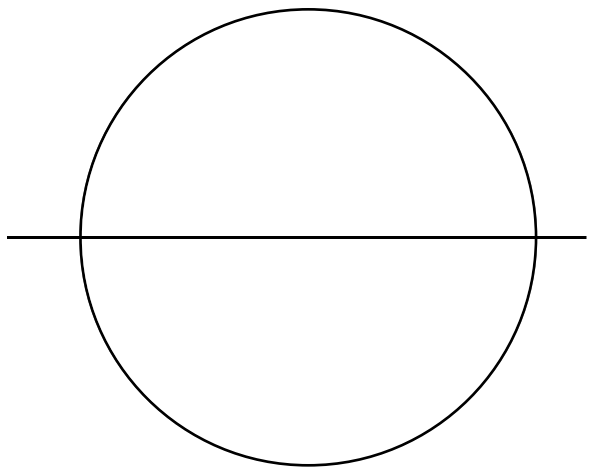
Example

Using Mohr's circle, determine the stresses on a surface that is rotated 30° clockwise. Units: MPa



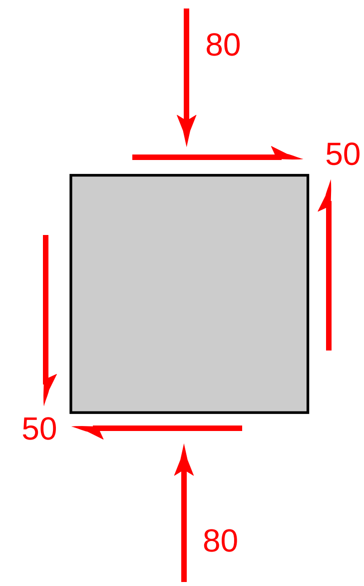
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



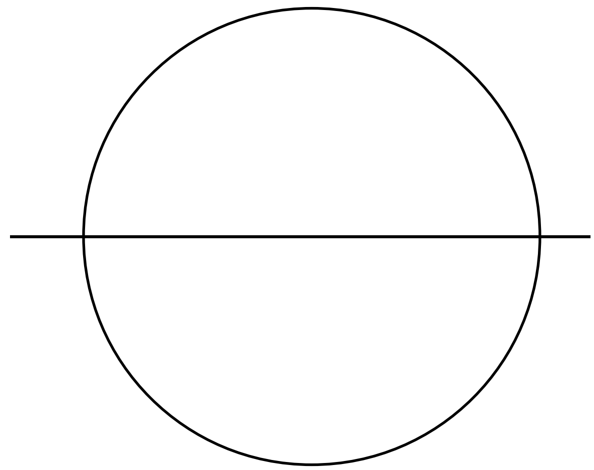
Example

Using Mohr's circle, determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa



$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



Example

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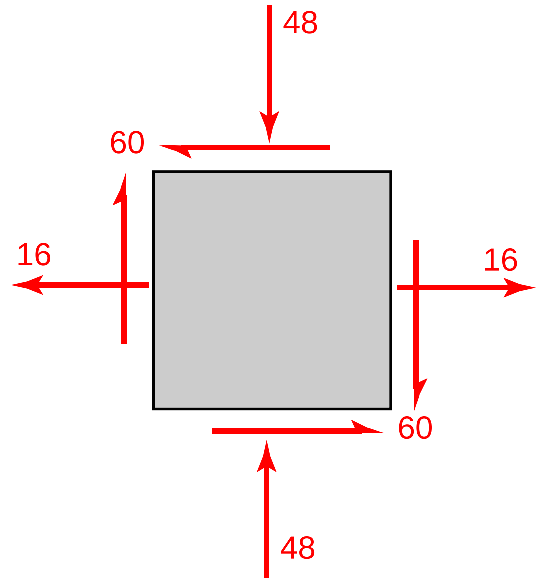
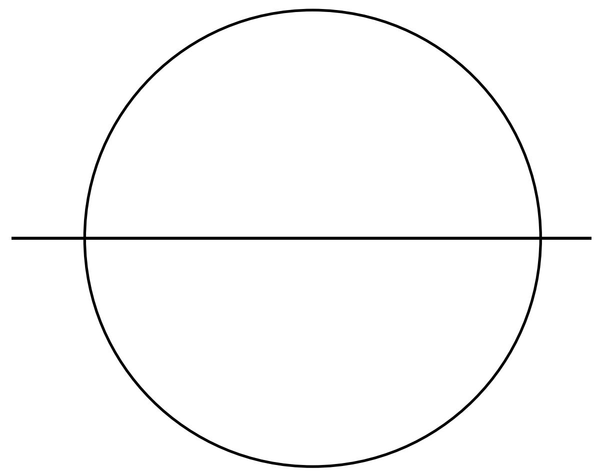
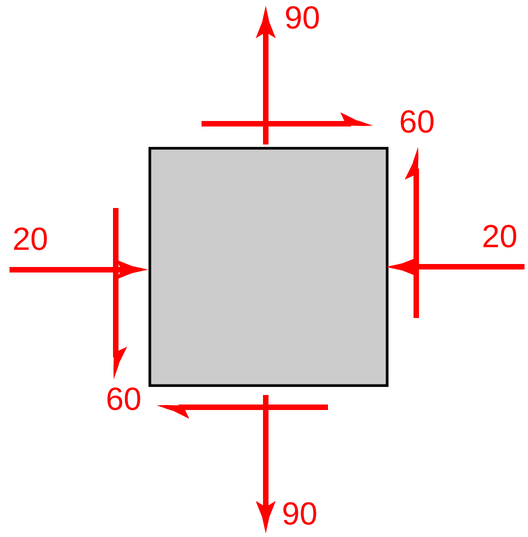


$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



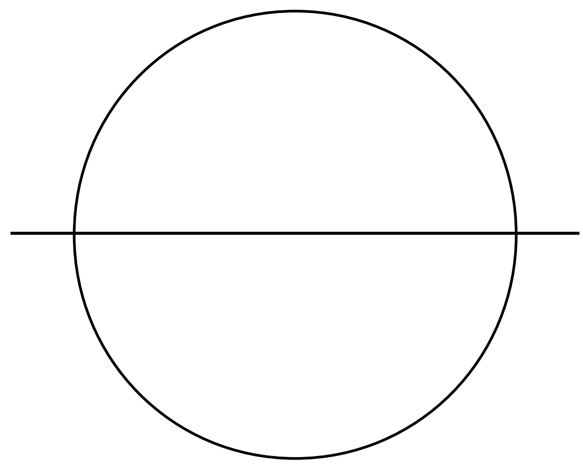
Example

Using Mohr's circle, determine the (a) principal stresses, (b) maximum in-plane shear stress and the associated normal stress. Units: MPa



$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

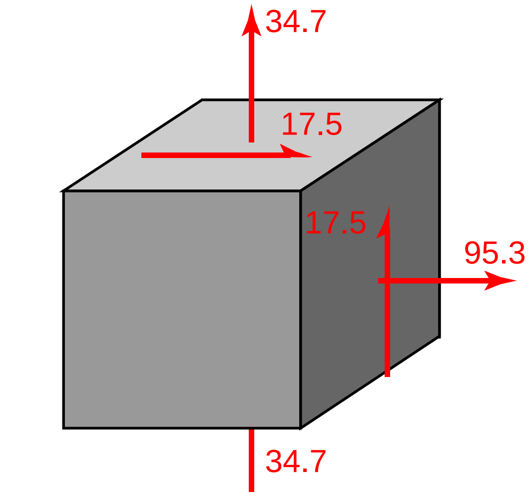
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

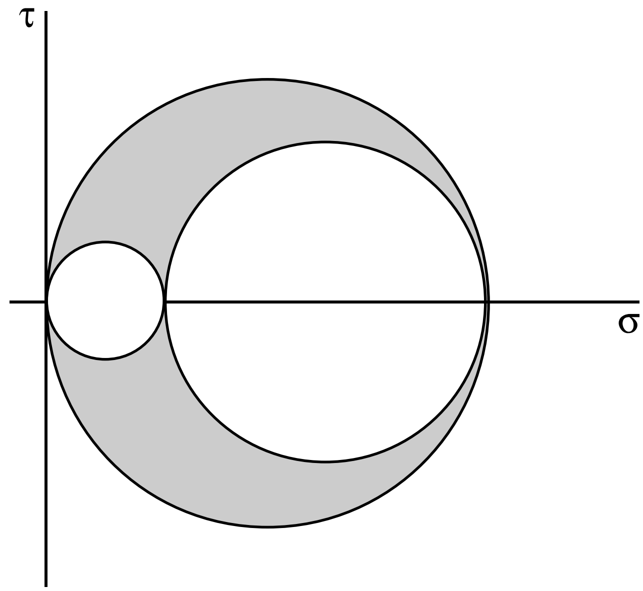


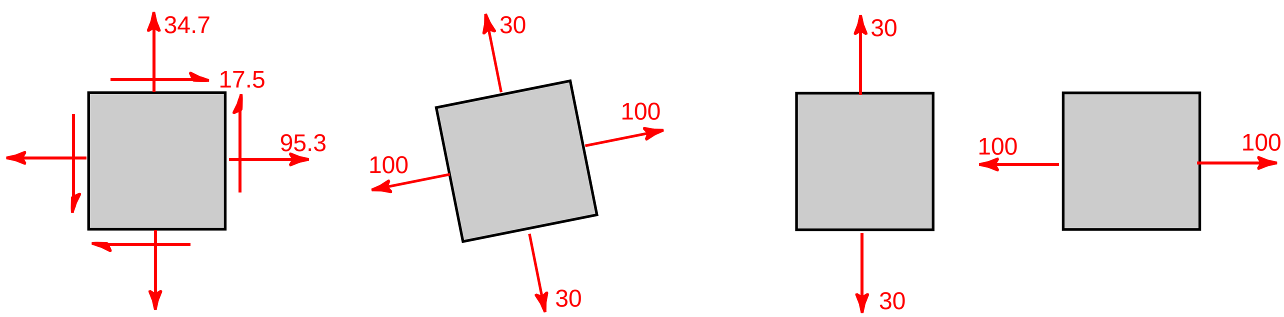
3D APPLICATIONS OF MOHR'S CIRCLE

Example

Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa

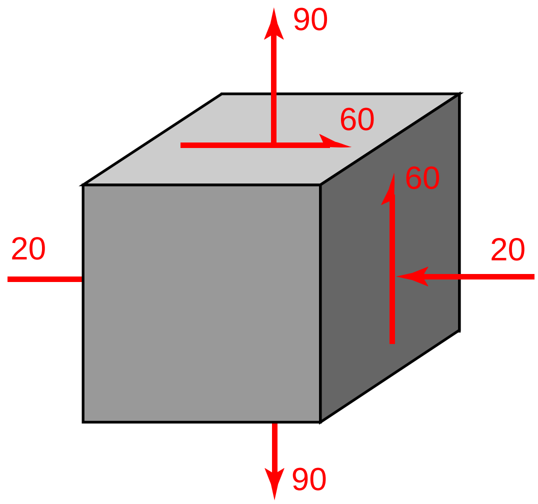


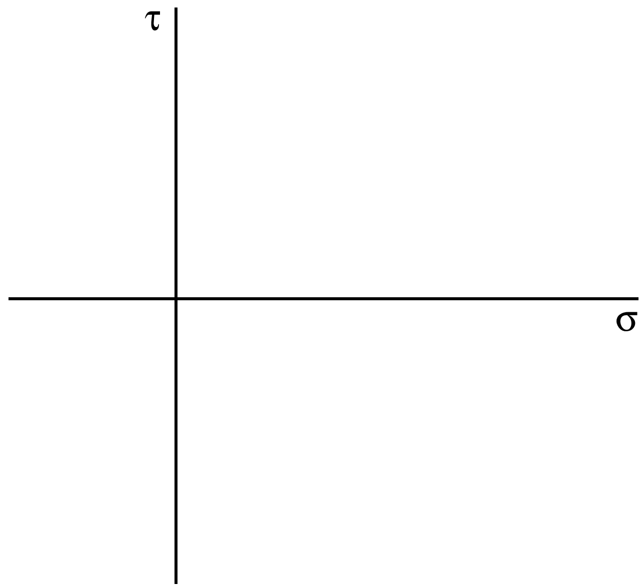


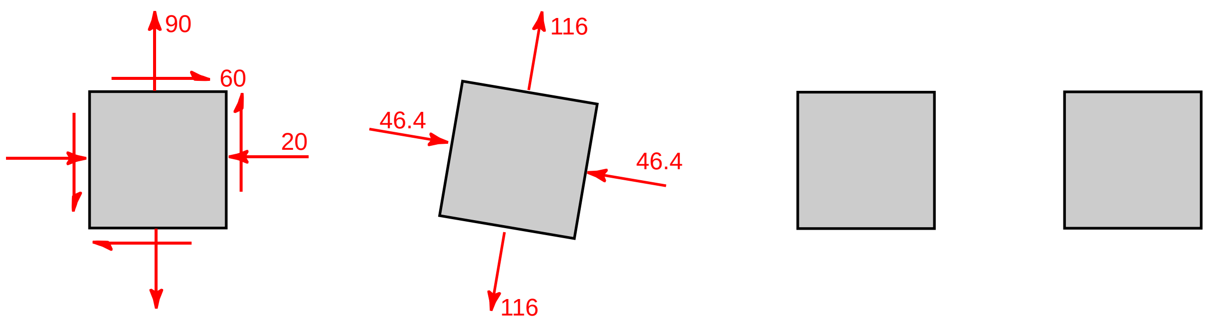


Example

Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa

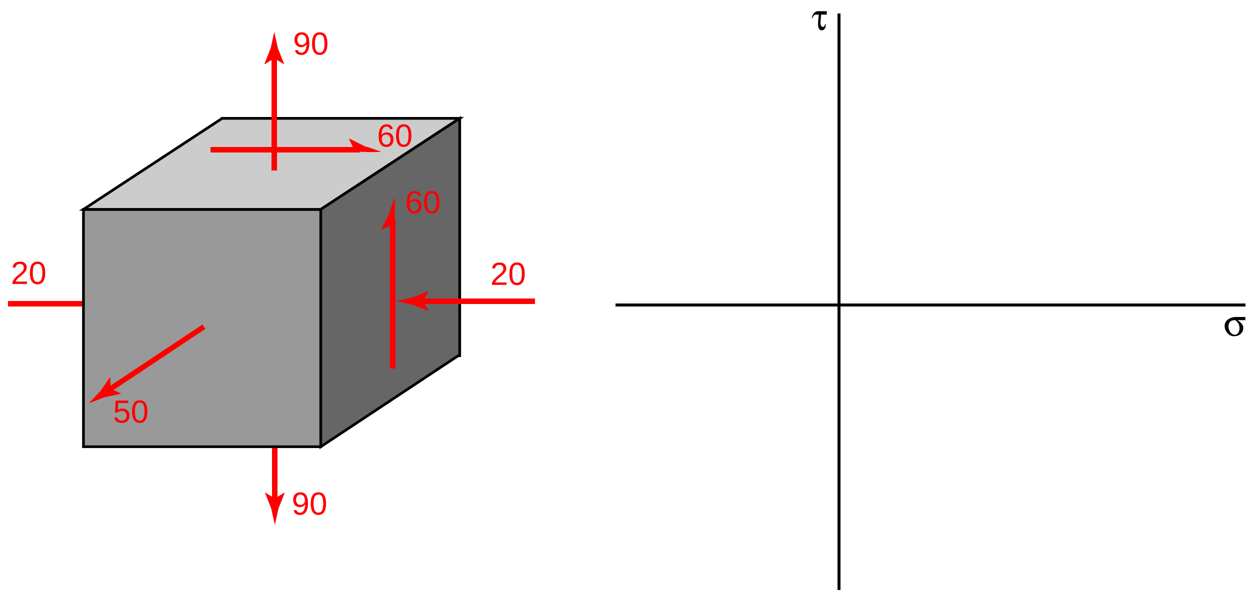


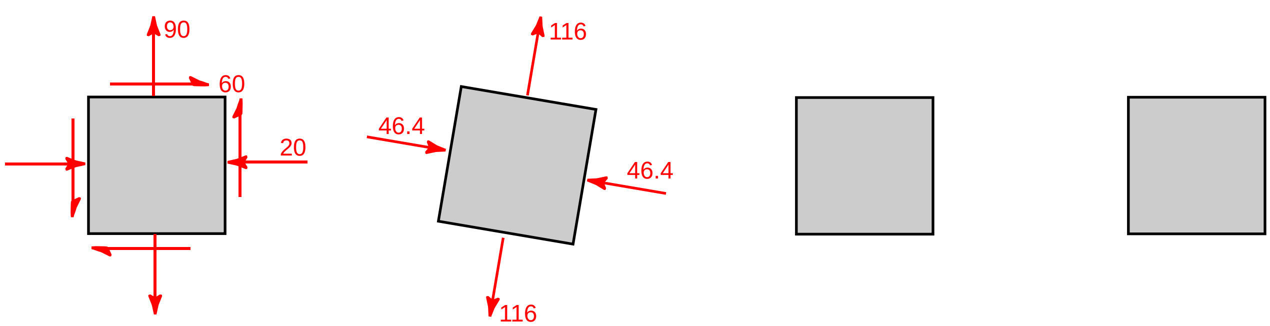




Example

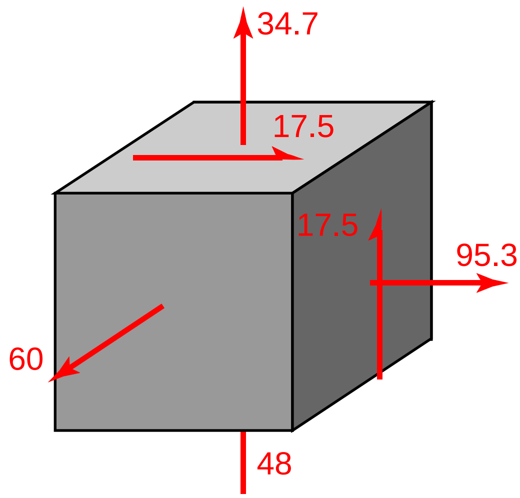
Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa

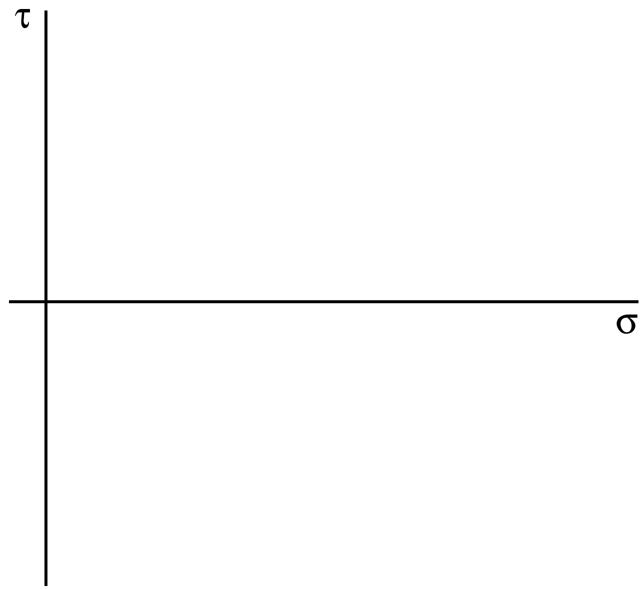


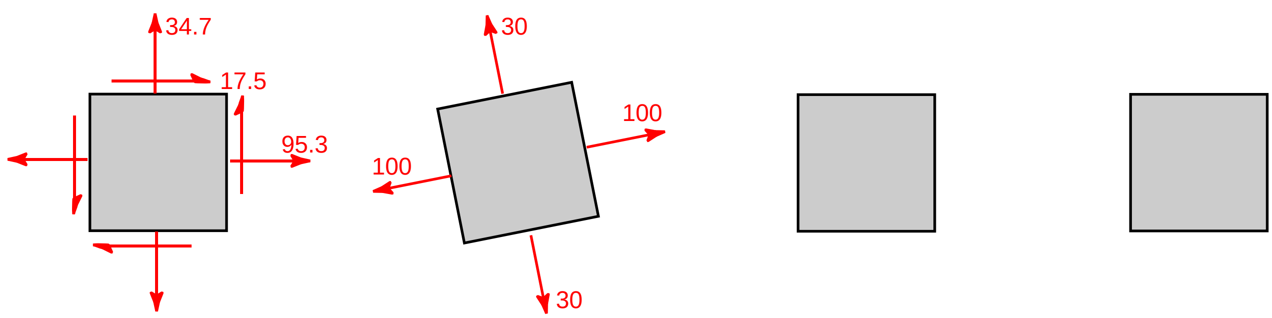


Example

Using Mohr's circle, determine the maximum shear stress.
(Hint: Consider both in-plane and out-of-plane shearing stresses).
Units: MPa

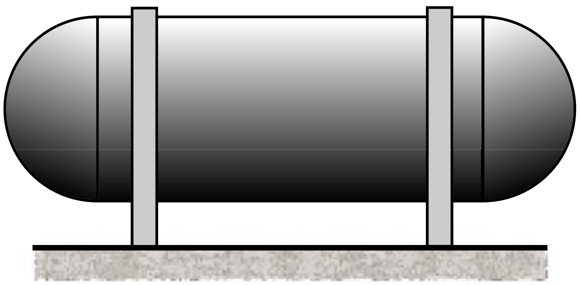




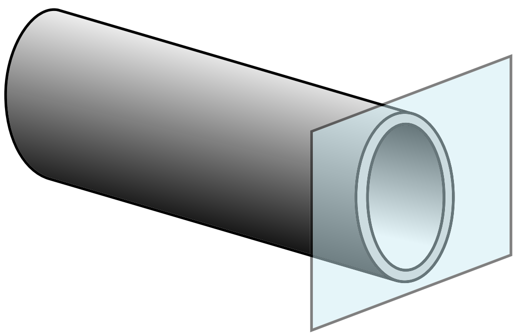


STRESSES IN THIN-WALLED PRESURE VESSELS

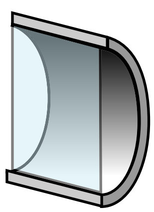
Cylindrical Pressure Vessels



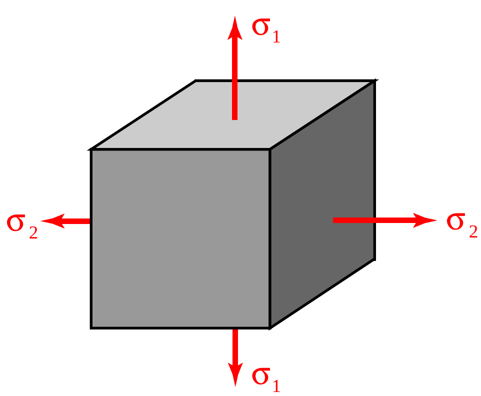
Cylindrical Pressure Vessel

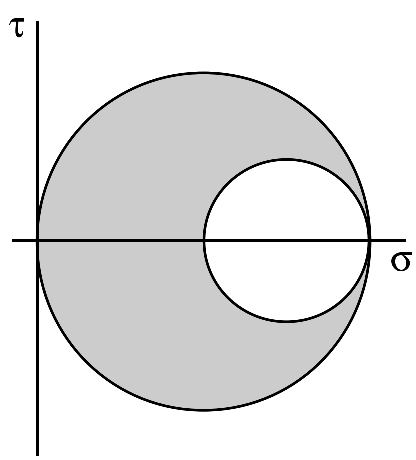


$$\sigma_2 = \frac{pr}{2t}$$

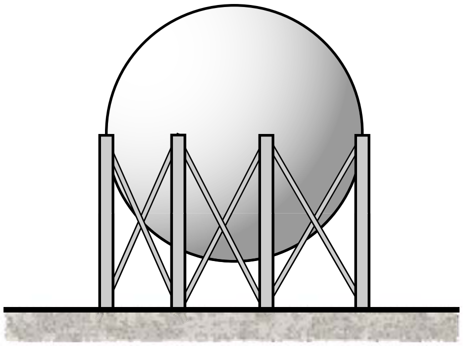


$$\sigma_1 = \frac{pr}{t}$$

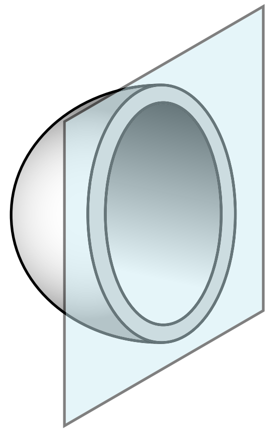




Spherical Pressure Vessels

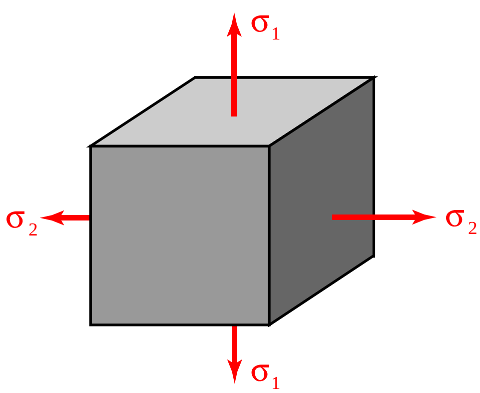


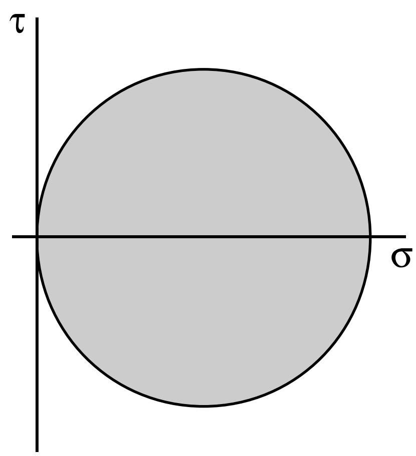
Spherical Pressure Vessel



$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

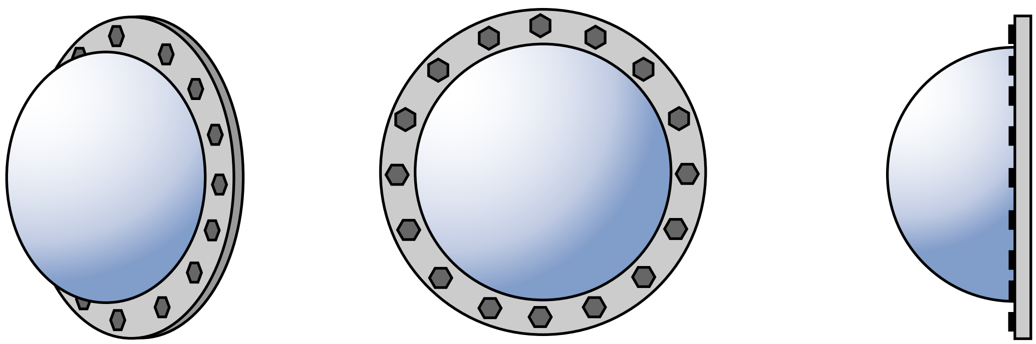
$$\tau_{max} = \frac{pr}{4t}$$





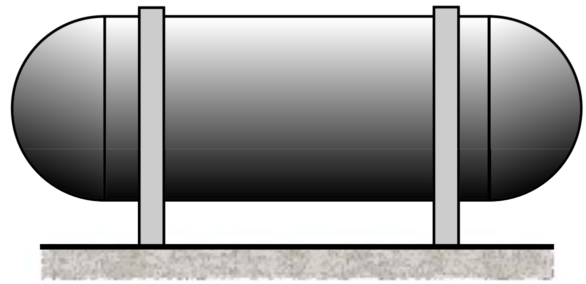
Example

The viewport is attached to the submersible with 16 bolts and has an internal air pressure of 95 psi. The viewport material used has an allowable maximum tensile and shear stress of 700 and 400 psi respectively. The inside diameter of the viewport is 18". Determine the force in each bolt and the wall thickness of the viewport.
Units: in



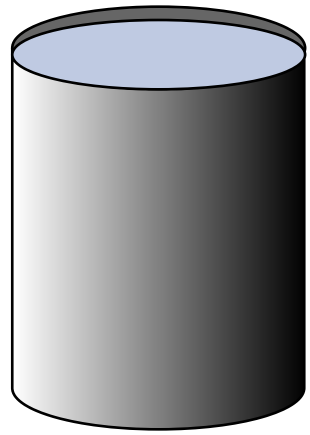
Example

The pressure vessel has an inside diameter of 2 meters and an internal pressure of 3 MPa. If the spherical ends have a wall thickness of 10 mm and the cylindrical portion has a wall thickness of 30 mm, determine the maximum normal and shear stress in each section.



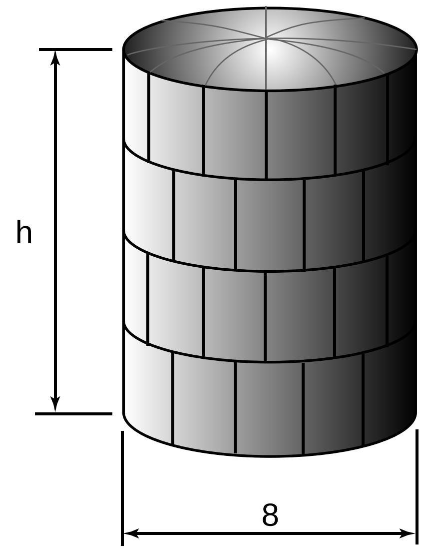
Example

The open water tank has an inside diameter of 50 ft and is filled to a height of 60 ft. Determine the minimum wall thickness due to the water pressure only if the allowable tensile stress is 24 ksi.



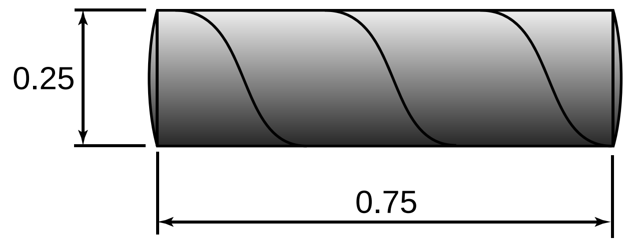
Example

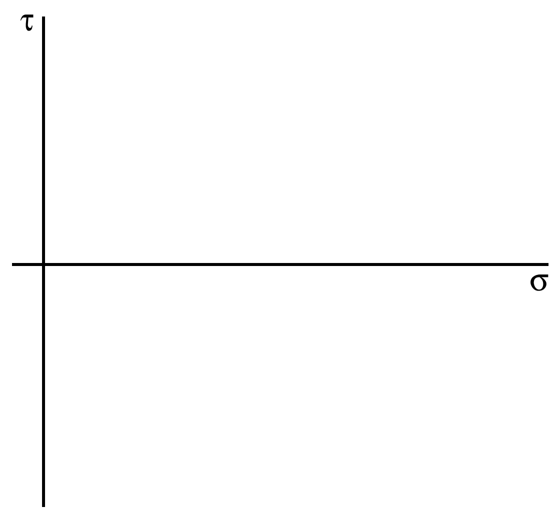
18 mm thick plates are welded as shown to form the cylindrical pressure tank. Knowing that the allowable normal stress perpendiculer to the weld is 60 MPa, determine the maximum allowable internal pressure and the height of the tank. Units: m.



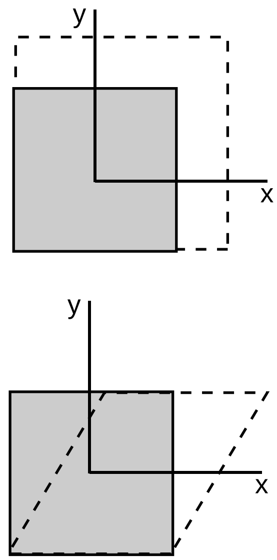
Example

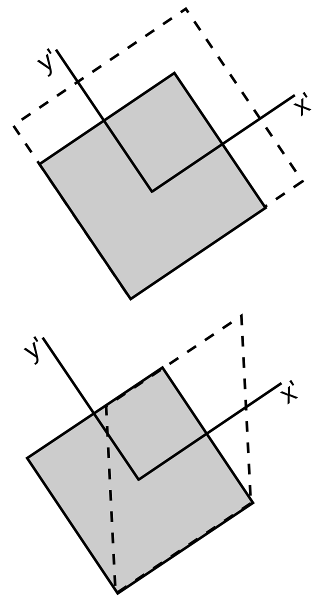
The cylindrical portion of the compressed air tank is made of 10 mm thick plate welded along a helix forming an angle of 45°. Knowing that the allowable stress normal to the weld is 80 MPa, determine the largest gage pressure that can be used in the tank. Units: m





TRANSFORMATION OF PLANE STRAIN





$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

PRINCIPAL STRAINS

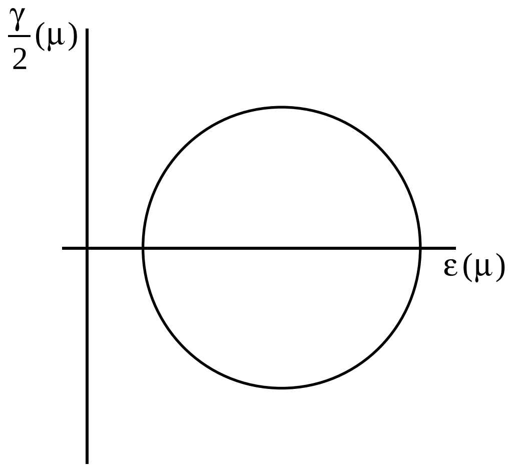
$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_{\max,\min} = \varepsilon_{a,b} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_c = -\frac{\upsilon}{1-\upsilon}(\varepsilon_{\max} + \varepsilon_{\min})$$

$$\gamma_{\max} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

MOHR'S CIRCLE FOR PLANE STRAIN



$$\varepsilon_{ave} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_{\max,\min} = \varepsilon_{a,b} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\max} = \varepsilon_{\max} - \varepsilon_{\min}$$

Example

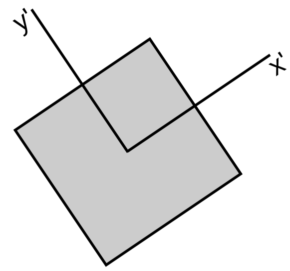
Given the strains below, determine the strains if the element is rotated 30° counterclockwise.

$$\varepsilon_x = -300\mu$$

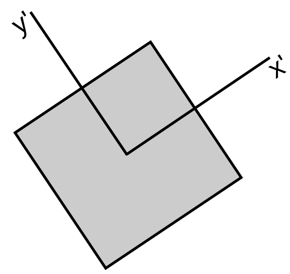
$$\varepsilon_y = -200\mu$$

$$\gamma_{xy} = +175\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$



$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$



$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

Example

Given the strains below, determine (a) the direction and magnitude of the principal strains, (b) the maximum in-plane shearing strain, (c) the maximum strain. Assume plane stress.

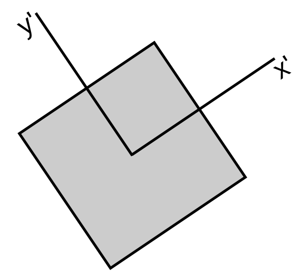
$$\upsilon = 1/3$$

$$\varepsilon_x = -300\mu$$

$$\varepsilon_y = -200\mu$$

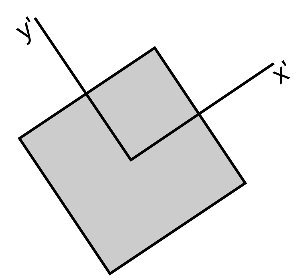
$$\gamma_{xy} = +175\mu$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$



$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

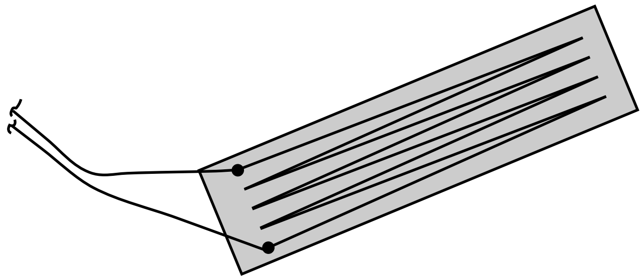
$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

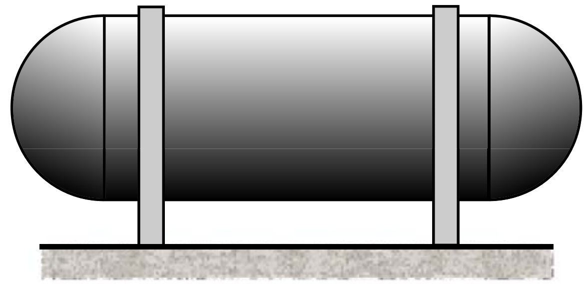


$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_c = -\frac{\upsilon}{1-\upsilon}(\varepsilon_1 + \varepsilon_2)$$

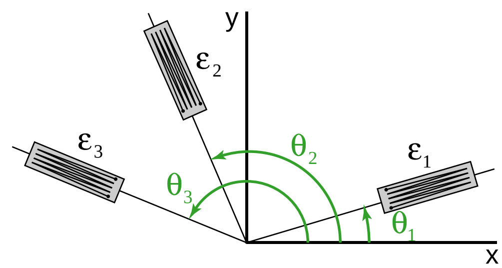
MEASUREMENTS OF STRAIN; STRAIN ROSETTE





Cylindrical Pressure Vessel

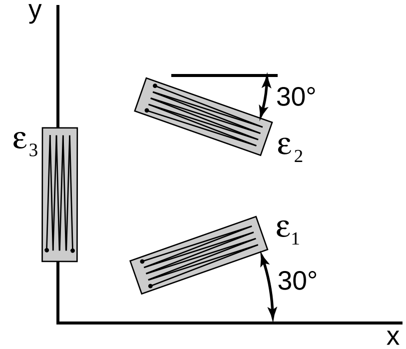
$$\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1$$
$$\varepsilon_2 = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2$$
$$\varepsilon_3 = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3$$



Example

Given the following strains, determine (a) the in-plane principal strains, (b) the in-plane maximum shearing strain.

$$\varepsilon_1 = +600\mu$$
$$\varepsilon_2 = +450\mu$$
$$\varepsilon_3 = -175\mu$$



$$\varepsilon_1 = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1$$

$$\varepsilon_2 = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2$$

$$\varepsilon_3 = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3$$

$$\varepsilon_{\max,\min} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\gamma_{\max} = 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Example

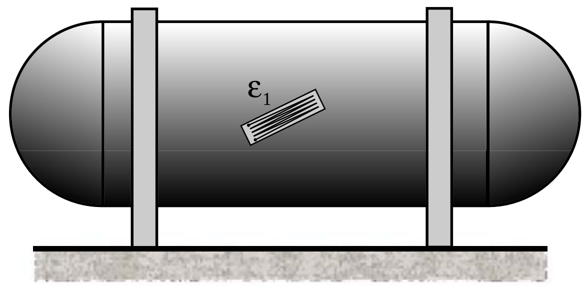
Given the strain measurements below for the 30" diameter, 0.25" thick tank, determine the gage pressure, (b) the principal stresses and the maximum in-plane shearing stress.

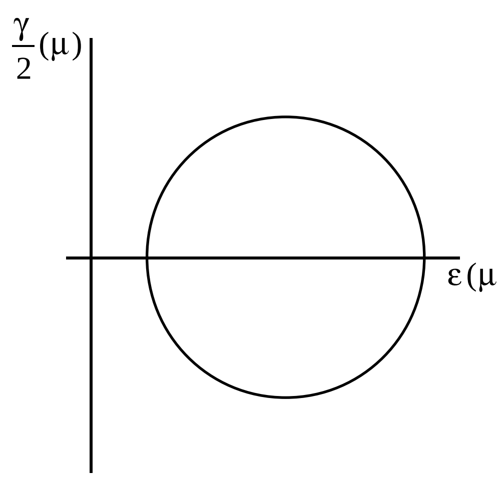
$$\varepsilon_1 = +160\mu$$

$$\upsilon = 0.3$$

$$\theta = 30°$$

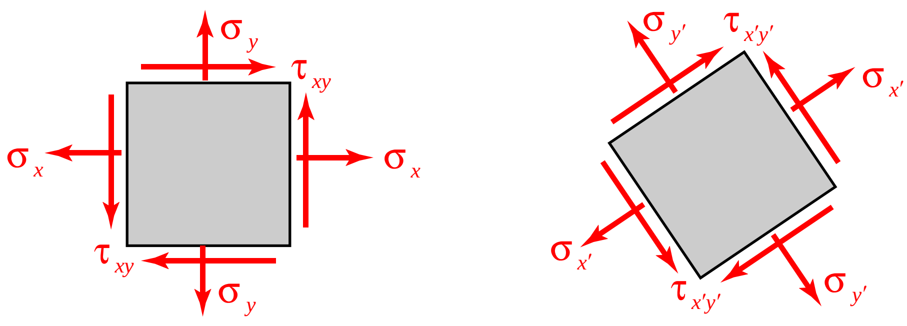
$$E = 29x10^{6\,psi}$$



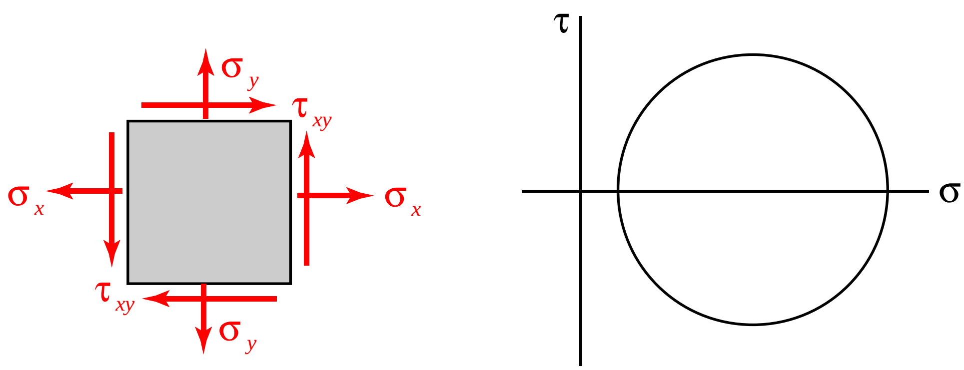


SUMMARY

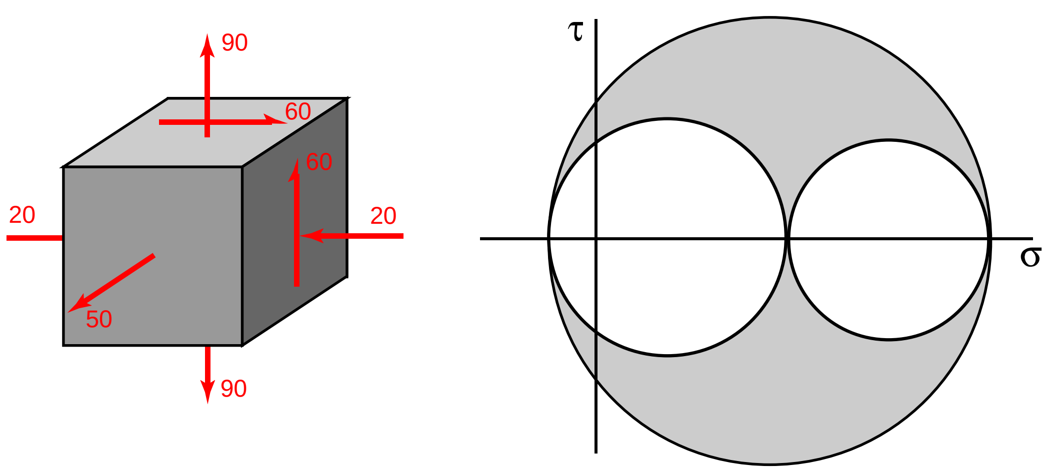
Transformation of Plane Stress



Mohr's Circle for Plane Stress

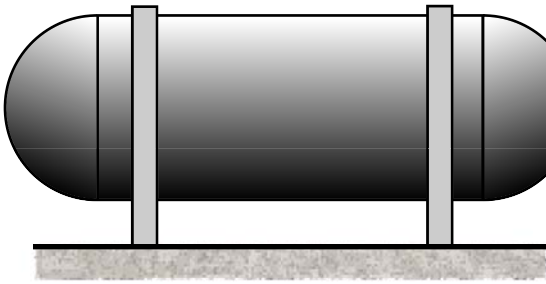


Application of Mohr's Circle to 3D Analysis

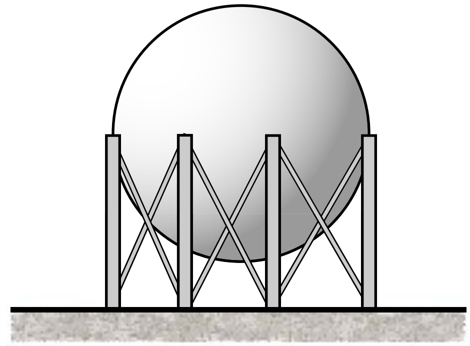


SUMMARY

Stresses in Thin-Walled Pressure Vessels

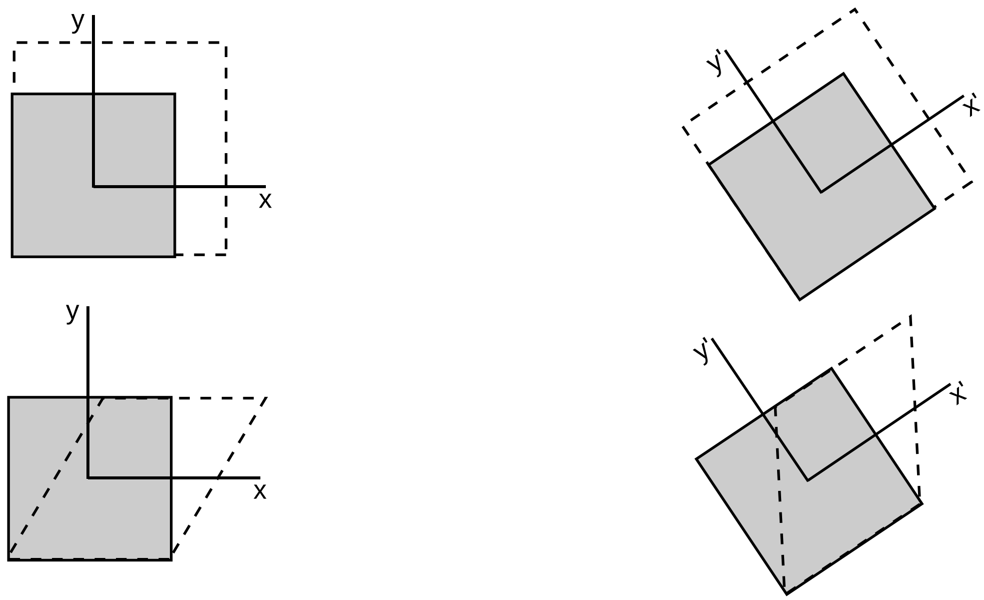


Cylindrical Pressure Vessel

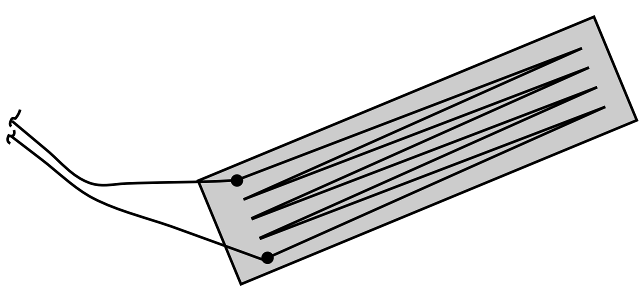


Spherical Pressure Vessel

Transformation of Plane Strain



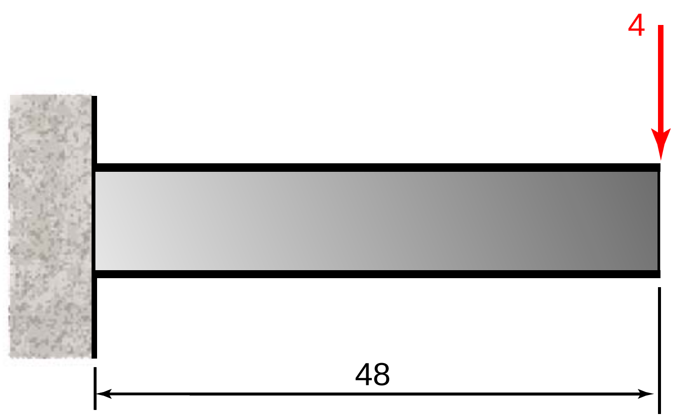
Measurements of Strain; Strain Rosette

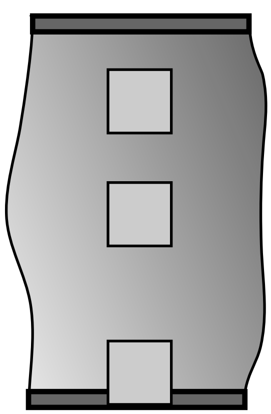


Principal Stresses under a Given Loading

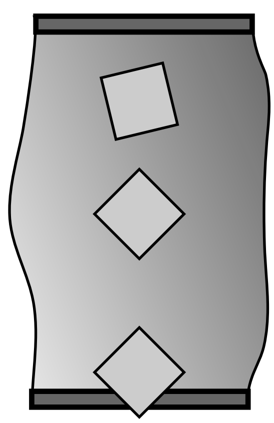
INTRODUCTION

PRINCIPAL STRESSES IN A BEAM

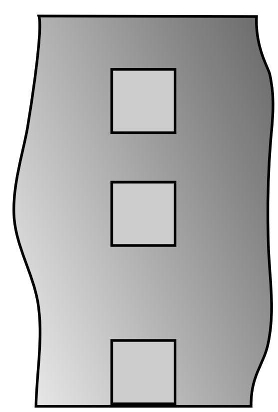




Wide Flange Stresses



Principal Wide Flange Stresses

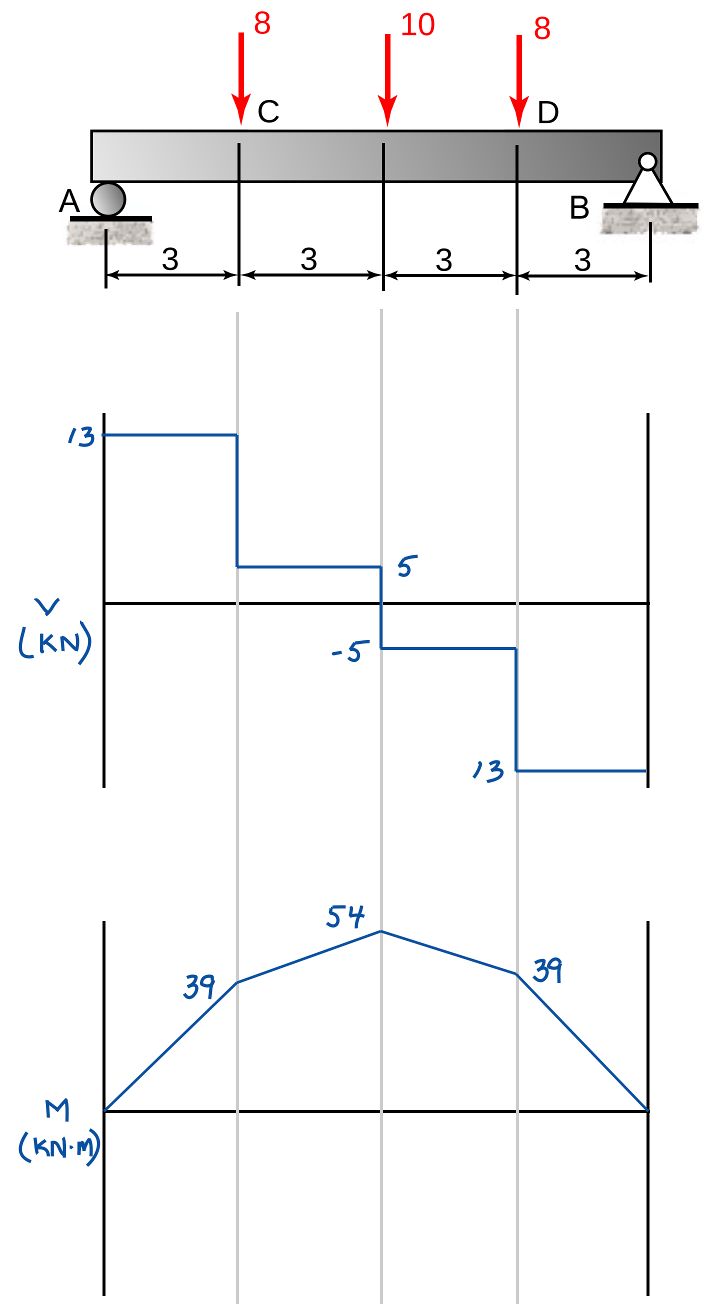


Rectangular Cross-section Stresses

Example

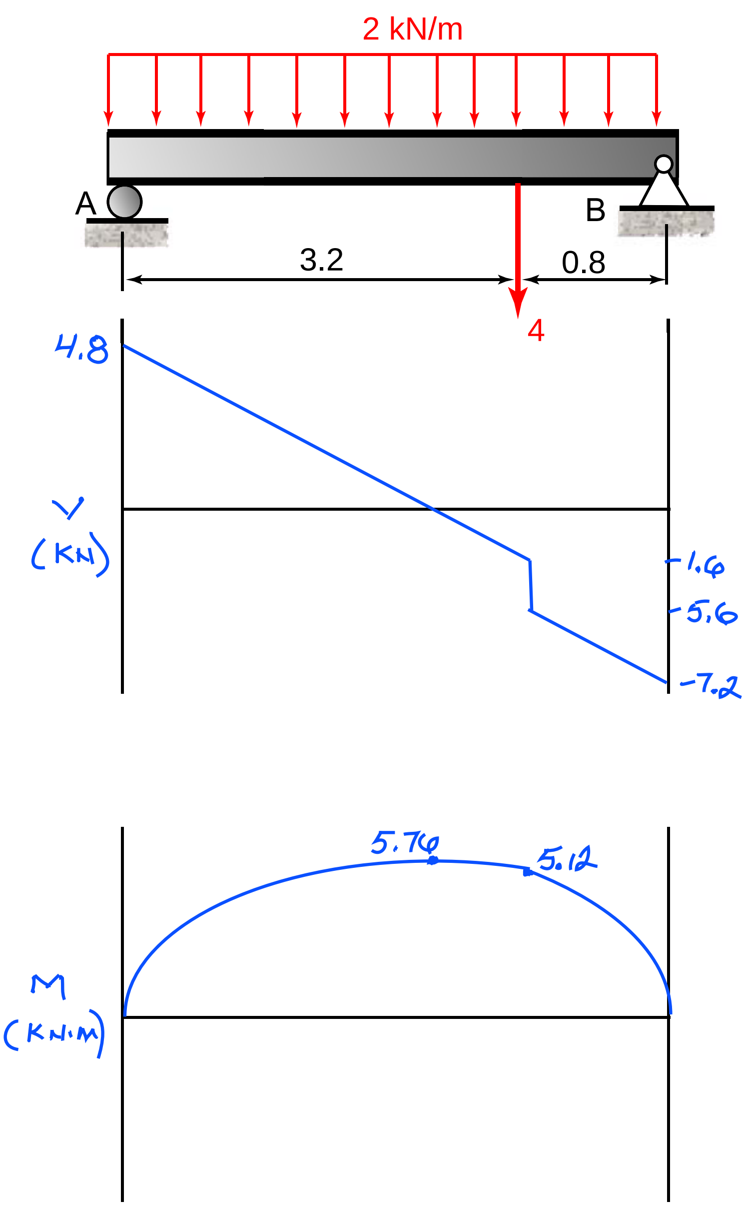
(a) Knowing that the allowable normal stress is 80 MPa and the allowable shear stress is 50 MPa, determine the height of the rectangular section if the width is 100 mm.

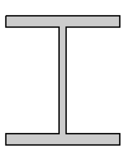
Units: kN, m

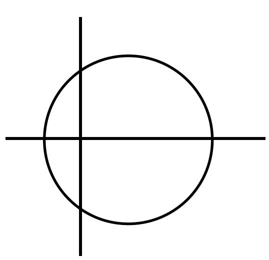


Example

(a) Knowing that the allowable normal stress is 80 MPa and the allowable shear stress is 50 MPa, select the most economical wide-flange shape that should be used to support the loading shown.
(b) Determine the principal stresses at the junction between the flange and web on a section just to the right of the 4 kN load. Units: kN, m

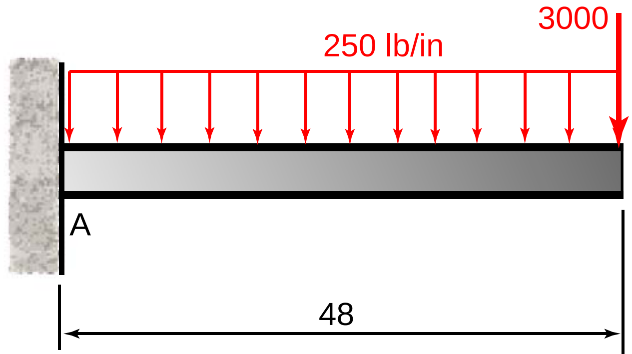


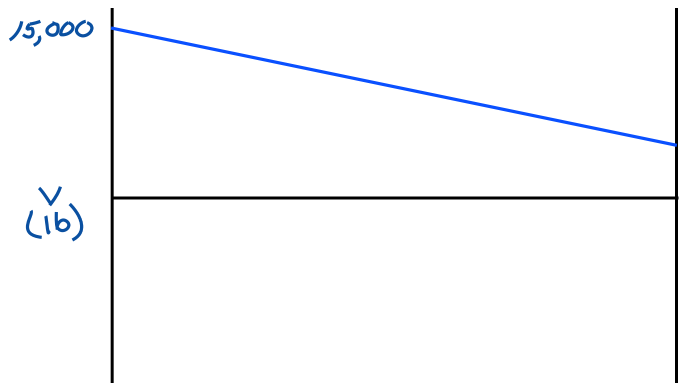


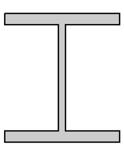


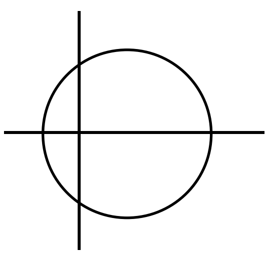
Example

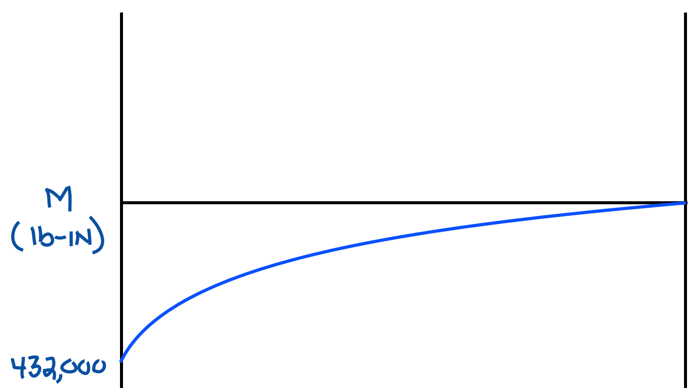
(a) Knowing that the allowable normal stress is 24 ksi and the allowable shear stress is 15 ksi, select the most economical W8 wide-flange shape that should be used to support the loading shown.
(b) Determine the principal stresses at the junction between the flange and web. Units: lb, in.







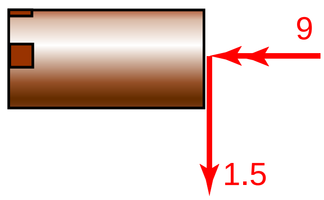


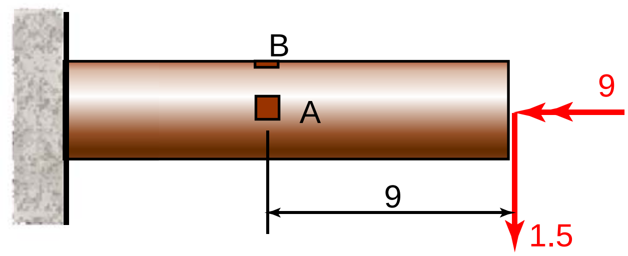


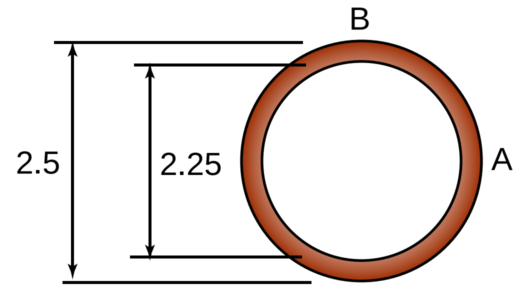
STRESSES UNDER COMBINED LOADINGS

Example

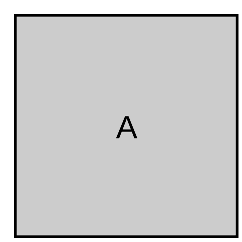
Determine the stresses at A and B. Units: k, k-in, in.

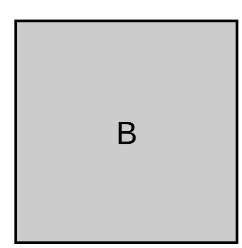






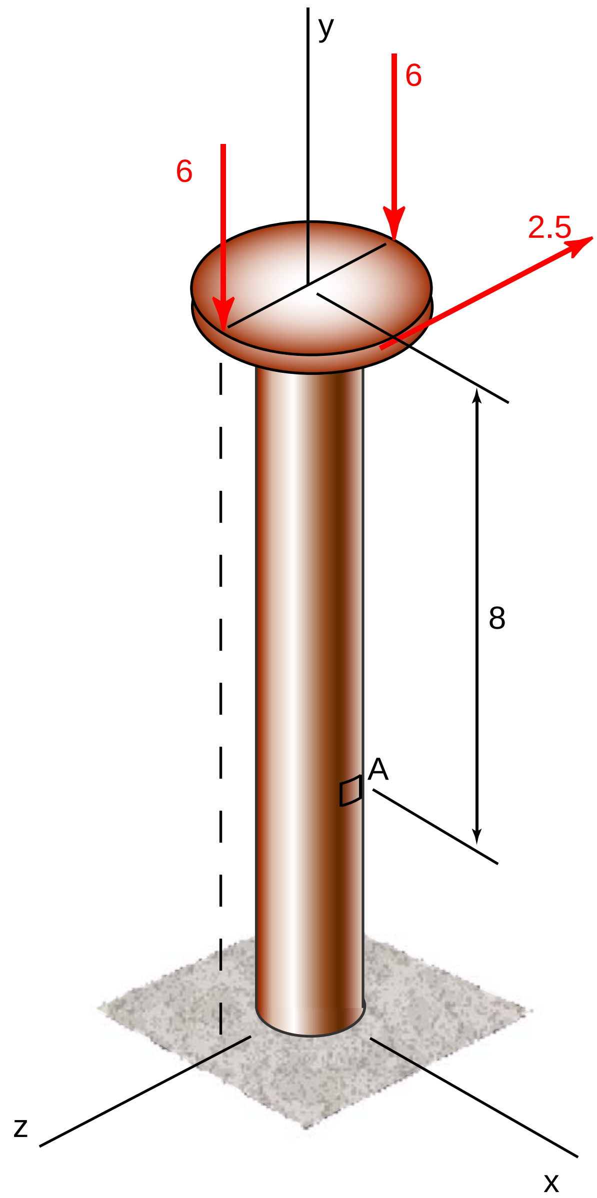
Cross-section

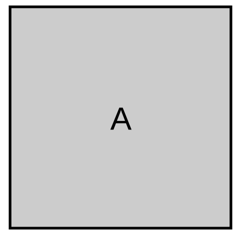




Example

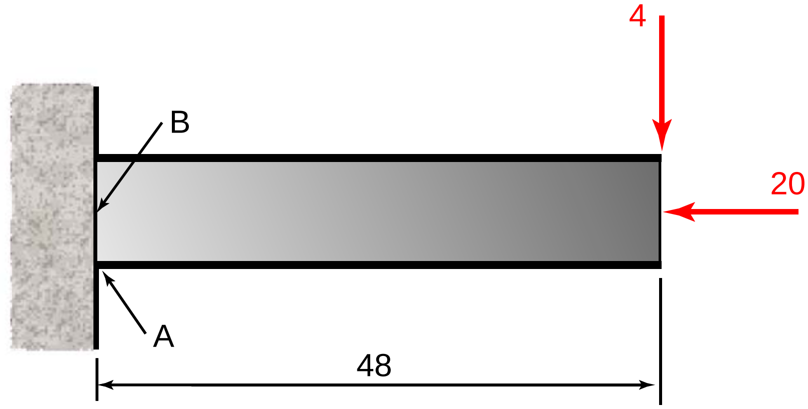
Determine the stresses at A. The disk has a diameter of 4" and the solid shaft has a diameter of 1.8". Units: kips, in.





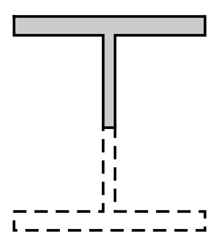
Example

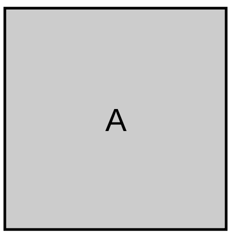
Determine the stresses at points A and B. The beam is a W6x20.
Units: kips, in.

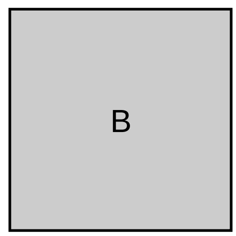


W6x20

Area, $A = 5.87^{in^2}$
Depth, $d = 6.20^{in}$
Flange Width, $b_f = 6.02^{in}$
Flange Thickness, $t_f = 0.365^{in}$
Web Thickness, $t_w = 0.260^{in}$
$I_x = 41.4^{in^4}$
$I_y = 13.3^{in^4}$
$S_x = 13.4^{in^3}$
$S_y = 4.41^{in^3}$



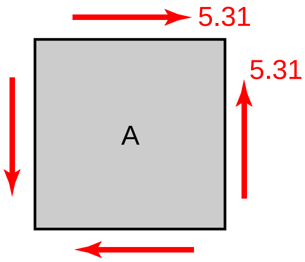


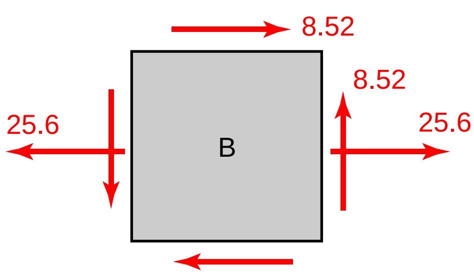


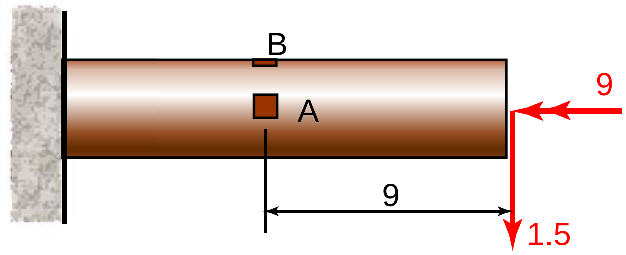
Example

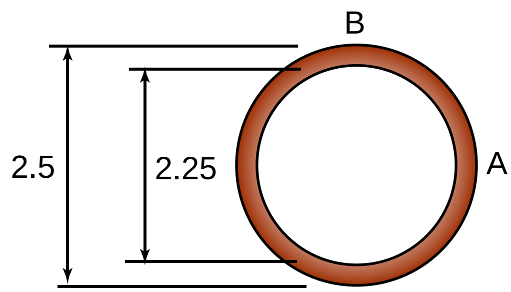
Determine the principal stresses and maximum in-plane shearing stress at A and B. Units: k, k-in.

From a previous solution:









Cross-section

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

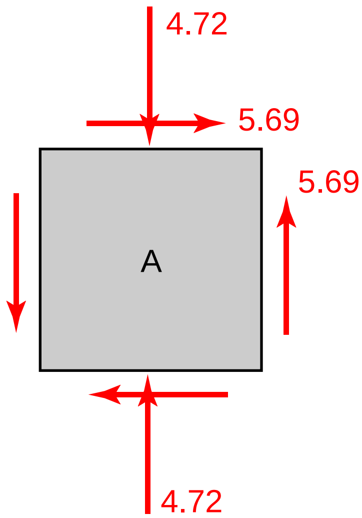
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

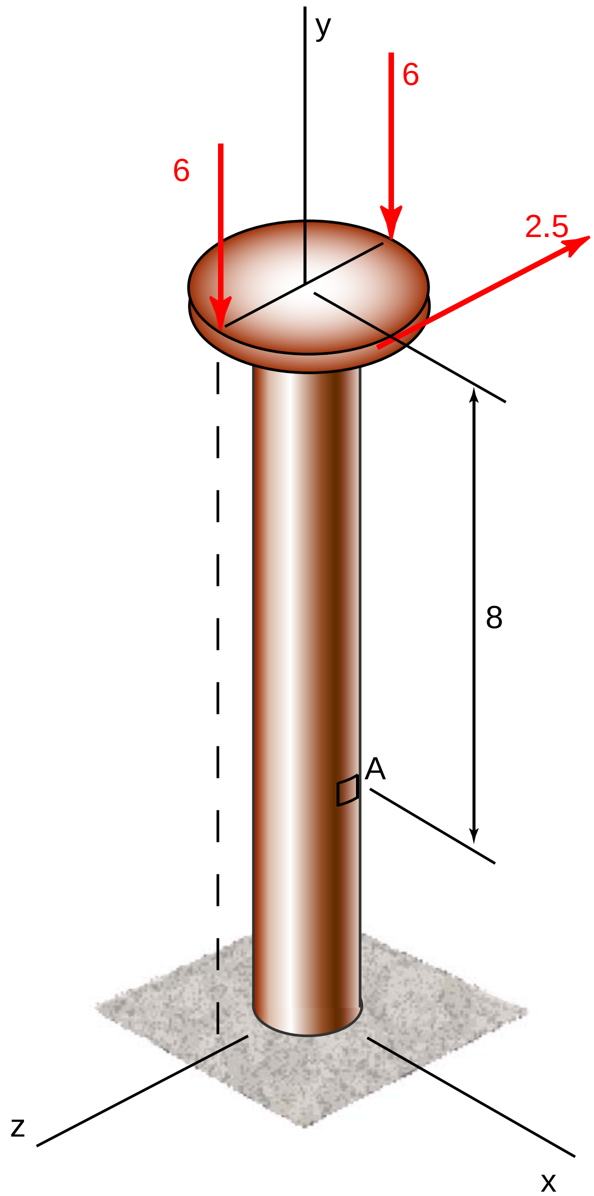
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Example

Determine the principal stresses and maximum in-plane shearing stress at A. The disk has a diameter of 4" and the solid shaft has a diameter of 1.8".

From a previous solution:





$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

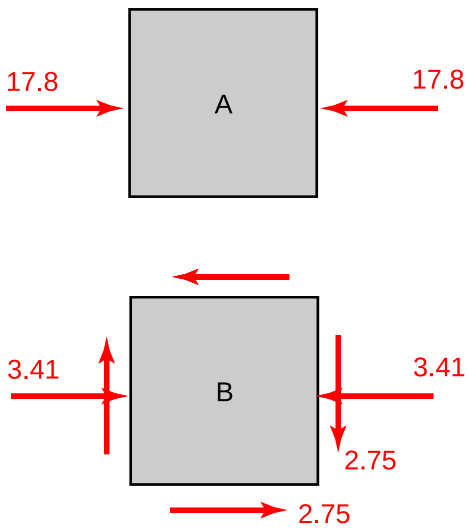
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

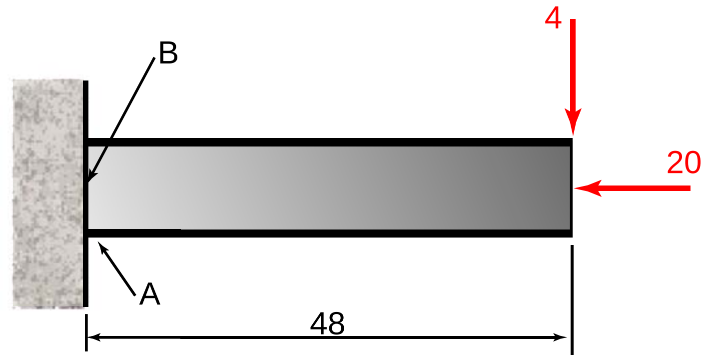
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Example

Determine the principal stresses and maximum in-plane shearing stress at A and B. The beam is a W6x20.

From a previous solution:





$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

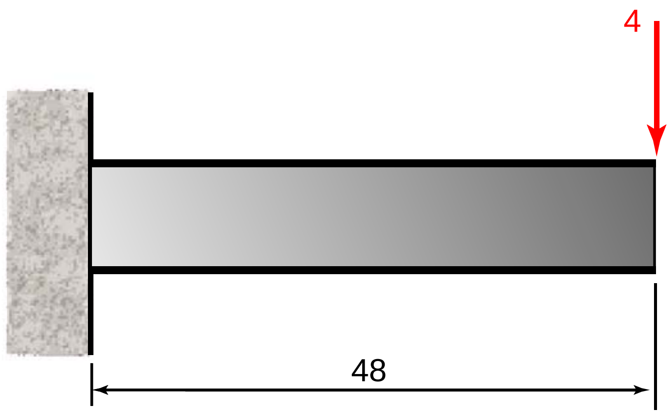
$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

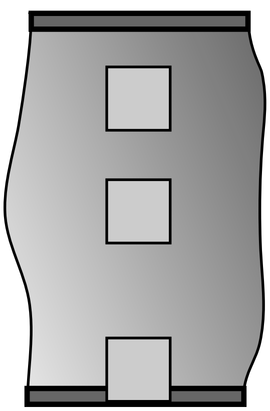
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

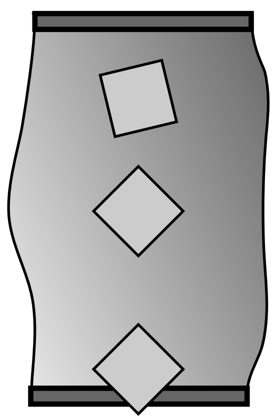
SUMMARY

Principal Stresses in a Beam

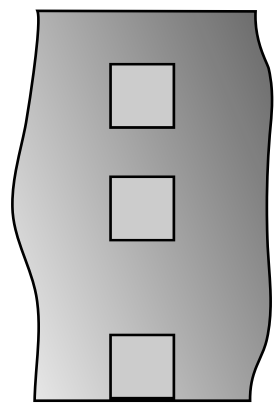




Wide Flange Stresses



Principal Wide Flange Stresses



Rectangular Cross-section Stresses