SIMPLIFYING RATIONAL EXPRESSIONS

Introduction

This lesson introduces **rational expressions**. Rational expressions are ratios that involve polynomials. Rational expressions can be used to model many real-world experiences. We'll look at a few examples later in this chapter.

SIMPLIFYING RATIONAL EXPRESSIONS

A rational expression is a ratio of polynomials. Here are a few examples:

$$\frac{x}{x+1},\quad \frac{x^2-3}{9},\quad \frac{5x^2y-15}{4x^3+12x-1}.$$

Every polynomial is a rational expression, since you can always divide it by one to make it a ratio of polynomials.

For example: $x^2-2x+5=\dfrac{x^2-2x+5}{1}$.

Recall that real numbers such as 1 can be thought of as 0-degree polynomials, so ordinary fractions are included in the definition of rational expressions.

As with ordinary fractions, the basic rules governing rational expressions are:

$$\frac{AC}{BC}=\frac{A\cancel{C}}{B\cancel{C}}=\frac{A}{B}$$

$$\frac{A}{B}\cdot\frac{C}{D}=\frac{AC}{BD}$$

$$\frac{A}{B}+\frac{C}{B}=\frac{A+C}{B}$$

The variables in the rules above now represent polynomials. Factoring is the key to effectively using these rules.

SIMPLIFYING RATIONAL EXPRESSIONS

Let's review how to reduce a fraction to lowest terms: you find the prime factors of the numerator and the denominator, and then cancel all the factors they have in common.

EXAMPLE A

Simplify $\frac{9}{15}$.

To reduce this fraction to lowest terms, replace each number with its prime factorization. Then cancel any factors common to the numerator and denominator.

$$\frac{9}{15} = \frac{\cancel{3} \cdot 3}{\cancel{3} \cdot 5} = \frac{3}{5}$$

Extended Example 1a

Simplify $\frac{196}{364}$.

SIMPLIFYING RATIONAL EXPRESSIONS

EXAMPLE B

Simplify $\dfrac{12x^2yz^3}{32xy^3z^2}$.

To reduce this fraction to lowest terms, replace each number with its prime factorization.

Then cancel any factors common to the numerator and the denominator.

$$\frac{12x^2yz^3}{32xy^3z^2} = \frac{\cancel{2}\cdot\cancel{2}\cdot 3\cdot\cancel{x}\cdot x\cdot\cancel{y}\cdot\cancel{z}\cdot\cancel{z}\cdot z}{\cancel{2}\cdot\cancel{2}\cdot 2\cdot 2\cdot 2\cdot\cancel{x}\cdot\cancel{y}\cdot y\cdot y\cdot\cancel{z}\cdot\cancel{z}} = \frac{3xz}{8y^2}$$

Notice that variables are treated just like the prime numeric factors.

Simplifying rational expressions can be made easier by using the rules for exponents. In the last example, there were three z's in the numerator and two z's in the denominator. We could have simply subtracted exponents as shown:

$$\frac{z^3}{z^2} = z^{3-2} = z^1 = z$$

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SIMPLIFYING RATIONAL EXPRESSIONS

Recall the following useful exponent rules:

$$\boxed{a^0 = 1 \qquad a^1 = a \qquad \frac{a^m}{a^n} = a^{m-n} \qquad a^{-n} = \frac{1}{a^n}}$$

For our next example, we'll redo Example B using the exponent rules above, and this rule for multiplying rational expressions: $\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$.

EXAMPLE C

Simplify $\frac{12x^2yz^3}{32xy^3z^2}$.

$$\begin{aligned}
\frac{12x^2yz^3}{32xy^3z^2} &= \frac{2^2 \cdot 3x^2yz^3}{2^5xy^3z^2} \\
&= \frac{2^2}{2^5} \cdot \frac{3}{1} \cdot \frac{x^2}{x^1} \cdot \frac{y^1}{y^3} \cdot \frac{z^3}{z^2} \\
&= 2^{2-5} \cdot 3 \cdot x^{2-1} \cdot y^{1-3} \cdot z^{3-2} \\
&= 2^{-3} \cdot 3 \cdot x^1 \cdot y^{-2} \cdot z^1 \\
&= \frac{1}{2^3} \cdot 3 \cdot x \cdot \frac{1}{y^2} \cdot z \\
&= \frac{3xz}{8y^2}
\end{aligned}$$

This method may seem like too many steps at first. However, when you get used to this, you can accomplish it with far fewer steps. Also, when the exponents are very large, it's impractical to write out all the factors. Exponent notation turns the cancellation process into simple subtraction of exponents.

SIMPLIFYING RATIONAL EXPRESSIONS

Extended Example 2a

Simplify $\dfrac{72x^3y^4z^5}{168x^5y^4z^3}$.

SIMPLIFYING RATIONAL EXPRESSIONS

As with fractions, to reduce a rational expression to lowest terms, we factor the numerator and denominator. Then we cancel any common factors.

EXAMPLE D

Simplify $\dfrac{5xy}{5x+10}$.

First, factor the denominator (the numerator is as factored as it can be!):

$$\frac{5xy}{5x+10} = \frac{5xy}{5(x+2)}.$$

Then cancel any common factors:

$$= \frac{\cancel{5}xy}{\cancel{5}(x+2)}$$

$$= \frac{xy}{x+2}$$

WARNING! Do not cancel the x's in the numerator and denominator. You can only cancel a common factor, and x is not a factor of the denominator. The only factors in the denominator of $\dfrac{xy}{x+2}$ are $(x+2)$ and 1. Only common factors can be cancelled; common terms should never be cancelled. A **factor** is multiplied; a **term** is added (or subtracted).

SIMPLIFYING RATIONAL EXPRESSIONS

EXAMPLE E

Simplify $\dfrac{x^2-1}{x^2-2x+1}$.

First, factor the numerator and the denominator.

$$\frac{x^2-1}{x^2-2x+1}=\frac{(x+1)(x-1)}{(x-1)(x-1)}$$

Then cancel any common factors:

$$=\frac{(x+1)\cancel{(x-1)}}{(x-1)\cancel{(x-1)}}$$

$$=\frac{x+1}{x-1}$$

You cannot simplify this expression further. The numerator has factor $(x+1)$, while the denominator has factor $(x-1)$. These factors are not equal, so nothing more can be cancelled.

SIMPLIFYING RATIONAL EXPRESSIONS

EXAMPLE F

Simplify $\dfrac{x^2 - 2x - 15}{x^2 - 5x - 24}$.

First, factor the numerator and the denominator:

$$\frac{x^2 - 2x - 15}{x^2 - 5x - 24} = \frac{(x+3)(x-5)}{(x+3)(x-8)}$$

Then cancel any common factors:

$$= \frac{\cancel{(x+3)}(x-5)}{\cancel{(x+3)}(x-8)}$$

$$= \frac{x-5}{x-8}$$

Note again that you cannot cancel the x's since they are terms, not factors. Only common factors can be cancelled; common terms should never be cancelled.

SIMPLIFYING RATIONAL EXPRESSIONS

Extended Example 3a

Simplify $\dfrac{x^2 - 6x + 9}{x^2 - 9}$.

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SIMPLIFYING RATIONAL EXPRESSIONS

Recall that a negative sign can be placed on the left side of the numerator, on the left side of the denominator, or to the left of an entire fraction. These placements are all equivalent. The same is true of rational expressions.

Sign Rule for Rational Expressions

$$-\frac{N}{D} = \frac{-N}{D} = \frac{N}{-D}$$

A Word to the Wise

When there is more than one term in the numerator (or denominator), you must use parentheses when placing the negative sign:

$$-\frac{a+b}{c+d} = \frac{-(a+b)}{c+d} = \frac{a+b}{-(c+d)}.$$

Recall that $A - B = -(B - A)$. Whether you realize it or not, you use this rule every time you subtract a larger number from a smaller one. Study the examples below to see the application of this rule.

$$1 - 4 = -(4 - 1) = -3$$

$$3 - 10 = -(10 - 3) = -7$$

$$20 - 44 = -(44 - 20) = -24$$

You probably solved these problems without thinking about the formula $A - B = -(B - A)$. In algebra, this property frequently proves to be quite handy.

SIMPLIFYING RATIONAL EXPRESSIONS

EXAMPLE G

Simplify $\dfrac{2-x}{x^2-4}$.

First, factor the denominator:

$$\frac{2-x}{x^2-4} = \frac{(2-x)}{(x+2)(x-2)}$$

Notice that in the numerator you have factor $(2-x)$, while in the denominator you have a very similar factor $(x-2)$. If we rewrite the numerator using the rule $A-B=-(B-A)$, we can then cancel the identical factors that result:

$$\begin{aligned}\frac{(2-x)}{(x+2)(x-2)} &= \frac{-(x-2)}{(x+2)(x-2)} \\ &= \frac{-\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} \\ &= \frac{-1}{x+2} \overset{\text{or}}{=} -\frac{1}{x+2}\end{aligned}$$

Note how the sign rule for rational expressions was used to move the negative from the 1 in the numerator to the left of the entire rational expression.

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SIMPLIFYING RATIONAL EXPRESSIONS

EXAMPLE H

Simplify $\dfrac{a^2 - b^2}{b^2 + 2ab + a^2}$.

First, factor the numerator and denominator:

$$\frac{a^2 - b^2}{b^2 + 2ab + a^2} = \frac{(a+b)(a-b)}{(b+a)^2}$$
$$= \frac{(a+b)(a-b)}{(a+b)^2}$$

Notice above that $(b+a) = (a+b)$. Finally, cancel the common factor:

$$= \frac{\cancel{(a+b)}(a-b)}{\cancel{(a+b)}(a+b)}$$
$$= \frac{a-b}{a+b}$$

SIMPLIFYING RATIONAL EXPRESSIONS

Extended Example 4a

Simplify $\dfrac{w^2 - 2w - 24}{-24 + 10w - w^2}$.

SIMPLIFYING RATIONAL EXPRESSIONS

EXAMPLE I

Simplify $\dfrac{4x^2-6xy+10xy-15y^2}{2x-3y}$.

There are four terms in the numerator. In such situations, try to factor by grouping:

$$\frac{\left(4x^2-6xy\right)+\left(10xy-15y^2\right)}{2x-3y}=\frac{2x\left(2x-3y\right)+5y\left(2x-3y\right)}{2x-3y}$$

$$=\frac{2x\left(2x-3y\right)+5y\left(2x-3y\right)}{2x-3y}$$

$$=\frac{\left(2x+5y\right)\left(2x-3y\right)}{\left(2x-3y\right)}$$

$$=\frac{\left(2x+5y\right)\cancel{\left(2x-3y\right)}}{\cancel{\left(2x-3y\right)}}$$

$$=2x+5y$$

SIMPLIFYING RATIONAL EXPRESSIONS

Extended Example 5a

Simplify $\dfrac{6t^2 - 10tw + 9tw - 15w^2}{2t^2 - 6tw + 3tw - 9w^2}$.

A Word to the Wise

When factoring, always check to see if there are any factors common to all the terms of a polynomial as a first step!

Extended Example 6a

Simplify $\dfrac{5xz^2 - 15xz - 440x}{5xz^2 - 10xz - 495x}$.

END OF LESSON

Simplify: $\dfrac{x^2 - 25}{x^2 + 10x + 25}$

Simplify: $\dfrac{xy + 2xz - 2y - 4z}{2z + y}$

Simplify: $\dfrac{-5b + 4a}{16a^2 - 25b^2}$

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Introduction

Multiplying and dividing rational expressions is done the same way as multiplying and dividing fractions. To multiply rational expressions, you simply multiply all the numerators together and all the denominators together. Division is accomplished by multiplying the first rational expression by the reciprocal of the second rational expression. Remember to simplify whenever possible!

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MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

This entire lesson can be summarized by the following two formulas:

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD}$$

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}$$

In arithmetic, the variables in the formulas above represent integers. For this lesson, they represent polynomials.

Multiplying Rational Expressions

EXAMPLE A

Find the product $\frac{5x}{15} \cdot \frac{8x}{12x}$.

First simplify each rational expression:

$$\frac{5x}{15} \cdot \frac{8x}{12x} = \frac{\cancel{5} \cdot x}{\cancel{5} \cdot 3} \cdot \frac{\cancel{2 \cdot 2} \cdot 2 \cdot \cancel{x}}{\cancel{2 \cdot 2} \cdot 3 \cdot \cancel{x}} = \frac{x}{3} \cdot \frac{2}{3}$$

Then multiply:

$$= \frac{x \cdot 2}{3 \cdot 3}$$

$$= \frac{2x}{9}$$

A Word to the Wise

Always cancel before you multiply!

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MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Extended Example 1a

Find the product $\frac{6z}{15a} \cdot \frac{25a}{2z}$.

Hint

STEP 1

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MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

EXAMPLE B

Find the product $\frac{13ab}{34cd} \cdot \frac{17ad}{26bc}$.

First factor all the coefficients.

$$\frac{13ab}{34cd} \cdot \frac{17ad}{26bc} = \frac{13ab}{17 \cdot 2cd} \cdot \frac{17ad}{2 \cdot 13bc}$$

Then multiply, canceling any common factors.

$$= \frac{13ab \cdot 17ad}{17 \cdot 2cd \cdot 2 \cdot 13bc}$$

$$= \frac{\cancel{13}a\cancel{b} \cdot \cancel{17}a\cancel{d}}{\cancel{17} \cdot 2c\cancel{d} \cdot 2 \cdot \cancel{13}\cancel{b}c}$$

$$= \frac{a^2}{4c^2}$$

EXAMPLE C

Multiply: $\dfrac{x^2+2x+1}{x^2+5x+6} \cdot \dfrac{x+2}{x+1}$.

Factor each polynomial in the rational expression on the left:

$$= \frac{(x+1)(x+1)}{(x+2)(x+3)} \cdot \frac{(x+2)}{(x+1)}$$

Then multiply and simplify:

$$= \frac{\cancel{(x+1)}(x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x+3)\cancel{(x+1)}}$$

$$= \frac{x+1}{x+3}$$

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Extended Example 2a

Multiply: $\dfrac{x^2-3x-40}{x^2+7x+12}\cdot\dfrac{x+3}{x-8}$.

Hint

STEP 1

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EXAMPLE D

Multiply: $\frac{x^2}{4x+16} \cdot \frac{x^2-16}{x^2-4x}$.

Factor and simplify:

$$= \frac{x^2}{4(x+4)} \cdot \frac{(x+4)(x-4)}{x(x-4)}$$

$$= \frac{x^2}{4(x+4)} \cdot \frac{(x+4)\cancel{(x-4)}}{x\cancel{(x-4)}}$$

$$= \frac{x \cdot x}{4(x+4)} \cdot \frac{(x+4)}{x}$$

Then multiply and simplify:

$$= \frac{x \cdot x \cdot (x+4)}{4(x+4) \cdot x}$$

$$= \frac{\cancel{x} \cdot x \cdot \cancel{(x+4)}}{4\cancel{(x+4)} \cdot \cancel{x}}$$

$$= \frac{x}{4}$$

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EXAMPLE E

Multiply: $\dfrac{2x^2+4x+2}{3x^2-6x}\cdot\dfrac{9x^3-18x^2}{2x+2}$.

Factor out the greatest common factors from each polynomial:

$$=\frac{2\left(x^2+2x+1\right)}{3x(x-2)}\cdot\frac{9x^2(x-2)}{2(x+1)}$$

Factor each polynomial. Notice that the trinomial in the first numerator is a binomial squared and can be factored:

$$=\frac{2(x+1)^2}{3x(x-2)}\cdot\frac{3\cdot 3x^2(x-2)}{2(x+1)}$$

Multiply and simplify:

$$=\frac{2(x+1)\cdot(x+1)\cdot 3\cdot 3x\cdot x(x-2)}{3x(x-2)\cdot 2(x+1)}$$

$$=\frac{\cancel{2}\cancel{(x+1)}\cdot(x+1)\cdot 3\cdot\cancel{3x}\cdot x\cancel{(x-2)}}{\cancel{3x}\cancel{(x-2)}\cdot\cancel{2}\cancel{(x+1)}}$$

$$=3x(x+1)$$

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Extended Example 3a

Multiply: $\dfrac{4x^3 - 49x}{2x^2 + 24x + 40} \cdot \dfrac{2x + 20}{2x^2 - 7x}$.

Hint

STEP 1

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Dividing Rational Expressions

To divide rational expressions you multiply by the reciprocal:

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C} = \frac{AD}{BC}.$$

EXAMPLE F

Divide: $\frac{4x}{6x} \div \frac{12}{15x}$.

Restate the problem as multiplication by the reciprocal of the second rational expression: $\frac{4x}{6x} \cdot \frac{15x}{12}$. Simplify $\frac{4x}{6x} \cdot \frac{15x}{12}$:

$$= \frac{2 \cdot 2x}{2 \cdot 3x} \cdot \frac{3 \cdot 5x}{2 \cdot 2 \cdot 3}$$

$$= \frac{\cancel{2} \cdot 2\cancel{x}}{\cancel{2} \cdot 3\cancel{x}} \cdot \frac{\cancel{3} \cdot 5x}{2 \cdot 2 \cdot \cancel{3}}$$

$$= \frac{2}{3} \cdot \frac{5x}{2 \cdot 2}$$

Then multiply and simplify:

$$= \frac{2 \cdot 5x}{3 \cdot 2 \cdot 2}$$

$$= \frac{\cancel{2} \cdot 5x}{3 \cdot \cancel{2} \cdot 2}$$

$$= \frac{5x}{6}$$

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MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Extended Example 4a

Divide: $\dfrac{2ab^2}{8bc^2} \div \dfrac{4a^2b}{16ac^2}$.

Hint

STEP 1

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EXAMPLE G

Divide: $\dfrac{x^2-5x+6}{x^2-x-6} \div \dfrac{x^2-4}{x^2+4x+4}$.

Invert the second rational expression, factor, and simplify:

$$= \frac{x^2-5x+6}{x^2-x-6} \cdot \frac{x^2+4x+4}{x^2-4}$$

$$= \frac{x^2-5x+6}{x^2-x-6} \cdot \frac{x^2+4x+4}{x^2-2^2}$$

$$= \frac{(x-2)(x-3)}{(x-3)(x+2)} \cdot \frac{(x+2)(x+2)}{(x-2)(x+2)}$$

$$= \frac{(x-2)\cancel{(x-3)}}{\cancel{(x-3)}(x+2)} \cdot \frac{(x+2)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$

$$= \frac{(x-2)}{(x+2)} \cdot \frac{(x+2)}{(x-2)}$$

Multiply and simplify:

$$= \frac{(x-2)\cdot(x+2)}{(x+2)\cdot(x-2)}$$

$$= \frac{\cancel{(x-2)}\cancel{(x+2)}}{\cancel{(x+2)}\cancel{(x-2)}}$$

$$= 1$$

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MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Extended Example 5a

Divide: $\dfrac{x^2 - 6x - 27}{x^2 - 14x + 49} \div \dfrac{x^2 - 14x + 45}{x^2 - 5x - 14}$.

Hint

STEP 1

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MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

EXAMPLE H

Simplify: $\dfrac{4x^2-9}{9x-x^3}\cdot\dfrac{x^2-x}{2x^2-x-3}\div\dfrac{2x^2+x-3}{x^2+3x}$.

Invert the last expression and change the division to multiplication. Then factor each polynomial. This time nothing cancels out:

$$=\frac{4x^2-9}{9x-x^3}\cdot\frac{x^2-x}{2x^2-x-3}\cdot\frac{x^2+3x}{2x^2+x-3}$$

$$=\frac{(2x)^2-3^2}{x\left(9-x^2\right)}\cdot\frac{x(x-1)}{(2x-3)(x+1)}\cdot\frac{x(x+3)}{(2x+3)(x-1)}$$

$$=\frac{(2x+3)(2x-3)}{x\left(3^2-x^2\right)}\cdot\frac{x(x-1)}{(2x-3)(x+1)}\cdot\frac{x(x+3)}{(2x+3)(x-1)}$$

$$=\frac{(2x+3)(2x-3)}{x(3+x)(3-x)}\cdot\frac{x(x-1)}{(2x-3)(x+1)}\cdot\frac{x(x+3)}{(2x+3)(x-1)}$$

Multiply and simplify:

$$=\frac{(2x+3)(2x-3)x(x-1)x(x+3)}{x(3+x)(3-x)(2x-3)(x+1)(2x+3)(x-1)}$$

$$=\frac{\cancel{(2x+3)}\,\cancel{(2x-3)}\,\cancel{x}\,\cancel{(x-1)}\,x\,\cancel{(x+3)}}{\cancel{x}\,\cancel{(x+3)}\,(3-x)\,\cancel{(2x-3)}\,(x+1)\,\cancel{(2x+3)}\,\cancel{(x-1)}}$$

$$=\frac{x}{(3-x)(x+1)}\quad\text{or}\quad=\frac{x}{(3-x)(1+x)}$$

MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Extended Example 6a

Simplify: $\dfrac{4y^2 - 4z^2}{3y^2 + 6yz + 3z^2} \cdot \dfrac{y^2 + yz}{2y - 2z} \div \dfrac{2y^2 - 2yz}{3y + 3z}$.

Hint

STEP 1

RESTART END OF LESSON BACK

Operate and simplify: $\dfrac{a^2 - b^2}{(ab)^2 + ab} \cdot \dfrac{3ab}{a - b}$

Operate and simplify: $\dfrac{2x + 8}{2x^2 + 3x - 20} \div \dfrac{16 - x^2}{x - 4}$

Operate and simplify: $\dfrac{18r^2 - 8s^2}{9r^2 - 4s^2} \div \dfrac{18rs - 12s^2}{9rs + 6s^2}$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Introduction

Multiplying fractions is easy; adding fractions is trickier. The same is true of rational expressions. As with ordinary fractions, the key is in factoring and in finding common denominators. In this section, we'll explore the process of adding and subtracting rational expressions.

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Recall that to add or subtract ordinary fractions that have a **common denominator**, you simply add or subtract the numerators. The common denominator is unchanged:

$$\frac{A}{B}+\frac{C}{B}=\frac{A+C}{B} \quad \text{and} \quad \frac{A}{B}-\frac{C}{B}=\frac{A-C}{B}$$

Always simplify the result, if possible. For example:

$$\frac{8}{7}+\frac{6}{7}=\frac{8+6}{7}=\frac{14}{7}=\frac{2\cdot 7}{7}=\frac{2\cdot \cancel{7}}{\cancel{7}}=2$$

The same process also applies to rational expressions.

EXAMPLE A

Add: $\dfrac{2x^2-6}{x-2}+\dfrac{2-x^2}{x-2}$.

Since these rational expressions have the same denominator, we add the numerators and then combine like terms:

$$=\frac{2x^2-6+2-x^2}{x-2}=\frac{x^2-4}{x-2}.$$

Notice that the numerator is a difference of squares. Factor it and simplify:

$$=\frac{x^2-2^2}{x-2}=\frac{(x+2)(x-2)}{(x-2)}=\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}=x+2$$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

EXAMPLE B

Subtract: $\dfrac{x^2-2}{x-1}-\dfrac{2x-3}{x-1}$.

Since these rational expressions both have the same denominator, we can subtract the numerators and keep the common denominator. Add parentheses to show that the minus sign applies to the entire numerator.

$$=\frac{x^2-2-(2x-3)}{x-1}$$

Distribute the minus sign to eliminate parentheses, and combine like terms:

$$=\frac{x^2-2-2x+3}{x-1}=\frac{x^2-2-2x+3}{x-1}=\frac{x^2-2x+1}{x-1}$$

Factor the numerator and simplify:

$$=\frac{(x-1)(x-1)}{(x-1)}=\frac{(x-1)\cancel{(x-1)}}{\cancel{(x-1)}}=x-1$$

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ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Extended Example 1a

$$\frac{2x^2+9}{x-5}-\frac{x^2+8x-6}{x-5}=?$$

Hint

STEP 1

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ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

To add or subtract fractions with different denominators, we start by factoring the denominators. Then use the property

$$\frac{A}{B} = \frac{AC}{BC}$$

to make the denominators the same. Note that this property is really the cancellation rule, $\frac{AC}{BC} = \frac{A}{B}$, but backwards. An equation is equally valid when read from left-to-right or when read from right-to-left.

Once the denominators are the same, we can add the fractions as in the previous examples. For example:

$$\begin{aligned} \frac{2}{3} + \frac{1}{5} &= \frac{2 \cdot 5}{3 \cdot 5} + \frac{1 \cdot 3}{5 \cdot 3} \\ &= \frac{10}{15} + \frac{3}{15} \\ &= \frac{10+3}{15} \\ &= \frac{13}{15} \end{aligned}$$

The same process applies to rational expressions.

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EXAMPLE C

$$\frac{x}{yz} - \frac{2y}{xz} + \frac{3z}{xy} = ?$$

First, find the least common multiple (LCM) of all of the denominators—each denominator must have the same factors. Remember, whatever you use to multiply the denominator you must also use to multiply the numerator:

$$\frac{x}{yz} - \frac{2y}{xz} + \frac{3z}{xy} = \frac{xx}{yzx} - \frac{2yy}{xzy} + \frac{3zz}{xyz}$$

$$= \frac{x^2 - 2y^2 + 3z^2}{xyz}$$

EXAMPLE D

Simplify: $\frac{4}{3x} + \frac{2}{6x} - \frac{3}{2x^2}$.

First, factor all the denominators:

$$= \frac{4}{3 \cdot x} + \frac{2}{2 \cdot 3 \cdot x} - \frac{3}{2 \cdot x \cdot x}$$

continued...

Example D, continued...

$$= \frac{4}{3 \cdot x} + \frac{2}{2 \cdot 3 \cdot x} - \frac{3}{2 \cdot x \cdot x}$$

Write in the factors needed to make all the denominators have their least common multiple, $2 \cdot 3 \cdot x \cdot x = 6x^2$:

$$= \frac{4 \cdot 2 \cdot x}{3 \cdot x \cdot 2 \cdot x} + \frac{2 \cdot x}{2 \cdot 3 \cdot x \cdot x} - \frac{3 \cdot 3}{2 \cdot x \cdot x \cdot 3}$$

$$= \frac{8x}{6x^2} + \frac{2x}{6x^2} - \frac{9}{6x^2}$$

Combine the numerators over their common denominator, and combine like terms:

$$= \frac{8x + 2x - 9}{6x^2}$$

$$= \frac{10x - 9}{6x^2}$$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Extended Example 2a

$$\frac{2a}{3ac}-\frac{5b}{2ab}-\frac{8c}{6bc}=?$$

Hint

STEP 1

RESTART

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ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

EXAMPLE E

Add: $\dfrac{3}{x} + \dfrac{2}{x-1}$.

The least common multiple of these denominators is $x(x-1)$. Start by multiplying the numerators and denominators by the factors needed to make the denominators equal to $x(x-1)$:

$$= \frac{3(x-1)}{x(x-1)} + \frac{2x}{(x-1)x}$$

$$= \frac{3(x-1)}{x(x-1)} + \frac{2x}{x(x-1)}$$

Add the numerators over their common denominator and simplify (distribute and combine like terms):

$$= \frac{3(x-1)+2x}{x(x-1)}$$

$$= \frac{3x-3+2x}{x(x-1)}$$

$$= \frac{5x-3}{x(x-1)}$$

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A Word to the Wise

In a problem like that in Example E, students often make the mistake of thinking you can get common denominators by simply adding -1 to the first denominator (x) to turn it into ($x-1$), like this:

$$\frac{3}{x}+\frac{2}{x-1}=\frac{3}{x-1}+\frac{2}{x-1}$$ **This is wrong!**

Common denominators are created from factors, not terms—you need to multiply, not add or subtract. What does work is to multiply both the numerator and the denominator by the same factor, as shown in Example E above. Remember, the word factor implies multiplication, not addition or subtraction.

Note:

- With ordinary fractions, we perform the multiplication in the numerator and the denominator at the end. However, as the factored form is considered simpler in algebra, you should ordinarily leave polynomials in factored form.

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

EXAMPLE F

Subtract: $\dfrac{4}{x+3}-\dfrac{2x-1}{x^2+5x+6}$.

Factor the denominator on the right:

$$=\frac{4}{x+3}-\frac{2x-1}{(x+2)(x+3)}$$

The LCM of these denominators is $(x+2)(x+3)$. Start by multiplying by the factor needed to make the first denominator equal to $(x+2)(x+3)$:

$$=\frac{4(x+2)}{(x+3)(x+2)}-\frac{2x-1}{(x+3)(x+2)}$$

Subtract the numerators over their common denominator and simplify (distribute and combine like terms):

$$=\frac{4(x+2)-(2x-1)}{(x+3)(x+2)}$$

$$=\frac{4x+8-2x+1}{(x+3)(x+2)}$$

$$=\frac{2x+9}{(x+3)(x+2)}$$

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Extended Example 3a

$$\frac{1}{z-11}+\frac{1}{z^2-5z-66}=?$$

Hint

STEP 1

RESTART

BACK 12 of 17 NEXT

EXAMPLE G

Simplify: $\dfrac{t}{t+1}-\dfrac{1}{t-2}+\dfrac{3}{t+1}+\dfrac{6}{(t+1)(t-2)}$.

The least common multiple of these denominators is $(t+1)(t-2)$. Start by multiplying by factors that will make the denominators all equal to the LCM:

$$=\frac{t\cdot(t-2)}{(t+1)\cdot(t-2)}-\frac{1\cdot(t+1)}{(t-2)\cdot(t+1)}+\frac{3\cdot(t-2)}{(t+1)\cdot(t-2)}+\frac{6}{(t+1)(t-2)}$$

$$=\frac{t(t-2)}{(t+1)(t-2)}-\frac{(t+1)}{(t+1)(t-2)}+\frac{3(t-2)}{(t+1)(t-2)}+\frac{6}{(t+1)(t-2)}$$

Combine the numerators over their common denominator and simplify:

$$=\frac{t(t-2)-(t+1)+3(t-2)+6}{(t+1)(t-2)}$$

$$=\frac{t^2-2t-t-1+3t-6+6}{(t+1)(t-2)}=\frac{t^2-2t-t-1+3t-6+6}{(t+1)(t-2)}$$

$$=\frac{t^2-1}{(t+1)(t-2)}$$

Notice that the numerator is the difference of two squares which can be factored. The expression can then be reduced:

$$=\frac{t^2-1^2}{(t+1)(t-2)}=\frac{(t+1)(t-1)}{(t+1)(t-2)}=\frac{\cancel{(t+1)}(t-1)}{\cancel{(t+1)}(t-2)}=\frac{t-1}{t-2}$$

RESTART BACK 13 of 17 NEXT

ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Extended Example 4a

$$\frac{1}{a-b}-\frac{a}{a^2-b^2}-\frac{b}{a^2+2ab+b^2}=?$$

Hint

STEP 1

RESTART

BACK 14 of 17 NEXT

EXAMPLE H

Subtract: $\dfrac{3}{x^2-25}-\dfrac{2}{5-x}$.

First, factor the denominators:

$$=\frac{3}{(x+5)(x-5)}-\frac{2}{(5-x)}.$$

Notice that the first fraction above has factor $(x-5)$ while the second has a very similar factor $(5-x)$. Recall that $(5-x)=-(x-5)$, so we can rewrite the second fraction as follows. Notice how the two negatives then make a positive, since $\dfrac{a}{-b}=-\dfrac{a}{b}$:

$$\frac{3}{(x+5)(x-5)}-\frac{2}{(5-x)}=\frac{3}{(x+5)(x-5)}-\frac{2}{-(x-5)}$$
$$=\frac{3}{(x+5)(x-5)}+\frac{2}{(x-5)}$$

continued...

Example H, continued…

$$= \frac{3}{(x+5)(x-5)} + \frac{2}{(x-5)}$$

Now make each denominator equal to the LCM of both denominators, $(x+5)(x-5)$:

$$= \frac{3}{(x+5)(x-5)} + \frac{2\cdot(x+5)}{(x-5)\cdot(x+5)}$$

$$= \frac{3}{(x+5)(x-5)} + \frac{2(x+5)}{(x+5)(x-5)}$$

Finally, add the numerators over their common denominator and simplify the numerator:

$$= \frac{3+2x+10}{(x+5)(x-5)}$$

$$= \frac{2x+13}{(x+5)(x-5)}$$

Extended Example 5a

Subtract: $\frac{1}{x^2-64}-\frac{3}{8-x}$.

Hint

STEP 1

RESTART

END OF LESSON

BACK

Operate and simplify: $\dfrac{2}{x+3}+\dfrac{5x}{x-2}+\dfrac{x}{x-6+x^2}$

Operate and simplify: $\dfrac{3}{x-5}+\dfrac{x}{x^2-10x+25}$

Operate and simplify: $\dfrac{2}{xy^3}-\dfrac{5}{x^2z^2}$

COMPLEX RATIONAL EXPRESSIONS

Introduction

In this lesson, you will learn how to simplify complex rational expressions. Recall that a **complex fraction** is a fraction that has a fraction in the numerator and/or in the denominator. Similarly, a **complex rational expression** is a rational expression that has a rational expression in the numerator and/or in the denominator.

$$\boxed{\frac{4+\frac{1}{3}}{3-\frac{1}{2}}}$$

A complex fraction.

$$\boxed{\frac{x+\frac{2}{x-2}}{1-\frac{3}{x}}}$$

A complex rational expression.

1 of 15

NEXT ›

COMPLEX RATIONAL EXPRESSIONS

Complex rational expressions are simplified in the same way as complex fractions. One way to transform a complex fraction into a simple fraction is to first simplify the numerator and denominator independently. Then, invert the fraction in the denominator, and multiply it by the fraction in the numerator. However, our first example will demonstrate a much simpler way to accomplish this.

EXAMPLE A

Simplify: $\dfrac{\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{5}{6}}$.

The goal is to eliminate all of the denominators in the numerator and the denominator of the complex fraction. To accomplish this, find the least common multiple of all the denominators.

In this example, 6 is the smallest number that 2, 3, and 6 all divide into without a remainder. So we multiply both the numerator and the denominator by 6, then distribute and simplify. First, factor using the prime factorization of 6:

$$=\frac{\left(\frac{1}{2}+\frac{1}{3}\right)\cdot 2\cdot 3}{\left(\frac{2}{3}+\frac{5}{2\cdot 3}\right)\cdot 2\cdot 3}=\frac{\frac{1\cdot 2\cdot 3}{2}+\frac{1\cdot 2\cdot 3}{3}}{\frac{2\cdot 2\cdot 3}{3}+\frac{5\cdot 2\cdot 3}{2\cdot 3}}=\frac{\frac{2\cdot 3}{2}+\frac{2\cdot 3}{3}}{\frac{2\cdot 2\cdot 3}{3}+\frac{5\cdot 2\cdot 3}{2\cdot 3}}$$

$$=\frac{\frac{\cancel{2}\cdot 3}{\cancel{2}}+\frac{2\cdot\cancel{3}}{\cancel{3}}}{\frac{2\cdot 2\cdot\cancel{3}}{\cancel{3}}+\frac{5\cdot\cancel{2\cdot 3}}{\cancel{2\cdot 3}}}=\frac{3+2}{4+5}=\frac{5}{9}$$

RESTART BACK 2 of 15 NEXT

COMPLEX RATIONAL EXPRESSIONS

When simplifying a complex rational expression, follow the same procedure that was demonstrated in Example A.

EXAMPLE B

Simplify: $\dfrac{\dfrac{1}{A}-\dfrac{2}{AB}}{\dfrac{3}{AB}+\dfrac{1}{B}}$.

Notice that the least common multiple of all the denominators is AB . So, we multiply the numerator and denominator of the expression by AB :

$$=\frac{\left(\dfrac{1}{A}-\dfrac{2}{AB}\right)\cdot AB}{\left(\dfrac{3}{AB}+\dfrac{1}{B}\right)\cdot AB}=\frac{\dfrac{1\cdot AB}{A}-\dfrac{2\cdot AB}{AB}}{\dfrac{3\cdot AB}{AB}+\dfrac{1\cdot AB}{B}}=\frac{\dfrac{AB}{A}-\dfrac{2AB}{AB}}{\dfrac{3AB}{AB}+\dfrac{AB}{B}}$$

$$=\frac{\dfrac{\cancel{A}B}{\cancel{A}}-\dfrac{2\cancel{AB}}{\cancel{AB}}}{\dfrac{3\cancel{AB}}{\cancel{AB}}+\dfrac{A\cancel{B}}{\cancel{B}}}=\frac{B-2}{3+A}\overset{\text{or}}{=}\frac{B-2}{A+3}$$

With practice, you'll be able to do much of this "in your head," jumping to the simplification in only one or two steps. Just ask yourself what's left after all matching factors have been cancelled.

RESTART BACK 3 of 15 NEXT

COMPLEX RATIONAL EXPRESSIONS

Extended Example 1a

Simplify: $\dfrac{\dfrac{3}{x}+\dfrac{5}{y}}{\dfrac{4}{xy}-\dfrac{2}{x}}$.

Hint

STEP 1

RESTART

BACK 4 of 15 NEXT

COMPLEX RATIONAL EXPRESSIONS

If there is only one fraction in the numerator and one in the denominator, it is easiest to just invert and multiply.

EXAMPLE C

Simplify: $\dfrac{\dfrac{4x}{5}}{\dfrac{7x}{3}}$.

$$= \frac{4x}{5} \div \frac{7x}{3} = \frac{4x}{5} \cdot \frac{3}{7x} = \frac{4x \cdot 3}{5 \cdot 7x} = \frac{4\cancel{x} \cdot 3}{5 \cdot 7\cancel{x}} = \frac{12}{35}$$

As with ordinary fractions, be sure to simplify whenever possible when working with rational expressions!

EXAMPLE D

Simplify: $\dfrac{\dfrac{3}{x+3}}{\dfrac{x}{x+3}}$.

Invert and multiply:

$$= \frac{3}{x+3} \cdot \frac{x+3}{x} = \frac{3(x+3)}{(x+3)x} = \frac{3\cancel{(x+3)}}{\cancel{(x+3)}x} = \frac{3}{x}$$

RESTART BACK 5 of 15 NEXT

COMPLEX RATIONAL EXPRESSIONS

Extended Example 2a

Simplify: $\dfrac{\dfrac{3A}{10Z}}{\dfrac{6A}{5Z}}$.

Hint

STEP 1

RESTART

COMPLEX RATIONAL EXPRESSIONS

EXAMPLE E

Simplify: $\dfrac{2-\dfrac{3}{x}}{3+\dfrac{4}{x}}$.

Multiply both the numerator and the denominator by the least common multiple, x:

$$= \frac{\left(2-\dfrac{3}{x}\right)\cdot x}{\left(3+\dfrac{4}{x}\right)\cdot x}$$

Distribute and simplify:

$$= \frac{2\cdot x-\dfrac{3\cdot x}{x}}{3\cdot x+\dfrac{4\cdot x}{x}} = \frac{2x-\dfrac{3\cancel{x}}{\cancel{x}}}{3x+\dfrac{4\cancel{x}}{\cancel{x}}} = \frac{2x-3}{3x+4}$$

RESTART BACK 7 of 15 NEXT

COMPLEX RATIONAL EXPRESSIONS

EXAMPLE F

Simplify: $\dfrac{x}{x+\dfrac{x}{x+1}}$.

Multiply the numerator and denominator by $(x+1)$ and distribute. Then cancel and simplify:

$$=\frac{x\cdot(x+1)}{\left(x+\dfrac{x}{x+1}\right)\cdot(x+1)}$$

$$=\frac{x(x+1)}{x\cdot(x+1)+\dfrac{x}{x+1}\cdot(x+1)}$$

$$=\frac{x^2+x}{x^2+x+\dfrac{x\cdot\cancel{(x+1)}}{\cancel{(x+1)}}}$$

$$=\frac{x^2+x}{x^2+x+x}=\frac{x^2+x}{x^2+2x}$$

Factor out an x from the numerator and denominator and cancel this common factor:

$$=\frac{x(x+1)}{x(x+2)}=\frac{\cancel{x}(x+1)}{\cancel{x}(x+2)}=\frac{x+1}{x+2}$$

RESTART BACK 8 of 15 NEXT

Extended Example 3a

Simplify: $\dfrac{1+\dfrac{x}{x-1}}{1-\dfrac{x}{x+1}}$.

Hint

STEP 1

COMPLEX RATIONAL EXPRESSIONS

EXAMPLE G

$$\frac{\frac{1}{r}-\frac{2r}{r-1}}{\frac{1}{r}+\frac{2r}{r-1}}=?$$

The least common multiple is the product of the denominators, $r(r-1)$. So, multiply both the numerator and the denominator by $r(r-1)$.

$$=\frac{\left(\frac{1}{r}-\frac{2r}{r-1}\right)\cdot r(r-1)}{\left(\frac{1}{r}+\frac{2r}{r-1}\right)\cdot r(r-1)}$$

$$=\frac{\frac{1\cdot r(r-1)}{r}-\frac{2r\cdot r(r-1)}{r-1}}{\frac{1\cdot r(r-1)}{r}+\frac{2r\cdot r(r-1)}{r-1}}$$

$$=\frac{\frac{r(r-1)}{r}-\frac{2r\cdot r(r-1)}{(r-1)}}{\frac{r(r-1)}{r}+\frac{2r\cdot r(r-1)}{(r-1)}}$$

continued...

Example G, continued...

$$= \frac{\dfrac{r(r-1)}{r} - \dfrac{2r \cdot r(r-1)}{(r-1)}}{\dfrac{r(r-1)}{r} + \dfrac{2r \cdot r(r-1)}{(r-1)}}$$

Cancel:

$$= \frac{\dfrac{\cancel{r}(r-1)}{\cancel{r}} - \dfrac{2r \cdot r\cancel{(r-1)}}{\cancel{(r-1)}}}{\dfrac{\cancel{r}(r-1)}{\cancel{r}} + \dfrac{2r \cdot r\cancel{(r-1)}}{\cancel{(r-1)}}} = \frac{(r-1)-2r^2}{(r-1)+2r^2} = \frac{r-1-2r^2}{r-1+2r^2}$$

Rewrite these polynomials in descending order:

$$= \frac{-2r^2+r-1}{2r^2+r-1}$$

Note:

- Usually at this point you would factor the numerator and denominator to see if any factors cancel out. In this problem, the numerator does not factor, so no further simplification is possible.

COMPLEX RATIONAL EXPRESSIONS

Extended Example 4a

$$\frac{\dfrac{2}{u+2}-\dfrac{u}{u-2}}{\dfrac{u}{u+2}+\dfrac{2}{u-2}}=?$$

Hint

STEP 1

RESTART

BACK 12 of 15 NEXT

EXAMPLE H

Simplify: $$\frac{\dfrac{1}{x-1}-\dfrac{2}{x^2+4x-5}}{\dfrac{3}{x^2+4x-5}+\dfrac{1}{x+5}}.$$

First, factor the denominators:

$$=\frac{\dfrac{1}{(x-1)}-\dfrac{2}{(x+5)(x-1)}}{\dfrac{3}{(x+5)(x-1)}+\dfrac{1}{(x+5)}}$$

Multiply the numerator and denominator by the least common multiple, $(x+5)(x-1)$:

$$=\frac{\left(\dfrac{1}{x-1}-\dfrac{2}{(x+5)(x-1)}\right)\cdot(x+5)(x-1)}{\left(\dfrac{3}{(x+5)(x-1)}+\dfrac{1}{x+5}\right)\cdot(x+5)(x-1)}$$

$$=\frac{\dfrac{1\cdot(x+5)(x-1)}{x-1}-\dfrac{2\cdot(x+5)(x-1)}{(x+5)(x-1)}}{\dfrac{3\cdot(x+5)(x-1)}{(x+5)(x-1)}+\dfrac{1\cdot(x+5)(x-1)}{x+5}}$$

continued...

COMPLEX RATIONAL EXPRESSIONS

Example H, continued...

$$= \frac{\dfrac{1\cdot(x+5)(x-1)}{x-1} - \dfrac{2\cdot(x+5)(x-1)}{(x+5)(x-1)}}{\dfrac{3\cdot(x+5)(x-1)}{(x+5)(x-1)} + \dfrac{1\cdot(x+5)(x-1)}{x+5}} = \frac{\dfrac{(x+5)(x-1)}{(x-1)} - \dfrac{2(x+5)(x-1)}{(x+5)(x-1)}}{\dfrac{3(x+5)(x-1)}{(x+5)(x-1)} + \dfrac{(x+5)(x-1)}{(x+5)}}$$

Cancel out the common factors and simplify:

$$= \frac{\dfrac{(x+5)\cancel{(x-1)}}{\cancel{(x-1)}} - \dfrac{2\cancel{(x+5)(x-1)}}{\cancel{(x+5)(x-1)}}}{\dfrac{3\cancel{(x+5)(x-1)}}{\cancel{(x+5)(x-1)}} + \dfrac{\cancel{(x+5)}(x-1)}{\cancel{(x+5)}}}$$

$$= \frac{(x+5)-2}{3+(x-1)}$$

$$= \frac{x+5-2}{3+x-1}$$

$$= \frac{x+3}{x+2}$$

COMPLEX RATIONAL EXPRESSIONS

Extended Example 5a

Simplify: $$\frac{\dfrac{1}{y+11}-\dfrac{y}{y^2+5y-66}}{\dfrac{y}{y^2+5y-66}-\dfrac{1}{y-6}}.$$

Hint

STEP 1

RESTART END OF LESSON BACK 15 of 15

Simplify: $\dfrac{\dfrac{a+b}{a-b}}{a^2-b^2}$

Simplify: $\dfrac{\dfrac{xy}{3}+\dfrac{xy}{2}}{\dfrac{xy}{3}}$

Simplify.

$$\frac{1+\dfrac{1}{2w}}{1-\dfrac{1}{2w}}$$

Simplify: $\dfrac{2-\dfrac{3}{4y^2}}{5y-\dfrac{1}{y^2}}$

Simplify.

$$\frac{\dfrac{2}{x-y}+\dfrac{3}{x+y}}{\dfrac{2}{x+y}+\dfrac{3}{x-y}}=$$

Simplify.

$$\frac{\dfrac{2}{x+4}-\dfrac{1}{x-5}}{\dfrac{1}{x-5}-\dfrac{3}{x^2-x-20}}=$$

RATIONAL EQUATIONS

Introduction

In this lesson we'll use all that you have learned in the previous sections to solve rational equations. A **rational equation** is an equation involving only rational expressions. A slight modification of the technique that you learned to simplify complex rational expressions can now be used to solve rational equations. Recall that solving an equation consists of finding values which, when substituted in place of the variables in the equation, make the equation true.

As was the case with complex rational expressions, first we factor all the denominators and find their least common multiple (LCM). Next, instead of multiplying the numerator and denominator by the LCM, we will multiply both sides of the equal sign by the LCM. This step changes all the rational expressions into polynomials and eliminates all fractions.

 NEXT ›

IMPORTANT WARNING! When solving rational equations, you must always check your solution. You can substitute a solution into the original equation and verify that both sides of the equation are equal. But there is an easier way: you may simply check that the solution you have found does not make any denominator in the original equation equal to zero. Assuming your algebra is correct, the answer you get is valid unless it makes a denominator zero in the original equation. If your answer results in a zero denominator, then instead of a valid solution, your "answer" is an **extraneous solution**—a value for which the equation is not defined (since division by zero is undefined). You may also say there is no solution in such cases.

EXAMPLE A

Solve: $\dfrac{2}{x}+\dfrac{x}{x-3}=1$.

Look at the result when we multiply both sides of the equal sign by the least common multiple of the two denominators:

$$x(x-3)\cdot\left(\frac{2}{x}+\frac{x}{x-3}\right)=x(x-3)\cdot 1$$

$$\frac{x(x-3)\cdot 2}{x}+\frac{x(x-3)\cdot x}{x-3}=x(x-3)$$

$$\frac{\cancel{x}(x-3)\cdot 2}{\cancel{x}}+\frac{x\cancel{(x-3)}\cdot x}{\cancel{(x-3)}}=x^2-3x$$

$$(x-3)\cdot 2+x^2=x^2-3x$$

$$2x-6+x^2=x^2-3x$$

continued...

RATIONAL EQUATIONS

Example A, continued...

Subtracting x^2 from both sides of the equal sign removes it from both sides:

$$\begin{array}{r} 2x-6+x^2 = x^2-3x \\ -x^2 \quad -x^2 \\ \hline 2x-6 = -3x \end{array}$$

Now we can solve for x:

$$\begin{array}{r} 2x-6=-3x \\ -2x \qquad -2x \\ \hline -6=-5x \end{array}$$

$$\frac{-6}{-5}=\frac{-5x}{-5}$$

$$\frac{6}{5}=\frac{\cancel{-5}x}{\cancel{-5}}$$

$$\frac{6}{5}=x$$

continued...

RATIONAL EQUATIONS

Example A, continued...

Now we must check. Is this really a solution to the original equation? Substitute and see:

$$\boxed{\frac{2}{x}+\frac{x}{x-3}=1 \quad \text{and} \quad x=\frac{6}{5}}$$

$$\frac{2}{\frac{6}{5}}+\frac{\frac{6}{5}}{\frac{6}{5}-3}\overset{?}{=}1 \to 2\cdot\frac{5}{6}+\frac{5\cdot\frac{6}{5}}{5\cdot\left(\frac{6}{5}-3\right)}\overset{?}{=}1 \to \frac{2\cdot 5}{2\cdot 3}+\frac{\frac{\cancel{5}\cdot 6}{\cancel{5}}}{\frac{\cancel{5}\cdot 6}{\cancel{5}}-5\cdot 3}\overset{?}{=}1$$

$$\to \frac{\cancel{2}\cdot 5}{\cancel{2}\cdot 3}+\frac{6}{6-15}\overset{?}{=}1 \to \frac{5}{3}+\frac{6}{-9}\overset{?}{=}1 \to \frac{5}{3}-\frac{2\cdot\cancel{3}}{3\cdot\cancel{3}}\overset{?}{=}1 \to 1\overset{yes}{=}1$$

Substituting a value into the original equation can take time! Instead, you may simply check that the solution you have found does not make any denominator in the original equation equal to zero. So instead of all the work above, we could have simply found that $x = 6/5$ does not make any of the denominators equal to zero in the original equation.

The original equation, $\frac{2}{x}+\frac{x}{x-3}=1$, has two denominators, x and $x-3$.

The first denominator is zero only when $x = 0$, while the second denominator is zero only when $x = 3$.

Thus our solution, $x=\frac{6}{5}$, is a valid solution.

RESTART BACK 4 of 23 NEXT

RATIONAL EQUATIONS

EXAMPLE B

Solve: $5+\dfrac{10}{x-1}=\dfrac{10x}{x-1}$.

This time we need to multiply both sides of the equal sign by the LCM of the denominators, 1 and $(x-1)$. Using the LCM, $(x-1)$, we distribute this factor and cancel common factors in the denominators:

$$
\begin{aligned}
(x-1)\cdot\left(5+\frac{10}{x-1}\right) &= (x-1)\cdot\frac{10x}{x-1}\\
(x-1)\cdot 5+\frac{(x-1)\cdot 10}{x-1} &= \frac{(x-1)\cdot 10x}{x-1}\\
5(x-1)+\frac{\cancel{(x-1)}\cdot 10}{\cancel{(x-1)}} &= \frac{\cancel{(x-1)}\cdot 10x}{\cancel{(x-1)}}\\
5(x-1)+10 &= 10x
\end{aligned}
$$

Next, use the distributive property to eliminate parentheses and combine like terms:

$$
\begin{aligned}
5x-5+10 &= 10x\\
5x+5 &= 10x
\end{aligned}
$$

continued...

Example B, continued…

Solve for x:

$$\begin{aligned} 5x + 5 &= 10x \\ -5x \quad & \quad -5x \\ \hline 5 &= 5x \\ \frac{5}{5} &= \frac{5x}{5} \\ \frac{\cancel{5}}{\cancel{5}} &= \frac{\cancel{5}x}{\cancel{5}} \\ 1 &= x \end{aligned}$$

However, this value of x would make a denominator equal to 0 in the original equation:

$$5 + \frac{10}{x-1} = \frac{10x}{x-1} \xrightarrow{\boxed{x=1}} 5 + \frac{10}{1-1} = \frac{10 \cdot 1}{1-1}$$

$$5 + \frac{10}{0} = \frac{10}{0}$$

Since division by zero is undefined, the only possible solution, $x = 1$, does not make the equation true (just undefined). So, this rational equation has no solution.

RATIONAL EQUATIONS

EXAMPLE C

Solve: $1-\dfrac{t-3}{t}=\dfrac{3}{t}$.

We first multiply both sides of the equal sign by the LCM of the denominators, which is t. Distributing this factor and canceling out the denominators, we get:

$$t\cdot\left(1-\frac{t-3}{t}\right)=t\cdot\frac{3}{t}$$

$$t\cdot 1-t\cdot\frac{t-3}{t}=\frac{t\cdot 3}{t}$$

$$t-\frac{\cancel{t}\cdot(t-3)}{\cancel{t}}=\frac{\cancel{t}\cdot 3}{\cancel{t}}$$

$$t-(t-3)=3$$

Next, we distribute the negative to eliminate parentheses. Then we combine like terms:

$$t-t+3=3$$

$$t-t+3=3$$

$$0+3=3$$

$$3=3$$

continued...

RESTART BACK 7 of 23 NEXT

Example C, continued...

The solution $3 = 3$ tells us more than the obvious fact that 3 is 3; this is true no matter what number t happens to be.

The original equation is true, no matter what t is. So the solution is all real numbers... except any that would result in a zero denominator in the original equation.

Looking again at the original equation,

$$1 - \frac{t-3}{t} = \frac{3}{t},$$

we can see that the only problem occurs when $t = 0$. So the solution consists of all real numbers, <u>except</u> $t = 0$.

RATIONAL EQUATIONS

Extended Example 1a

Solve: $\frac{5}{a}+\frac{2a}{a+3}=2$.

Hint

STEP 1

RESTART BACK 9 of 23 NEXT

EXAMPLE D

Solve: $\dfrac{5}{x+2}+\dfrac{2}{x-6}=\dfrac{x-3}{x^2-4x-12}$.

First, factor the denominator on the right side of the equation. Then multiply both sides of the equal sign by the least common multiple of all the denominators:

$$\frac{5}{x+2}+\frac{2}{x-6}=\frac{(x-3)}{(x+2)(x-6)}$$

$$(x+2)(x-6)\cdot\left(\frac{5}{x+2}+\frac{2}{x-6}\right)=(x+2)(x-6)\cdot\frac{(x-3)}{(x+2)(x-6)}$$

$$\frac{(x+2)(x-6)\cdot 5}{x+2}+\frac{(x+2)(x-6)\cdot 2}{x-6}=\frac{(x+2)(x-6)\cdot(x-3)}{(x+2)(x-6)}$$

Cancel all matching factors.

$$\frac{\cancel{(x+2)}(x-6)\cdot 5}{\cancel{(x+2)}}+\frac{(x+2)\cancel{(x-6)}\cdot 2}{\cancel{(x-6)}}=\frac{\cancel{(x+2)(x-6)}\cdot(x-3)}{\cancel{(x+2)(x-6)}}$$

$$(x-6)\cdot 5+(x+2)\cdot 2=x-3$$

continued...

RATIONAL EQUATIONS

Example D, continued ...

$(x-6)\cdot 5+(x+2)\cdot 2=x-3 \rightarrow 5(x-6)+2(x+2)=x-3$

Eliminate parentheses and solve for x:

$$\begin{aligned} 5x-30+2x+4 &= x-3 \\ 7x-26 &= x-3 \\ -x \qquad &\; -x \\ \hline 6x-26 &= -3 \\ +26 &\; +26 \\ \hline 6x &= 23 \\ \frac{6x}{6} &= \frac{23}{6} \\ \frac{\cancel{6}x}{\cancel{6}} &= \frac{23}{6} \\ x &= \frac{23}{6} \end{aligned}$$

Take a look at the denominators in the original equation, and you will see that only the values -2 and 6 make the denominators zero. So our solution is valid.

Even though a solution is valid, it still is a good idea to check that the solution is actually correct by substituting it into the original equation (not shown here). If a computational error was made, it would be possible to end up with a "solution" that does not result in a zero denominator, but which also is <u>not</u> a solution to the original equation.

RESTART BACK 11 of 23 NEXT

RATIONAL EQUATIONS

Extended Example 2a

Solve: $\dfrac{1}{t+1} - \dfrac{3}{t-3} = \dfrac{t-1}{t^2 - 2t - 3}$.

Hint

STEP 1

RESTART BACK 12 of 23 NEXT

RATIONAL EQUATIONS

EXAMPLE E

Solve: $\dfrac{6}{x+5} = \dfrac{11}{x}$.

Multiplying both sides of the equation by the LCM, $x(x+5)$, clears the fractions:

$$\frac{x(x+5)\cdot 6}{x+5} = \frac{x(x+5)\cdot 11}{x}$$

$$\frac{x\cancel{(x+5)}\cdot 6}{\cancel{(x+5)}} = \frac{\cancel{x}(x+5)\cdot 11}{\cancel{x}}$$

$$6x = (x+5)\cdot 11$$

Note that the last line is exactly what we would have obtained by simply **cross-multiplying** the original equation:

$$\frac{6}{x+5} = \frac{11}{x}$$

continued...

RESTART BACK 13 of 23 NEXT

Example E, continued...

Solving for x, we get:

$$\begin{aligned} 6x &= (x+5)\cdot 11 \\ 6x &= 11x+55 \\ -11x &\quad -11x \\ \hline -5x &= 55 \\ \frac{-5x}{-5} &= \frac{55}{-5} \\ \frac{\cancel{-5}x}{\cancel{-5}} &= -11 \\ x &= -11 \end{aligned}$$

This value does not make any of the denominators zero (only -5 and 0 do), so $x=-11$ is a valid solution.

RATIONAL EQUATIONS

EXAMPLE F

Solve: $\dfrac{k^2+9k}{k+10} = \dfrac{10}{k+10}$.

Multiplying both sides of the equation by the LCM, $k+10$, clears the fractions:

$$\frac{(k+10)\cdot\left(k^2+9k\right)}{k+10} = \frac{(k+10)\cdot 10}{k+10}$$

$$\frac{\cancel{(k+10)}\cdot\left(k^2+9k\right)}{\cancel{(k+10)}} = \frac{\cancel{(k+10)}\cdot 10}{\cancel{(k+10)}}$$

$$k^2+9k=10$$

Solving for k, we get:

$$\begin{array}{rcl} k^2+9k & = & 10 \\ -10 & & -10 \\ \hline k^2+9k-10 & = & 0 \end{array}$$

$$(k+10)(k-1)=0$$

continued...

RATIONAL EQUATIONS

Example F, continued...

$$(k+10)(k-1)=0$$

$$\begin{array}{rcl} k+10 & = & 0 \\ -10 & & -10 \\ \hline & k = & -10 \end{array} \qquad \begin{array}{rcl} k-1 & = & 0 \\ +1 & & +1 \\ \hline & k = & 1 \end{array}$$

$$\downarrow \qquad\qquad \downarrow$$

$$k=-10 \quad \text{or} \quad k=1$$

Looking at the original equation,

$$\frac{k^2+9k}{k+10}=\frac{10}{k+10},$$

we see that a zero denominator occurs when $k+10=0$. This happens only when $k=-10$. Since $k=-10$ is one of our possible solutions, we know that we cannot use that value. The only valid solution then is $k=1$.

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RATIONAL EQUATIONS

Extended Example 3a

Solve: $\dfrac{2x}{9} = \dfrac{3x-1}{7}$.

Hint

STEP 1

RESTART

BACK 17 of 23 NEXT

RATIONAL EQUATIONS

EXAMPLE G

Simplify: $2-\frac{6}{x}=\frac{x}{x-4}$.

This time we need to multiply both sides of the equal sign by the LCM of the denominator, $x(x-4)$. Distributing this factor, and canceling out the denominator, we get:

$$x(x-4)\cdot\left(2-\frac{6}{x}\right)=x(x-4)\cdot\frac{x}{x-4}$$

$$x(x-4)\cdot 2-\frac{x(x-4)\cdot 6}{x}=\frac{x(x-4)\cdot x}{x-4}$$

$$2x(x-4)-\frac{6x(x-4)}{x}=\frac{x^2(x-4)}{(x-4)}$$

$$2x(x-4)-\frac{6\cancel{x}(x-4)}{\cancel{x}}=\frac{x^2\cancel{(x-4)}}{\cancel{(x-4)}}$$

$$2x(x-4)-6(x-4)=x^2$$

Next, use the distributive property to eliminate parentheses. Then combine like terms:

$$2x^2-8x-6x+24=x^2$$

$$2x^2-14x+24=x^2$$

continued...

RESTART BACK 18 of 23 NEXT

RATIONAL EQUATIONS

Example G, continued...

Subtract x^2 from both sides of the equation, and factor the resulting trinomial:

$$\begin{array}{rcl} 2x^2 - 14x + 24 & = & x^2 \\ \underline{-x^2 \qquad\qquad} & & \underline{-x^2} \\ x^2 - 14x + 24 & = & 0 \\ (x-12)\cdot(x-2) & = & 0 \\ \downarrow \qquad \downarrow & & \\ x = 12 \text{ or } x = 2 & & \end{array}$$

Only 0 and 4 make the denominators 0, so our solutions are both valid.

RESTART   BACK NEXT

RATIONAL EQUATIONS

Extended Example 4a

Solve: $\dfrac{y-1}{7} = \dfrac{8}{y}$.

Hint

STEP 1

RESTART BACK 20 of 23 NEXT

EXAMPLE H

Suppose Julie can paint a room in 6 hours, while Joe can paint the room in 8 hours. How long will it take them to paint the room if they work together?

Suppose Julie works for t hours. In that time she can paint $\frac{t}{6}$ of the room.

To convince yourself of this, try substituting $t = 6$ (hours) into the expression: $\frac{t}{6} = \frac{6}{6} = 1$, one house. Now try substituting $t = 3$: $\frac{t}{6} = \frac{3}{6} = \frac{1}{2}$. Indeed, it makes sense that if she can paint the whole room in 6 hours she can paint half the room in 3 hours.

Similarly, if Joe works for t hours he can paint $\frac{t}{8}$ of the room.

So, if they both work together for t hours, they can paint $\frac{t}{6} + \frac{t}{8}$ of a room.

How long will it take them to paint one whole room? We need to solve the equation:

$$\frac{t}{6} + \frac{t}{8} = 1$$

Factor the denominators:

$$\frac{t}{2 \cdot 3} + \frac{t}{2 \cdot 2 \cdot 2} = 1$$

The LCM of these denominators is $2 \cdot 2 \cdot 2 \cdot 3$.

continued...

Example H, continued ...

Multiplying both sides of the equation by the LCM, $2 \cdot 2 \cdot 2 \cdot 3$, to cancel the denominators, we get:

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot \left(\frac{t}{2 \cdot 3} + \frac{t}{2 \cdot 2 \cdot 2} \right) = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 1$$

$$\frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot t}{2 \cdot 3} + \frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot t}{2 \cdot 2 \cdot 2} = 24$$

$$\frac{2 \cdot 2 \cdot \cancel{2 \cdot 3} \cdot t}{\cancel{2 \cdot 3}} + \frac{\cancel{2 \cdot 2 \cdot 2} \cdot 3 \cdot t}{\cancel{2 \cdot 2 \cdot 2}} = 24$$

$$4t + 3t = 24$$

Solving for t we get:

$$7t = 24 \quad \rightarrow \quad \frac{\cancel{7}t}{\cancel{7}} = \frac{24}{7} \quad \rightarrow \quad t = \frac{24}{7}$$

To make sense of this answer, we need to convert it to hours and minutes (there is no need to calculate to the nearest second). Dividing, we get:

$$t = \frac{24}{7} \cong 3.428571429$$

So it will take 3 hours, plus an additional 0.428571429 of an hour. Convert that fractional hour to minutes by multiplying it times 60 (since there are 60 minutes in each hour):

$$0.428571429 \left[\cancel{\text{hours}} \right] \cdot 60 \left[\frac{\text{minutes}}{\cancel{\text{hour}}} \right] = 25.71428574 \text{ minutes}$$

It will take about 3 hours and 26 minutes for Julie and Joe to paint the room if they work together.

RESTART BACK 22 of 23 NEXT

RATIONAL EQUATIONS

Extended Example 5a

Suppose Sam can mow a lawn in 4 hours, while Tom can mow the same lawn in 3 hours. How long will it take both of them working together to mow the lawn?

Hint

STEP 1

RESTART END OF LESSON BACK

Solve: $\dfrac{3}{12x} - \dfrac{2}{x} = \dfrac{7}{6}$

Solve: $\dfrac{3}{x} - \dfrac{5}{x-1} = 16$

Solve.

$$\frac{12}{x-4} - \frac{7}{2} = \frac{11}{x}$$

Solve: $\frac{4}{x} - \frac{x-1}{x+1} = \frac{5}{x+1}$

If it takes Katie 7 hours to construct a lamp, and it takes Fernando 9 hours to construct a lamp, how long will it them to construct a lamp working together? (Round your answer to the nearest minute.)

DIRECT AND INVERSE VARIATION

Introduction

In this lesson, you will learn how to solve problems involving **direct and inverse variation**. Many real-world quantities exist in certain fixed proportions. For example, the ingredients in a recipe are in fixed proportions. As another example, consider a molecule of water. A molecule of water is made of one atom of oxygen and two atoms of hydrogen. Whatever the quantity of water, the number of hydrogen atoms is always double the number of oxygen atoms. Proportional relations like this are examples of **direct variation**.

Other quantities behave in such a way that the larger one quantity grows, the smaller the other becomes (and vice versa). For example, the faster you go, the less time it takes to get to your destination. Such a relation is an example of **inverse variation**. Numerous applications of direct and inverse variation are found in the real world.

DIRECT AND INVERSE VARIATION

Direct Variation

Variable y is said to be **directly proportional** to x (also stated as "y varies directly as x"), provided that $y = Kx$ for some constant K, the **constant of proportionality**. To solve variation problems, we must first solve for the **constant of proportionality.** The constant of proportionality allows us to answer any question concerning a given relation of variation.

EXAMPLE A

Suppose that y is directly proportional to x, and $y = 10$ when $x = 5$. What is y when $x = 7$?

We are told that "y is directly proportional to x." This translates into the equation:

$$y = Kx\,,$$

for some constant K. The next information we are told is precisely what we need to solve for the constant of proportionality, K. We are told that "$y = 10$ when $x = 5$." Substitute these values into the equation above, and then solve for K:

$$\begin{array}{ccc} & y = Kx & \\ \swarrow & & \searrow \\ y = 10 & & x = 5 \\ \searrow & & \swarrow \\ & 10 = K \cdot 5 & \\ & 10 = 5K & \end{array}$$

continued...

DIRECT AND INVERSE VARIATION

Example A, continued...

$$10 = 5K$$
$$\frac{10}{5} = \frac{5K}{5}$$
$$2 = \frac{\cancel{5}K}{\cancel{5}}$$
$$2 = K$$

Now substitute this constant of proportionality back into the original variation equation:

$$y = Kx$$
$$y = 2x$$

So y is always twice x. Now we can answer the question that was posed, "What is y when $x = 7$?":

$$y = 2x = 2 \cdot 7 = 14$$

The previous example shows exactly what the constant of proportionality really is. In that example, we found it to be a magnification factor—quantity y is always twice quantity x. So the magnification factor K is set equal to 2. With direct variation, the constant of proportionality is always a magnification factor (or a reducing factor). For example, when you set the magnification factor on a copy machine to reduce or enlarge a document, you are assigning a value to a constant of proportionality.

RESTART   BACK   3 of 18   NEXT

DIRECT AND INVERSE VARIATION

EXAMPLE B

Suppose that y is directly proportional to x, and $y = 14$ when $x = 10$. What is y when $x = 3$?

We are told that "y is directly proportional to x." This translates into the equation:

$$y = Kx,$$

for some constant K, whose value we must find. Next we know that we need to solve for our constant of proportionality, K. We are told that "$y = 14$ when $x = 10$." Substitute these values into the above equation, and then solve for K:

$$\begin{array}{ccccc} & & y = Kx & & \\ & \swarrow & & \searrow & \\ y = 14 & & & & x = 10 \\ & \searrow & & \swarrow & \\ & & 14 = K \cdot 10 & & \end{array}$$

$$14 = 10K$$

$$\frac{14}{10} = \frac{10K}{10}$$

$$\frac{\cancel{2} \cdot 7}{\cancel{2} \cdot 5} = \frac{\cancel{10}K}{\cancel{10}}$$

$$\frac{7}{5} = K$$

continued...

DIRECT AND INVERSE VARIATION

Example B, continued...

Substitute this constant of proportionality back into the original variation equation:

$$y = Kx$$

$$y = \frac{7}{5} \cdot x$$

Now we can answer the question, "What is y when $x = 3$?":

$$\begin{aligned} y &= \frac{7}{5} \cdot 3 \\ &= \frac{7 \cdot 3}{5} \\ y &= \frac{21}{5} \overset{or}{=} 4.2 \end{aligned}$$

All variation problems are solved in the same way. First find the constant of proportionality, K, and then answer the question that was posed.

RESTART BACK 5 of 18 NEXT

DIRECT AND INVERSE VARIATION

Extended Example 1a

Suppose that y is directly proportional to x, and $y = 23$ when $x = 9$. What is y when $x = 5$?

Hint

STEP 1

RESTART BACK 6 of 18 NEXT

EXAMPLE C

Suppose that W is directly proportional to m, and $W = 6.4$ when $m = 47.2$. What is W when $m = 103.6$?

We are told that $W = Km$. Substituting $W = 6.4$ and $m = 47.2$ into this equation, we can solve for the constant of proportionality, K:

$$W = Km$$

$$6.4 = K(47.2)$$

$$\frac{6.4}{47.2} = \frac{47.2K}{47.2}$$

$$\frac{6.4}{47.2} = \frac{\cancel{47.2}K}{\cancel{47.2}}$$

$$\frac{6.4}{47.2} = K$$

When a problem results in decimal numbers like this, it is best to express our solution in decimal form. Using a calculator, we compute:

$$K = 0.1355932203$$

Now substitute this constant of proportionality back into the original variation equation:

$$W = Km$$

$$W = 0.1355932203\,m$$

continued...

DIRECT AND INVERSE VARIATION

Example C, continued...

Use all the digits of precision you can at this stage of your calculation. Rounding should only be done after all calculation has been completed. Otherwise the rounding error may become greatly magnified by the calculation itself and cause the final answer to be greatly inaccurate.

We can now answer the question, "What is W when $m = 103.6$?":

$$W = 0.1355932203(103.6)$$
$$W = 14.04745762$$

Finally, we should round our answer. Since the numbers given in this problem are all given to the nearest tenth, we will round the answer to the nearest tenth:

$$W \cong 14.0$$

Note: Remember that the symbol $\cong$ means "approximately."

RESTART   BACK 8 of 18 NEXT

DIRECT AND INVERSE VARIATION

Extended Example 2a

Suppose that Q is directly proportional to p , and $Q = 13.71$ when $p = 11.22$. What is Q when $p = 55.55$?

Hint

STEP 1

RESTART

BACK 9 of 18 NEXT

EXAMPLE D

Suppose that y is directly proportional to x, and $y = 1.7$ when $x = 7.3$. What is x when $y = 5.2$?

We are told that $y = Kx$. Substituting $y = 1.7$ and $x = 7.3$ into this equation, we can solve for the constant of proportionality, K:

$$y = Kx$$

$$1.7 = K(7.3)$$

$$\frac{1.7}{7.3} = \frac{7.3K}{7.3}$$

$$\frac{1.7}{7.3} = \frac{\cancel{7.3}K}{\cancel{7.3}}$$

$$\frac{1.7}{7.3} = K \rightarrow K \cong 0.2328767123$$

Now substitute this constant of proportionality back into the original variation equation:

$$y = Kx$$
$$y = 0.2328767123\,x$$

continued...

RESTART BACK 10 of 18 NEXT

DIRECT AND INVERSE VARIATION

Example D, continued...

Rounding should only be done <u>after</u> all calculation has been completed. Otherwise, a rounding error could cause the final answer to be inaccurate.

We can now answer the question, "What is x when $y = 5.2$?".
NOTICE!! This time we are asking for x instead of y.

$$y = 0.2328767123\,x$$

$$5.2 = 0.2328767123\,x$$

$$5.2 = 0.2328767123\,x$$

$$\frac{5.2}{0.2328767123} = \frac{0.2328767123\,x}{0.2328767123}$$

$$\frac{5.2}{0.2328767123} = \frac{\cancel{0.2328767123}\,x}{\cancel{0.2328767123}}$$

$$22.3294118 \cong x$$

Finally, we round the answer. Since the numbers given in this problem are all given to the nearest tenth, we will round our answer to the nearest tenth:

$$x \cong 22.3$$

DIRECT AND INVERSE VARIATION

Extended Example 3a

Suppose that Q is directly proportional to p, and $Q = 32.1$ when $p = 23.4$. What is p when $Q = 12.3$?

Hint

STEP 1

RESTART BACK 12 of 18 NEXT

Inverse Variation

Variable y is said to be **inversely proportional** to x (also stated as "y varies inversely as x"), provided that $y = \frac{K}{x}$ for some constant K, the **constant of proportionality**.

As with direct variation problems, a good rule to follow is to give exact answers when the problem is stated using integers. If the problem is stated using decimals, round to the same apparent unit as the given decimal numbers.

EXAMPLE E

Suppose that y is inversely proportional to x, and $y = 14$ when $x = 10$. What is y when $x = 3$?

We are told that $y = \frac{K}{x}$. Substitute $y = 14$ and $x = 10$ into this equation to solve for the constant of proportionality, K:

$$y = \frac{K}{x}$$

$$14 = \frac{K}{10}$$

$$10 \cdot 14 = 10 \cdot \frac{K}{10}$$

$$140 = \frac{\cancel{10}K}{\cancel{10}}$$

$$140 = K$$

continued...

Example E, continued...

Substitute this constant of proportionality back into the original inverse variation equation:

$$y = \frac{K}{x}$$

$$y = \frac{140}{x}$$

Now we can answer the question, "What is y when $x = 3$?":

$$y = \frac{140}{3} \overset{or}{=} 46.\overline{6}$$

Again, note how similar the above example is to the previous ones - first use the given information to find K, and then answer the question that was posed.

DIRECT AND INVERSE VARIATION

Extended Example 4a

Suppose that z is inversely proportional to t, and $z = 12.83$ when $t = 2.79$. What is z when $t = 5.20$?

Hint

STEP 1

RESTART

BACK 15 of 18 NEXT

EXAMPLE F

Suppose that y is inversely proportional to x, and $y = 12.9$ when $x = 48.7$. What is x when $y = 33.3$?

We are told that $y = \frac{K}{x}$.

Substituting $y = 12.9$ and $x = 48.7$ into this equation, we can solve for the constant of proportionality, K:

$$y = \frac{K}{x}$$

$$12.9 = \frac{K}{48.7}$$

$$48.7 \cdot 12.9 = 48.7 \cdot \frac{K}{48.7}$$

$$628.23 = \frac{\cancel{48.7} \cdot K}{\cancel{48.7}}$$

$$628.23 = K$$

Substitute this constant of proportionality back into the original inverse variation equation:

$$y = \frac{K}{x} \quad \rightarrow \quad y = \frac{628.23}{x}$$

continued...

DIRECT AND INVERSE VARIATION

Example F, continued...

Now we can answer the question, "What is x when $y = 33.3$?". Substitute in this value for y:

$$y = \frac{628.23}{x}$$

$$33.3 = \frac{628.23}{x}$$

Multiply both sides of the equation by x to eliminate the fraction:

$$x \cdot 33.3 = x \cdot \frac{628.23}{x}$$

$$33.3x = \frac{\cancel{x} \cdot 628.23}{\cancel{x}}$$

$$33.3x = 628.23$$

Divide both sides of the equation by 33.3 to solve for x:

$$\frac{33.3x}{33.3} = \frac{628.23}{33.3}$$

$$\frac{\cancel{33.3}x}{\cancel{33.3}} = \frac{628.23}{33.3}$$

$$x = 18.86576577$$

Rounding this answer to the nearest tenth, we get $x \cong 18.9$.

DIRECT AND INVERSE VARIATION

Extended Example 5a

Suppose that A is inversely proportional to B, and $A = 24$ when $B = 36$.
What is B, if $A = 30$?

Hint

STEP 1

RESTART END OF LESSON BACK 18 of 18

If y is directly proportional to x, and $y = 12.2$ when $x = 3.1$, find x when $y = 2.8$

If y is inversely proportional to x, and $y = 87.3$ when $x = 29.1$, find y when $x = 4.3$. (Round off your answer to the nearest tenth.)

If y is inversely proportional to x, and $y = 32.1$ when $x = 21.2$, find x when $y = 100$. (Round off your answer to the nearest tenth.)

If y is inversely proportional to x, and $y = 51.9$ when $x = 11.3$, find y when $x = 123.4$.
(Round your answer to the nearest tenth.)